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TECHNICAL MEMORANDUM 1287

DEPENDENCE OF THE ELASTIC STRAIN COEFFICIENT OF  
COPPER ON THE PRE-TREATMENT

By W. Kuntze

Translation of "Abhängigkeit der elastischen Dehnungszahl  $\alpha$  des  
Kupfers von der Vorbehandlung."  
Zeitschrift für Metallkunde, 20. Jahrgang, Heft 4, 1928



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## TECHNICAL MEMORANDUM 1287

DEPENDENCE OF THE ELASTIC STRAIN COEFFICIENT OF  
COPPER ON THE PRE-TREATMENT\*<sup>1</sup>

By W. Kuntze

Communication from the National Material-Testing Office, Berlin-Dahlem

This communication includes the following points: Systematic determination of the strain coefficient and of its variation with increasing stretch and test stress; influence of drawing, storing, and heat treatment; proportionality of strains; hysteresis; pure (stress free) deformation; internal stresses; conclusions for the test methods.

## INTRODUCTION

The conditions under which the elastic coefficients have hitherto been determined are probably quite varied and, therefore, the coefficients thus found, particularly those of the nonferrous metals, differ considerably. It is known<sup>2</sup> that the elastic strain coefficient  $\alpha$  varies with the degree of loading and the degree of plastic strain (stretch); unknown, however, is the magnitude of this variation. A systematic investigation was therefore undertaken with pure (99.84%) copper; the degree of stretching and, after every stretching, the test stress were continually increased; moreover the stretched specimen was stored at low temperature, thus artificially aged.

The test method was controlled very carefully so as to eliminate external temperature influences as well as, in particular, the effects of time (influence of the elastic after-effect), thus to produce always

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\*"Abhängigkeit der elastischen Dehnungszahl  $\alpha$  des Kupfers von der Vorbehandlung." Zeitschrift für Metallkunde, 20. Jahrgang, Heft 4, 1928, pp. 145-150.

<sup>1</sup>The complete original report will be published soon in a special issue of the Communications from the National Material-Testing Office and of the KWI for metal research.

<sup>2</sup>Grüneisen, Annal. d. Phys. bc. 22, 1907, p. 801.

Bach and Baumann, Elastizität und Festigkeit.

Körber and Rohland, Mitt. KWI für Eisenforschung, Düsseldorf, Bd. 5, 1924, pp. 37-54; see also Sachs-Fiek "Der Zugversuch", 1926, p. 134.

comparable conditions in this respect. The details of the test performance cannot be discussed at this point.

### TEST EVALUATION

#### 1. Influence of Loading and Test Stretching on the Experimental Results

In order to present a survey of the tests, figure 1a shows the stress-strain diagram of the copper tensile rod (16 mm diameter) with all unloading points, which are designated by the numbers 1 to 36. Figure 1b represents the elastic strain curves found by means of a Martens' mirror extensometer, the single values of which resulted from retrogressive strain values for the degrees of stretching investigated. The differences ( $\lambda - 0.4P$ ) for the selected load steps were plotted in order to obtain - in view of the very high sensitivity of the representation - sufficiently steep curves. The faired curves agree very well with the test points. They deviate only up to two test units, that is,  $2 \times 10^{-5}$  cm.

From the tangents at the point of origin of all curves in figure 1b the slopes of the curves at the origin were determined and, hence,  $\alpha_0$  (for infinitely small loading) was calculated. (Compare Grüneisen, footnote 2). From the chords between corresponding curve points and the point of origin resulted the  $\alpha$ -values for the higher load ( $\alpha = \frac{\lambda}{l_0}$ , for  $l = 10\text{cm}$ )<sup>3</sup>. The stresses required for the calculation of  $\alpha$  referred to the respective cross section  $f$  obtained by the stretching can be found in figure 1a.

For every test, the values of  $\alpha$  were plotted as functions of the stress in figure 2. The curves obtained are approximately straight lines, that is,  $\alpha$  increases in proportion to the stress. The abscissa is intersected by these straight lines at points which correspond to the value  $\alpha_0$  and must agree with the values calculated from the initial tangents. The 13 straight lines in figure 2 correspond to the 13 tests with different degrees of stretching. The slopes of the straight lines

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<sup>3</sup>Strain coefficient  $\alpha = \frac{1}{E}$ ;  $E$  = modulus of elasticity. The strain coefficient  $\alpha$  indicates the strain for unit stress, thus  $\alpha = \frac{\epsilon}{\sigma}$ , with  $\epsilon$  signifying the extension of unit length, thus  $\epsilon = \frac{\lambda}{l}$ .  
( $\lambda$  = extension of the gage length  $l$ )

are a measure of the hysteresis, that is, of the degree of curvature of the elastic strain curves in figure 1b.

The continuous variation of the slopes of the lines 3 to 13 may be seen from figure 3a. Three slope values were plotted here for the annealed specimen, corresponding to the tests 1, 18, and 26; 1 and 18 (also test 2 in fig. 2) do not correspond to complete annealings. This will be discussed in more detail later. The lines in figure 2 shift their position first to the right and then again to the left. Thus  $\alpha$ , for all loading stresses, at first increases with the stretching and then decreases. The variation of  $\alpha_0$  with the degree of stretching is represented in figure 3b. It is pointed out that the\* . . . . . only amounts to a fraction of one percent of  $\alpha$  and that, nevertheless, position and direction could be ascertained. The determination of  $\alpha$  with an accuracy of, on the average, 0.2% was made possible by the use of the faired curves in figure 1b. The  $\alpha$ -values in figure 2 were plotted only up to a loading of approximately 2/3 to 3/4 of the respective yield point. When this range is exceeded, the  $\alpha$ -line pronouncedly deviates to the right, that is, the  $\alpha$ -values determined by unloadings increase considerably. This is represented in figure 2 for test 8 only.

## 2. Behavior of the Drawn Material in the Investigation

In like manner several tests were performed with copper rods of 8 mm diameter drawn through dies; hard and half-hard drawn were compared to annealed material. The variation of  $\alpha$  with the stress and the degree of drawing is shown, corresponding to the former tests 1 to 13, in figures 4a to c from which  $\alpha_0$  may be observed for all degrees of drawing of the three types. The variation of  $\alpha_0$  of the two rods drawn different amounts in comparison with the annealed rod can be seen in figure 3c to e. In this representation the attempt was made to bring the degree of drawing, that is, the reduction of the cross section of the copper rods in the die, into agreement with the cross-sectional reduction by the stretching in the machine; the degree of drawing was converted into longitudinal stretching and plotted. In spite of the considerable deformation which took place in the die, the maximum of  $\alpha$  in stretching again occurs. However, even the degree of deformation produced in the die alone seems to cause a similar course of  $\alpha$ , as is shown by the curve f in figure 3; f goes through the starting points of

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\*Translator's note: Here one line of printing was obviously omitted in the original German paper, and the continuity is, therefore, interrupted.

the three curves c, d, and e, and therefore, does not contain any additional test stretchings.

### 3. Variation of the Experimental Results with Storing, Tempering,<sup>4</sup> and Annealing

In the following, the influence of storing and heat treatment on the elastic properties is treated. In the tests described so far, the specimen was stored for one day after having been stretched, and only then were the elastic properties determined. If the test is performed immediately after the stretching, after-effects that may appear would probably take place comparatively rapidly and the test duration would probably be of influence. Actually tests executed immediately after the stretching showed less uniform results than those performed some time after the stretching. Figure 5a compares the  $\lambda$ -values of four specimens stretched different amounts (tests 3 to 6), each of which was tested once immediately, and again after one day had elapsed. Due to the storing through 24 hours,  $\lambda$ , and therewith  $\alpha$ , decreased, the reduction amounting, on the average, to 1 to 2%. By short-time storing after the stretching, permanent residual strains appeared frequently even for the smallest load ranges (fig. 5b); however, after their elimination they did not have any further influence on the unloading values or, therewith, on  $\alpha$ .

After the copper specimen had been pre-stretched to 41.1%, it was subjected to a longer aging process by storing it for 12 days and heating (tempering) it for about 7 hours daily at 100° C. Thereby  $\alpha_0$  decreased by  $5\frac{1}{2}\%$  (test 14, figs. 6 and 3b), and the specimen followed Hooke's law up to a loading of 4 kg/mm<sup>2</sup>, that is,  $\alpha$  remained constant. After an insignificantly small plastic stretching of 0.005% (test 15, fig. 6) the proportionality of the strains was lost again, and hysteresis occurred even for minute loads, that is, the vertical  $\alpha$  line changed into an inclined one. Next, the specimen was stretched 0.5% (test 16) and then 1.55% further (test 17). As before, the stretching caused an increase of  $\alpha$ . Further stretching failed to increase this  $\alpha$  to a higher value than the original value of  $\alpha$  of specimen 13, stretched 41.1% and not tempered. This value, almost completely attained by only 0.5% stretching, could hardly be increased by further stretching up

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<sup>4</sup>The term "tempering" as used herein is defined as a heating of more or less duration at temperatures below the recrystallization temperature.

to 2.05% (figs. 6 and 3b). The extremely small stretching of 0.005% was, therefore, sufficient to cancel the validity of Hooke's law caused by the aging while the slight stretching of 0.5% completely cancelled the effect of the tempering.

Thereupon the copper rod - having thus been stretched 41.15% - was once more annealed at 400° C (test 18), then in 2 steps stretched 2.74% and tempered at increasing duration and temperature (tests 19 to 25). On the basis of the assumption that the first annealing of the copper rod supplied (drawn, thus slightly deformed) had not brought about a complete softening, the rod was now, after a stretching of 2.74%, annealed at 650° C (test 26)<sup>5</sup>. The results of all three annealings were different, as clearly shown in figure 7a (strain diagram) and figure 7b ( $\alpha$  values for increasing stress). Even though with the increase of the loadings the  $\alpha$ -values in the 3 tests differ more and more, all three lines still coincide in  $\alpha_0 = 8.16 \frac{\text{mm}^2}{\text{kg}} \times 10^{-6}$ , that is, for the  $\alpha_0$  determined at infinitely small loading the first and second annealing also were equivalent to the third at 650° C. Now there arises the question as to whether an unobjectionable determination of this seemingly excellent value  $\alpha_0$  for the annealed material is at all possible; for we know that  $\alpha_0$  can only be found by extrapolation and that the test values from which  $\alpha_0$  is extrapolated are observed under loads which already cause permanent residual strains. We know further that after even the slightest plastic deformation  $\alpha$  increases disproportionately rapidly, thus is always too large. In order to make a reasonable extrapolation possible, the following procedure was adopted: The annealed specimen was, at first, not subjected to any initial loading. Each load step in the test was, therefore, preceded only by the previous load step which was each time smaller than the actual one. The experimental method can be seen from table I.

TABLE I

Loading kg	Retrospective elastic strain values in $\text{cm} \cdot 10^{-5}$ under the loads (horizontal column headings) and after the initial loadings (vertical column headings) in kg						
	100	200	300	400	500	600	700
200	73	---	---	---	---	---	---
300	---	114	---	---	---	---	---
400	---	---	155	---	---	---	---
500	---	---	---	196	---	---	---
600	---	---	---	---	241	---	---
700	---	---	---	---	---	287	---
800	---	---	---	---	---	---	334

<sup>5</sup>According to R. Karnop and G. Sachs, Z. f. Phys., vol. 42, 1927, pp. 283-301 recrystallization occurs, after 2.7% drawing at approximately 600° C, whereas for drawings over 5% only approximately 350° C is required.

It is true that in this test method the test values never correspond to the annealed material but always to the material already slightly drawn by the preceding load step. Since, however, under these conditions for infinitely small loads the preceding initial loading also is infinitely small, as is also the stretching, the  $\alpha$ -line thus found tends toward the actual  $\alpha_0$  value, that is, the extrapolation results in the actual value for  $\alpha_0$  of the annealed material.

Additionally, the specimen was then drawn in four steps up to 3.6% and tempered at various lengths of time (tests 27 to 36).

## RESULTS

### 1. Variability of the Elastic Properties

It is of importance to be aware that in the case of copper pure elasticity which follows Hooke's law appears as well as hysteresis. Whereas the former was observed only on strongly deformed (41.1% stretched) and then tempered material, (fig. 6), hysteresis is a characteristic of a merely deformed material. The "pure" hysteresis (as long as no permanent residual strains appear in combination with it or, having appeared, have been eliminated, we will call it "pure") has complete reversability in common with the pure elasticity. However, the curved loading and unloading curve encloses a surface, the area of which is a measure of the used-up expenditure of energy. The law of curvature is expressed in the proportionality between  $\alpha$  and  $\sigma$  found for all degrees of stretching (fig. 2), and the slope of the straight line

$\cot \beta = \frac{\alpha_{\sigma} - \alpha_0}{\sigma}$  becomes steeper for increasing stretching with

decreasing hysteresis (fig. 3a). These conditions of pure hysteresis were satisfied in a loading region which amounted to about 2/3 to 3/4 of the respective yield point produced by prestretching.

In the region between this limit and the yield point the strain curves were altered by permanent residual strains - which crept in again and again even after repeated loading up to the yield point - in the sense of a greater curvature so that unloading and loading curves no longer show a uniform distribution of curvature. The curve of the elastic strains also assumes a more pronounced curvature so that  $\alpha$ , too, increases in this range more than in proportion to  $\sigma$  (fig. 2, curve 8). Recurrent regularities of the elastic phenomena appearing in connection with loading and deformation may, therefore, be determined only in the range of "pure" hysteresis, because above this range incalculable disturbances of plastic character occur.

Within the range of pure hysteresis the influence of cold-stretching was shown to be as follows:

First,  $\alpha_0$  (or  $\alpha$  in case of comparable loading stresses) varied with the degree of stretching and decreased particularly between 3 and 41% pre-stretching (fig. 3b);

Second,  $\alpha$  increased in proportion to the loading stress, but to a different extent, according to the degree of the preceding plastic deformation (fig. 2). These two phenomena, (a) the decrease of  $\alpha$  with the deformation and (b) the increase with the stress, seem, for instance, according to figure 8, to add up to the curve c, the shape of which corresponds to the test results in figure 3b. This variation of  $\alpha$  appears only in simultaneous combination with hysteresis which decreased first more strongly than the stretching, then (from 3 to 41% stretching) about proportionally to it (fig. 3a).

To all appearances we are dealing with simulated elasticity phenomena if a material has been cold stretched. A stretched rod contains hidden stresses which naturally increase with the deformation. These hidden stresses probably produce the part of the curve b in figure 8. The initially indiscernably hidden stress influences simulate elasticity coefficients which do not really correspond to the material. By tempering<sup>6</sup> these stresses are eliminated or reduced without being detrimental to the strength characteristics (cf. fig. 1a), and  $\alpha$  is lowered by an amount corresponding to that produced by the eliminated stresses (cf. test 14, fig. 3b). Simultaneously - at least when very strong stretchings had preceded - Hooke's law attains validity for the new state; thus the hypothesis suggests itself that hysteresis is caused by hidden stresses which, in turn, originate only in case of plastic deformation. In fact, the stresses are produced again after the tempering by renewed, extremely slight stretching of 0.005% (test 15, fig. 6), and hysteresis immediately reappears. By further stretching of 0.5% (test 16)  $\alpha$ , too, again noticeably increases; only as long, however, as the degree of deformation (which is known to decrease the magnitude of  $\alpha$ ) does not predominate. Thus it was not possible to increase  $\alpha$ , by means of further stretching of 1.55% (test 17), beyond its original value which it had reached in test 13 for a stretching of 41.1% (fig. 3b). If the variation of  $\alpha$  according to figure 8c were merely a result of pure deformation, the material drawn in the die would probably, in case of further stretching in the test machine, not again exceed a maximum,

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<sup>6</sup>Heyn, E.: Mittlg. a. d. Königl. Materialprüfungsamt 1917.  
Masing, G. and Haase, C., Wissensch. Ver. Siemenskonzern Bd. 3, 2, 1924, pp. 22-38.



since its deformation caused by drawing already has progressed further than corresponds to the position of the maximum of  $\alpha$  in figure 3c. According to figures 3d and e, however, a maximum, nevertheless, appeared. Half-hard and hard drawn copper rods behaved under the influence of the tensile stresses imposed in the testing machine essentially exactly like annealed ones, that is, they again exceeded a maximum of  $\alpha$ . The internal stresses produced by the test stretching are, therefore, more effective with respect to the variation of  $\alpha$  than those imparted to the material by drawing (cf. fig. 3f). Therewith the fact should be clarified that the variation of  $\alpha$  is not caused by the pure deformation alone but by the combination of the pure deformation and the remaining internal (hidden) stresses. That, however, the pure deformation in itself also causes a decrease of  $\alpha$  is shown by test 14 (fig. 3b); according to that test, by the tempering, which eliminated the stresses,  $\alpha_0$  in the specimen stretched 41.1% became considerably smaller (4.6%) than the  $\alpha_0$  of the annealed specimen<sup>7</sup> whereas the  $\alpha_0$  of the three annealed specimens (1, 18, 26) remained constant (deviation  $\pm 0\%$ ).

Slightly deformed specimens show after the tempering no region, or a very small one, of pure proportionality. All the  $\alpha$ -lines have, in contrast to the rectilinearity of the stretched specimens, the characteristic curved shape, open on top, which is also to be found above the region of proportionality in the specimen stretched 41.1% and tempered (test 14).

In order to clarify the intensity of the tempering, on the one hand specimens were compared which had been stored at the same temperature, but for different durations, on the other hand those stored at different temperatures. Figure 9 shows for two examples the decrease of  $\alpha_0$  with the duration of the tempering. It seems to make a difference in the test results whether or not the specimen is cold-stored for a length of time after the cooling-off. A longer storing after the cooling seems to again increase the value of  $\alpha$ . This was the case in test 23 which resulted, in spite of longer tempering, in a somewhat larger  $\alpha$  than test 22.

In figure 10 we see the decrease of  $\alpha_0$  with the degree of temperature at which the tempering was done. The duration of tempering or, respectively, annealing was one hour for the temperatures 310°, 400°, 650°. For 100° the value corresponding to one hour was obtained from figure 9.

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<sup>7</sup>This does not agree with the findings of Körber and Rohland (footnote 2) according to whom the tempering of a stretched specimen restores the  $\alpha$  of the annealed specimen.

Figures 9 and 10 show that even a one-hour's tempering accomplishes a considerable effect, that that effect with time still increases gradually, and that, for a shorter duration, a greater effect is obtained if the temperature is selected as high as possible while still remaining below the recrystallization temperature. The shaded area then indicates the entire possibility of variation of  $\alpha_0$  with the degree of temperature and the duration for a specimen stretched 2.74 and 41.1%. According to the position of the  $\alpha$ -lines in figure 6, however, the effect of tempering on the  $\alpha$ -values determined for small loading stresses is much greater than the effect on those determined at higher loads (compare specimen 13 with 14)<sup>8</sup>.

Similar behavior is noticeable in the three annealings (tests 1, 18, and 26). The supplied specimen, that is, the specimen drawn in the die, furthermore, the specimen stretched 41.1% and the one stretched 2.74%, was annealed at 400°, at 400°, and at 650°, respectively, for one hour. Each time, the annealing temperature was above recrystallization temperature. According to figure 7b the effect  $\alpha_0$  had already been attained in all 3 tests although the  $\alpha$ -values determined at higher stresses varied widely, because for tests 1 and 18 the full degree of softening (cf. fig. 7a) was not yet attained as it was in test 26.

In figure 11 we find a compilation of the variation of  $\alpha$  with the stretch, test stress, and tempering. The curves  $r$  and  $r'$  show the variation of  $\alpha$  with increasing stretch. For  $r'$  the preceding softening was, due to the annealing, more complete than for  $r$ . The series of lines  $t$  indicate the variation of  $\alpha$  obtained by the tempering at 100° C for 24 hours duration following the stretching. The indices 0 to 4 correspond to the test stresses for which  $\alpha$  was determined. For 41.1% stretching  $t_0$  coincides with  $t_1$  to  $t_4$ ; this signifies that up to the stress 4 kg/mm<sup>2</sup> the  $\alpha$  values are constant and Hooke's law is therefore valid. One may see from the diagram that Hooke's law can come into force only after extensive stretchings and following tempering, and that it represents (as shown by the continuity of the curves) probably only a limiting case of minimum variation of  $\alpha$  with the test stress.

Three states, deformation, tempering (aging) and annealing, thus produce entirely different elasticity coefficients. In addition, their variation with the test loading has to be taken into consideration. Only the strain coefficient  $\alpha_0$  of the annealed material was found to be a recurrent constant value; it did not matter to what extent the material had been deformed beforehand.

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<sup>8</sup>This was true also for the specimens 22, 23, and 35 to 36.

## 2. Conclusions Concerning the Test Performance and Practice from the Variation of the Elastic Properties

So far we have considered to what influences the elastic strain coefficient  $\alpha$  is subject; we shall now discuss how such influences can unaccountably come into play in the conducting of a test. The strong effect of the most minute strains on the numerical value of  $\alpha$  has been emphasized.

If an annealed material is to be investigated and permanent residual strains appear even for very small loads, one usually eliminates the latter by a preloading, thus - according to established opinion - increasing the E limit. One then has eliminated the permanent residual strains, but brought out the hysteresis. By the preloading a small stretching has been produced; in the case of copper the latter has probably been sufficient to change the figure  $\alpha_0$  by several percent, compared to the  $\alpha_0$  of entirely undeformed material (for 0.26% stretching approximately 6% increase of  $\alpha_0$ ). In the static determination it is, furthermore, not possible to ascertain the strain coefficient of the annealed copper, since for every loading material that has already been stretched is investigated; and since with the loading the stretching increases;  $\alpha$ , too, increases correspondingly with the loading. (For a loading stress of 1 kg/mm<sup>2</sup> about 5%). Since only  $\alpha_0$  can be considered a characteristic value for the annealed material, it is necessary to find  $\alpha_0$  by extrapolation. Annealed specimens do not turn out uniformly if the annealing temperature has not been selected corresponding to the preceding deformation. However, as the tests 1, 18, and 26 in figure 7 show, it is precisely this  $\alpha_0$ -value which is insensitive to the degree of annealing so that it may yield a reliable result for the annealed material. However, the determination of  $\alpha_0$  may be accomplished only with very accurate measurements for different stepwise increasing loadings, with account taken of the after effects, and then by extrapolation.

Testing deformed and untempered materials with respect to elasticity seems completely without purpose; for one thing, the variation of  $\alpha_0$  with the cold-deformation is not a fixed value, due to the not easily perceptible and variable stress influences; second, the hysteresis produces a considerable variation of  $\alpha$  with the test stress which means an additional complication. Moreover, the duration of a storage period affects the results in an entirely incalculable manner.

Tempering or storing only partly eliminates these incalculable influences as far as slightly deformed materials are concerned. The elastic properties pertinent to pure deformation made apparent by tempering appear only in very considerably deformed materials. Since in such materials  $\alpha$  remains, after the tempering, constant with the load,

the static determination of  $\alpha$  is very simple, provided the existing, usually very low limit of pure elasticity is not incautiously exceeded; for even the slightest plastic stretching will destroy the invariability of  $\alpha$  with load.

Anyway, in general the degree of variability of  $\alpha$  will be the conclusive factor in deciding whether to make allowance for fluctuations in order to avoid a complicated investigation.

$\alpha_0$  was, for instance, for copper:

$$\alpha_0 = 77.8 \times 10^{-6} \text{ mm}^2/\text{kg} \text{ (41.1\% stretched, afterward tempered)}$$

$$\alpha_0 = 81.6 \times 10^{-6} \text{ mm}^2/\text{kg} \text{ (annealed)}$$

$$\alpha_0 = 83.5 \times 10^{-6} \text{ mm}^2/\text{kg} \text{ (0.02\% stretched)}$$

$$\alpha_0 = 90.5 \times 10^{-6} \text{ mm}^2/\text{kg} \text{ (2.6\% stretched)}$$

$$\alpha = 101.0 \times 10^{-6} \text{ mm}^2/\text{kg} \text{ (4.7\% stretched, load 12.1 kg/mm}^2\text{)}$$

The values therefore vary up to 25%.

According to experience so far and to the literature, iron and steel seem to show exactly the same behavior as copper, the only difference being that the fluctuations will frequently vary within such narrow limits that they may be practically neglected.

#### SUMMARY

In the elastic straining of copper both Hooke's law and hysteresis may appear, according to the pretreatment.

After plastic deformation without subsequent storing hysteresis always occurred. The latter appeared as "pure" hysteresis up to test loads which remained 1/4 to 1/3 below the yield point produced in the respective case by stretching; that is, within this range no permanent residual strains were present, and  $\alpha = \frac{\epsilon}{\sigma}$  increased linearly with the test load; above the range of "pure" hysteresis, however,  $\alpha$  increases more than proportionally as from the very beginning for the annealed specimen.

With the degree of stretching  $\alpha$  first (up to 3% strain) rapidly increased, then continuously decreased to the maximum load strain. This behavior occurred when the specimen was annealed as well as when it was drawn in the die before the stretching. This phenomenon is the consequence of two overlapping single effects:

1.  $\alpha$  decreases with the pure deformation for which the influence of internal stresses will be assumed ineffective
2.  $\alpha$  increases with the appearance of internal stresses.

By tempering at low temperatures - whereby internal stresses are known to disappear -  $\alpha$  decreased by a certain amount. However, very small deformations immediately caused  $\alpha$  to increase again. Highly stretched and tempered specimens showed a lower  $\alpha$  than annealed ones. Storing of stretched specimens at room temperature resulted in both a decrease of  $\alpha$  and in a reappearance of permanent residual strains.

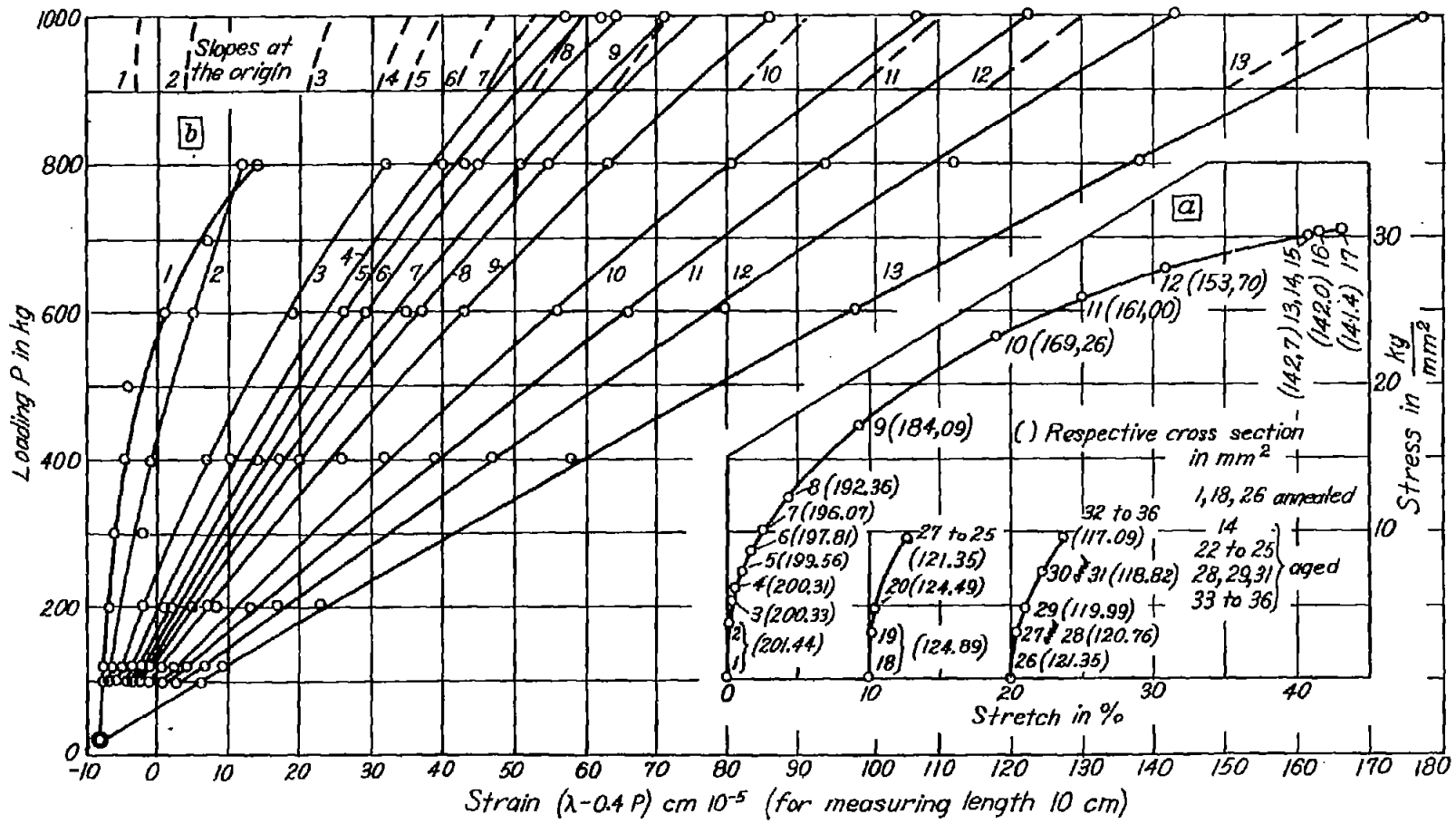
The increase of  $\alpha$  with the stress was greatest in the annealed state; it decreased with the degree of stretching. Highly stretched and thereupon tempered specimens showed, up to a load of 1/7 to 1/8 of the yield point, no variation of  $\alpha$ ; that is, Hooke's law became manifest. Extremely minute deformations, however, immediately cancelled this phenomenon again.

It may be concluded from the continuity of the occurrences that Hooke's law represents a practical limiting case of minimum variation of  $\alpha$  with the test stress.

From the increase of  $\alpha$  with the test stress one must draw the conclusion that the only sensible course, particularly for the annealed state, is to determine  $\alpha_0$  (for infinitely small loading), by extrapolation. For the tempered state an existing range of proportional strain must not be exceeded by loading, since otherwise, due to the slightest deformation, hysteresis and an increase of  $\alpha$  would immediately reappear.

I should like here to express my gratitude to Professor Memmler for his support of my work. For manifold stimulation regarding evaluation and interpretation of the test results I am indebted to Dr. Sachs who discussed them with me.

Translated by Mary L. Mahler  
National Advisory Committee  
for Aeronautics



Figures 1(a) and (b).- Stress-strain diagram and elastic strain curves of copper.

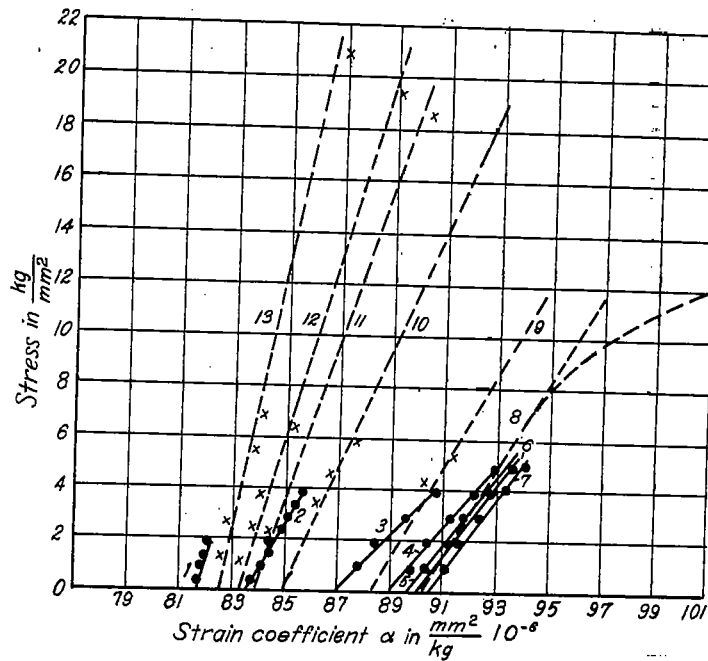


Figure 2.- The strain coefficient for annealed and stretched copper as a function of stress.

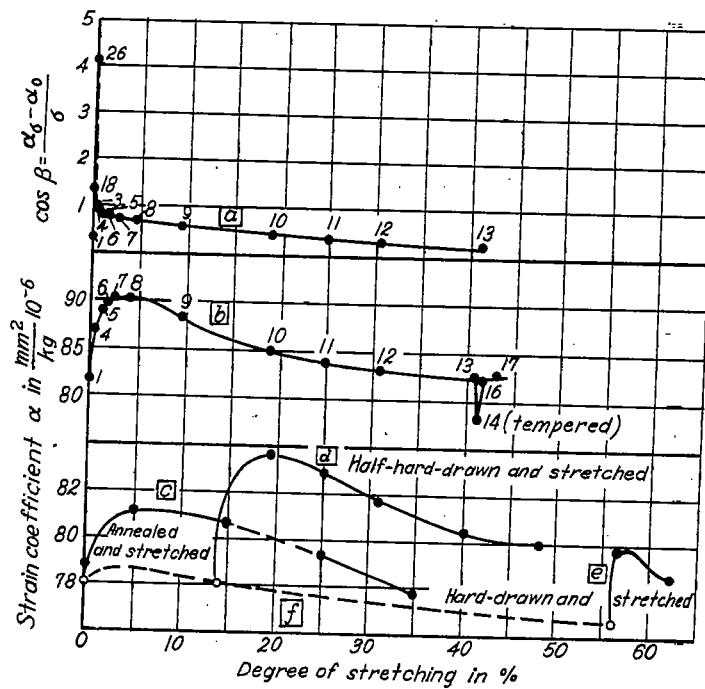
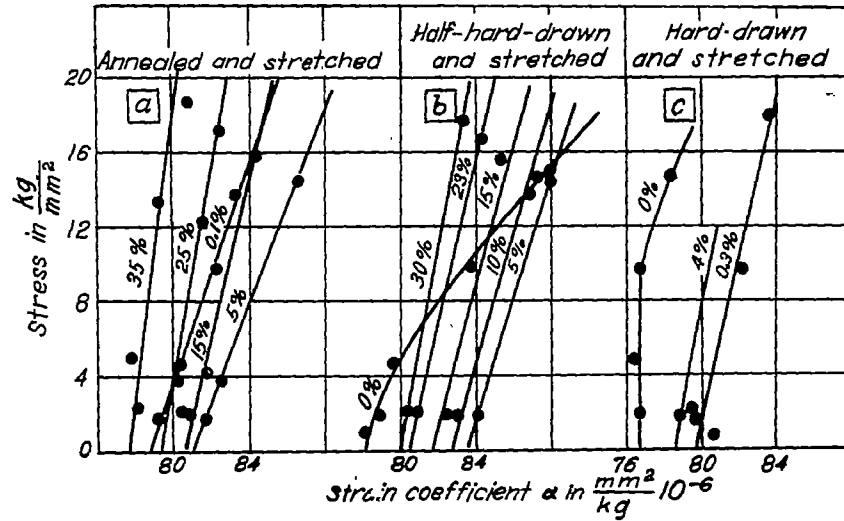
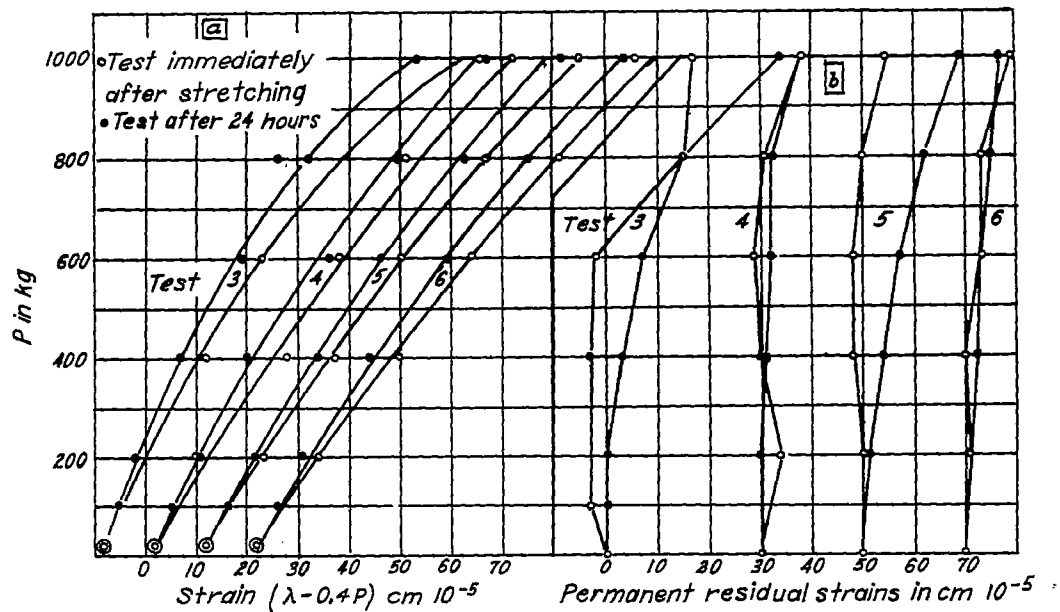


Figure 3(a).- Continuous variation of the direction constant.

Figures 3(b) through (f).- The strain coefficient as a function of the degree of stretching.



Figures 4(a) through (c).- The strain coefficient as a function of stress for drawn copper in comparison to annealed copper.



Figures 5(a) and (b).- Influence of storing on the strain coefficient.



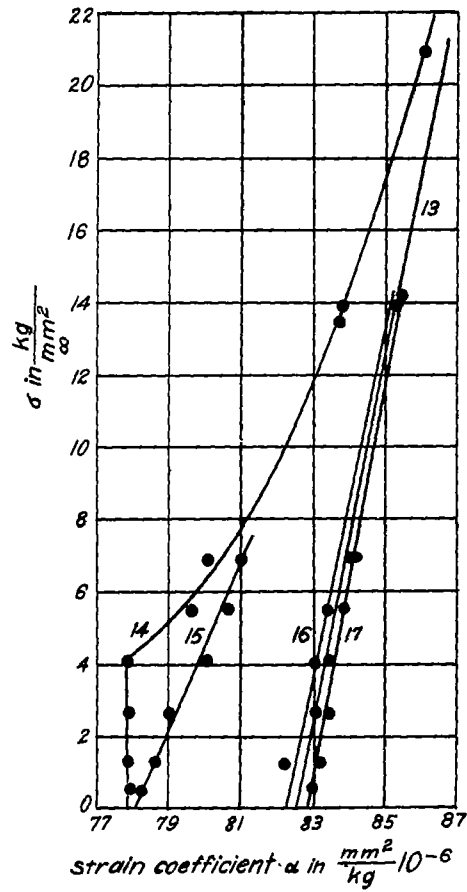
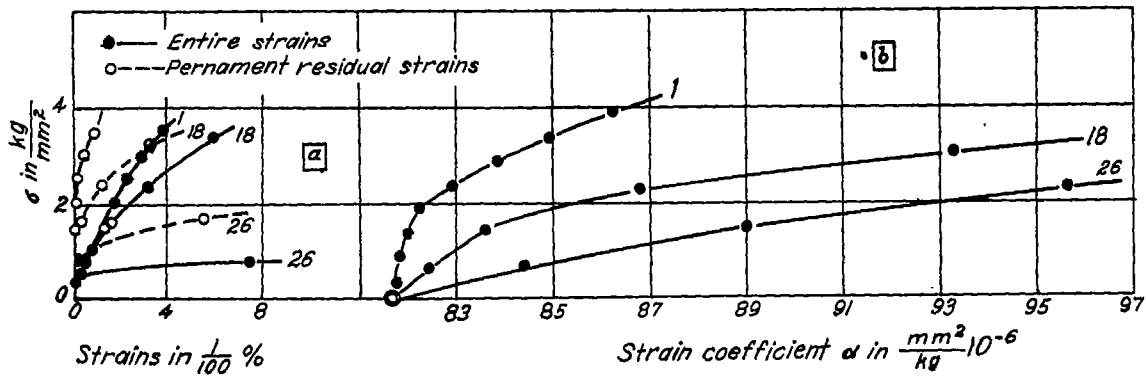


Figure 6.- Influence of tempering on the strain coefficient.



Figures 7(a) and (b).- Elastic strain curves and dependence of the strain coefficient on the stress for copper of various degrees of softening.

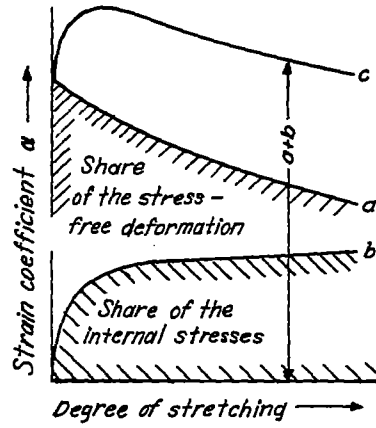


Figure 8.- Influence of deformation and internal stresses on the strain coefficient.

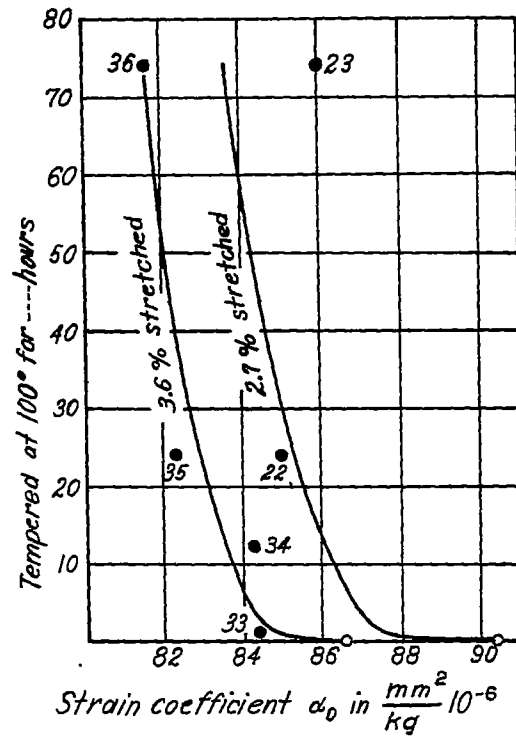
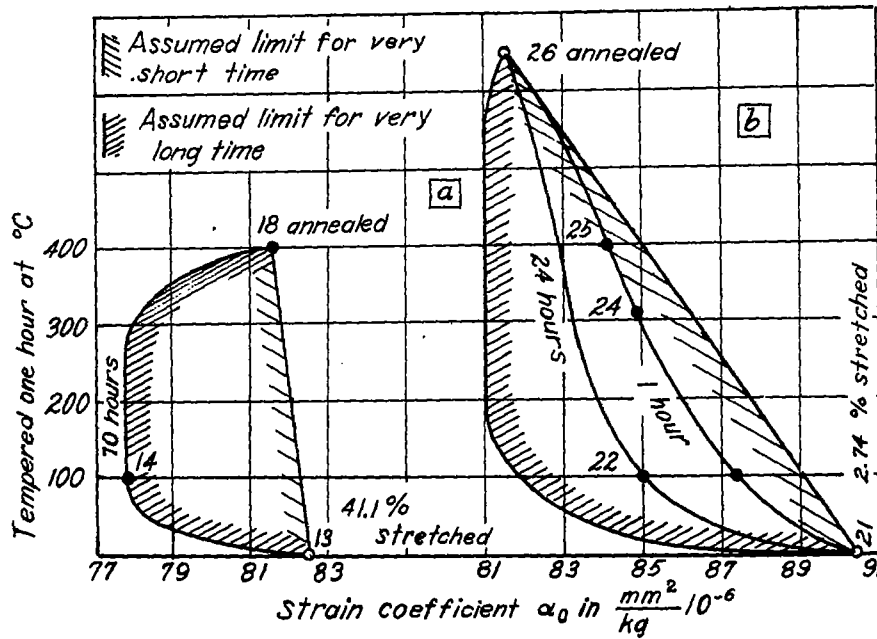


Figure 9.- Reduction of the strain coefficient with the duration of tempering.



Figures 10(a) and (b).- Reduction of the strain coefficient with the magnitude of the temperatures at which the tempering took place.

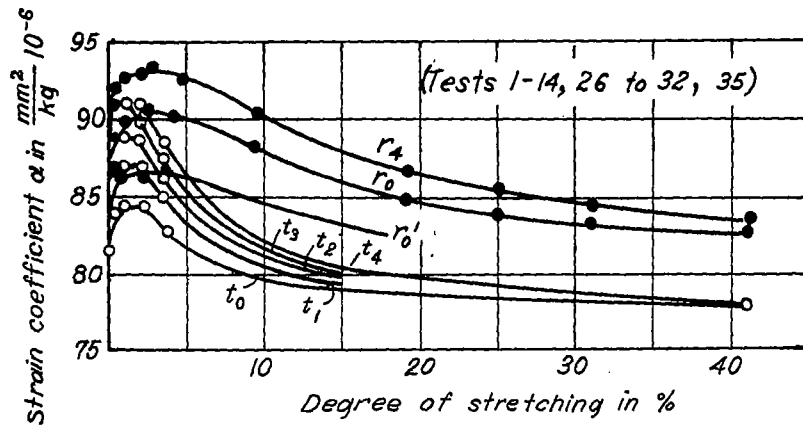


Figure 11.- Survey of the variation of the strain coefficient as a function of stretch, test stress, and tempering.  $r$  = drawn, 400° annealed, stretched and stored 24 hours;  $r'$  = 2.74 percent stretched, 650° annealed, stretched and stored 24 hours;  $t$  = stretched and tempered 24 hours;

index 0 to 4 = test stress in  $\frac{\text{kg}}{\text{mm}^2}$ .