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# RESEARCH MEMORANDUM

## DYNAMIC RESPONSE AT ALTITUDE OF A TURBOJET ENGINE WITH VARIABLE AREA EXHAUST NOZZLE

By Gene J. Delio and Solomon Rosenzweig

Lewis Flight Propulsion Laboratory
Cleveland, Ohio



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# DYNAMIC RESPONSE AT ALTITUDE OF A TURBOJET ENGINE WITH VARIABLE AREA EXHAUST NOZZLE

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## SUMMARY

The dynamic characteristics of a turbojet engine with variable exhaust nozzle area were investigated over a range of altitudes and flight Mach numbers. These characteristics generalized to standard static sea-level conditions thereby permitting the prediction of engine dynamic behavior at all altitudes and flight Mach numbers.

Data resulting from approximate step disturbances in either independent variable suggested transfer functions derivable from basic functional relationships. The minimum data required to define the transfer functions were: experimentally determined dynamic characteristics (engine time constant and initial rise ratio) obtained from indicial responses; static characteristics determined from steady-state performance curves for each of the independent variables. The engine time constant and initial rise ratios could be obtained from a step change in either of the independent variables.

The constants of the transfer functions, which describe engine dynamic behavior, varied throughout the engine-speed range and for different exhaust nozzle areas. These constants, generalized to standard static sea-level conditions, are plotted as functions of generalized speed. These constants are also tabulated for five particular combinations of engine speed and exhaust nozzle area.

## INTRODUCTION

Control system design for turbojet engines is critical because full realization of engine potentialities requires operation at or near the maximum temperature and rotational speed of the engine. It is also desirable to have a controller that (1) prevents power-plant damage; (2) operates in a stable manner under varying conditions of altitude, flight Mach number, and thrust setting; (3) requires no attention from the operator except for thrust setting; and (4) has good response time at all operating conditions. These requirements can be met only by closed loop control systems, which permit much greater accuracy than is possible with open loop systems.

In a closed loop control system for a turbojet engine, the power plant is one element of the control system, and any variation in the engine dynamic behavior is reflected throughout the system. Previous theoretical and experimental analyses (references 1 to 3) have indicated that engine dynamics vary with altitude, flight Mach number, and operating point. Therefore, a knowledge of these variations in engine behavior must be incorporated into the controller to insure satisfactory operation of the control system over the complete operating range. It has been shown (reference 4) that the steady-state characteristics of a turbojet engine generalize for varying altitude and flight Mach number conditions. Because engine dynamics also vary, a generalization similar to steady-state generalization would be desirable.

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The objects of this investigation, carried out at the NACA Lewis laboratory, are: (1) to present the dynamic characteristics of the engine dependent variables with respect to the independent variables (engine fuel flow and exhaust nozzle area); (2) to show the variation of engine dynamic characteristics with altitude, flight Mach number, and engine speed; (3) to demonstrate a means of generalization of the dynamic characteristics to sea-level conditions; and (4) to determine the minimum amount of experimental data necessary to completely describe turbojet engine dynamic characteristics.

A turbojet engine was operated over a range of altitudes varying from 15,000 to 40,000 feet at flight Mach numbers ranging from 0.22 to 0.88. The transient responses of the dependent engine variables (speed, compressor-discharge total pressure, turbine-discharge total pressure, turbine-discharge temperature, and jet thrust) to approximate step disturbances of the independent variables (fuel flow and exhaust nozzle area) were recorded. Steady-state calibration data were obtained over the range investigated.

Data are presented in graphical form to indicate the trends of the variation in engine dynamic characteristics with altitude, flight Mach number, and engine speed. Generalization of the data to sea-level conditions is also shown.

## THEORETICAL ANALYSIS

A knowledge of engine dynamic characteristics results from the study of engine responses to certain test inputs. In practice, the engine may be subjected to either a sinusoidal or a transient input because of the ease with which these inputs are applied and the resulting responses can be analyzed.

The value of determining the response to a transient input has been pointed out (for example, reference 5). Specifically, any linear system is completely determined if its response to some type of transient input is specified. Reference 6 demonstrates that indicial responses (step disturbances of the independent variables) yield the basic dynamic characteristics of a turbojet engine with minimum engine and equipment operating time. Data resulting from step function inputs indicate that engine dynamic behavior can be described by elementary transfer function forms, and the nature of the indicial responses hint that certain functional relationships exist. With the use of only the basic assumption of linearity, functional relationships can be used to derive the forms of the engine transfer functions. These transfer functions fit experimental data which are limited in accuracy by various instrumentation lags and supply source regulation. In addition, the gas turbine engine is primarily a single capacity system. Therefore, the method of transient analysis produces results that can be interpreted simply.



Engine Speed Response

Typical responses to a step change in fuel flow, exhaust nozzle area being held constant, are shown in figure 1. The speed response is exponential in nature, and the transfer function for this particular form is (reference 5)

$$KG(s) = \frac{K_{nw}}{1 + \tau s} \tag{1}$$

(All symbols are defined in appendix A.) The gain term $K_{nw}$ is a measure of engine sensitivity, or the speed change due to a change in fuel flow, $\frac{dN}{dW}$. The engine time constant $\tau$ is a measure of the acceleration time of the engine and is the time for the engine speed to increase or decrease by an amount $\left(1 - \frac{1}{e}\right)$ of the final change $\frac{dN}{dW}\Delta W$. Figure 2(a) illustrates an indicial speed response to fuel flow.

From the basic assumptions and the mathematical development shown in reference 6, the transfer function relating speed to changes in both the fuel flow and exhaust nozzle area is

$$\Delta N(s) = \frac{K_{nw}}{1 + \tau s}\Delta W(s) + \frac{K_{na}}{1 + \tau s}\Delta A(s) \tag{2}$$

The engine time constant is

$$\tau = \frac{-J}{\left(\frac{\partial Q}{\partial N}\right)_{W,A}} \tag{2a}$$

The gain terms $K_{nw}$ and $K_{na}$, defined in appendix A, are a measure of speed sensitivity in equilibrium to changes in fuel flow and exhaust nozzle area, respectively. 2379

In the design of gas turbine engine controls, other engine parameters are of importance. Compressor-discharge total pressure may be used to limit fuel flow, thereby preventing compressor stall or surge. Turbine-discharge temperature and pressure limits are needed to prevent overtemperature and engine damage. Jet thrust is a measure of the power output of the engine. Therefore, the dynamic responses of these variables should also be known.

### Compressor-Discharge Total Pressure

The response of compressor-discharge total pressure to an approximate step disturbance in fuel flow is illustrated in figure 1. A transfer function describing this type of response is (reference 5)

$$KG(s) = K_{p_c w} \frac{(1 + d\tau s)}{1 + \tau s} \tag{3}$$

The gain term $K_{p_c w}$ is the sensitivity of pressure response to changes of fuel flow. Figure 2(b) illustrates an indicial pressure response to fuel flow. As developed in reference 6,

$$K_{p_c w} = \left(\frac{\partial P_c}{\partial W}\right)_{N,A} + K_{nw}\left(\frac{\partial P_c}{\partial N}\right)_{W,A} \tag{4}$$

$$d = \frac{\left(\frac{\partial P_c}{\partial W}\right)_{N,A}}{K_{p_c w}} \tag{4a}$$

In figure 2(b), the initial pressure rise is due to the fuel increase only and it is at constant speed. The additional pressure rise is due to the speed change only and it is at constant fuel flow. The initial rise ratio $d$ is the ratio of the pressure rise at constant speed to the entire pressure rise. When the pressure rise at constant speed is less than the entire pressure rise, the initial rise ratio $d$ is less than unity.

From the basic assumptions and the mathematical development shown in reference 6, the transfer function relating compressor-discharge
2379 total pressure to changes in both the fuel flow and exhaust nozzle area is

$$\Delta P_c(s) = K_{p_c w} \frac{(1 + d\tau s)}{1 + \tau s} \Delta W(s) + K_{p_c a} \frac{(1 + e\tau s)}{1 + \tau s} \Delta A(s) \qquad (5)$$

Again, the gain terms $K_{p_c w}$ and $K_{p_c a}$ are the amount of pressure changes resulting from a change in fuel flow and exhaust nozzle area, respectively. The values for the initial rise ratio are

$$d = \frac{\left(\frac{\partial P_c}{\partial W}\right)_{N,A}}{K_{p_c w}} \qquad (5a)$$

$$e = \frac{\left(\frac{\partial P_c}{\partial A}\right)_{N,W}}{K_{p_c a}} \qquad (5b)$$

The initial rise ratio $d$ is the ratio of the initial change to the total change in pressure at constant exhaust nozzle area, and it is the immediate effect noted from a change in fuel flow at constant speed. Similarly, the initial rise ratio $e$ is the ratio of the initial change to the total change in pressure at constant fuel flow, and it is the immediate effect noted from a change in area at constant fuel flow.

## Turbine-Discharge Total Temperature

## and Total Pressure

The responses of turbine-discharge total temperature and total pressure to a step change in fuel flow at constant area are also shown in figure 1. The pressure response indicates an initial rise ratio less than one; that is, the pressure increase due to fuel flow only is less than the pressure increase due to both fuel flow and speed. The temperature response, however, indicates that the temperature increase due to fuel flow only is greater than the temperature increase due to both fuel flow and speed; that is, the initial rise ratio for the temperature response is greater than one. Similar data at maximum speed shows that, for this particular engine, the reverse is true. The pressure overshoots its final value whereas the temperature does not. It follows that at some speed less than maximum, both the temperature and pressure responses have the same initial rise ratio. There is, therefore, one operating point of the turbojet engine at which the turbine-discharge total pressure and total temperature respond identically to fuel-flow changes.

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The form of the transfer functions for temperature and pressure responses to fuel-flow changes (fig. 1) is of the same form as the transfer function describing the compressor-discharge total pressure response to fuel flow. From the basic assumptions and the mathematical development shown in reference 6, the transfer function relating turbine-discharge total pressure to changes in both the fuel flow and exhaust nozzle area is

$$\Delta P_t(s) = K_{p_t w} \frac{(1 + f\tau s)}{1 + \tau s} \Delta W(s) + K_{p_t a} \frac{(1 + g\tau s)}{1 + \tau s} \Delta A(s) \tag{6}$$

The initial rise ratios are

$$f = \frac{\left(\frac{\partial P_t}{\partial W}\right)_{N,A}}{K_{p_t w}} \tag{6a}$$

and

$$g = \frac{\left(\frac{\partial P_t}{\partial A}\right)_{N,W}}{K_{p_t a}} \tag{6b}$$

The explanation of the gain terms and initial rise ratios follows that for the compressor-discharge total pressure response.

Similarly, the transfer function relating turbine-discharge temperature to changes in both fuel flow and exhaust nozzle area is

$$\Delta T(s) = K_{tw} \frac{(1 + h\tau s)}{1 + \tau s} \Delta W(s) + K_{ta} \frac{(1 + j\tau s)}{1 + \tau s} \Delta A(s) \tag{7}$$

The initial rise ratios are

$$h = \frac{\left(\frac{\partial T}{\partial W}\right)_{A,N}}{K_{tw}} \tag{7a}$$



and

$$j = \frac{\left(\frac{\partial T}{\partial A}\right)_{W,N}}{K_{ta}} \tag{7b}$$

Again, the explanation for the gain terms and initial rise ratios follows that for the compressor-discharge total pressure responses.

### Jet Thrust

Comparison of the jet thrust response with the turbine-discharge total pressure and total temperature responses indicates a similarity of form. Therefore, the same type of transfer function describes the jet thrust response to fuel flow changes and, as developed in reference 6, is

$$\Delta F(s) = K_{fw} \frac{(1 + k\tau s)}{1 + \tau s} \Delta W(s) + K_{fa} \frac{(1 + l\tau s)}{1 + \tau s} \Delta A(s) \tag{8}$$

where the constants of the equation are described in a manner similar to the preceding transfer functions.

### Minimum Data Necessary to Completely Describe Dynamic Behavior

With the assumption of a constant inlet Mach number and constant altitude, a turbojet engine with a variable area nozzle has two degrees of freedom. However, the recorded response of a particular dependent variable with respect to simultaneous changes of fuel flow and exhaust nozzle area is difficult to interpret. The use of linear analysis allows a simplification. As shown by the transfer functions describing engine behavior, the response to one input may be analyzed separately while the other input is held constant. The response to two inputs then is the superposition of the two separate responses. Therefore, the dynamic response of any dependent engine variable may be obtained by varying the fuel flow and area separately and superimposing the results of both responses.

All engine gains are static characteristics and may be determined by measuring the slope of the steady-state relationships. The engine time constant $\tau$ is a dynamic characteristic and must be determined experimentally. From its mathematical definition, it is proportional to the negative reciprocal of $\left(\frac{\partial Q}{\partial N}\right)$ at constant fuel flow and constant exhaust nozzle area. Therefore, the determination of the engine time constant using either fuel flow or exhaust nozzle area as a forcing function should produce the same result. 2579

The initial rise ratios, using compressor-discharge pressure as an example, are also dynamic characteristics and can be expressed as

$$d = \frac{\left(\frac{\partial P_c}{\partial W}\right)_{N,A}}{K_{p_c w}} = 1 - \frac{K_{nw}}{K_{p_c w}}\left(\frac{\partial P_c}{\partial N}\right)_{W,A} \tag{9}$$

$$e = \frac{\left(\frac{\partial P_c}{\partial A}\right)_{N,W}}{K_{p_c a}} = 1 - \frac{K_{na}}{K_{p_c a}}\left(\frac{\partial P_c}{\partial N}\right)_{W,A} \tag{10}$$

obtained by using the identities

$$K_{p_c w} = K_{nw}\left(\frac{\partial P_c}{\partial N}\right)_{W,A} + \left(\frac{\partial P_c}{\partial W}\right)_{N,A}$$

$$K_{p_c a} = K_{na}\left(\frac{\partial P_c}{\partial N}\right)_{W,A} + \left(\frac{\partial P_c}{\partial A}\right)_{N,W}$$

The quantities $d$ and $e$ are not independent and one can be calculated from the other, the single unknown quantity being $\left(\frac{\partial P_c}{\partial N}\right)_{W,A}$. This effect, common for transients caused by fuel flow or area changes, is the effect of engine speed on the variable $(P_c)$ at constant fuel flow and area.

As a result, the minimum data needed to completely describe the compressor-discharge total pressure response to changes in both fuel flow and exhaust nozzle area are:

(1) Static characteristics determined from steady-state curves: $K_{nw}$, $K_{na}$, $K_{p_c w}$, $K_{p_c a}$, and so forth

(2) Experimentally determined dynamic characteristics

(a) Engine time constant $\tau$

(b) An initial rise ratio, or $\left(\frac{\partial}{\partial N}\right)_{W,A}$, for each variable

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## APPARATUS AND INSTRUMENTATION

### Engine Installation

A turbojet engine was mounted on a wing section which spanned the test section of the NACA Lewis laboratory altitude wind tunnel. The altitude and flight Mach number were simulated in the following manner. Air from a climatic control source was supplied to the engine through a ram pipe. The engine exhausted into the altitude wind tunnel maintained at constant altitude. The wind-tunnel static pressure $p_0$ and static temperature $t_0$ determined the simulated altitude conditions. The ratio of engine-inlet total pressure $P_1$ to $p_0$, or the ram pressure ratio, was a measure of the engine simulated flight speed.

Engine. - The turbojet engine used in this investigation consisted of an 11-stage axial-flow compressor, eight through-flow combustion chambers, a single-stage gas turbine, and a variable area exhaust nozzle. A sketch of this engine is shown in figure 3.

Fuel system. - The fuel system was of standard design except for the fuel metering valve. This valve was designed to maintain a fixed fuel flow rate for each fuel valve position independent of the pump-discharge pressure or combustion-chamber pressure.

A voltage signal was amplified and fed into a positional servomotor which was coupled to the fuel valve through a gear reducer. The DC level determined the engine operating point and a switching of the DC levels produced approximate step disturbances in the fuel flow. The lagging of the fuel flow behind fuel valve position was small compared with the dominant lag, engine speed.

Exhaust nozzle. - The variable area exhaust nozzle was positioned in essentially the same manner as the fuel valve. A voltage signal was amplified and fed into a positional servomotor which was coupled to the exhaust nozzle with a gear reducer and ball-bearing screw jack. The DC voltage level determined the position of the exhaust nozzle.

### Instrumentation

The measurement and recording of engine transients required instrumentation and recording facilities that possessed the following characteristics; (1) linear phase shift and flat frequency response, (2) sensing elements that had good dynamic characteristics, and (3) signal amplification about any operating point. The recording system was used to obtain the correct form of the transient, and the absolute values of the recorded variables were obtained from steady-state instrumentation. 2379

The recording system used to obtain the correct wave shapes of the transients consisted basically of a multichannel recorder with associated amplifiers, and a sensing element for each channel. A complete description of the design of the instrumentation is given in reference 7. The typical trace shown in figure 1 was obtained from this type of instrumentation.

The instrumentation for steady-state values was used for the calibration of the recorded transient data. All pressures were measured by mercury manometers and photographically recorded. Temperatures were obtained from chromel-alumel and iron-constantan thermocouples and recorded on self-balancing potentiometers. Engine thrust was measured by balance scales, engine speed by a chronometric tachometer, and fuel flow by rotameters. Exhaust nozzle area was indicated by a potentiometer voltage.

## PROCEDURE AND METHODS

### Experimental Procedure

The engine used in this investigation was operated over a range of engine speeds at the following simulated altitudes and flight Mach numbers:

| Altitude (ft) | Flight Mach number |
|---|---|
| 15,000 | 0.22, 0.63 |
| 25,000 | .22, .85 |
| 40,000 | .22, .63 |

The dynamic characteristics of the engine were first evaluated for fuel-flow changes at a constant exhaust nozzle area. The power plant was subjected to a sudden change in fuel flow which resulted in a speed change of 400 rpm. The fuel-flow changes were initiated at speeds ranging from idling to maximum engine speed. The transient behavior of the fuel valve position (fuel flow), exhaust nozzle area, engine speed, compressor-discharge total pressure, turbine-discharge total temperature and pressure, and jet thrust were recorded. Steady-state calibration values of each recorded parameter were observed at the initial and final points of each run.

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The same experimental procedure was used to determine the dynamic response of the engine to changes in exhaust nozzle area at constant fuel flow.

## Procedure for Processing Data

Determination of engine time constants and initial rise ratios. - A method of plotting the transient response recorded data was devised to compensate for the inherent lag contained in the fuel flow. As developed in appendix B

$$ln\left[1 - \frac{\Delta N}{(\Delta N)_f}\right] = -\frac{t}{\tau} \tag{B4}$$

and

$$ln\left[1 - \frac{\Delta P_c}{(\Delta P_c)_f}\right] = ln\ (1 - d) - \frac{t}{\tau} \tag{B7}$$

Figure 4 is a plot of the preceding equations as applied to figure 1. In this manner the initial rise ratio of the increase of compressor-discharge total pressure to fuel-flow changes, and the engine time constant can be determined.

The deviation from a straight line at zero time is due to the fact that fuel flow was not a true step function. The straight line portion of the speed response was extended to the initial steady-state value, and this intersection is the theoretical zero time that would have corresponded to a true step input. This procedure was also followed in the evaluation of turbine-discharge total pressure, turbine-discharge total temperature, and jet thrust responses to approximate steps in fuel flow.

The various gain terms of the transfer functions were determined from the slope of the steady-state curves relating the various parameters about the desired operating point.

<u>Correction for changes of inlet total pressure</u>. - The characteristics of the altitude-wind-tunnel control of inlet total pressure was such that during engine acceleration, the inlet total pressure varied in the same manner as speed (fig. 1). This effect essentially changed the flight Mach number during engine acceleration. Therefore, correction factors were applied to the data to determine the true values of the constants of the transfer functions for a constant flight Mach 2379 number. The development of these correction factors is given in appendix B. Corrections were applied to the responses of engine speed, compressor-discharge total pressure, and turbine-discharge total temperature.

The turbulence contained in the turbine-discharge pressure response masked the speed effect (fig. 1). For this variable the initial pressure rise was noted directly from the data because the inlet total pressure had not yet changed. The total change of pressure was obtained from the slope of the steady-state curve at the desired operating point. The ratio of the two pressure rises thus determined the initial rise ratio.

A similar procedure was followed for determining the initial rise ratio of the jet thrust response. Here it was necessary because the engine vibration produced force changes of the order of the thrust increase. Furthermore, the decrease in inlet total pressure produced an equivalent positive thrust which amplified the speed effect on the jet thrust response.

## Generalization of Data

It is shown in reference 4 that generalized parameters permit the results of specific tests with a specific engine to be used for estimating performance at other conditions. The results obtained from steady-state performance at altitude conditions generalize to standard sea-level conditions. Thus, the engine gains determined from the slope of the steady-state data would generalize because they are static characteristics.

If the variation of viscosity, combustion efficiency, and compressibility is small for small deviations from engine equilibrium, it may be possible to generalize dynamic characteristics in the same manner that steady-state characteristics generalize.

The following corrections, listed in reference 4, have been applied to all altitude data to generalize to standard sea-level conditions: $\frac{N}{\sqrt{\theta}}$, $\frac{W}{\delta\sqrt{\theta}}$, $\frac{F}{\delta}$, $\frac{T}{\theta}$, $\frac{P}{\delta}$, $\frac{Q}{\delta}$, where

$$\delta = \frac{P_1}{2116\ (\mathrm{lb/sq\ ft})}$$

$$\theta = \frac{T_1}{519^{\circ}\ \mathrm{R}}$$



## RESULTS AND DISCUSSION

The results of the application of the indicial response method (step function input) for determining the transfer functions of a turbojet engine are presented in two parts. First, the results are shown for a step input of fuel flow at constant exhaust nozzle area. Second, steady-state relationships at constant fuel flow are shown that aid in determining the gains associated with constant fuel-flow performance. Also, a comparison is made of the results of determining the engine time constant at either constant area or constant fuel flow.

### Transfer Functions at Constant Exhaust Nozzle Area

Engine speed. - The engine-speed transfer function can be described by the engine time constant $\tau$ and the speed gain $K_{nw}$. The time constant is shown as a function of engine speed in figure 5 for the various altitudes and flight Mach numbers investigated. At altitudes of 15,000 and 40,000 feet there is approximately 25 percent decrease in time constant resulting from an increase in Mach number from 0.22 to 0.63. At 25,000 feet there is approximately 35 percent decrease in the time constant resulting from an increase in Mach number from 0.22 to 0.85. At a constant Mach number, there is approximately 87 percent increase in engine time constant from 15,000 to 40,000 feet.

In reference 4 it is stated that generalization factors could be used to correlate performance characteristics of the engine except for the characteristics involving fuel flow explicitly. Generalized fuel flow is shown as a function of generalized speed in figure 6, for the altitudes and Mach numbers considered. The fuel flow characteristics of this engine are such that combustion efficiency does not permit generalization. However, for the analysis of engine dynamics, the interest lies in the slope of the engine-speed - fuel-flow curves and not in the absolute values. The data, generalized to static standard sea-level conditions, appear as parallel curves thereby having identical slopes. It is this feature of the engine-speed - fuel-flow curves which allows generalization of the gain $\overline{K}_{nw}$ as shown in figure 7.

The generalized time constant is also shown as a function of generalized speed in figure 7. This generalization accounts for the effect of altitude and flight Mach number on the engine time constant. The use of both generalized curves consequently, allows the prediction of the engine acceleration characteristics for any altitude or flight Mach number. 2379

Compressor-discharge total pressure. - The compressor-discharge total pressure transfer function can be described by the engine time constant $\tau$, the pressure gain $K_{p_c w}$, and the initial rise ratio d. The time constant has already been discussed.

The initial rise ratio d generalized to standard static sea-level conditions, appears to vary linearly with speed (fig. 8) above 5500 rpm. It approaches unity at maximum speed, which indicates that the pressure increase due to speed alone (constant fuel flow) approaches zero for the condition of maximum speed.

The pressure gain $\overline{K}_{p_c w}$ is also shown in figure 8 generalized to standard sea-level conditions. The gain diminishes with increasing engine speed and is maximum at the lowest speed. The variation in gain with speed is approximately 700 percent.

Turbine-discharge total temperature. - The turbine-discharge total temperature transfer function can also be described by the engine time constant, temperature gain, and initial rise ratio. The generalized temperature gain $\overline{K}_{tw}$ (fig. 9) rises sharply with speed to 6500 rpm and remains essentially constant at a value 0.160 from 6500 rpm to maximum speed.

The initial rise ratio h (fig. 9) drops rapidly with speed to 6500 rpm and remains fairly constant at 1.2 to a speed of 7700 rpm. In this speed range the initial rise ratio is greater than unity indicating that the temperature initially overshoots its final value. At maximum speed, however, the temperature does not overshoot its final value.

Turbine-discharge total pressure. - The generalized constants of the transfer function describing the response of turbine-discharge pressure to changes in fuel flow are plotted in figure 10. The pressure gain $\overline{K}_{p_t w}$ decreases with increasing speed and approaches a minimum at maximum speed. The initial rise ratio f increases with increasing speed and it reaches unity at a generalized speed of 8100 rpm. At this operating point the turbine-discharge pressure follows exactly the change in fuel flow. Above this speed, this turbine-discharge pressure overshoots its final value.

Jet thrust. - The thrust response to changes in fuel flow can be described by a transfer function whose generalized constants are plotted in figure 11, and, including the generalized time constant of figure 7, the dynamic behavior of thrust at standard static sea-level conditions is completely described. Generally, the thrust gain decreases with increasing speed, whereas the initial rise ratio increases with increasing speed, but always remains below unity.

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Generalized transfer functions. - Figures 7 to 11 indicate that the dynamic characteristics of a turbojet engine generalize and these data can be used to predict engine dynamic behavior at any altitude and flight Mach number included in the range investigated. With the use of the correction factors listed in appendix B and the data in figures 7 to 11, the various responses at an arbitrary flight Mach number and altitude become:

Engine speed

$$\Delta N(s) = \frac{\left(\frac{1}{\delta}\right) \overline{K}_{nw}}{1 + \frac{\sqrt{\theta}}{\delta} \overline{\tau} s} \Delta W(s) \tag{11}$$

Compressor-discharge total pressure

$$\Delta P_c(s) = \overline{K}_{p_c w} \left(\frac{1}{\sqrt{\theta}}\right)\left(\frac{1 + d \frac{\sqrt{\theta}}{\delta} \overline{\tau} s}{1 + \frac{\sqrt{\theta}}{\delta} \overline{\tau} s}\right) \Delta W(s) \tag{12}$$

Turbine-discharge total pressure

$$\Delta P_t(s) = \overline{K}_{p_t w} \left(\frac{1}{\sqrt{\theta}}\right)\left(\frac{1 + f \frac{\sqrt{\theta}}{\delta} \overline{\tau} s}{1 + \frac{\sqrt{\theta}}{\delta} \overline{\tau} s}\right) \Delta W(s) \tag{13}$$

Turbine-discharge total temperature

$$\Delta T(s) = \overline{K}_{tw} \left(\frac{\sqrt{\theta}}{\delta}\right)\left(\frac{1 + h \frac{\sqrt{\theta}}{\delta} \overline{\tau} s}{1 + \frac{\sqrt{\theta}}{\delta} \overline{\tau} s}\right) \Delta W(s) \tag{14}$$

[redacted]

Jet thrust

$$\Delta F(s) = \overline{K}_{fW} \left(\frac{1}{\sqrt{\theta}}\right)\left(\frac{1 + k\,\frac{\sqrt{\theta}}{\delta}\,\overline{\tau}\,s}{1 + \frac{\sqrt{\theta}}{\delta}\,\overline{\tau}\,s}\right)\Delta W(s) \tag{15}$$

Transfer Functions at Constant Fuel Flow

In the section, Minimum Data Necessary to Completely Describe 2379
Dynamic Behavior, it is indicated that the dynamic characteristics resulting from changes in either fuel flow or nozzle area are not mutually independent. The dynamic characteristics resulting from fuel-flow changes can be used to evaluate the dynamic characteristics resulting from area changes. The only additional data necessary are the static characteristics describing the steady-state performance.

Engine time constant. - The generalized time constant obtained for step changes in exhaust nozzle area is shown in figure 12. It is compared with the engine time constant obtained from a step input in fuel flow. The agreement indicates that the engine time constant may be obtained by either input.

The curves presented in figure 13 were obtained by cross-plotting the data shown in figures 14(a), 14(b), and 14(c). These data were obtained by operating the engine at a constant speed and varying the exhaust nozzle area and fuel flow to maintain the specified speed.

The gain terms for compressor-discharge total pressure, turbine-discharge total pressure, turbine-discharge total temperature, and jet thrust can be similarly determined from figures 15 to 18. A constant corrected fuel-flow line of 5000 pounds per hour is shown in figures 14 to 18, and the respective gains were obtained from the slope of the constant fuel-flow lines at the various speeds.

Engine gains. - Figure 13 presents the steady-state performance of engine speed against exhaust nozzle area at constant corrected fuel flows of 5000 and 3600 pounds per hour. For a constant corrected fuel flow of 5000 pounds per hour the slopes are equal at any speed. Thus the gain term $\overline{K}_{na}$ is independent of altitude conditions. Similarly, the engine gain generalizes at a corrected fuel flow of 3600 pounds per hour.

Initial rise ratios. - The initial rise ratio for a particular dependent engine variable may be calculated as outlined in the section, Theoretical Analysis. With the use of the response of compressor-discharge total pressure as an example, equation (9) gives

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$$e = 1 + (d - 1) \frac{K_{na}}{K_{nw}} \frac{K_{p_c w}}{K_{p_c a}} \tag{16}$$

Generalized transfer functions. - The values presented in figures 14 to 18 are generalized. With the use of these values, the responses at prescribed altitude and flight Mach number can be determined similarly to the responses for changes in fuel flow. The response for changes in both exhaust nozzle area and fuel flow then is the linear sum of each response. As an example, the response at prescribed altitude and flight Mach number of speed to simultaneously varying exhaust nozzle area and fuel flow is

$$\Delta N = \frac{\overline{K}_{nw}\left(\frac{1}{\delta}\right)}{1 + \frac{\sqrt{\theta}}{\delta}\,\overline{\tau} s}\,\Delta W(s) + \frac{\overline{K}_{na}\,(\sqrt{\theta})}{1 + \frac{\sqrt{\theta}}{\delta}\,\overline{\tau} s}\,\Delta A(s) \tag{17}$$

## Dynamic Characteristics at Sea-Level Rated Conditions

With the use of the methods described in this paper and the values determined from the figures presented, the dynamic characteristics of the turbojet engine are summarized as follows:

| Rating: | Equivalent steady-state conditions: |
|---|---|
| $N = 7950$ rpm | $W = 5400$ pounds per hour |
| $T = 1600^{\circ}$ R | $P_c = 10{,}900$ pounds per square foot |
| $F = 5200$ pounds | $P_t = 3880$ pounds per square foot |
| | $A = 2.40$ square feet |

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[Engine time constant = 1.8 sec]

| | N | T | F | $P_c$ | $P_t$ |
|---|---|---|---|---|---|
| $\bar{K}_{xw}$ | 0.38 | 0.62 | 0.66 | 0.47 | 0.59 |
| $\left(\frac{\partial}{\partial W}\right)_{N,A}$ | ---- | 0.60 | 0.31 | 0.29 | 0.57 |
| $\frac{\left(\frac{\partial}{\partial W}\right)_{N,A}}{\bar{K}_{xw}}$ | ---- | 0.96 | 0.47 | 0.62 | 0.97 |
| $\left(\frac{\partial}{\partial N}\right)_{W,A}$ | ---- | ≈ 0 | 0.97 | 0.47 | ≈ 0 |
| $\bar{K}_{xa}$ | 0.75 | ≈ 0 | -0.69 | 0.34 | -0.40 |
| $\left(\frac{\partial}{\partial A}\right)_{N,W}$ | ---- | ≈ 0 | -1.40 | 0 | -0.37 |
| $\frac{\left(\frac{\partial}{\partial A}\right)_{N,W}}{\bar{K}_{xa}}$ | ---- | ---- | 2.0 | 0 | 0.97 |

The listed values are ratios, percent per percent. As an example, the variation of turbine-discharge total pressure with a simultaneous increase of 1 percent fuel flow and 1 percent exhaust area is as follows. Initially there would be 0.57 percent increase due to fuel-flow change and 0.37 percent decrease due to nozzle-area change. At the final value, there is 0.59 percent increase due to fuel-flow change and 0.40 percent decrease due to nozzle-area change. The initial rise ratio for changes of either independent variable is the same, 0.97.

For the conditions listed, the area has negligible effect on the turbine-discharge temperature T. Also, the nozzle area does not directly affect the compressor-discharge total pressure because the engine is in a choked condition. There is, however, an indirect effect on compressor-discharge total pressure by the exhaust nozzle area. This effect is the increase due to speed changes, 0.34 percent increase per percent increase of area. The initial rise ratio is zero, or there is no instantaneous change in compressor-discharge total pressure.

The values listed for jet thrust are measured thrust values. Although the data have been corrected for the effect of changing inlet total pressure on engine performance, another error exists. This error is due to the forces caused by the changing ram pressure.

In some cases this error amounted to 50 percent of the measured thrust increase. The values listed for turbine-discharge total pressure are more indicative of actual thrust response.

The following table provides a measure of the variation of engine gain over a range of engine speed and exhaust nozzle area:

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| Engine gain | Engine speed $\overline{N}$ (rpm) | Exhaust nozzle area A (sq ft) | Parameter x | | | | |
|---|---|---|---|---|---|---|---|
| | | | N | $P_c$ | $P_t$ | T | F |
| $\overline{K}_{xw}$ | 6440 | 3.0 | 0.64 | 1.16 | 0.12 | 0.33 | 1.40 |
| | 8350 | 3.0 | .39 | .41 | .65 | .70 | .67 |
| | 6440 | 2.2 | .53 | 1.08 | .62 | .35 | 1.46 |
| | 8350 | 2.3 | .31 | .38 | .70 | .56 | .45 |
| $\overline{K}_{xa}$ | 6440 | 3.0 | 0.34 | 0.52 | -0.14 | -0.17 | 0.09 |
| | 8350 | 3.0 | .25 | .13 | .13 | .14 | -.50 |
| | 6440 | 2.2 | .80 | 1.13 | -.21 | -.76 | .31 |
| | 8350 | 2.3 | .70 | 0 | -.39 | -.82 | -1.30 |

The gains, percent per percent, range from low speed small area to high speed large area. As an example, at the constant large area (3.0 sq ft) the turbine-discharge total pressure gain increases from 0.12 to 0.65 percent per percent change in fuel flow; also, it varies from a decrease of 0.14 percent to an increase of 0.13 percent per percent increase in exhaust nozzle area.

## SUMMARY OF RESULTS

In this investigation, the dynamic characteristics of a turbojet engine with variable area nozzle were investigated and are presented. From this investigation the following results were obtained:

(1) Data resulting from approximate step disturbances in either independent variable suggested transfer functions whose forms can be derived from functional relationships.

(2) The minimum data necessary to completely describe dynamic behavior were: the dynamic characteristics (engine time constant and initial rise ratio) determined from changes of either independent variable; and the steady-state characteristics determined for each of the independent variables.

(3) The engine time constants and initial rise ratios were obtained from transients initiated by either of the independent variables.

(4) The dynamic characteristics varied with altitude, flight Mach number, and engine speed. The constants of the transfer functions describing engine behavior generalized to one curve at standard static sea-level conditions. This generalization permitted the prediction of engine dynamic behavior at any altitude and flight Mach number.

Lewis Flight Propulsion Laboratory 2379
National Advisory Committee for Aeronautics
Cleveland, Ohio, October 26, 1951

# APPENDIX A

## SYMBOLS

The following symbols are used in this report:

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| | |
|---|---|
| $A$ | exhaust nozzle area, sq ft |
| $d,e,f,g$ <br> $h,j,k,l$ | initial rise ratios |
| $e$ | base of natural logarithms |
| $F$ | jet thrust, lb |
| $G$ | functional relationship |
| $G_1,G_2,K_1,K_2$ <br> $L_1,L_2,k_2,l_2$ <br> $g_2, \tau_2$ | correction factors |
| $J$ | rotor moment of inertia, (lb)(ft)(sec$^2$) |
| $K$ | engine gain |
| $K_{xa}$ | engine gain, change in parameter $x$ $(N, P_c, P_t, F, \text{or } T)$ due to change in area |
| $K_{xw}$ | engine gain, change in parameter $x$ $(N, P_c, P_t, F, \text{or } T)$ due to change in fuel flow |
| $N$ | engine speed, rpm |
| $P_c$ | compressor-discharge total pressure, lb/sq ft |
| $P_t$ | turbine-discharge total pressure, lb/sq ft |
| $Q$ | engine torque, lb-ft |
| $s$ | Laplacian operator |
| $T$ | turbine-discharge total temperature, $^{\circ}$R |
| $t$ | time, sec |
| $W$ | engine fuel flow, lb/hr |

[redacted]

$\delta$ ratio of total pressure at engine inlet to absolute pressure at standard sea-level conditions

$\Delta$ incremental change

$\theta$ ratio of total temperature at engine inlet to absolute temperature at standard sea-level conditions

$\tau$ engine time constant, sec

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Subscripts:

0 ambient

1 compressor inlet

Barred symbols indicate generalized to standard static sea-level conditions.

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# APPENDIX B

## DERIVATION OF EQUATIONS FOR ANALYSIS OF DATA

### Method of Plotting Data

The engine speed response to a step change in fuel flow at constant exhaust nozzle area is developed in reference 6. This dynamic characteristic is described in the complex domain as,

$$\Delta N(s) = \left[\left(\frac{1}{1 + \tau s}\right)\right] \left(\frac{\Delta W}{s}\right) K_{nw} \tag{B1}$$

Transforming to the real domain

$$\Delta N(t) = \Delta W(1 - e^{-t/\tau}) K_{nw} \tag{B2}$$

For the initial conditions at zero

$$1 - \frac{\Delta N}{(\Delta N)_f} = e^{-t/\tau} \tag{B3}$$

where

$\Delta N$ speed change at time $t$

$(\Delta N)_f$ total change in speed

Therefore,

$$\ln\left(1 - \frac{\Delta N}{(\Delta N)_f}\right) = -\frac{t}{\tau} \tag{B4}$$

Similarly, the compressor-discharge total pressure response to a step change in fuel flow at constant exhaust nozzle area becomes,

$$\Delta P_c = \Delta W \left[1 - (1-d)e^{-t/\tau}\right] K_{p_c w} \tag{B5}$$

For the initial conditions at zero

$$1 - \frac{\Delta P_c}{(\Delta P_c)_f} = (1-d)e^{-t/\tau} \tag{B6}$$

therefore

$$\ln\left[1 - \frac{\Delta P_c}{(\Delta P_c)_f}\right] = \ln(1-d) - \frac{t}{\tau} \tag{B7}$$

and at $t = 0$ 2379

$$\frac{\Delta P_c}{(\Delta P_c)_f} = d$$

where

$\Delta P_c$ pressure rise at time $t$

$(\Delta P_c)_f$ final pressure rise

$d$ initial rise ratio

It can be shown that turbine-discharge total pressure and temperature, and jet thrust have similar indicial responses because the same functional relationship has been assumed.

$$\ln\left[1 - \frac{\Delta P_t}{(\Delta P_t)_f}\right] = \ln(1-f) - \frac{t}{\tau} \tag{B8}$$

$$\ln\left[1 - \frac{\Delta T}{(\Delta T)_f}\right] = \ln(1-h) - \frac{t}{\tau} \tag{B9}$$

$$\ln\left[1 - \frac{\Delta F}{(\Delta F)_f}\right] = \ln(1-k) - \frac{t}{\tau} \tag{B10}$$

### Correction Factors for Changes in Inlet Total Pressure

The air supply to the engine was regulated with a valve in the ram pipe. During engine acceleration the valve was maintained fixed, thus the simulated flight Mach number varied. The pressure drop across the valve depended on mass air flow, which varied with engine speed. As a result, the poor regulation of the air supply source was reflected in the data (fig. 1). Correction factors are necessary so that simulated inlet conditions can be assumed constant during transient operation. The effect of engine discharge on simulated altitude was insignificant.

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The inlet total pressure $P_1$ is assumed to vary linearly with speed for the magnitude of speed change considered. This variation can be considered another engine input. For constant exhaust nozzle area operation the following analysis results.

$$Q = Q(N, W, P_1) \tag{B11}$$

$$P_c = P_c(N, W, P_1) \tag{B12}$$

$$\Delta Q = J\Delta \dot{N} \tag{B13}$$

Expanding equation (B11) about an equilibrium point gives

$$\Delta Q = \left(\frac{\partial Q}{\partial N}\right)_{W,P_1} \Delta N + \left(\frac{\partial Q}{\partial W}\right)_{N,P_1} \Delta W + \left(\frac{\partial Q}{\partial P_1}\right) \Delta P_1 \tag{B14}$$

When equations (B13) and (B14) are equated,

$$\Delta N(s) = \frac{K_{nw}}{1 + \tau s} \Delta W(s) + \frac{L_1}{1 + \tau s} \Delta P_1(s) \tag{B15}$$

Similarly, expanding equation (B12) and substituting equation (B15) yield

$$\Delta P_c(s) = K_{P_c W} \frac{(1 + d\tau s)}{1 + \tau s} \Delta W(s) + L_2 \frac{(1 + l_2 \tau s)}{1 + \tau s} \Delta P_1(s) \tag{B16}$$

Equations (B15) and (B16) describe the speed and compressor-discharge total pressure responses, respectively, when both the fuel flow and inlet total pressure vary simultaneously. However, it is noted that the actual recorded data for the engine speed and compressor-discharge total pressure responses are of the form (fig. 1).

$$\Delta N(s) = \frac{G_1}{1 + \tau_2 s} \Delta W(s) \tag{B17}$$

where $\tau_2$ is the time constant determined from the data, and

$$\Delta P_c(s) = G_2 \frac{(1 + g_2 \tau_2 s)}{1 + \tau_2 s} \Delta W(s) \tag{B18}$$

where $G_1$ and $G_2$ are the actual gains determined from the initial and final equilibrium points. The initial rise ratio $g_2$ and engine time constant $\tau_2$ are also determined from the actual data.

The data show a static correspondence between inlet total pressure and engine speed (fig. 1),

$$P_1 = -kN \tag{B19}$$

where $k$ is a proportionality factor. The substitution of equation (B19) into equations (B15) and (B16), and comparison with equations (B17) and (B18) result in the following correction factors:

$$\tau = \frac{K_{nw}}{G_1} \tau_2 \tag{B20}$$

and

$$d = \frac{G_2}{K_{p_c w}} g_2 \tag{B21}$$

The gains $K_{nw}$ and $K_{p_c w}$ are evaluated from steady-state data and correspond to theoretical gains resulting from a constant inlet total pressure. The gains $G_1$ and $G_2$ are determined from the actual data (initial and final equilibrium points) and include the variation of total inlet pressure with engine speed.

The initial rise ratio $g_2$ and engine time constant $\tau_2$, measured from actual data, also include the variation of total inlet pressure with speed. The values of these dynamic characteristics for constant inlet total pressure can be determined from the relationships, equations (B20) and (B21).

## Generalization of Transient Data

According to reference 4, the steady-state relationships obtained at various altitudes and flight Mach numbers should generalize to one relationship at standard sea-level conditions. The variation of Reynolds number is negligible. Those performance parameters that involve fuel flow, however, do not generalize because combustion efficiency varies with operating conditions. The resulting curves, although not coincident, may have identical slopes.

Corrected parameters appearing in reference 4 are used to generalize the transfer functions developed in this appendix.

Generalized engine speed $\frac{N}{\sqrt{\theta}} = \overline{N}$

Generalized fuel flow $\frac{W}{\delta\sqrt{\theta}} = \overline{W}$



Generalized thrust $\frac{F}{\delta} = \overline{F}$

Generalized temperature $\frac{T}{\theta} = \overline{T}$

Generalized pressure $\frac{P}{\delta} = \overline{P}$

In order to generalize the data obtained at various altitudes, flight Mach numbers, and engine speeds, to standard sea-level conditions, the following relationships are used:

$$\delta = \frac{P_1}{2116\ \text{lb/sq ft}}$$

$$\theta = \frac{T_1}{519^{\circ}\ \text{R}}$$

The measured dynamic characteristics are generalized as follows:

$$\overline{K}_{nw} = \delta\ K_{nw}$$

$$\overline{K}_{na} = \frac{1}{\sqrt{\theta}}\ K_{na}$$

$$\overline{K}_{p_c w} = \sqrt{\theta}\ K_{p_c w}$$

$$\overline{K}_{p_c a} = \frac{1}{\delta}\ K_{p_c a}$$

$$\overline{K}_{p_t w} = \sqrt{\theta}\ K_{p_t w}$$

$$\overline{K}_{p_t a} = \frac{1}{\delta}\ K_{p_t a}$$

$$\bar{K}_{tw} = \frac{\delta}{\sqrt{\theta}} K_{tw}$$

$$\bar{K}_{ta} = \frac{1}{\theta} K_{ta}$$

$$\bar{K}_{fw} = \sqrt{\theta}\, K_{fw}$$

$$\bar{K}_{fa} = \frac{1}{\delta} K_{fa}$$

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$$\bar{\tau} = \frac{\delta}{\sqrt{\theta}} \tau$$

The initial rise ratios, being dimensionless, require no altitude correction.

REFERENCES

1. Otto, Edward W., and Taylor, Burt L., III: Dynamics of a Turbojet Engine Considered as a Quasi-Static System. NACA Rep. 1011, 1951. (Formerly NACA TN 2091.)

2. Feder, Melvin S., and Hood, Richard: Analysis for Control Application of Dynamic Characteristics of Turbojet Engine with Tail-Pipe Burning. NACA TN 2183, 1950.

3. Heidmann, Marcus F.: Analysis of Effect of Variations in Primary Variables on Time Constant and Turbine-Inlet-Temperature Overshoot of Turbojet Engine. NACA TN 2182, 1950.

4. Sanders, Newell D.: Performance Parameters for Jet-Propulsion Engines. NACA TN 1106, 1946.

5. Brown, Gordon S., and Campbell, Donald P.: Principles of Servomechanisms. John Wiley & Sons, Inc., 1948.

6. Delio, G. J.: Evaluation of Three Methods for Determining Dynamic Characteristics of a Turbojet Engine. NACA TN 2634, 1952.

7. Delio, Gene J., and Schwent, Glennon V.: Instrumentation for Recording Transient Performance of Gas-Turbine Engines and Control Systems. NACA RM E51D27, 1951.

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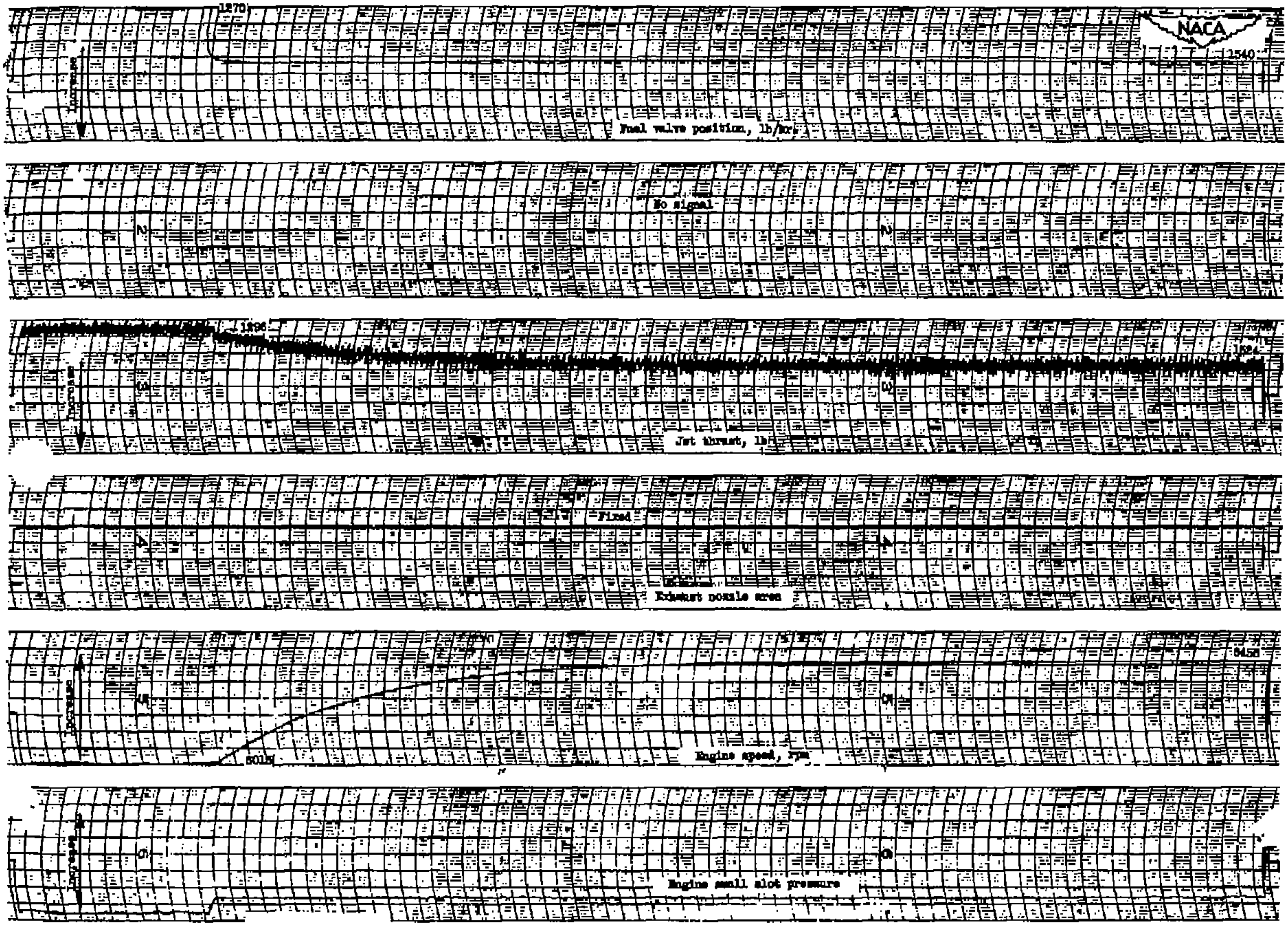


Figure 1. - Typical recording of engine variables on multichannel recorder.

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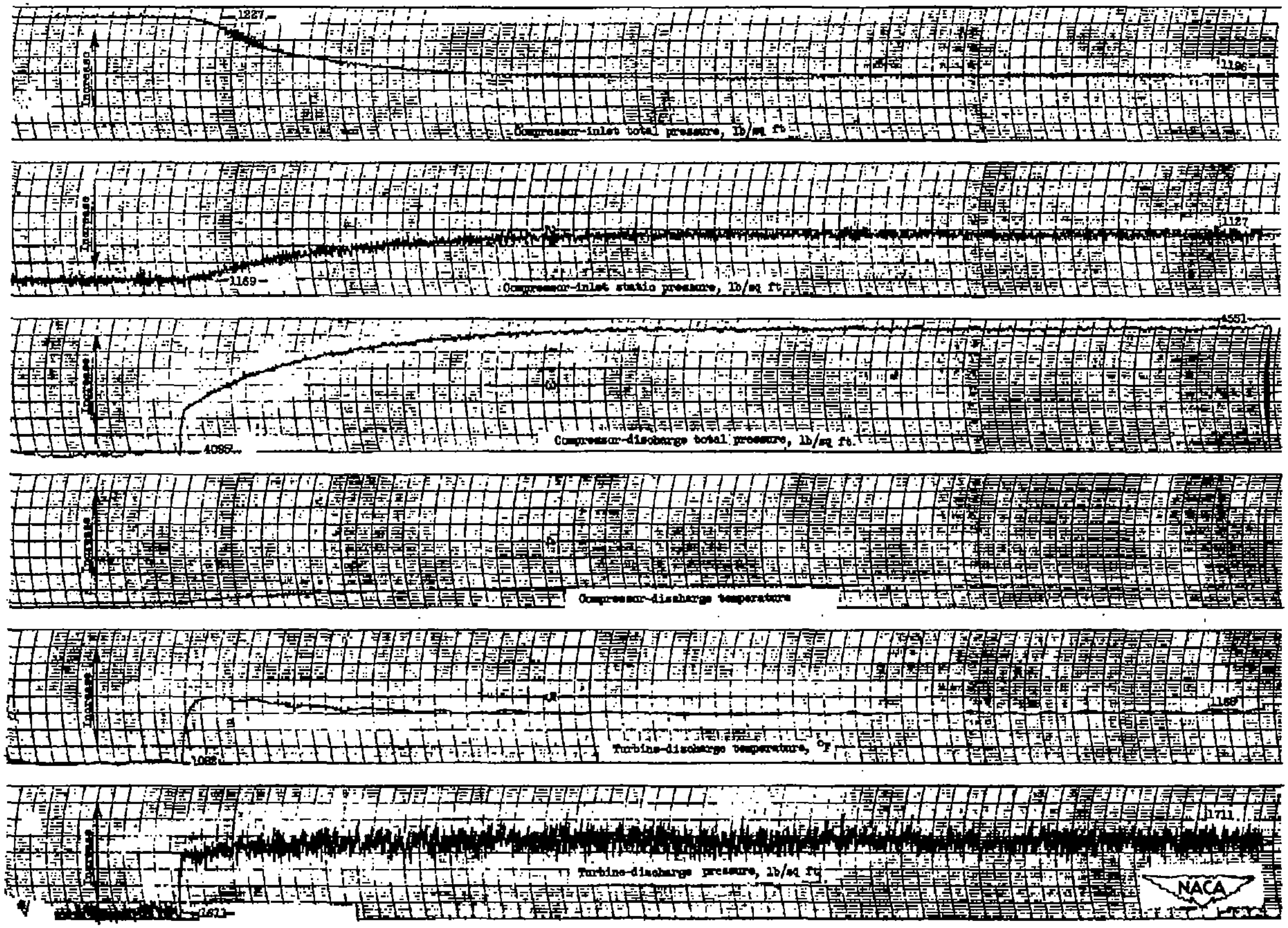


Figure 1. - Concluded. Typical recording of engine variables on multichannel recorder.

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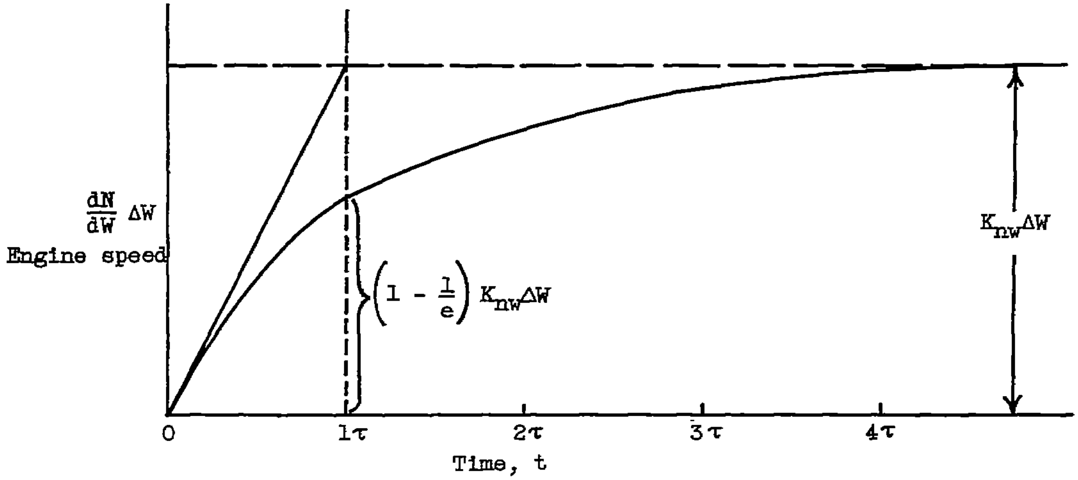
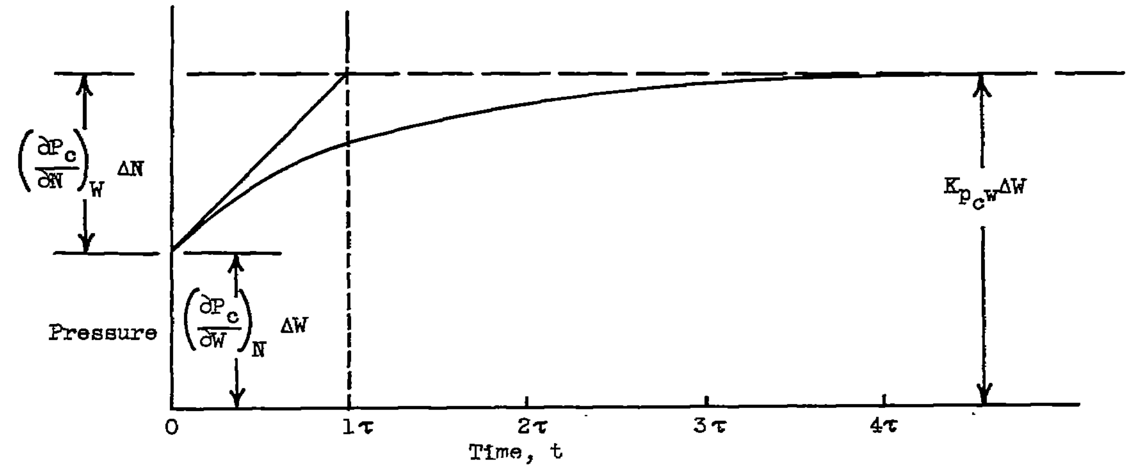



(a) Engine speed.




(b) Compressor-discharge total pressure.

Figure 2. - Theoretical indicial responses of pressure and engine speed to fuel flow.

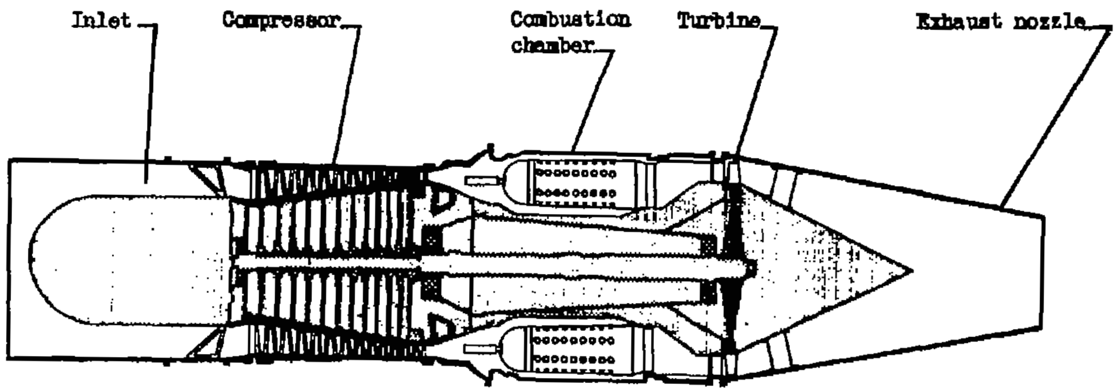



Figure 3. - Typical turbojet engine showing component units.



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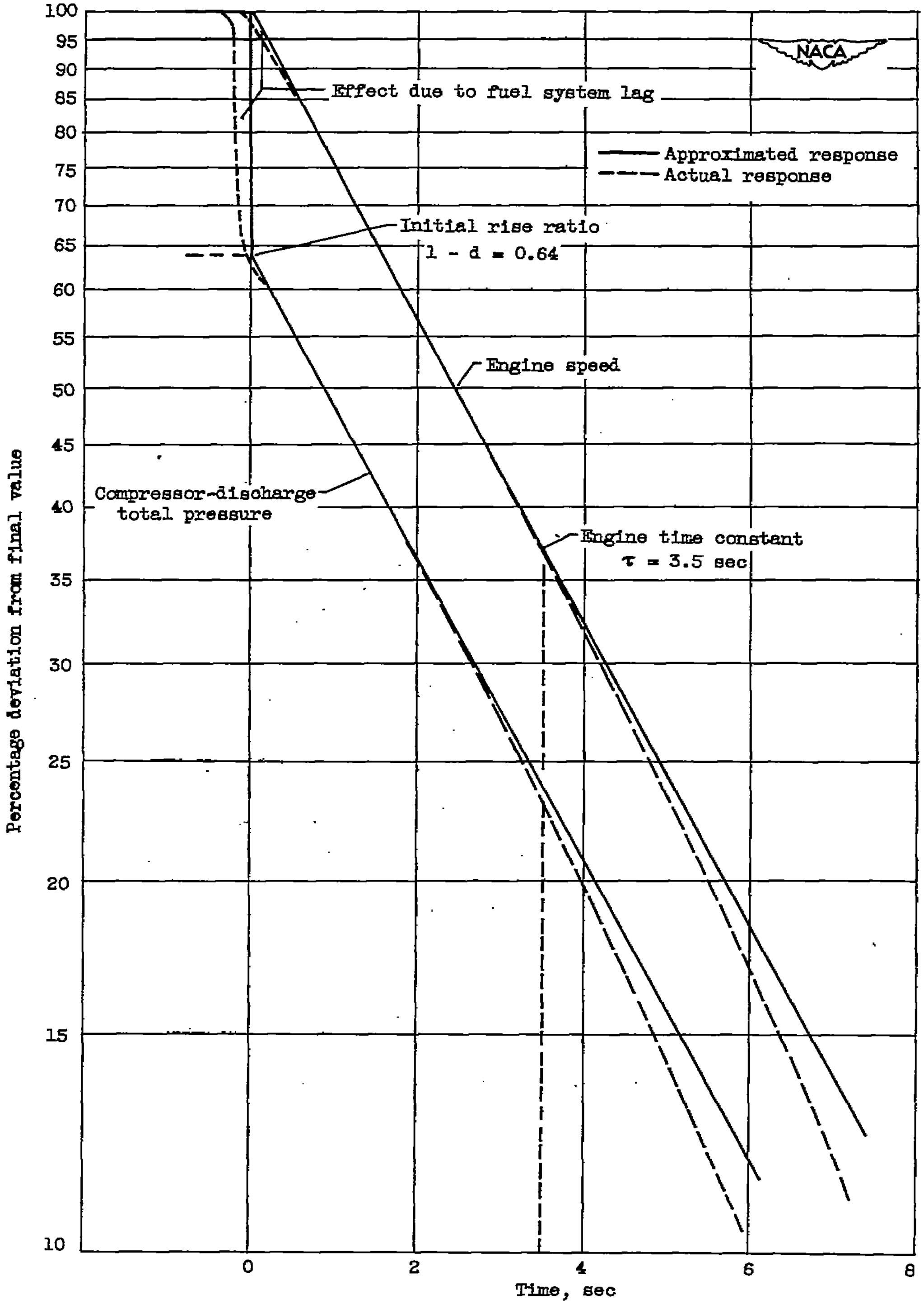


Figure 4. - Responses of compressor-discharge total pressure and engine speed to an approximate step change in fuel flow.

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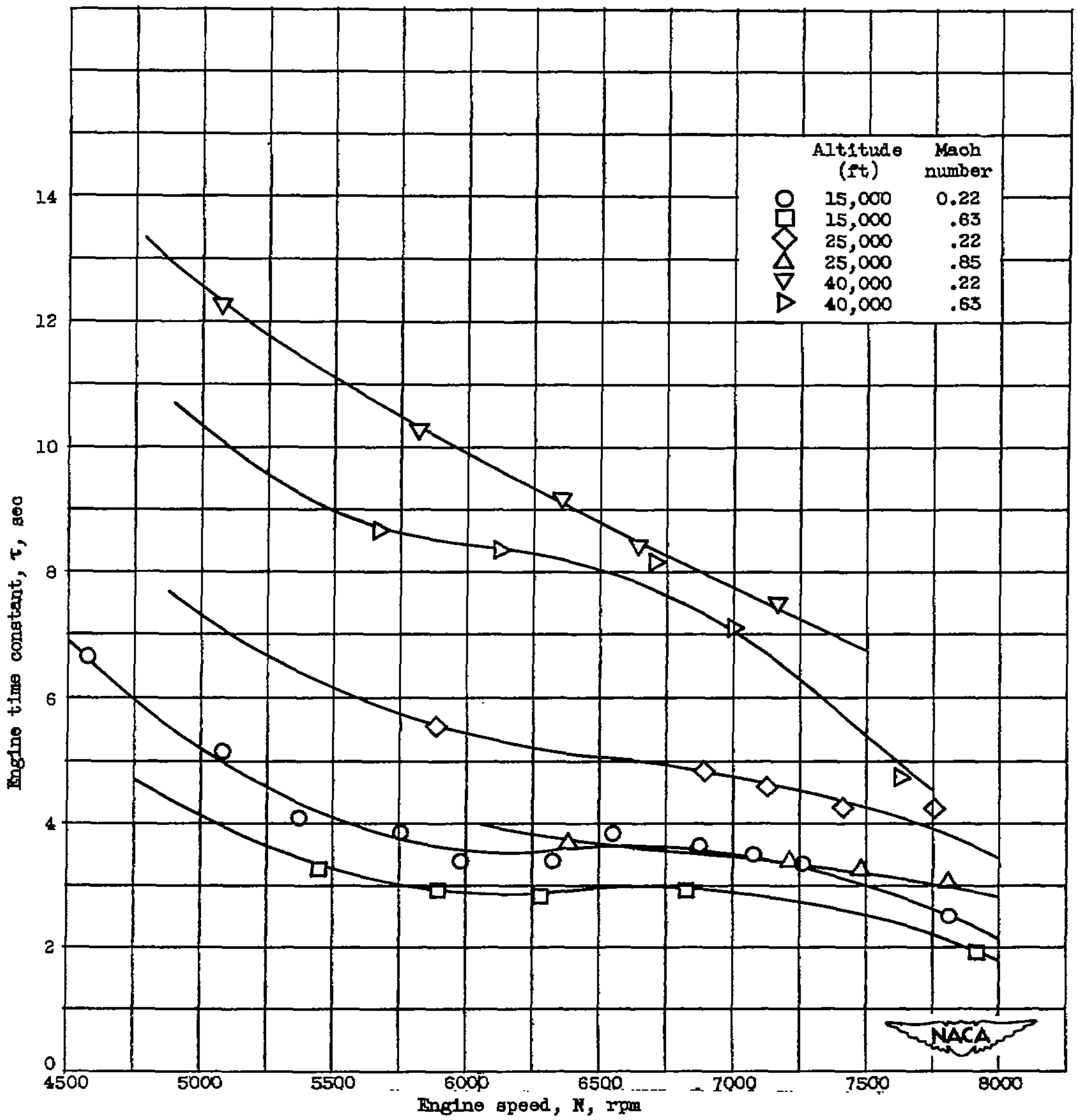


Figure 5. - Effect of altitude and flight Mach number on engine time constant.

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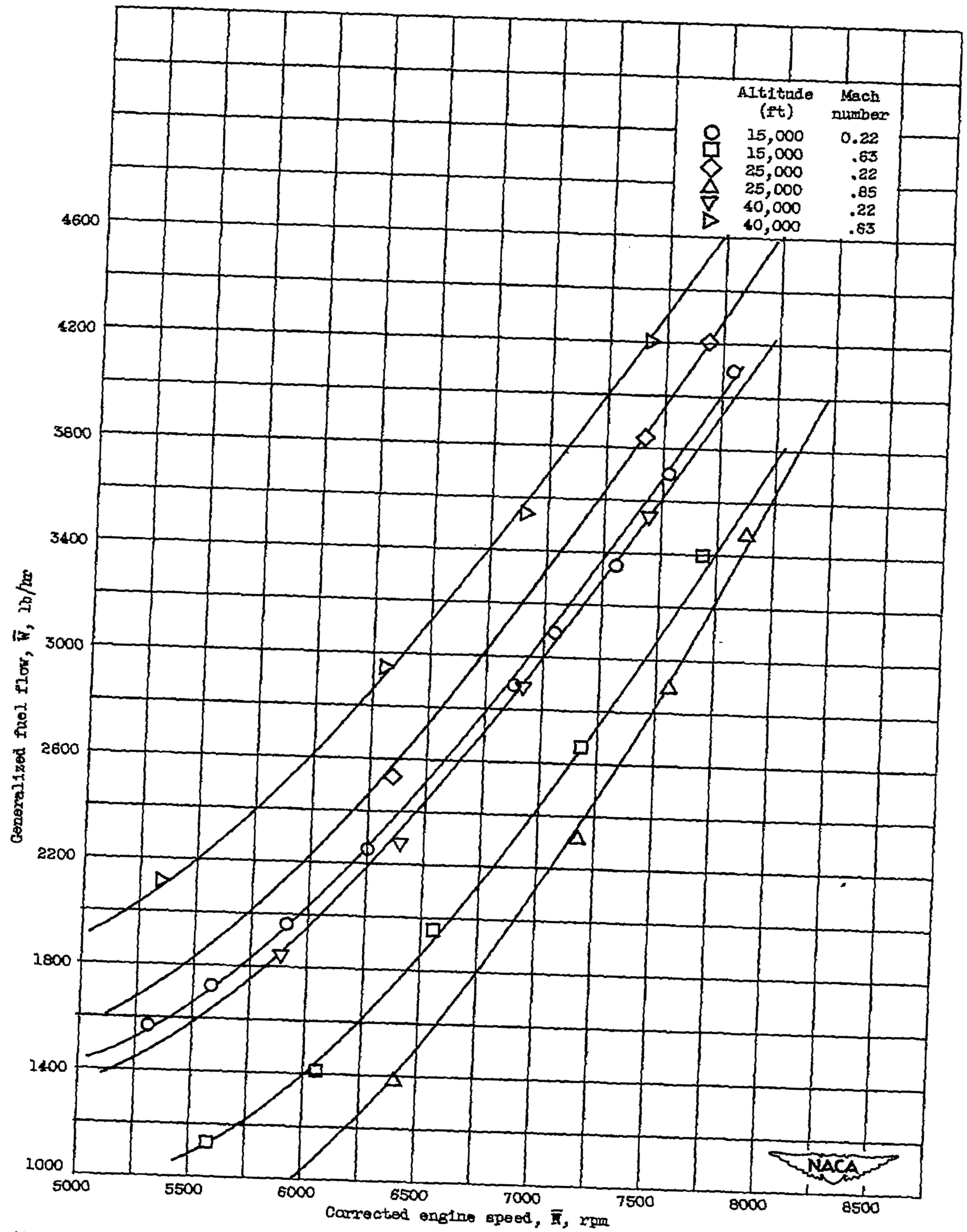


Figure 6. - Effect of combustion efficiency on generalization of fuel flow to static sea-level conditions.

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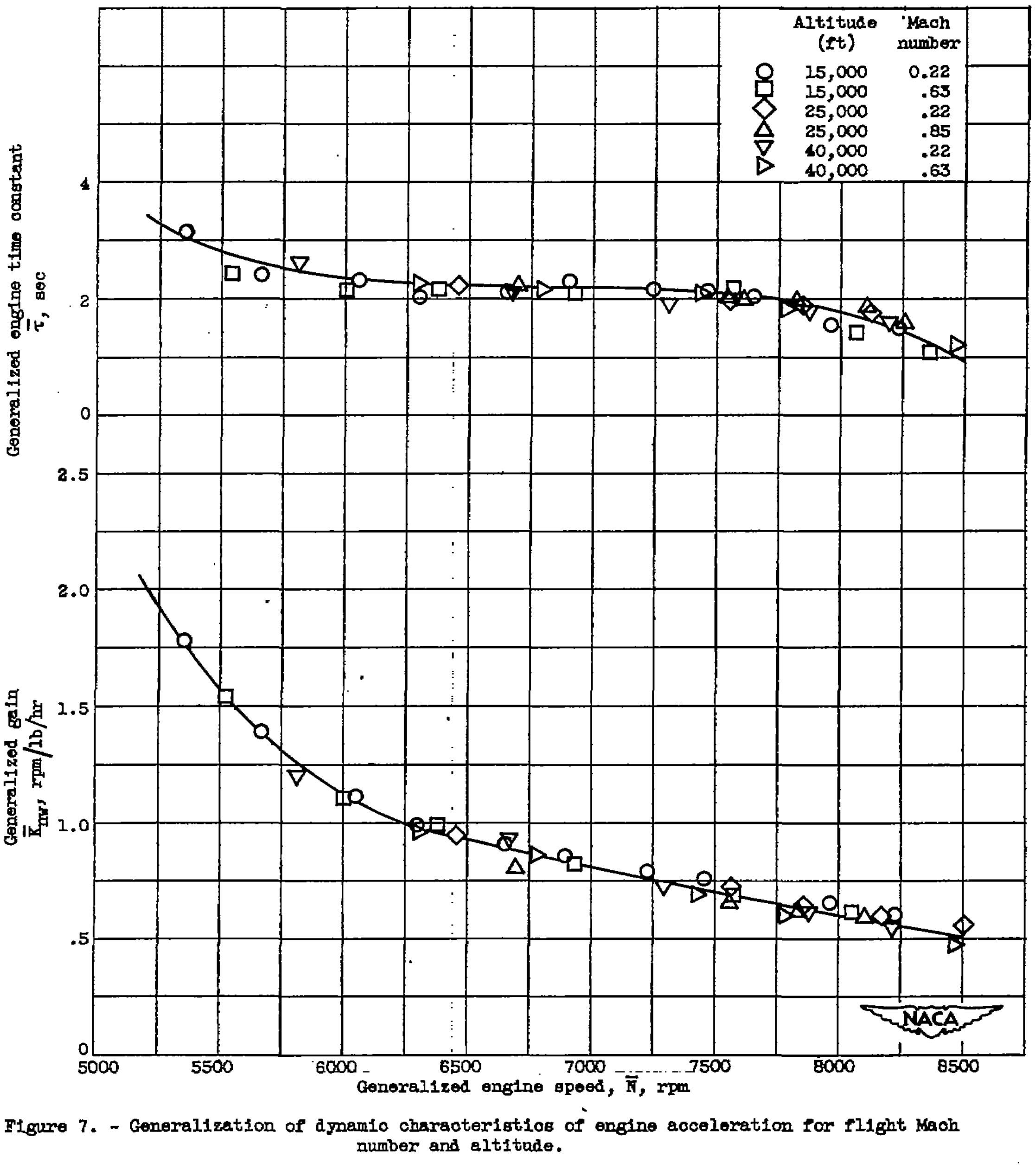


Figure 7. - Generalization of dynamic characteristics of engine acceleration for flight Mach number and altitude.

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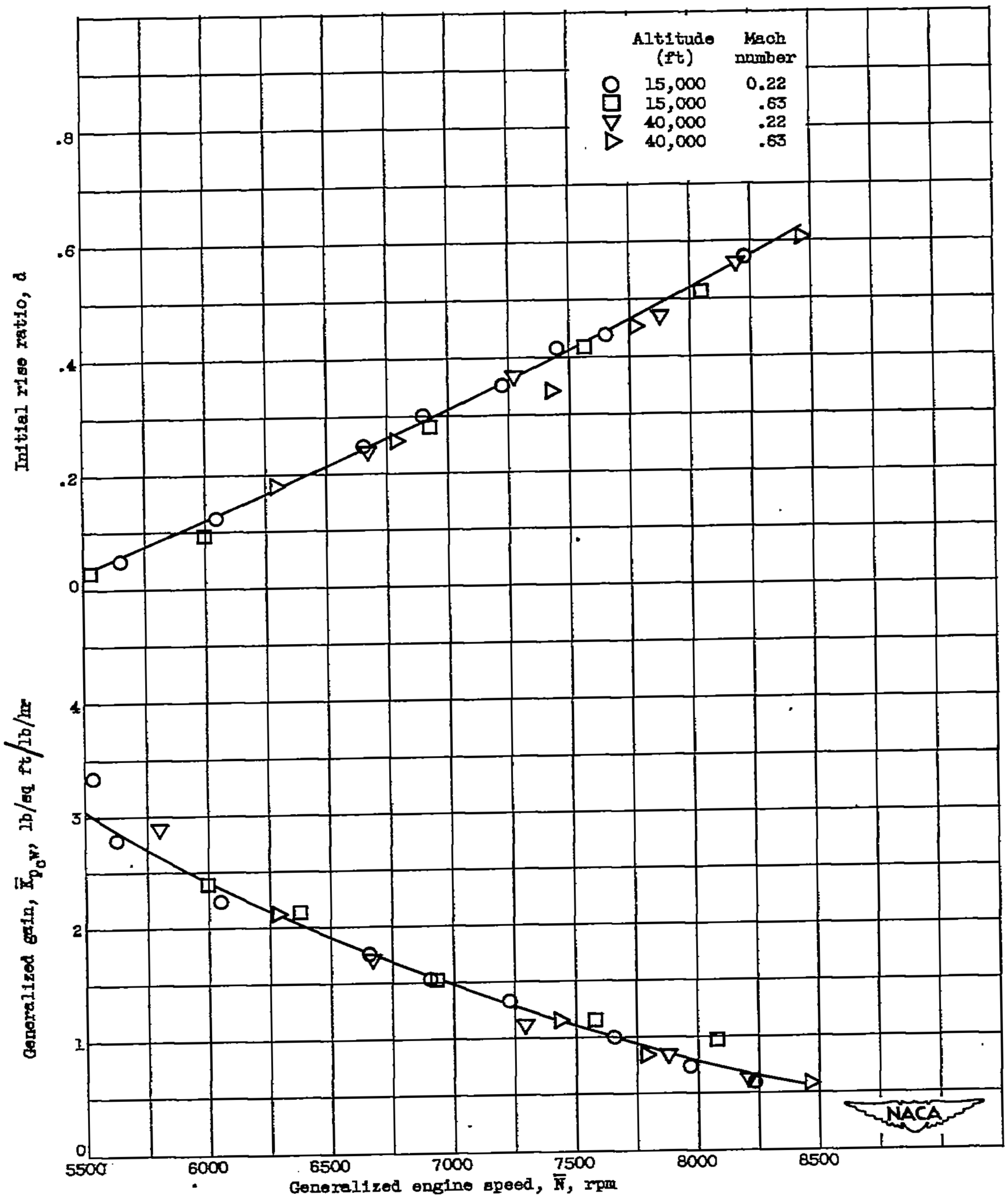


Figure 8. - Generalization of dynamic characteristics of compressor-discharge total pressure for flight Mach number and altitude.

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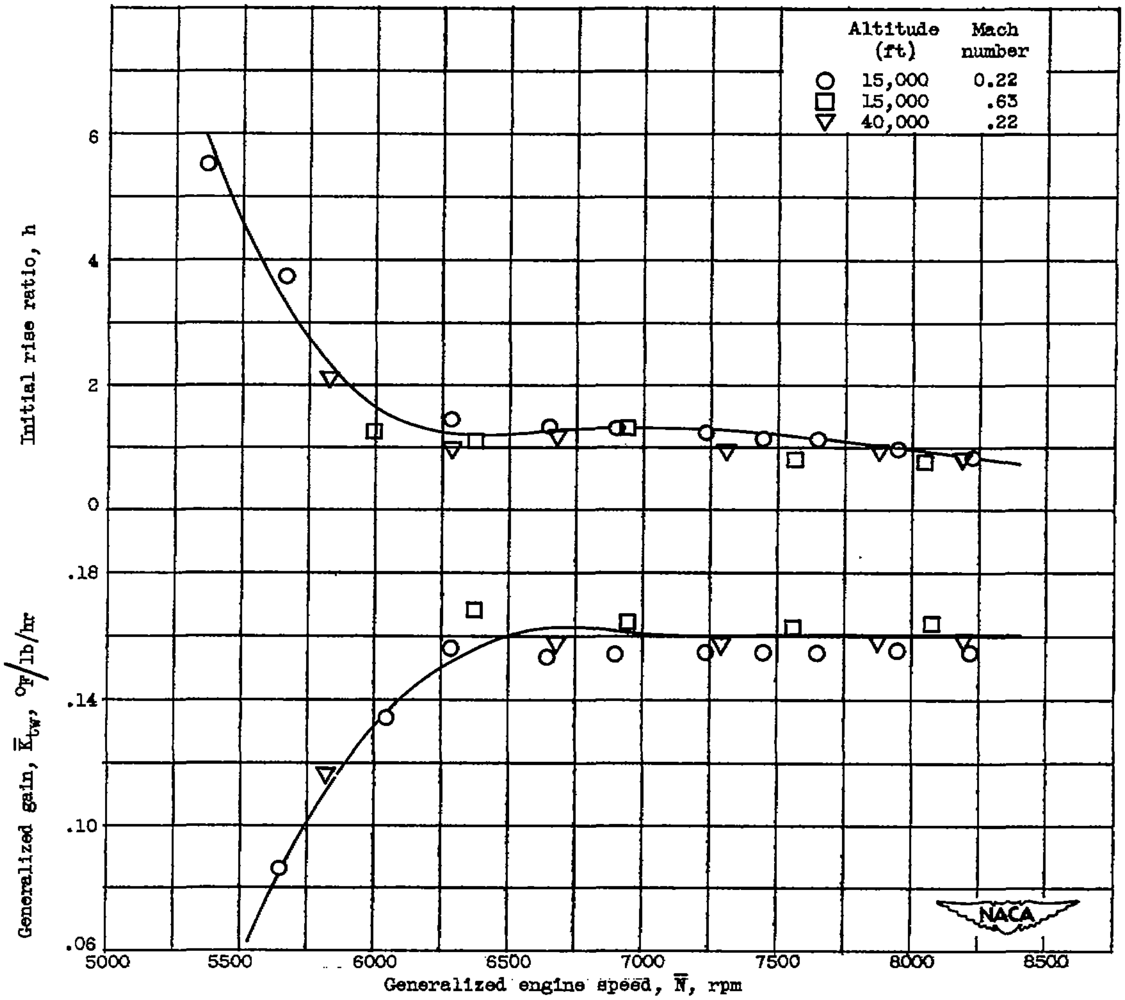


Figure 9. - Generalization of dynamic characteristics of turbine-discharge total temperature for flight Mach number and altitude.

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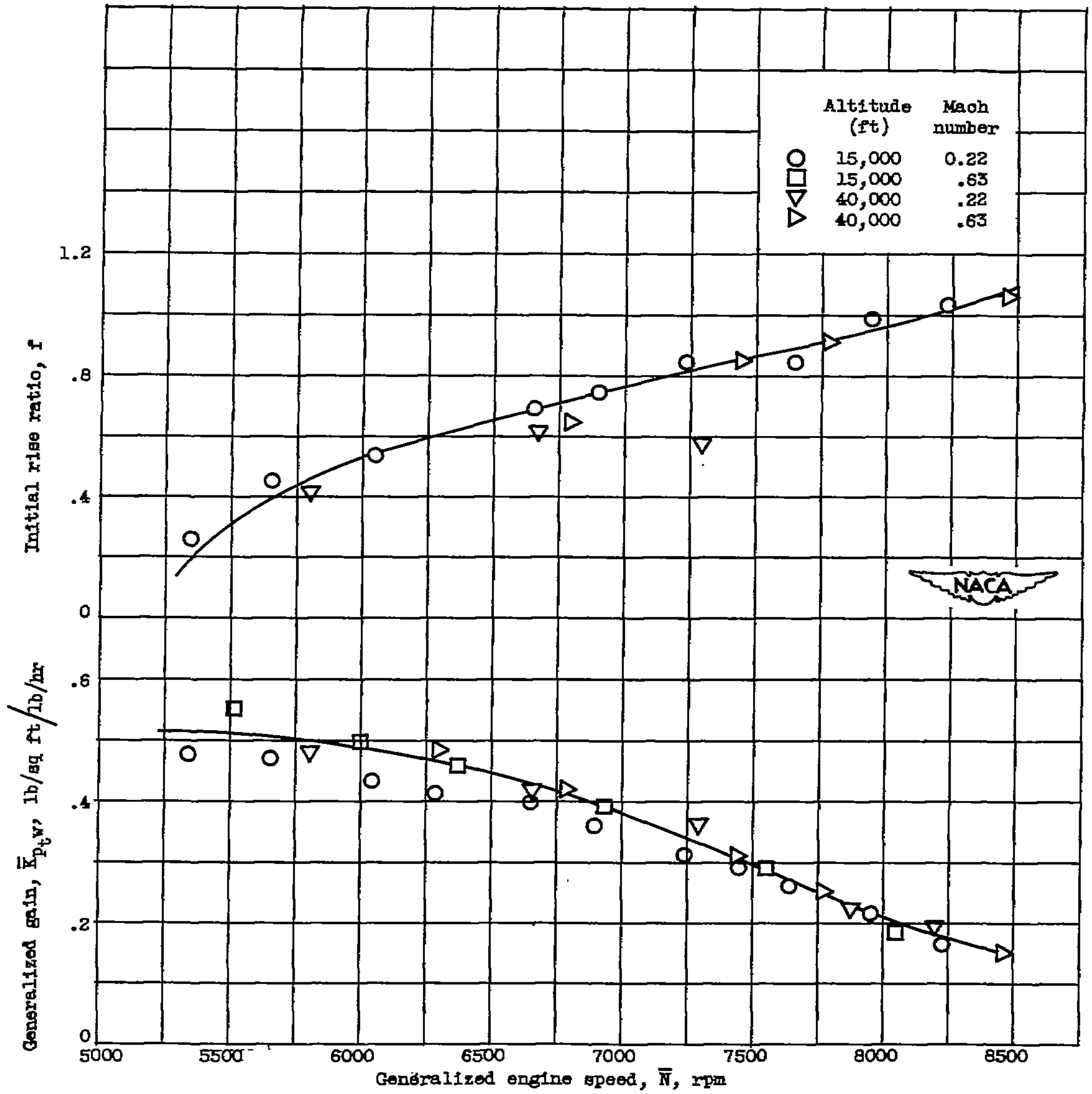


Figure 10. - Generalization of dynamics characteristics of turbine-discharge total pressure in flight Mach number and altitude.

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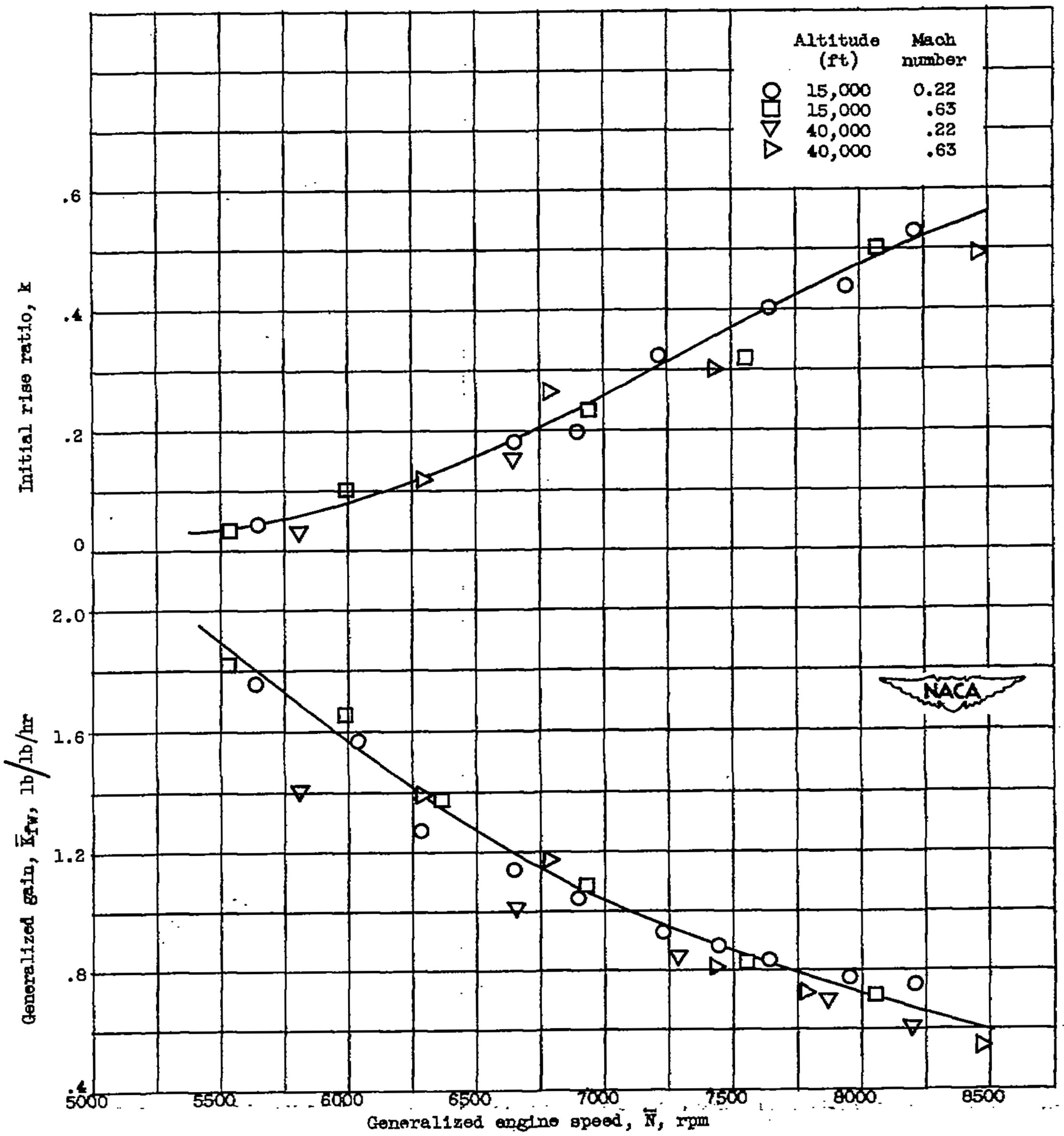


Figure 11. - Generalization of dynamic characteristics of jet thrust for flight Mach number and altitude.

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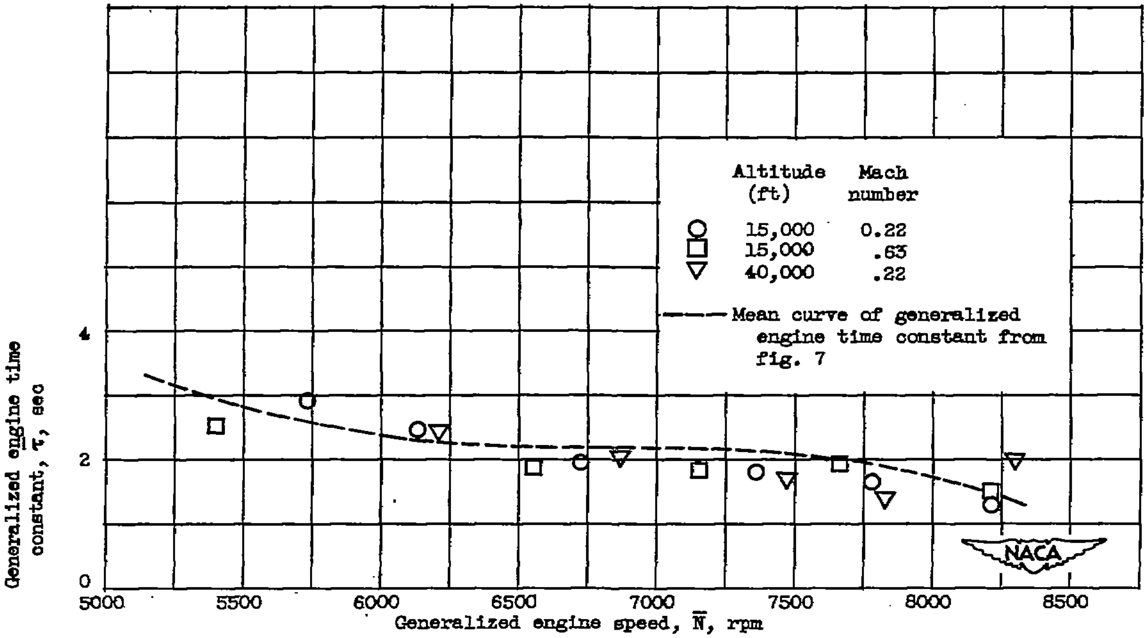



Figure 12. - Comparison of generalized engine time constant obtained with fuel-flow change and with exhaust-nozzle-area change.

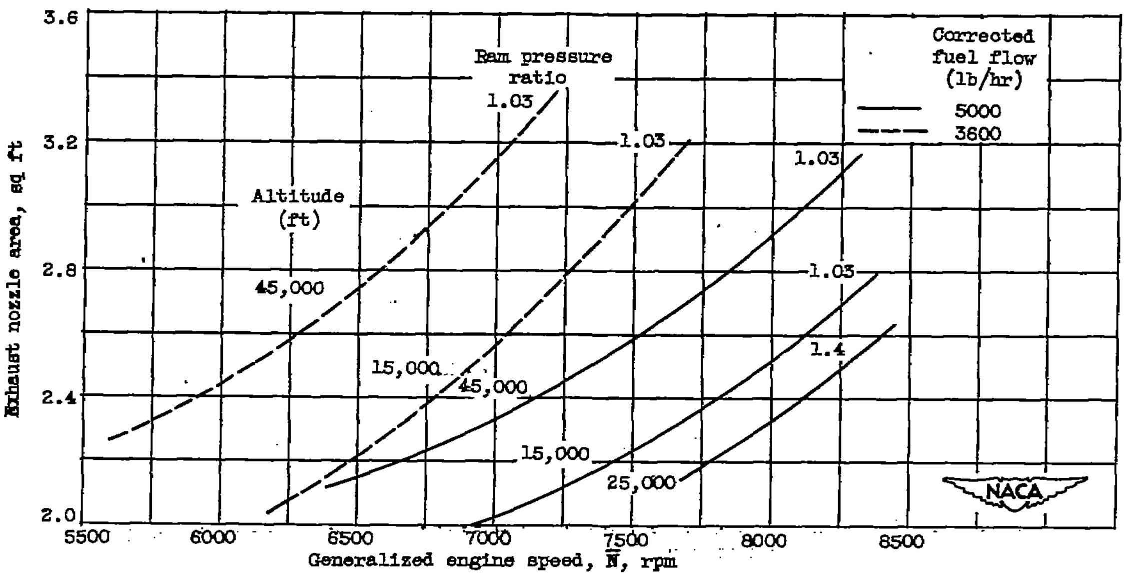
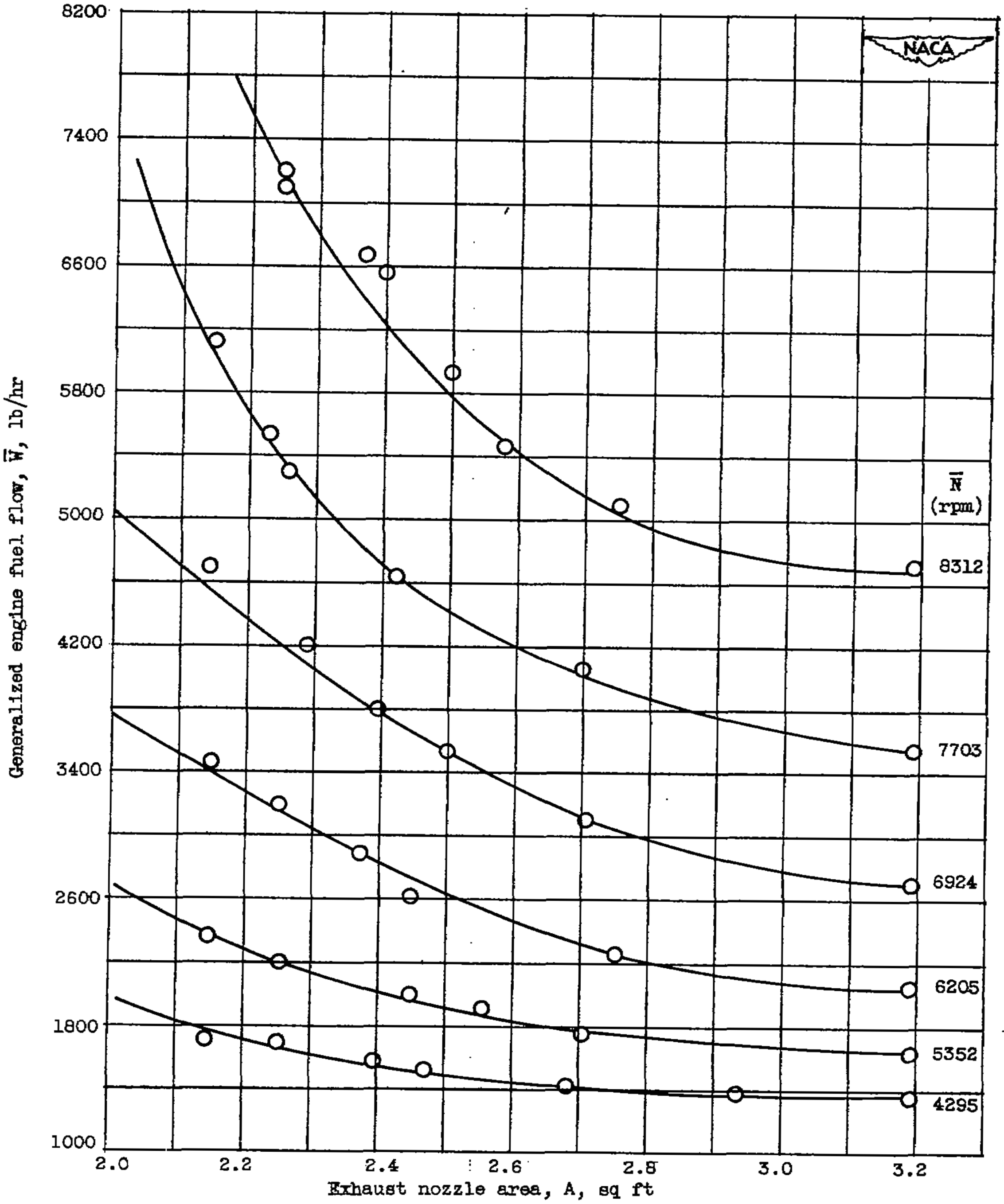



Figure 13. - Determination of generalized engine gain, speed to exhaust nozzle area, with changes in ram pressure ratio and altitude.




(a) Generalized fuel flow against exhaust nozzle area; altitude, 15,000 feet; ram pressure ratio, 1.03.

Figure 14. - Determination of generalized gain, speed to exhaust nozzle area, with changes in ram pressure ratio and altitude.

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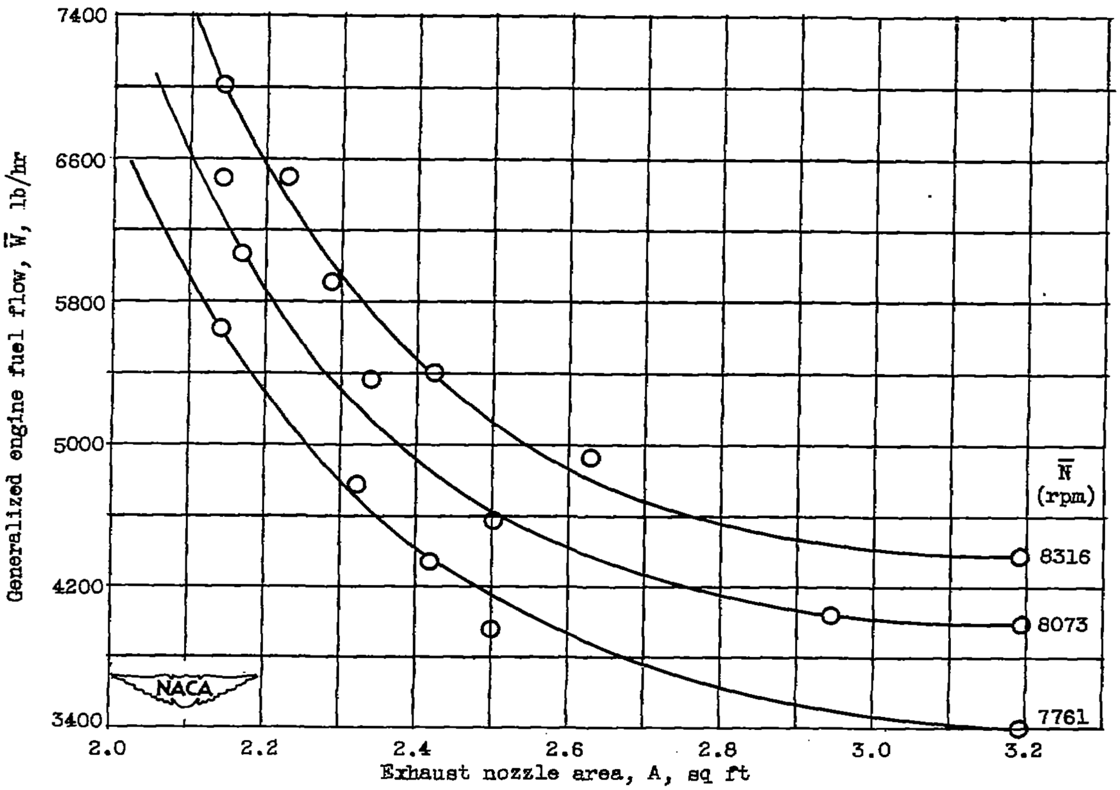


(b) Generalized fuel flow against exhaust nozzle area; altitude, 25,000 feet; ram pressure ratio, 1.14.

Figure 14. - Continued. Determination of generalized gain, speed to exhaust nozzle area, with changes in ram pressure ratio and altitude.

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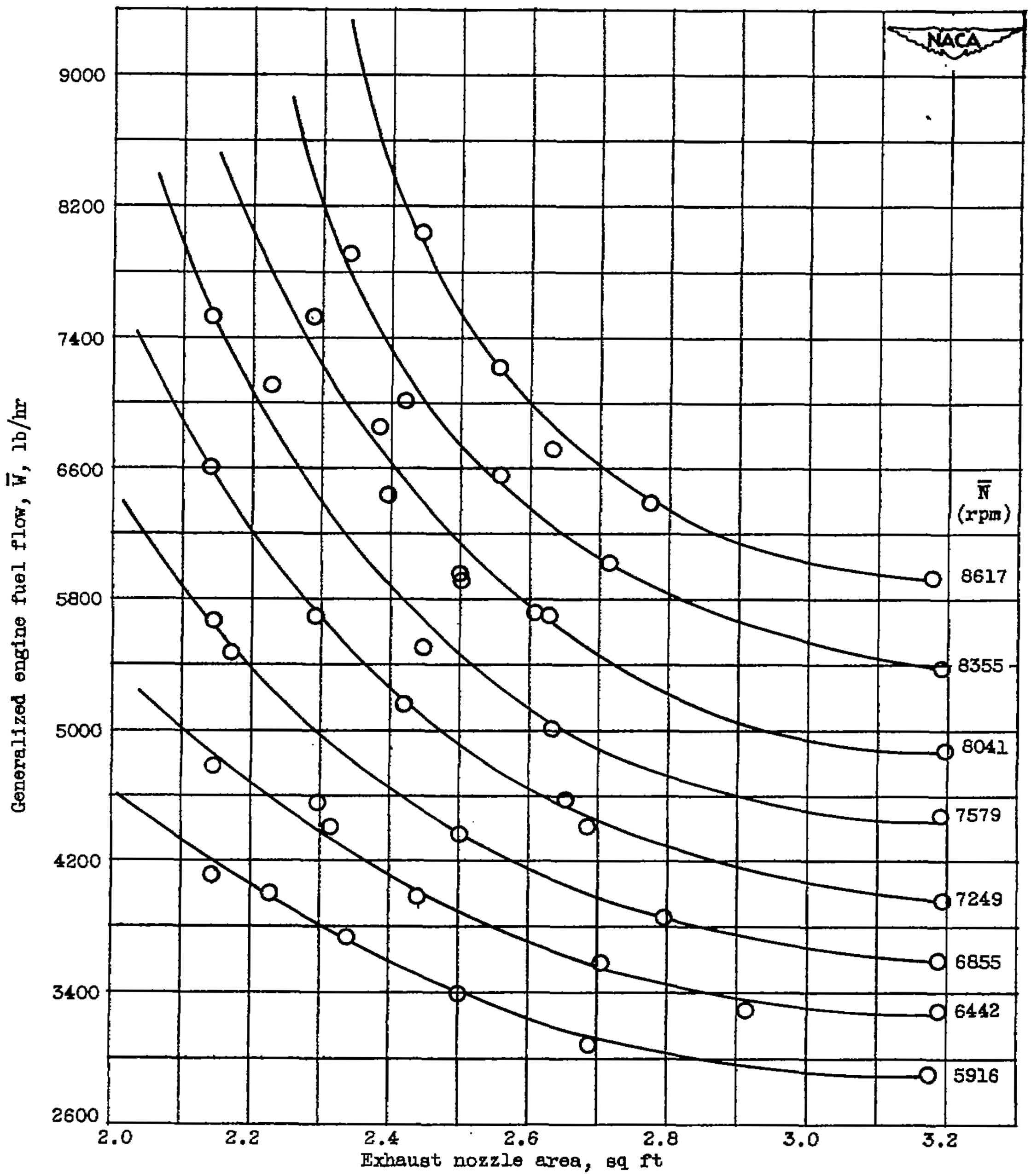


(c) Generalized fuel flow against exhaust nozzle area; altitude, 45,000 feet; ram pressure ratio, 1.03.

Figure 14. - Concluded. Determination of generalized gain, speed to exhaust nozzle area, with changes in ram pressure ratio and altitude.

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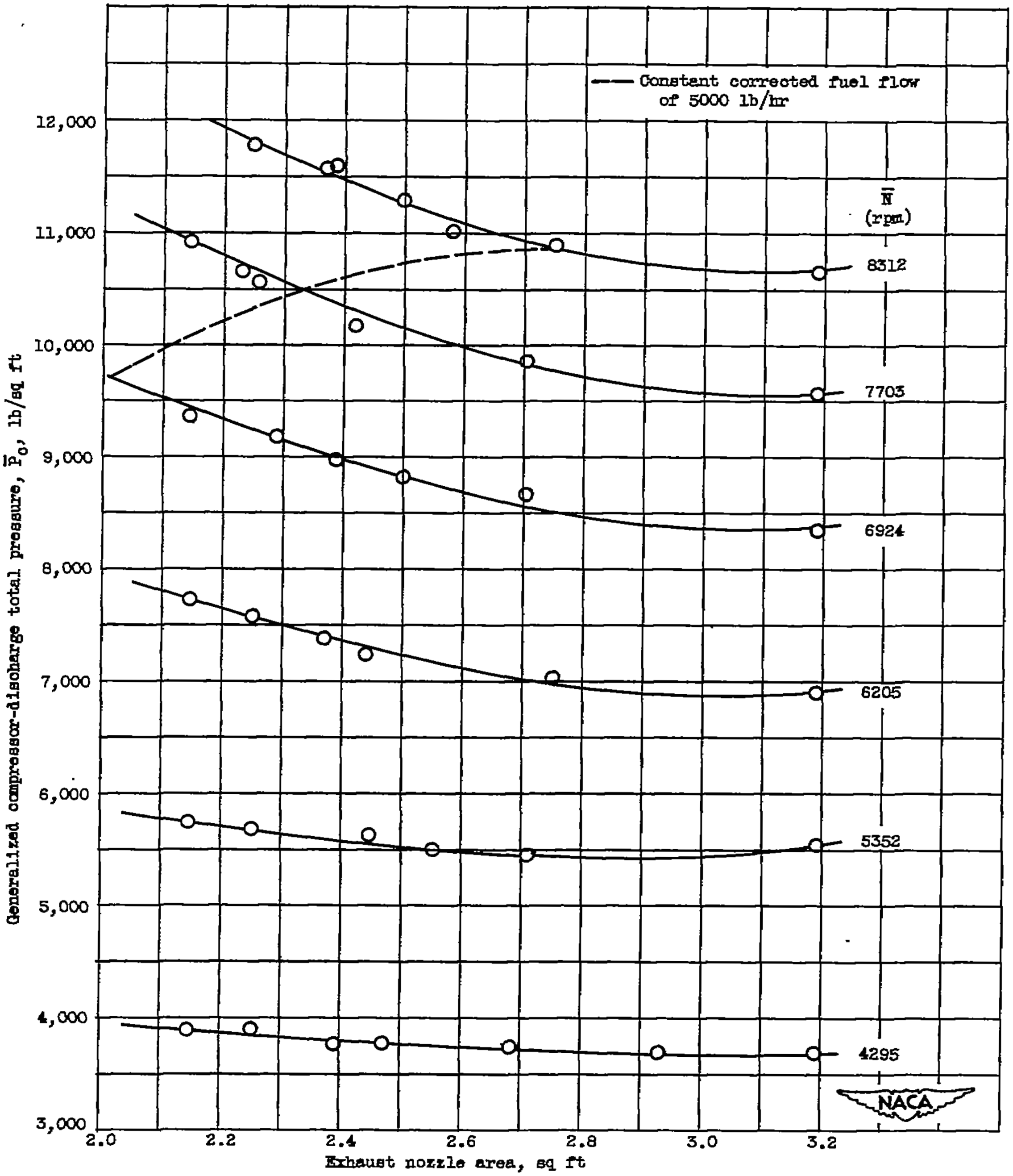


(a) Generalized compressor-discharge total pressure against exhaust nozzle area; altitude, 15,000 feet; ram pressure ratio, 1.03.

Figure 15. - Determination of generalized gain, compressor-discharge total pressure to exhaust nozzle area, with changes in ram pressure ratio and altitude.

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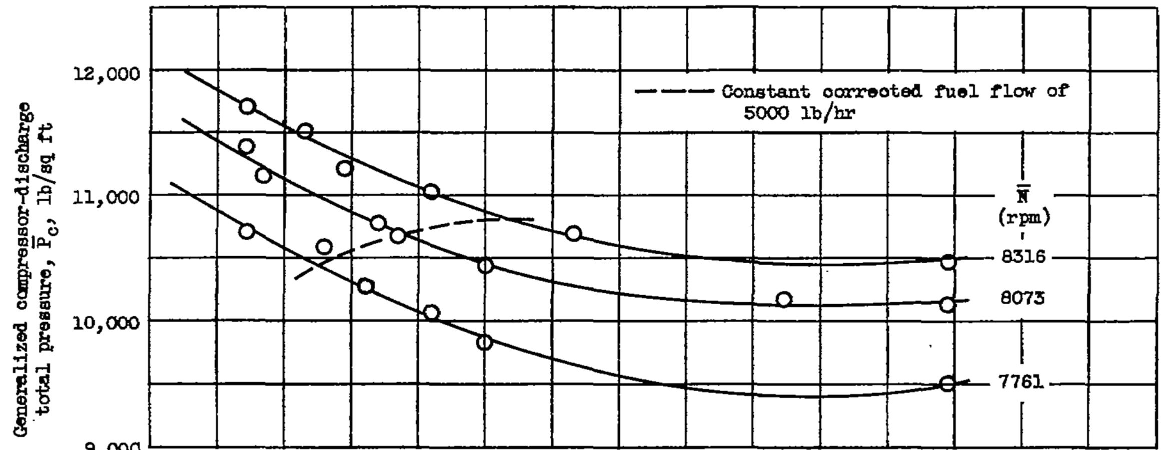
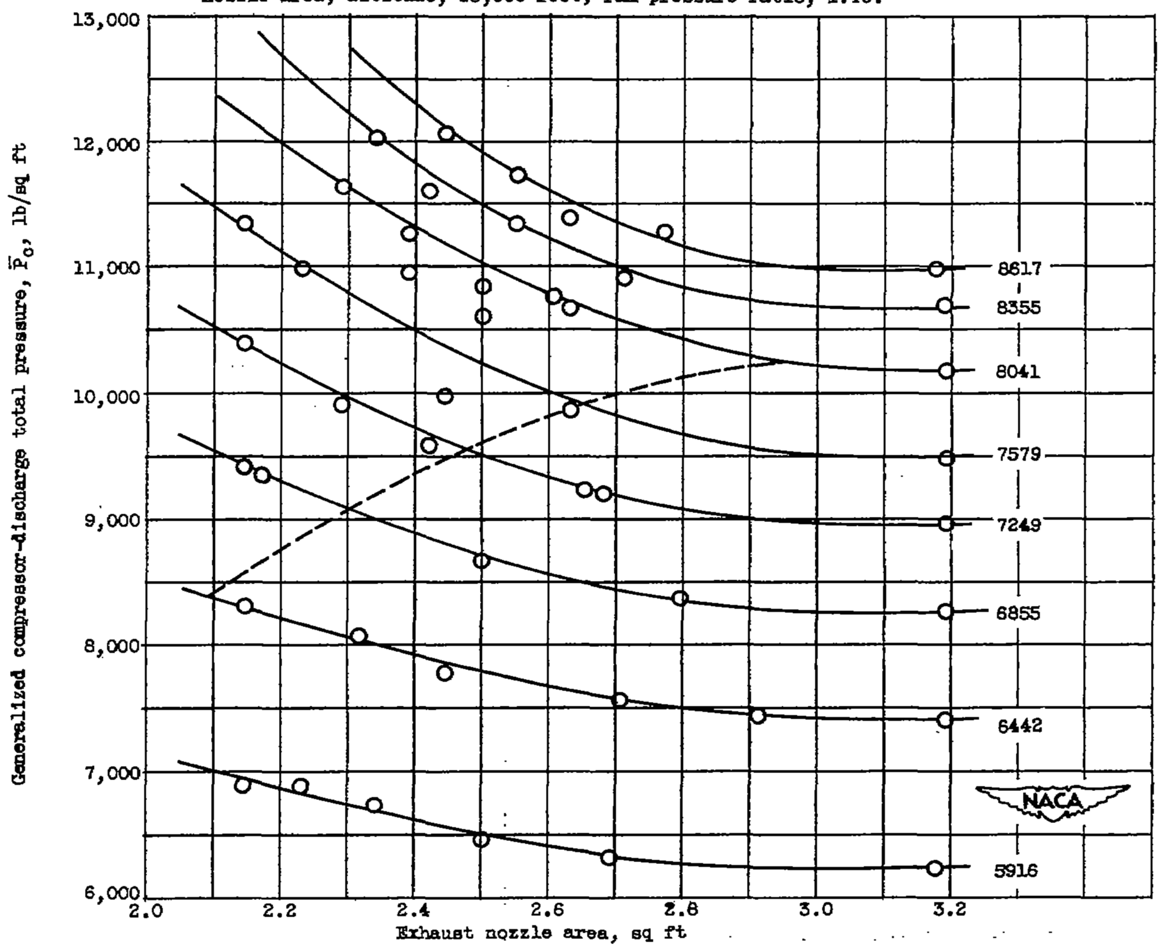



(b) Generalized compressor-discharge total pressure against exhaust nozzle area; altitude, 25,000 feet; ram pressure ratio, 1.40.




(c) Generalized compressor-discharge total pressure against exhuast nozzle area; altitude, 45,000 feet; ram pressure ratio, 1.03.

Figure 15. - Concluded. Determination of generalized gain, compressor-discharge total pressure to exhaust nozzle area, with changes in ram pressure ratio and altitude.

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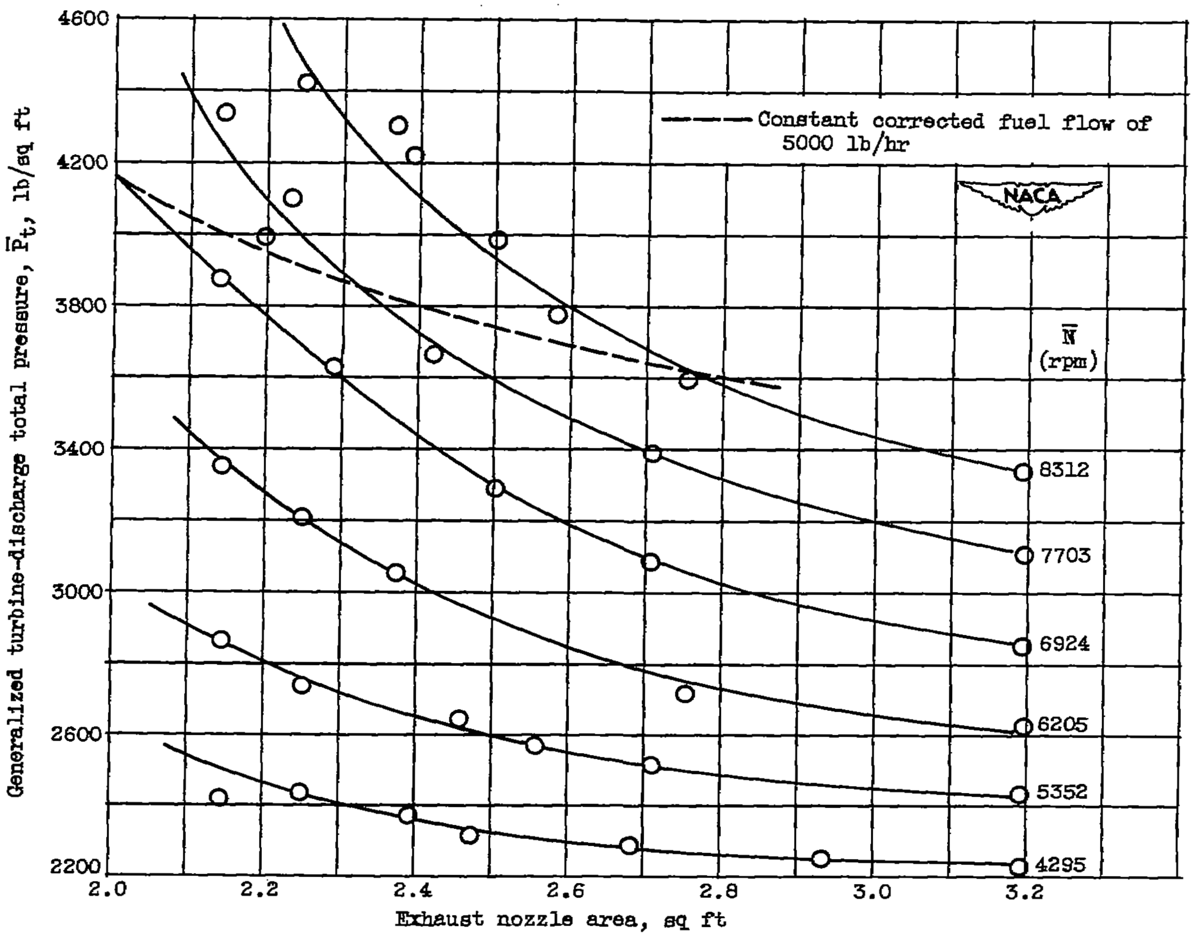


(a) Generalized turbine-discharge total pressure against exhaust nozzle area; altitude, 15,000 feet; ram pressure ratio, 1.03.

Figure 16. - Determination of generalized gain, turbine-discharge total pressure to exhaust nozzle area, with changes in ram pressure ratio and altitude.

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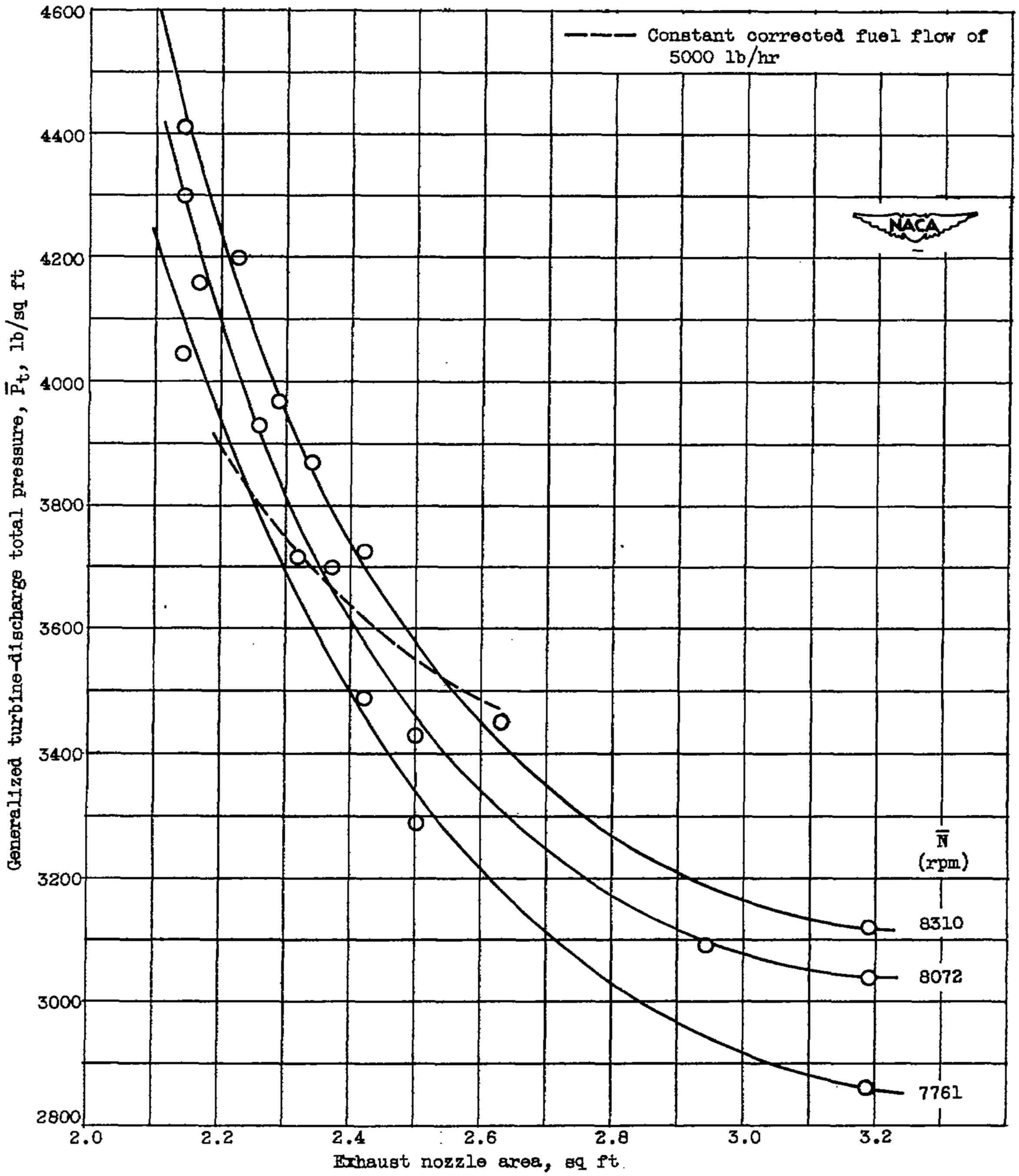


(b) Generalized turbine-discharge total pressure against exhaust nozzle area; altitude, 25,000 feet; ram pressure ratio, 1.40.

Figure 16. - Continued. Determination of generalized gain, turbine-discharge total pressure to exhaust nozzle area, with changes in ram pressure ratio and altitude.

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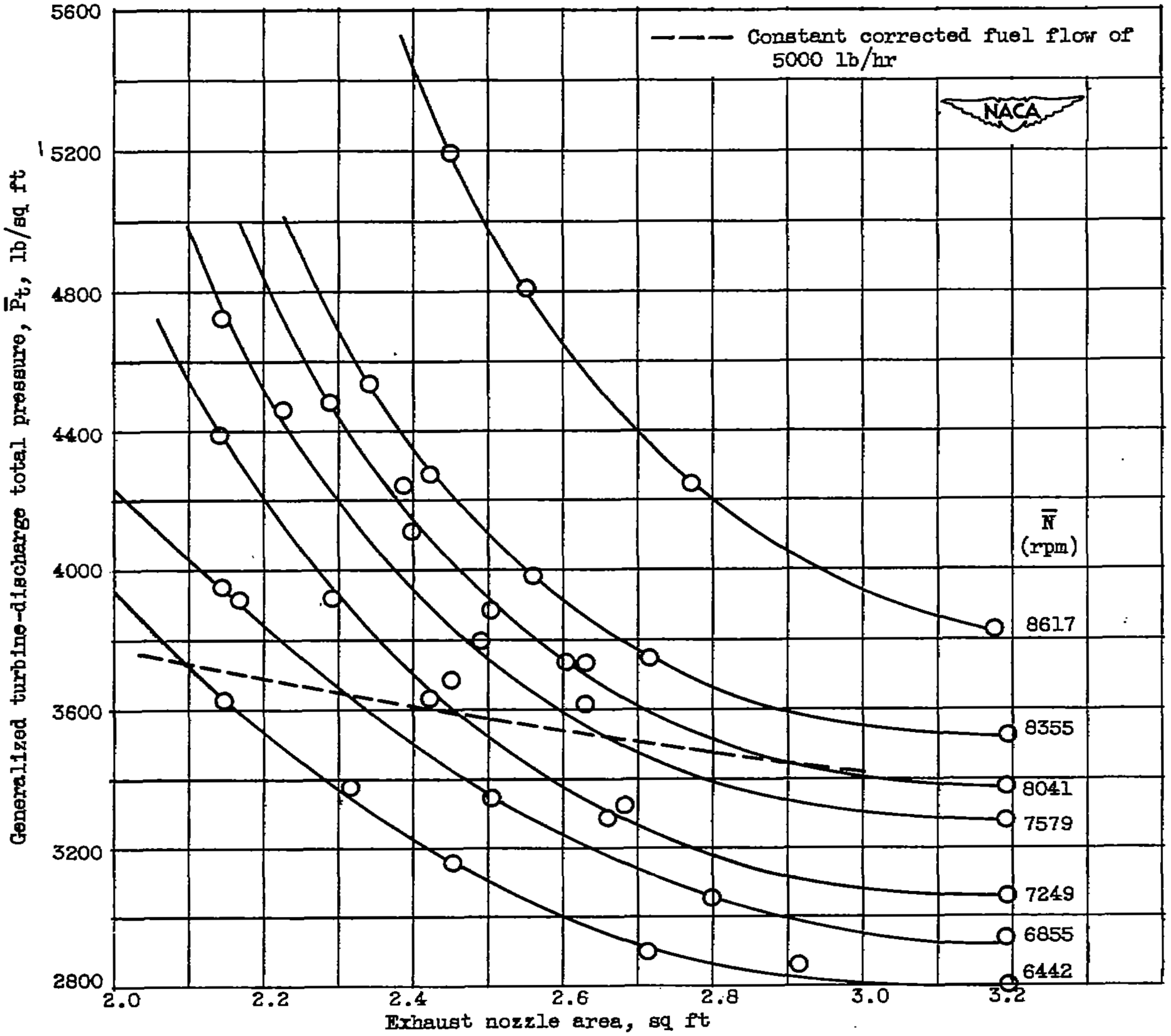


(c) Generalized turbine-discharge total pressure against exhaust nozzle area; altitude, 45,000 feet; ram pressure ratio, 1.03.

Figure 16. - Concluded. Determination of generalized gain, turbine-discharge total pressure to exhaust nozzle area, with changes in ram pressure ratio and altitude.

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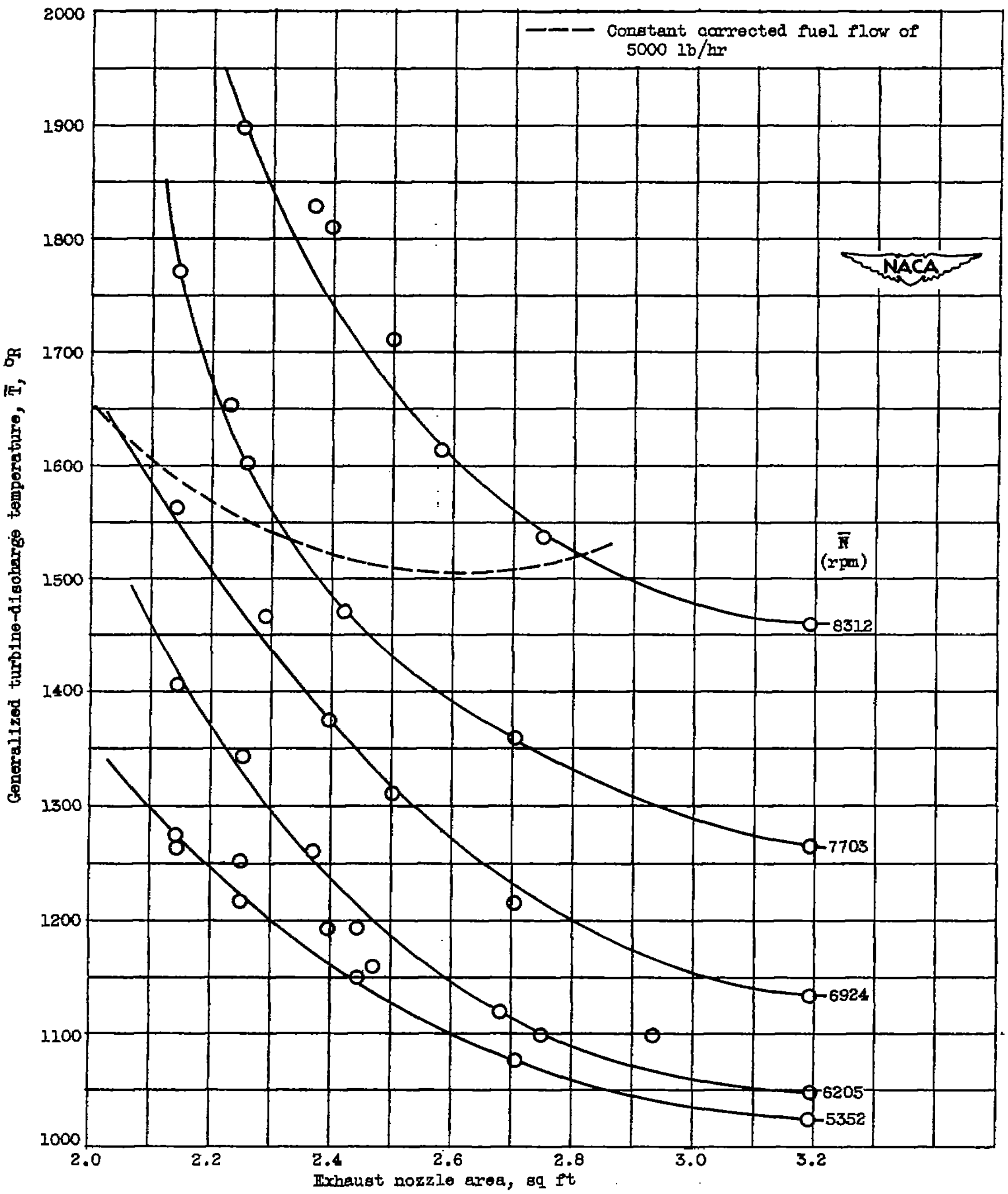


(a) Generalized turbine-discharge temperature against exhaust nozzle area; altitude, 15,000 feet; ram pressure ratio, 1.03.

Figure 17. - Determination of generalized gain, turbine-discharge temperature to exhaust nozzle area, with changes in ram pressure ratio and altitude.

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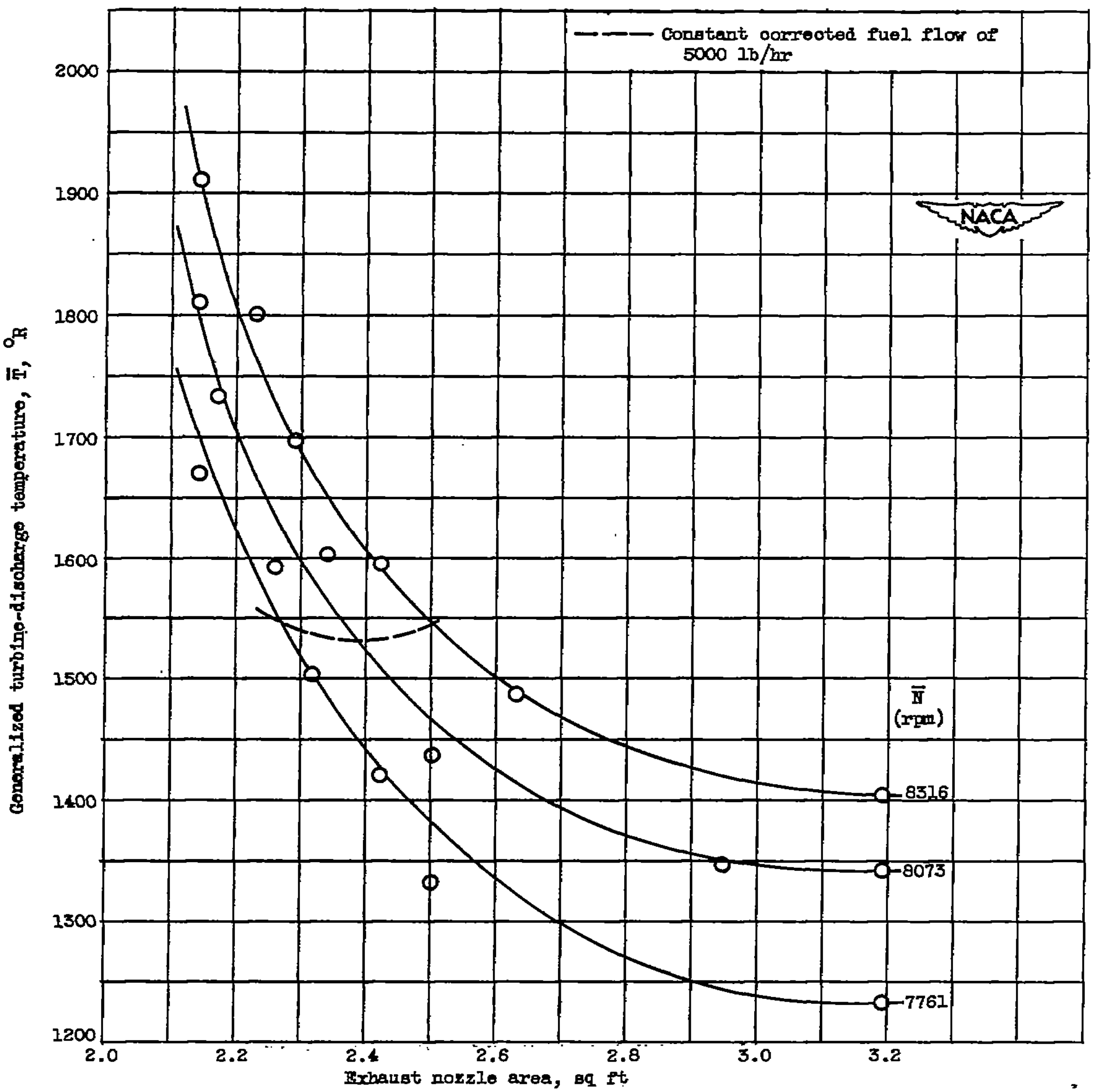


(b) Generalized turbine-discharge temperature against exhaust nozzle area; altitude, 25,000 feet; ram pressure ratio, 1.40.

Figure 17. - Continued. Determination of generalized gain, turbine-discharge temperature to exhaust nozzle area, with changes in ram pressure ratio and altitude.

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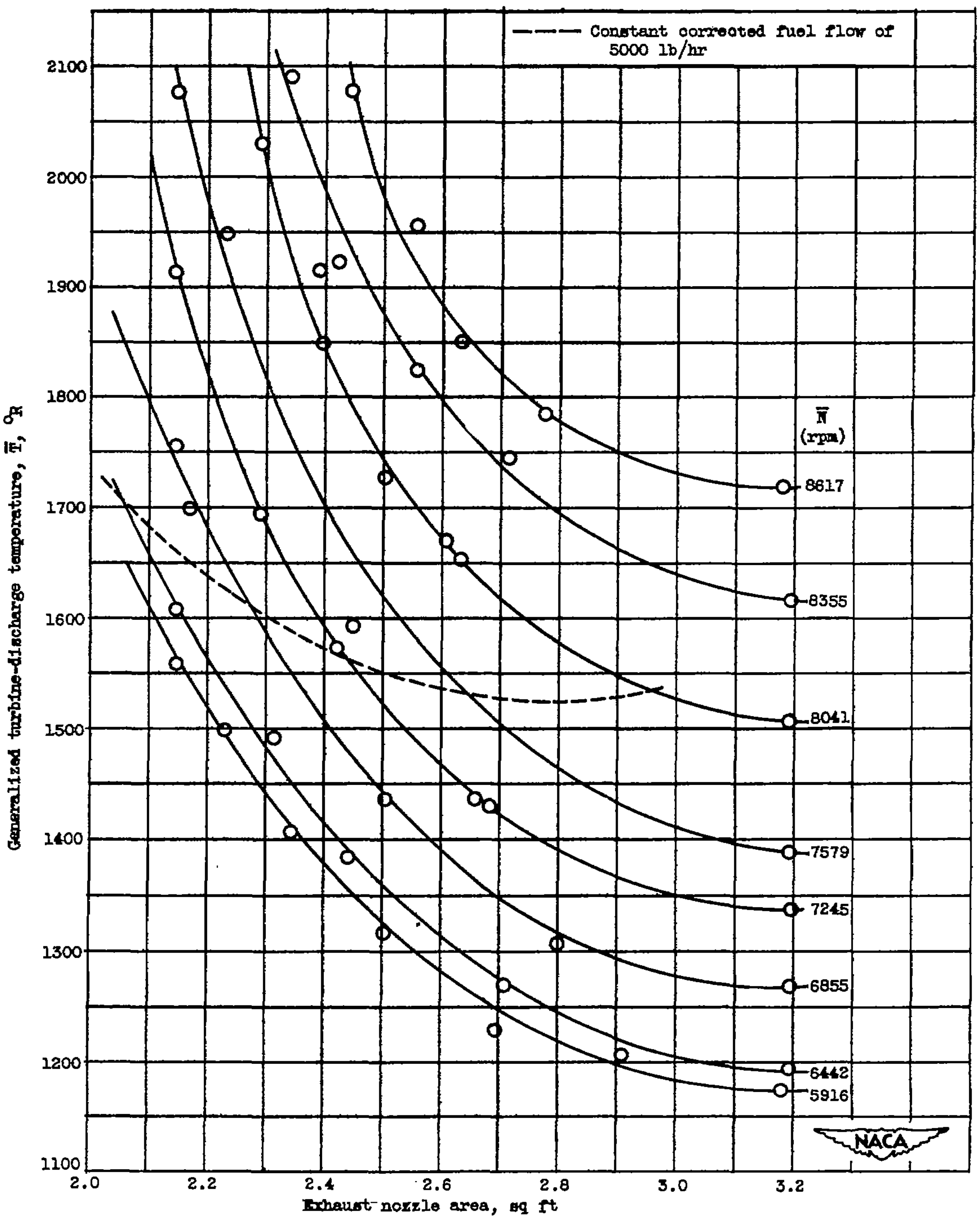


(c) Generalized turbine-discharge temperature against exhaust nozzle area; altitude, 45,000 feet; ram pressure ratio, 1.03.

Figure 17. - Concluded. Determination of generalized gain, turbine-discharge temperature to exhaust nozzle area, with changes in ram pressure ratio and altitude.

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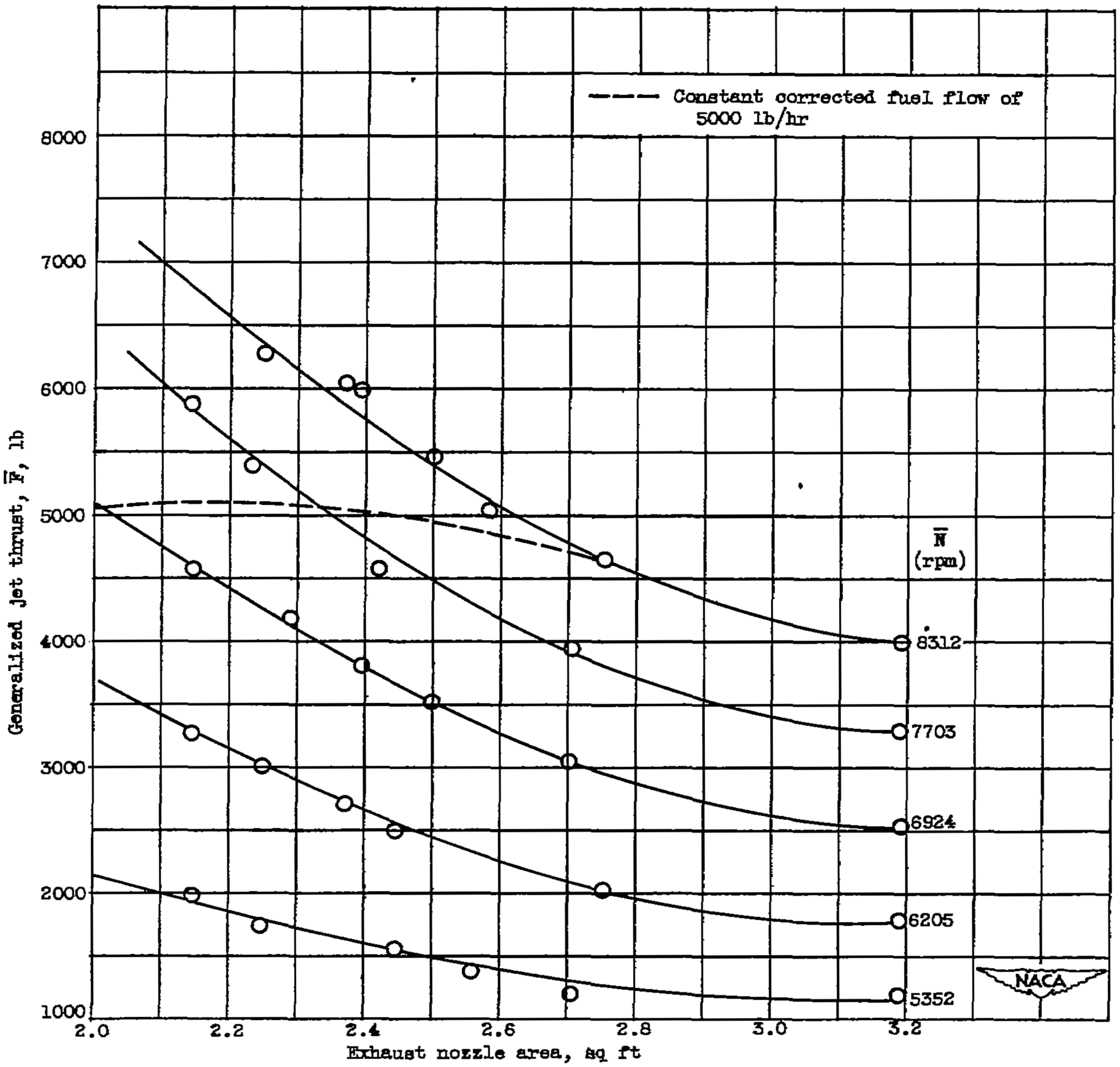


(a) Generalized jet thrust against exhaust nozzle area; altitude, 15,000 feet; ram pressure ratio, 1.03.

Figure 18. - Determination of generalized gain, jet thrust to exhaust nozzle area, with changes in ram pressure ratio and altitude.

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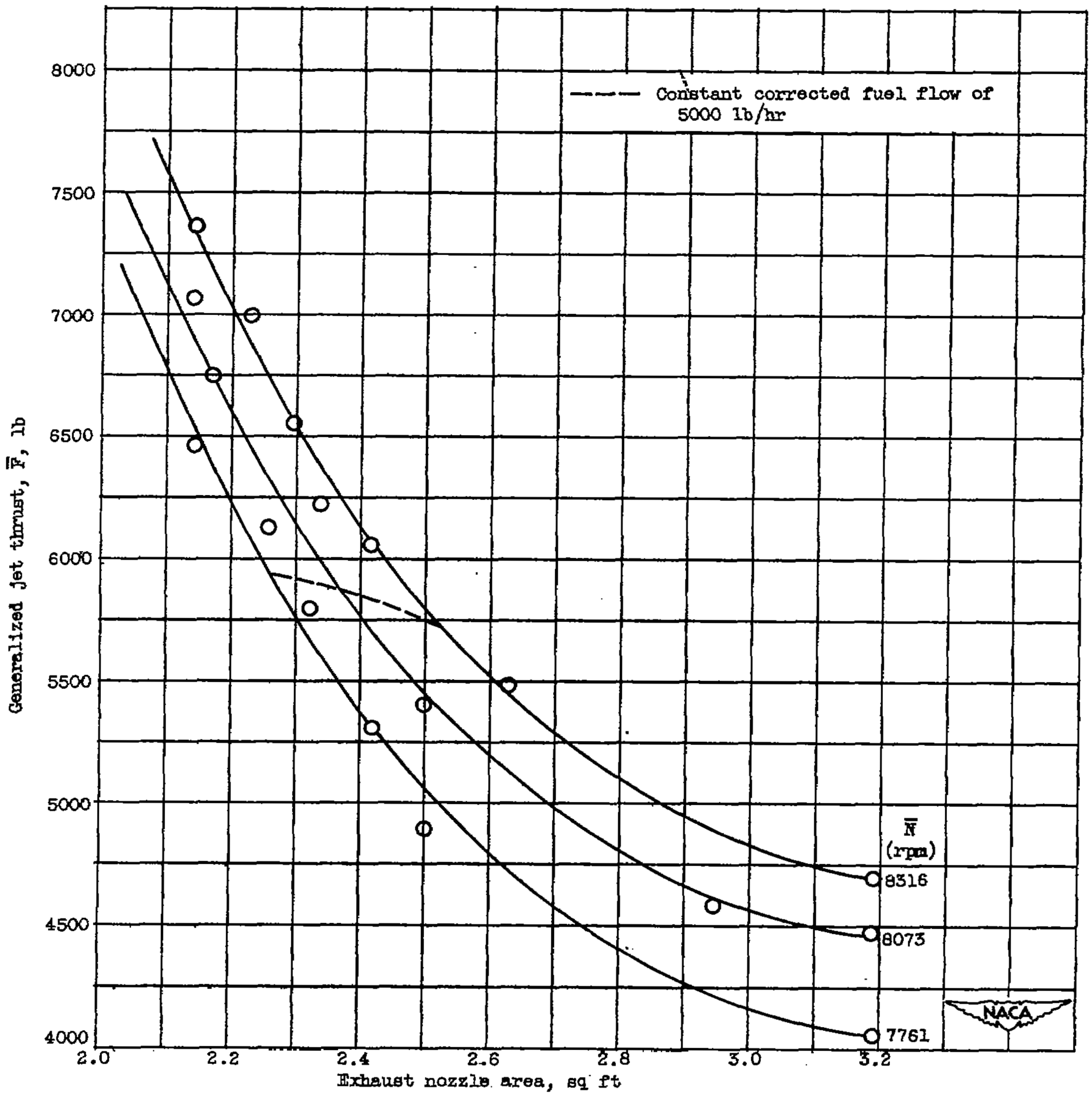


(b) Generalized jet thrust against exhaust nozzle area; altitude, 25,000 feet; ram pressure ratio, 1.4.

Figure 18. - Continued. Determination of generalized gain, jet thrust to exhaust nozzle area, with changes in ram pressure ratio and altitude.

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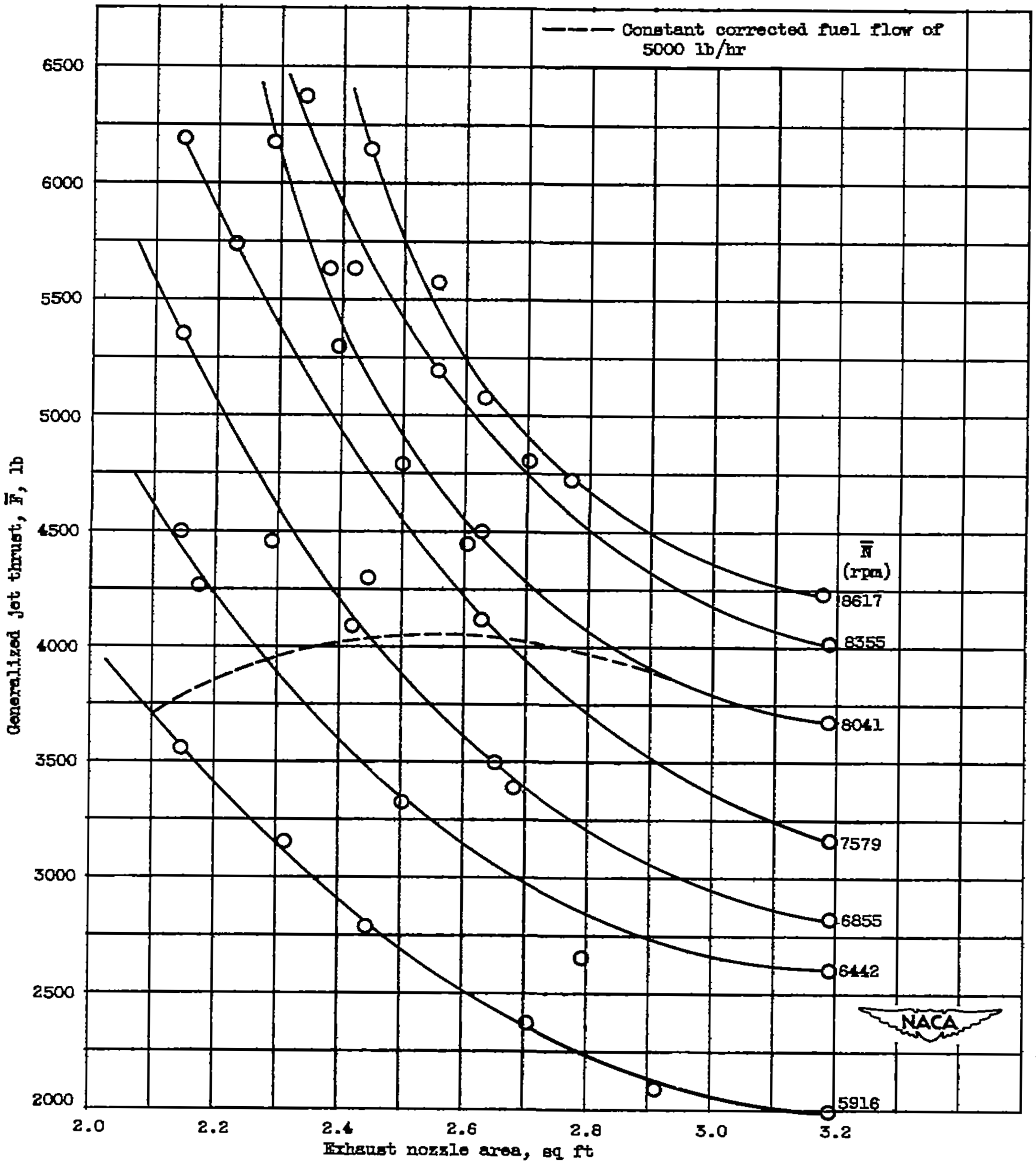


(c) Generalized jet thrust against exhaust nozzle area; altitude, 45,000 feet; ram pressure ratio, 1.03.

Figure 18. - Concluded. Determination of generalized gain, jet thrust to exhaust nozzle area, with changes in ram pressure ratio and altitude.

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SECURITY INFORMATION

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