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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM 1283

RESISTANCE OF A DELTA WING IN A SUPERSONIC FLOW

By E. A. Karpovich and F. I. Frankl

Translation

"Soprotivlenie Strelovidnogo Kryla pri Sverkhzvukovykh Skorostiakh."
Prikladnaya Matematika i Mekhanika, Vol. XI, No. 4, 1947.

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RESISTANCE OF A DELTA WING IN A SUPERSONIC FLOW*

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A plane wing of the plan form of an isosceles triangle (angle between the equal sides 2γ) symmetrically placed relative to the xz-plane is considered. The side opposite the angle 2γ serves as the trailing edge of the wing. The wing is inclined at the small angle of attack β in a flow of velocity V greater than the velocity of sound. The coordinate axes are fixed to the wing, the x-axis being directed along the flow, the z-axis vertically upward, and the y-axis perpendicular to the x- and z-axes (fig. 1). The flow will be conical (reference 1) and for the investigation of the motion it is sufficient to consider the velocity field in a certain plane perpendicular to the direction of the velocity of the basic flow, for example, in the plane $x=1$ with complex variable τ_1. After carrying out the transformation

$$\tau_1 = \frac{2\tau}{1 + \tau\bar{\tau}} \tan \alpha \tag{1}$$

where α is the Mach angle, the velocity components u, v, and w in the region $|\tau| < 1$ satisfy the equation of Laplace (reference 1) so that $u = \text{Re } f(\tau) \tan \alpha$, where $f(\tau)$ is a certain analytic function determined by the boundary conditions on the wing and on a unit circle.

The condition of nonvorticity gives

$$\omega(\tau) = v + iw = -\frac{1}{2}\int \tau\, df + \frac{1}{\bar{\tau}}\, d\bar{f} \tag{2}$$

If the wing is located entirely within the Mach cone ($\gamma < \alpha$), there may be expected at the forward edges, due to the infinite velocity, the occurrence of suction forces decreasing the resistance of the wing. The formula for computing the total resistance is given by M. I. Gurevich (reference 2) in the form

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$$X = \frac{\rho}{2}\iint (\mu^2 u^2 + v^2 + w^2) d\sigma \qquad \left(\mu = \frac{1}{\tan \alpha}\right) \tag{3}$$

where the integration is carried out over the entire plane τ_1 with a cut-out from $\tau_1 = -\tan\gamma$ to $\tau_1 = \tan\gamma$.

It is therefore clear that the total resistance will always be greater than zero.

Equation (3), however, is unsuitable for direct computation and with such representation the suction force is evidently not separated out. A different method of determining the resistance of a delta wing that avoids this difficulty is therefore indicated.

The momentum theorem is applied to the volume of air within the cones enclosing the leading edges. The suction force is then

$$X_1 = -2 \iint_{S+\sigma} (p - p_0) \cos (nx)\, dS - 2 \iint_{S+\sigma} \rho(u + V)v_n\, dS \tag{4}$$

where S is the lateral surface area of the cone, σ its base in the plane $x=1$, and n the outer normal to the surface.

The force X_1 is of the second-order magnitude relative to β, so that in the expressions under the integral sign second-order smallness terms must be taken into account. By use of the Bernoulli integral and the properties of conical flow, there is obtained

$$X_1 = \rho_0 \lim_{\delta_1 \to 0} \left\{ \int_0^{2\pi} (\mu^2 u^2 + v^2 + w^2) \cos \vartheta_1 \tan \gamma \delta_1\, d\theta_1 - 2 \int_0^{2\pi} u\,(v \cos \vartheta_1 + w \sin \vartheta_1)\delta_1\, d\vartheta_1 \right. \tag{5}$$

where δ_1 and ϑ_1 are polar coordinates in the plane τ_1 with center $\tau_1 = -\tan\gamma$.

Equation (5) gives the principal term of the suction force. For u, v, and w are determined with an error of the second-order smallness; that is, the error in the expression for X_1 will be of the third-order smallness. Equation (5) can easily be obtained directly from equation (3). It is necessary to differentiate equation (3) with respect to x and by using the equation

$$- \mu^2 \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad \left(\mu = \frac{1}{\tan \alpha}\right) \tag{6}$$

to reduce the surface integral to a contour integral. The equation is then integrated with respect to x after which the part corresponding to the suction force, which coincides with equation (5), can easily be separated out. For a symmetrical wing (references 2 and 3)

$$f(\tau) = \frac{\mu V \beta (1+\tau^2) \tan \gamma}{E(k') \sqrt{(b^2-\tau^2)(b^{-2}-\tau^2)}}$$

$$C_y = \frac{2\pi\beta}{E(k')} \tan \gamma \; * \tag{7}$$

where $E(k')$ is the complete elliptic integral of the second kind with modulus of $k' = \sqrt{1 - k^2}$ and $k = \mu \tan \gamma$, and b is determined from the relation $\tan \gamma / \tan \alpha = 2b/(1+b^2)$.

For computing the suction force, it is sufficient to have the principal terms $f(\tau)$ and $\omega(\tau)$:

$$f(\tau) = \frac{c_1}{\sqrt{\tau + b}} + O(1)$$

$$\omega(\tau) = \frac{1}{2} \left[bf + \frac{1}{2} \bar{f} \right] + O(1)$$

$$\left(c_1 = \sqrt{\frac{1 + b^2}{1 - b^2}} \, \frac{b}{2} \, \frac{\mu V \beta \tan \gamma}{E(k')} \right)$$

* Compare Ribner's (TR 908 pg 3)

$$C_{L_\alpha} = \frac{\pi A}{2E'(BC)}$$

The integrals entering equation (5) are readily computed:

$$X_1 = -\frac{\rho_0}{2}\pi V^2\beta^2 \frac{\tan^2\gamma}{E^2(k')}\frac{1-b^2}{1+b^2} \tag{8}$$

$$C_{x_1} = \frac{2X_1}{\rho_0 V^2 \tan\gamma} = -\beta^2\pi \frac{\tan\gamma}{E^2(k')}\frac{1-b^2}{1+b^2} = -\frac{k'}{4\pi}\cot\gamma C_y{}^2 \tag{9}$$

The drag coefficient will be

$$C_x = C_y\beta - \frac{k'}{4\pi}\cot\gamma C_y{}^2 \tag{10}$$

If $\gamma = \alpha$, that is, $b = 1$, then

$$C_x = C_y\beta$$

and the suction force vanishes. If $\alpha = \frac{1}{2}\pi$, $b = 0$,

$$C_x = C_y\beta - \frac{1}{4\pi}\cot\gamma\, C_y{}^2 = \pi\beta^2\tan\gamma \tag{11}$$

It is noted that $C_{x1}\mu/4\beta^2$ and $C_x\mu/4\beta^2$ are functions only of b or of $k = \mu\tan\gamma$. These relations are graphically shown in figure 2.

Translated by S. Reiss,
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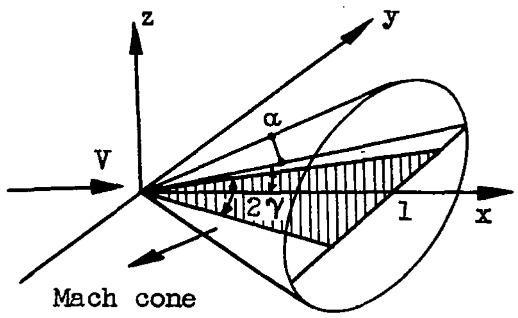

Figure 1.

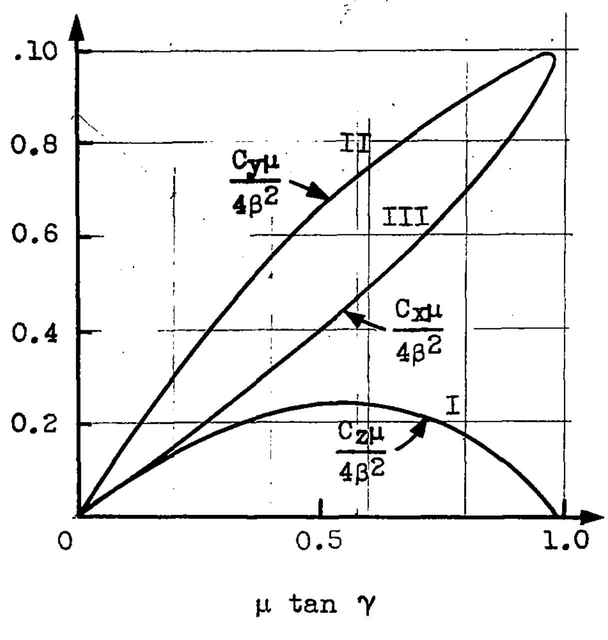

Figure 2.

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