

918

~~1136.5~~

~~23~~

~~Copy 1~~

~~1136.5~~

TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 966

1.8.2

COMPARISON OF AUTOMATIC CONTROL SYSTEMS

By W. Oppelt

Luftfahrtforschung
Vol. 16, No. 8, August 20, 1939
Verlag von R. Oldenbourg, München und Berlin

FOR REFERENCE

NOT TO BE TAKEN FROM THIS ROOM

Washington
February 1941



3 1176 01440 7168

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM NO. 966

COMPARISON OF AUTOMATIC CONTROL SYSTEMS*

By W. Oppelt

SUMMARY

This report deals with a reciprocal comparison of an automatic pressure control, an automatic rpm control, an automatic temperature control, and an automatic directional control. It shows the difference between the "faultproof" regulator and the actual regulator which is subject to faults, and develops this difference as far as possible in a parallel manner with regard to the control systems under consideration. Such an analysis affords, particularly in its extension to the faults of the actual regulator, a deep insight into the mechanism of the regulator process; for this purpose, certain important, practical cases require only cursory treatment, while others of less importance must be explained.

The results of the comparison of these four arrangements, even at the same steps in the consideration, are quite different from one another, since the systems to be regulated manifest differences. Differentiation between direct and indirect control is shown to be unnecessary for this analysis.

INTRODUCTION

This comparison of different control systems is unlike that of other similar reports, to the extent that the treatment proceeds in strictly parallel manner and finds its justification in the deep insight afforded of the mechanism of the control process. It indicates the differences and common actions of different control problems; it indicates to what extent and with what modifications the results obtained on one component system can be generalized and transferred, and so give the practical engineer the means for a quick grasp of dissimilar control systems, so as to make the correct selection and appraise any modifications necessary.

*"Vergleichende Betrachtung verschiedener Regelaufgaben hinsichtlich der geeigneten Regelgesetzmässigkeit." Luftfahrtforschung, vol. 16, no. 8, August 20, 1939, pp. 447-472.

To achieve these results, mathematical representation is unavoidable, but the results are so explained as to make their realization physically comprehensible. With this in view, the behavior of the "regulating system" is presented first. On the other hand, the "governing principle" is established which states how the deflection of the control link is related to the state of the regulating system. The combination of both gives the behavior of the "regulated system." This reduces the problem to "control with faultproof regulator." Subsequent treatment shows the effects of instrumental defects of the "actual" regulator. The most essential results are derived from an analysis of the continuous control; a study of the extreme transitions affords indications and conclusions for intermittent control systems. Presentation in tables assures a clear and convenient comparison. The diagrammatic sketch of a regulated system and the chosen terms are shown in figure 1.

The regulator ascertains with its "detector" the state of the regulating system and adjusts its control link. The detector gives the order to the amplifier. The constant switch in the amplifier connects an external auxiliary power supply and so actuates the machine. The machine adjusts the control link. On the direct regulator the adjusting force of the detector itself suffices to adjust the control link without the aid of an auxiliary force, but not on the indirect regulator.

The regulating problem can be divided into two parts: one, the finding of a suitable governing principle, the solution of which requires a knowledge of the regulating system; the second requires the instrumental realization of the established governing principle, a problem for the regulator design expert.

The attempts to establish a governing principle for the regulator to follow, fail: first, in the determination of the behavior of the regulating system; secondly, in the actual search for an adequate principle to which, thirdly, the study of the combined action of the regulating system and the governing principle must contribute.

1. ATTITUDE OF THE REGULATING SYSTEM

The identification of the state of the regulating system necessitates the following quantities:

The control link setting, η ;

The departure φ of the system from the theoretical state and, if necessary, its time rate-derivatives;

An interference quantity ξ which affords an indication for an outside disturbance of the equilibrium of the system.

Scale and scale coefficients of these quantities are read off later from the equations of the regulating system.

Since a perfectly exact representation of the behavior of the regulating system is generally not possible, approximations are resorted to which embrace its characteristic behavior and afford sufficient accuracy for the range of small vibrations. Out of the multitude of spheres of application of control systems, there are essentially four dissimilar types, whose attitudes are expressed by four differential equations of the following basic form:

$$\text{const } \ddot{\varphi} + \varphi = \eta \quad (1)$$

$$\text{const } \dot{\varphi} = \eta \quad (2)$$

$$\text{const } \ddot{\varphi} + \text{const } \dot{\varphi} + \varphi = \eta \quad (3)$$

$$\text{const } \ddot{\varphi} + \text{const } \dot{\varphi} = \eta \quad (4)$$

Of these four systems, the first appears principally on the pressure regulator, the second on the speed regulator, the third on the temperature regulator, and the fourth on the automatic directional control.

a) Pressure Regulator

A typical case of pressure control is shown in figure 2a. By way of a throttle D_1 , which represents the control link, the pressure medium enters chamber R in which the pressure is to be regulated. From chamber R the pressure medium is discharged by way of a fixed, adjustable throttle D_2 .

Under assumedly constant effects otherwise the system has, in the state of equilibrium, the tendency to adjust a certain pressure to each setting of the control link D_1 .

The system therefore disposes to some extent of a natural resetting ability, sometimes called "self-regulation" (Cf., for instance, Th. Stein: Selbstregelung ..., Z.V.D.I., vol. 72, 1928, p. 165). The mechanism of this process operates by way of the flow volume of the pressure medium. A pressure drop corresponding to this volume occurs at the two throttling points.

A change in throttle D_1 changes the pressure drop at that point; the equilibrium is then no longer possible by the same flow volume. But a change in volume necessarily implies a change in pressure on the fixed throttle D_2 and hence of the regulating pressure. Analytical study (reference 1) shows that the resulting stationary pressure φ is proportional to the adjustment η .

$$\varphi = \sim \eta \quad (1a)$$

If, however, the system is not in equilibrium, the momentary difference in flow volume at throttles D_1 and D_2 is equalized by the accumulator effect of the intermediate space. This accumulator ability may, for instance, lie in the nature of the space (if a pressure chamber or the like adjoins this space, fig. 2b), or it may lie in the nature of the pressure medium (as, for example, in the compressibility of gases). This accumulator ability signifies a gradual assimilation to the state of equilibrium. The rate at which the system tends toward equilibrium is proportional to the departure from it, which is expressed by an extension of equation (1a) to the form (reference 1):

$$T_{a1} \dot{\varphi} + \varphi = \eta \quad (1b)$$

where T_{a1} is a constant which represents an indication for this rate. Constant T_{a1} proves to be dependent on the loading z of the system. Necessarily, pressure and volume are coupled in the pressure-regulating system. For definite identification of the state, both pressure and volume should be indicated. However, since the behavior of the volume is singularly dependent upon the behavior of the pressure so long as no changes are effected in the volume adjustment of the regulating system, the amount of the pressure difference φ and a quantity z are sufficient. Quantity z indicates the setting which the volume adjustment assumes (for instance, through throttle D_2). It is designated as "load z "; "heavy load z " denotes

such an adjustment (great opening of throttle D_2) on the regulating system to which a great flow volume belongs. By increasing load x the proportionate quantity of the accumulator volume decreases. This becomes readily apparent when the great throttle openings under heavy load x are divided into several small ones with corresponding accumulator volumes and then compared with the conditions under light load x (fig. 2c).

On the other hand, the effect of the control link D_1 on the pressure is also dependent on the volume, i.e., on load x . By great flow volume the pressure drop at D_1 itself, is correspondingly greater. Both the effective accumulator volume and the effect η of the control link, show themselves to be inversely proportional to load x , which leads to the following equation (reference 1).

$$\frac{T_a}{x} \dot{\phi} + \phi = \frac{\eta}{x} \quad (1c)$$

or, after rewriting, to:

$$T_a \dot{\phi} + x \phi = \eta \quad (1d)$$

where T_a is a system constant. Outside disturbances such as changes in the preliminary pressure before throttle D_1 or in the pressure behind throttle D_2 shift the equilibrium position and are accounted for by an interference quantity ξ .

Herewith the equation for the behavior of the pressure regulating system takes the form

$$T_a \dot{\phi} + x \phi = \eta + \xi \quad (1)$$

where T_a is a system constant, which is an indication for the time in which equilibrium is established after a disturbance. T_a is usually termed the "starting time T_a ." The "load x " is a measure for the volume adjustment.

b) The rpm Regulator for Prime Movers

The prime movers fall into two groups: those in which the torque is largely dependent upon the setting of the control link and those in which the torque is, in addition, dependent upon the rpm itself.

The first group follows the law (reference 2):

$$M_d \sim \eta$$

The torque is, in part, absorbed by the driven engine; the rest is taken from the kinetic energy of the moved masses or absorbed by it, hence this latter part serves for a change $\dot{\phi}$ of the rpm. Herewith

$$M_d = \underbrace{M_{d_{\text{mod}}}} + \underbrace{M_{d\dot{\phi}}} = \eta \quad (2a)$$

$$- \xi + T_a \dot{\phi} = \eta \quad (2)$$

Quantity T_a is again a "starting time"; it gives an indication for the rate of change of the rpm by control-link adjustment. The steam engine is a typical example of the first group of prime movers; the direct-current, shunt-wound motor, of the other.

Its field being kept constant, its torque is solely dependent on the armature current i which, according to the ohmic law, depends upon the driving voltage and the armature resistance, R . The driving voltage consists of the terminal voltage less the dynamo effect of the rotating armature in the field. Since the field is constant, this dynamo effect is proportional to the rpm $\dot{\phi}$ of the motor:

$$i = \frac{E - \text{const } \dot{\phi}}{R} \sim M_d$$

which, with equation (2a) gives:

$$M_d = M_{d_{\text{mod}}} + M_{d\dot{\phi}} = - \xi + T_a \dot{\phi} = \frac{E}{R} - \text{const } \frac{\dot{\phi}}{R}$$

or, since the terminal voltage E represents the control link here:

$$R T_a \dot{\phi} + \text{const } \dot{\phi} = \eta + R \xi$$

or, with different scale units,

$$T_{ae} \dot{\phi} + \dot{\phi} = \eta_e + \xi_e \quad (2b)$$

This equation (2b), representing the behavior of the prime mover of the second group, therefore corresponds to equation (1) of the pressure-regulating system under constant load x , so will not be treated separately in the following. Its behavior appears here under the concept "pressure regulator."

c) Temperature Regulator

So long as the control link does not intervene, the behavior of the temperature-regulating system follows a relation similar to equation (1) of the pressure-regulating system (reference 3):

$$T_a \dot{\varphi} + \varphi = C \quad (3a)$$

Again the relation of volume of heat flow to temperature difference, which explains the presence of term φ , forms a state of equilibrium. Energy accumulators exist here also (heat accumulator ability of walls, etc.), which necessitates a steady approach to the equilibrium state and explains the presence of term $T_a \dot{\varphi}$. The starting process in this instance is not affected under load, the same specific accumulator volume being always available.

The contest between volume-of-heat input and heat volume required in the process, transpires within the term C of equation (3a); the remaining C changes the temperature state φ of the system until equilibrium exists (by way of the outwardly discharged heat volume, which is proportional to φ). The term C therefore can be divided into a part ξ corresponding to the heat volume utilized and a part $\bar{\eta}$ representing a measure for the volume of heat input:

$$T_a \dot{\varphi} + \varphi = \bar{\eta} + \xi \quad (3b)$$

In this case the term $\bar{\eta}$ cannot be directly tied in with the control link setting η because of the time differences existing between the accumulator ability of the thermal masses and those which are expressed by a different equation (3c) (reference 3):

$$T_v \dot{\eta} + \eta = \eta \quad (3c)$$

This equation signifies that in the stationary state ($\dot{\eta} = 0$) each control link position η defines a certain

energy input $\bar{\eta}$. But as long as this state has not been reached, a part of the released energy serves to bring the accumulator masses in the energy flow path to the new state ($T_V \bar{\eta}$). The time constant T_V is designated as the "deceleration time T_V ."

Combining (3b) and (3c) gives the behavior of the temperature-regulated system, equation (3).

$$T_V T_A \ddot{\varphi} + (T_V + T_A) \dot{\varphi} + \varphi = \bar{\eta} + \xi + T_V \dot{\xi} \quad (3)$$

This equation (3) indicates that a disturbance of the state of equilibrium acts differently, according to whether the disturbance arrives from the consumer side (ξ) or from the energy input side ($\bar{\eta}$). Following a sudden change of control link position $\bar{\eta}$, the state φ of the regulating system approaches the new equilibrium position after a damped harmonic oscillation. On the stated premises, the damping of this process is always superaperiodic.* By a disturbance from the consumer side (for instance, on removal of bread from an oven), however, the system does not follow a superaperiodic oscillation, but a simple aperiodic damping motion (differential equation of the first order (3b)) into the new equilibrium position** because the vibration susceptible deceleration in the energy input path does not then become effective.

d) Automatic Directional Control

In the systems dealt with so far, only the regulating quantity and its first-time derivative were concerned except in the temperature regulator, where the second derivative occurred; but it was still so far in the background compared to the first as to make it insufficient for bringing the system out of the superaperiodic state.

*The proof is as follows: The damping of the oscillation process according to (3) is: $D = \frac{1}{2} \frac{T_V + T_A}{\sqrt{T_V T_A}}$, whence $4D^2 = \frac{T_V}{T_A} + \frac{T_A}{T_V} + 2$, which is always greater than 4; hence D is always greater than 1, i.e., always superaperiodic.

**This process is more readily apparent with (3b) than equation (3), because of the interruption for ξ due to the abrupt changes of ξ .

In the automatic directional control, however, the effect of the second derivative predominates as expression of the kinetic energy which can be accumulated through the moment of inertia of the vehicle.

The forces of inertia are opposed by the moments exerted by an adjustment of the control link (here, of the rudder) as well as by the moments which correspond to the rate of rotation of the craft. The latter are created by the "damping" which the craft undergoes in its surrounding medium. Hence (reference 4):

$$\text{Inertia moment} \times \text{angular velocity} = \text{effective moment} \quad (4a)$$

$$J \ddot{\varphi} = \text{const} \dot{\varphi} + \text{const} \cdot \eta$$

or, rewritten as

$$\ddot{\varphi} + M \dot{\varphi} = N \eta \quad (4b)$$

N denotes a constant for the rudder effectiveness; in it are contained the speed of the airplane, the size of the rudder area, the distance of the rudder from the center of gravity of the craft, the reciprocal value of inertia moment of the craft, and a factor taking account of the form of the rudder. Quantity M gives an indication of the damping capacity of the craft; in it enters the reciprocal value of the inertia moment of the craft, the path velocity, and a factor taking the form of the craft into account. Outside influences disturbing the state of equilibrium are again expressed by an interference quantity ξ :

$$\ddot{\varphi} + M \dot{\varphi} = N \eta + \xi \quad (4)$$

in which form equation (4) characterizes the behavior of a craft. It thus holds for the automatic control of ships, airships, and the directional control of airplanes because for these cases the cycle of the forces comprised in equation (4) is the most essential of the problem.

The characteristic differences of these four regulating systems following an abrupt adjustment of the control link from η_a to η_b , are exemplified in table I.

The pressure-regulating system is in equilibrium $\varphi_a = \frac{\eta_a}{z}$ at control link setting η_a . On adjustment to

η_b , the state φ of the system approaches the new state of equilibrium $\varphi_b = \eta_b/z$, according to an e function. The time constant of this e function is small by heavy load z , hence the system reaches its new attitude more quickly by heavy, than by light load z . Moreover, the equilibrium change $\varphi_b - \varphi_a$ under light load z is greater than by heavy load, despite equal control-link adjustment. Under zero load the new state of equilibrium is infinitely remote; the e function has degenerated in a straight line.

The rpm regulating system is in a constant state of change $\dot{\varphi}_a = \eta_a/T_a$ during the control-link setting η_a . On changing the setting η_b , it abruptly assumes the new rate of change $\dot{\varphi}_b = \eta_b/T_a$. The rpm regulating system therefore acts like the pressure-regulating system under zero load; all processes of the rpm regulator then follow from corresponding ones of the pressure regulator through the limit transition $z \rightarrow 0$.

The temperature-regulated system is, during control-link setting η_a in a setting $\varphi_a = \eta_a$ and reaches, after adjustment to η_b , the new position φ_b after a superaperiodic oscillation.

Suppose the regulating system of the automatic directional control accomplishes a constant change $\dot{\varphi}_a = N \eta_a/M$ of its state during control-link setting η_a . It reaches, after adjustment to η_b , its new rate of change $\dot{\varphi}_b = N \eta_b/M$, according to a transition curve.

2. APPROPRIATE GOVERNING PRINCIPLE

The governing principle characterizes the action of the regulator. It expresses the relation of the regulators between the phase quantities of the regulating system and the control-link setting.

For the problems in question, several principles have been evolved empirically. They are:

$$- \eta = a \varphi$$

(5)

$$- \eta = a \varphi + c \int \varphi dt \quad (6)$$

$$- \dot{\eta} = c \int \varphi dt \quad (7)$$

These principles proceed from two critical points. Once, $-\eta = a \varphi$, the control-link setting η is made proportional to the departure φ from the theoretical state of the regulating system. That is, every departure φ defines a certain control-link setting η , which tries to remove this departure. In the second case, $-\dot{\eta} =$

$c \int \varphi dt$, every departure from the theoretical state defines a certain rate of change $\dot{\eta}$ of the control-link deflection, because the principle $-\dot{\eta} = c \int \varphi dt$ implies

$-\ddot{\eta} = c \varphi$. Here the control link is continuously in motion so long as the theoretical state has not been reached.

Lastly, they are used in combination, $-\eta = a \varphi + c \int \varphi dt$.

Occasionally the first (and second) time derivative of φ is employed in order to achieve certain actions:

$$- \eta = a \varphi + b \dot{\varphi} \quad (8a)$$

$$- \eta = a \varphi + b \dot{\varphi} + c \int \varphi dt \quad (8b)$$

The minus sign before η indicates that the control-link setting η opposes the departure φ of the state; i.e., tries to eliminate it.

Regulators, according to principle (5), $-\eta = a \varphi$, may be termed "regulators with conjugate setting," because every departure φ defines a conjugate control-link setting η .

Regulators, according to the principle (7), $-\dot{\eta} = c \int \varphi dt$, may be termed "regulators with conjugate running speed," because each departure φ defines one running speed $\dot{\eta}$ of the control link.

Regulators embodying both, $-\eta = a\varphi + c \int \varphi dt$, belong to no group. They are usually (their running speed portion $c \int \varphi dt$ being mostly small) modified regulators with conjugate setting, the zero position of the conjugate setting being shifted by the portion $c \int \varphi dt$; but they are also occasionally built up from running-speed regulators by adding the time derivative $\dot{\varphi}$ to the departure φ :

$$\dot{\eta} = a\dot{\varphi} + c\varphi, \text{ i.e., } \eta = a\varphi + c \int \varphi dt + \text{const}$$

With the action of the regulator on the regulating systems, two aims are pursued: First, a definite state of the regulating system is to be preserved; i.e., the regulation in its proper sense. Then, certain intentional changes of state over the regulator are aimed at, whereby the regulator removes departures from the plane of these changes - a problem designated as "directional control." If a timed schedule of change is involved, it is called "time-table regulation."

Equations (5), (6), and (7) contain only the relations necessary for a regulation. For the directional control of any process, the deflection η of the control link is also made dependent upon another quantity ξ which, if desired, is adjustable; the neutral setting of the detector is simply shifted for the corresponding amount.

Herewith, equations (5), (6), and (7) become:

$$-\eta = a(\varphi + \xi) \quad (5a)$$

$$-\eta = a(\varphi + \xi) + c \int (\varphi + \xi) dt \quad (6a)$$

$$-\dot{\eta} = c \int (\varphi + \xi) dt \quad (7a)$$

The constants a , b , and c in equations (5) to (8) indicate the extent to which the control-link deflection η depends upon the individual influences. Disclosure of the nature

of these influences is deferred to the combined action of regulator and regulating system, although the constants can already be identified by name: Quantity a is called "resetting effect a " because a reset of the control link proportional to a departure ϕ exerts a resetting moment on the regulating system. Quantity b is the "damping effect b ," since a control-link deflection corresponding to a rate of change $\dot{\phi}$ of the state has a damping effect on the regulating course. Quantity c is the "displacement effect c " because its action is, so to say, based upon a displacement of the neutral position of the control link.

Constants a , b , and c cannot be arbitrarily selected. Limiting conditions follow from the regulating process with faultproof regulator. Other limitations follow from the errors of the actual regulator.

Neither detector nor control link can arbitrarily execute great movements. Their range must be so chosen that normal regulating processes take place within it. Their limits are, in consequence, without (i.e., outside) the scope* of the present study.

3. COORDINATION OF GOVERNING PRINCIPLE AND REGULATING SYSTEM

The study of this combined action may be grouped as follows: First, the regulator is assumed to be faultproof and hence exactly complies with its governing principle (equations (5) to (8)). This case represents the best possible state obtainable with the chosen principle; it also contains all the requirements which the regulated system can satisfy theoretically and in the most favorable case with the chosen principle and the chosen magnitude of the constants a , b , and c .

Then the regulating process is analyzed with consideration of the error of the actual regulator. The term "error" includes all departures from the purposed governing

*The influence of a regulator is usually expressed in respect to such limits, or with reference to the load limits of the regulating system which, for example, introduces the concept of the so-called "degree of nonuniformity."

principle. These errors can be grouped according to certain characteristics.

The mathematical treatment consists of bringing equations (1) to (4) in relation with equations (5) to (8) and of obtaining therefrom the equations of the regulated system.* The solution of these equations leads to increasing or decreasing oscillations and aperiodic motions. The aperiodic motions, which by concurrent existence of oscillations represent their median position, are always damping. Mathematically, this is indicated by the fact that the coefficients of the differential equations are all positive; physically, it is indicated by the fact that by longer departures Φ from the theoretical position, the control link is ultimately always deflected in the restoring sense. Should a solution yield oscillations, a concurrently existent aperiodic motion is usually secondary, because the oscillation portion can in most cases exceed a damping limit and hence lead to impractical regulating processes, while the aperiodic motion is always damping.

The characteristic quantities of an oscillation are expressed by the damping D and the natural frequency ω_e . These values are assumed to be known from the pertinent differential equation.** If, in addition, increasing oscillations are possible by certain setting of the regulator, the relation for the damping limit itself is then indicated. For the damping limit the oscillations are then exactly undamped permanent oscillations. With $\varphi = \sin \omega t$, the differential equation of the process can be divided into two terms with \sin and \cos . These terms must each, by itself, be zero, since the equation must be valid for each instant. The result is two equations, from which the relation for the damping limit is obtained. In

*It should be remembered that by positive factors the linear homogeneous differential equation of the first order represents aperiodically vanishing motion forms; that of the second order, damped oscillation forms of motion; and that of the third order, increasing oscillation forms of motion by certain factors.

**Lehr, in Schwingungstechnik I, Berlin, 1930: uses the damping criterion D which, aside from convenient analytical calculation, has the advantage of including the super-aperiodic case with finite numbers; $D = \text{zero damping}$, $D = 1$ aperiodic damping.

simple cases this procedure is usually omitted. The statement of the damping limit indicates the zone within which damping oscillation occurs. For appraising the practical use of a regulating adjustment, the knowledge of the damping limit is usually insufficient; it should also include the damping D and the natural frequency ω_0 . However, the aspect of the damping limit itself, affords considerable information on the direction in which the regulating effects must be modified in order to assure satisfactory conditions.

For the clarification of the processes it often proves expedient either to assume an external interfering force, which somehow sets the regulated system into permanent oscillations, or to select the constants a , b , and c in such a manner as to preserve its state therein, and then to analyze the phase position of the control link. For this case the application of the data from the vibration theory is especially convenient.

No special treatment of the swinging-in processes is attempted. For the typical case of abrupt change of state (for instance, a disturbance ξ , or a directional control ζ), the time constant K is sufficient (in processes following a differential equation of the first order), or the damping D and natural frequency ω_0 (processes expressed by a differential equation of the second order). Processes expressed by a differential equation of the third order then manifest swinging-in processes by which the oscillation is formed about a damping e-function. This e-function is so placed in relation to the new state of equilibrium that the departures existing without them are even increased at first. This behavior has its physical cause in the appearance of a lagging phase displacement on the control link; for instance (as explained later on),

through the portion $c \int \varphi dt$ of the governing principle

or the "inertia δ " of the regulator, in consequence of which the state is forced beyond the new state of equilibrium. If, instead of an abrupt change in theoretical state, an arbitrary interference function with respect to time is involved, that of itself necessitates a separate analysis of the swinging-in action.

The graphical representation is made with a view to the most essential control influence, i.e., of the resetting effect a . By aperiodic motions their time con-

stant K , by damped oscillations their damping D in relation to the resetting effect a is presented. If the regulating course indicates a damping limit, the overstepping of which is followed by increasing oscillations of state φ , then this damping limit is presented, and the resetting effect a plotted as dependent quantity; for it must then be so chosen in relation to other quantities that the regulating course remains within the damping limit. The damped zone is emphasized by hatching. For the principle $-\eta = c \int \varphi dt$, the displacement effect c sub-

stitutes for the resetting effect a . While dealing with the individual cases, the appended graphical representation should be consulted. No special reference is made in the text.

a) Faultproof Regulator

The combined action of governing principle and regulating system for the different controls is now analyzed and the results compared.

.α) Pressure Regulator

The combination of the pertinent groups of equations gives the following:

<u>Governing Principle</u>	<u>Regulated System</u>	
$-\eta = a(\varphi + \xi)$	$T_a \ddot{\varphi} + (z+a)\dot{\varphi} = \xi - a\xi$	(9)
$-\eta = a(\varphi + \xi) + c \int (\varphi + \xi) dt$	$T_a \ddot{\varphi} + (z+a)\dot{\varphi} + c\varphi = \dot{\xi} - c\xi - a\xi$	(10)
$-\eta = c \int (\varphi + \xi) dt$	$T_a \ddot{\varphi} + z\dot{\varphi} + c\varphi = \dot{\xi} - c\xi$	(11)

The factor z appearing in equations (9) to (11) of the regulated system indicates that all regulating processes on the pressure regulator differ under different load z .

The use of a regulator with conjugate setting, $-\eta = a(\varphi + \xi)$, gives a linear differential equation of the first order for the regulating process. It means that after a sudden change in equilibrium (for instance, due to transition to a new theoretical state by means of change of directional control quantity ξ , or by a permanent

outside interference ξ) the new state is reached through an e-function. The time constant K of this e-function follows from equation (9) at:*

$$K = \frac{T_a}{z + a}$$

The new state of equilibrium therefore is reached sooner by short starting times T_a (because then smaller accumulator volumes need to be filled), heavy load z (because the involved accumulator volumes are small, fig. 2c), and great resetting effect a of the regulator (because the regulator then exerts a great effect even at small departures). Because accumulator volume and regulator resetting effect remain, a finite time constant of the process remains even by disappearing load $z = 0$.

For the regulator with conjugate running speed, $\omega = \eta = c \int (\varphi + \xi) dt$, the regulating process is represented by a linear differential equation of the second order. It implies that the regulating process after a disturbance of its state takes place in damped oscillations. An oscillation is possible now because of the continued existence of a control-link setting $c \int \varphi dt$ after a departure φ , which forces the system beyond its neutral position toward the other side. The damping and frequency of this oscillation process follow from equation (11) at**

* Equation $\varphi = \varphi_0 e^{-\frac{1}{K}t}$ inserted in equation (9) gives:

$$- T_a \varphi_0 \frac{1}{K} e^{-\frac{1}{K}t} + (z+a) \varphi_0 e^{-\frac{1}{K}t} = \xi - a \xi = \text{const}$$

and

$$- T_a \frac{1}{K} + z + a = \frac{\text{const}}{\varphi_0 e^{-\frac{1}{K}t}}, \quad \text{whereby } K = \frac{T_a}{z + a}$$

** See footnote, p. 18.

$$D = \frac{1}{2} \frac{z}{\sqrt{cT_a}}$$

$$\omega_e = \sqrt{\frac{c}{T_a} - \frac{z^2}{4T_a^2}}$$

Hence the shorter the starting time T_a , the better the damping D of the process (since at the then-accelerated processes the setting η of the control link has not yet reached such abnormal values as by gradual movements), the higher the load z (because then the accumulator volume involved is small), and the smaller is the regulator displacement effect c , chosen (because then the displacements of the control link are slight). The natural frequency ω_e increases with increasing displacement effect c (since it is the real cause of the vibration susceptibility), reduced starting time T_a , and increasing load z (for with both, the accumulator volume involved becomes smaller). The natural frequency ω_e becomes zero when $D > 1$ (i.e., enters the superaperiodic zone), as theoretically must be the case by every oscillation. By disappearing load, $z = 0$, the damping itself becomes zero. (The accumulator effect of the regulating system still exists but its inherent resetting ability is gone.)

Lastly, the application of the principle $\ddot{\varphi} + 2D\dot{\varphi} + \omega_e^2\varphi = \text{const}$ also affords a damped swing-in

** (From p. 17)

Equation (11) $T_a \ddot{\varphi} + z\dot{\varphi} + c\varphi = \xi - c\xi$ transformed to $\ddot{\varphi} + \frac{z}{T_a}\dot{\varphi} + \frac{c}{T_a}\varphi = \frac{\xi - c\xi}{T_a}$, corresponds to the basic equation

$$\ddot{\varphi} + 2D\omega_0\dot{\varphi} + \omega_0^2\varphi = \text{const}$$

of a damped oscillation. This comparison gives:

$$2D\omega_0 = \frac{z}{T_a} \quad \text{and} \quad \omega_0^2 = \frac{c}{T_a}$$

Lastly, because of the relation $\omega_e^2 = \omega_0^2 (1 - D^2)$ existing between the assumedly undamped frequency ω_0 and the actual frequency ω_e , the values are those cited in the text.

into the theoretical state (equation (10)), with the characteristic quantities:

$$D = \frac{1}{2} \frac{z + a}{\sqrt{cT_a}}$$

$$\omega_e = \sqrt{\frac{c}{T_a} - \frac{(z + a)^2}{4 T_a^2}}$$

The action of the regulator process again approaches the conditions for conjugate setting. Finite damping prevails even by vanishing load $z = 0$ (because of the remaining resetting effect of the regulator); the damping increases with increasing resetting effect a (approach to the aperiodic case of conjugate setting). The damping D is considerable by high load z ; it becomes less as the displacement effect c increases. The damping can become superaperiodic if the value

$$c = \frac{(z + a)^2}{4 T_a^2}$$

is not reached.

A permanent disturbance of the state of equilibrium (for instance, by a change in preliminary pressure) signifies a change in the interference quantity ξ by $\Delta\xi$; hence by conjugate setting, the new state of equilibrium is formed only by a departure $\Delta\varphi$ from the theoretical state:

$$\Delta\varphi = \frac{\Delta\xi}{z + a}$$

It requires this permanent departure to produce a permanent adjustment of the control organ, which is necessary in order to balance the permanent disturbance of the equilibrium. It is obvious that this departure $\Delta\varphi$ will be so much less as the action of the regulator (its resetting effect) is greater. It is less also under high load z , because then the action of the control link on the regulating system is greater. By conjugate running speed or

- $\eta = a (\varphi + \xi) + c \int (\varphi + \xi) dt$, there is no permanent departure $\Delta\varphi$. The control link is adjusted until the

theoretical state has been reached again, since the control link comes to rest only in the theoretical state. A temporary departure occurs here also. Mathematically, this action manifests itself in the appearance of its derivative $\dot{\xi}$ rather than in the interference quantity itself.

A directional control of the regulator process - that is, an intentional change of the theoretical state - is achieved by a change in the directional control quantity ξ . If the latter is changed by an amount $\Delta\varphi$, the theoretical state, by conjugate setting, itself changes to the amount

$$\Delta\varphi = \frac{a}{z + a} \Delta\xi$$

This change is not proportionate - it is less by great load z , despite the equal change $\Delta\xi$ of the directional control quantity. By great load z , this smaller change $\Delta\varphi$ suffices to re-establish equilibrium, because of the greater inherent resetting ability of the regulating system under great load. By conjugate running speed, the change $\Delta\varphi$ of the theoretical state corresponds exactly to the adjustment $\Delta\xi$; for the control link stops running only when the new state corresponds exactly to that prescribed. The same holds for $-\eta = a(\varphi + \xi) +$

$c \int (\varphi + \xi) dt$. The instantaneous adjustment of the control link by $a\xi$ (caused by the conjugate setting portion in the governing principle) speeds up the attainment of the new state. After an abrupt change of the theoretical state by directional control, the new state is reached conformable to the swinging-in action of the regulator.

β) rpm Regulator

The combination of governing principle and regulating system here affords the following equations:

Governing Principle

Regulated System

$$-\eta = a(\varphi + \xi) \quad T_a \ddot{\varphi} + a\varphi = \dot{\xi} - a\xi \quad (12)$$

$$-\eta = a(\varphi + \xi) + c \int (\varphi + \xi) dt \quad T_a \ddot{\varphi} + a\ddot{\varphi} + c\dot{\varphi} = \dot{\xi} - c\xi - a\dot{\xi} \quad (13)$$

$$-\eta = c \int (\varphi + \xi) dt \quad T_a \ddot{\varphi} + c\varphi = \dot{\xi} - c\xi \quad (14)$$

The equations of the rpm regulator, (12) to (14) follow from the equations of the pressure regulator, (9) to (11), by vanishing load z . The action of the rpm regulator is as that of a pressure regulator by zero load.

The regulator with conjugate setting, $- \eta = a (\varphi + \xi)$, again indicates, after a disturbance, an aperiodic approach to the theoretical state conformable to an e-function. Its time constant is, according to equation (12):

$$\tau = \frac{T_a}{a}$$

and becomes infinitely great by vanishing resetting effect of the regulator (since the rpm regulating system has no inherent resetting ability like the pressure-regulating system).

The regulator with conjugate running speed, $- \eta = c \int (\varphi + \xi) dt$, shows undamped oscillations, the regulating system has no resetting effect of its own, and the action of the regulator is undamping owing to its lagging phase position. This control system is therefore impractical.

The natural frequency of the undamped oscillations follows from equation (14) at

$$\omega = \sqrt{\frac{c}{T_a}}$$

Being caused by the regulator, it increases as its effect increases and as T_a decreases (because of the then-lowered accumulator effect).

The regulator with $- \eta = a (\varphi + \xi) + c \int (\varphi + \xi) dt$ also reaches the theoretical state by a damped swinging-in process, equation (13), with the characteristic quantities:

$$D = \frac{1}{2} \frac{a}{\sqrt{cT_a}}$$

$$\omega_e = \sqrt{\frac{c}{T_a} - \frac{a^2}{4T_a^2}}$$

Again the action resembles that for conjugate setting. The damping increases with increasing resetting effect a , decreasing displacement effect c , and diminishing starting time T_a : the frequency increases as c and a and T_a decrease for the same reasons as on the pressure regulator.

By conjugate setting, a permanent disturbance of the equilibrium (by a change of the torque absorbed by the driven machine, for instance) indicates that the new state of equilibrium can only be reached by a departure $\Delta\varphi$ from the theoretical state

$$\Delta\varphi = \frac{\Delta\xi}{a}$$

Here also the departure is so much less as the resetting effect a of the regulator is greater. By conjugate running speed, $-\eta = c \int (\varphi + \xi) dt$, or $-\eta = a (\varphi + \xi) + c \int (\varphi + \xi) dt$, there is no lasting departure because the control link remains at rest only when the theoretical state prevails.

A directional control of the regulator process, achieved by a change of quantity ξ , modifies the theoretical state - by conjugate setting - to the amount of

$$\Delta\varphi = \Delta\xi$$

This change is proportionate because the rpm regulating system has no resetting ability of its own. By conjugate running speed and governing principle $-\eta = a (\varphi + \xi) + c \int (\varphi + \xi) dt$, the same occurs.

γ) Temperature Regulator

Here the corresponding equations are:

Governing Principle

Regulated System

$$-\eta = a(\varphi + \xi) \quad T_V T_a \ddot{\varphi} + (T_V + T_a) \dot{\varphi} + (1+a)\varphi = \xi + T_V \dot{\xi} - a\xi \quad (15)$$

$$-\eta = a(\varphi + \xi) + c \int (\varphi + \xi) dt \quad T_V T_a \ddot{\varphi} + (T_V + T_a) \dot{\varphi} + (1+a)\varphi + c\varphi = \xi + T_V \dot{\xi} - c\xi - a\xi \quad (16)$$

$$-\eta = c \int (\varphi + \xi) dt \quad T_V T_a \ddot{\varphi} + (T_V + T_a) \dot{\varphi} + \varphi + c\varphi = \xi + T_V \dot{\xi} - c\xi \quad (17)$$

The use of a regulator with conjugate setting, $\eta = a(\varphi + \xi)$, changes the inherently superaperiodic vibratory system into one susceptible to periodic vibrations, equation (15). The characteristic quantities of this oscillation are:*

$$D = \frac{T_a + T_v}{2} \frac{1}{\sqrt{T_v T_a (1 + a)}}$$

$$\omega_0 = \sqrt{\frac{1 + a}{T_v T_a} - \frac{(T_v + T_a)^2}{4 T_v^2 T_a^2}}$$

The vibration susceptibility of the system is formed as follows: On swinging through the theoretical state φ , the regulator adjusts the setting of the control link exactly correct for the theoretical state φ as state of equilibrium. But shortly before that, a departure, and hence a different control-link setting, existed. This other setting had effected an energy input at that moment which did not correspond to the equilibrium state. However, owing to the accumulator masses (equation (5b)) still existent on the path of energy flow, this energy is belated and so forces the system beyond the theoretical state again. An increasing resetting effect amplifies this action, which is evidenced in rising natural frequency ω_0 and reduced damping D . The latter, however, always remains positive. If one of the time constants is small, the damping will be great (the system approaches that of the aperiodic pressure regulator); should one of the time constants be great, the damping will be great also (because the damping reducing decelerating effect of the other time constants is small also). If both time constants are of the same order of magnitude the damping will be small, being a minimum for

$$T_a = T_v \quad \text{of} \quad D = 1/\sqrt{1 + a}$$

On the regulator with conjugate running speed,

*As in the preceding footnote, the comparison between equation (15) and the basic equation of the damped oscillation, gives:

$$2D\omega_0 = \frac{T_v + T_a}{T_v T_a}$$

and

$$\omega_0^2 = \frac{1 + a}{T_v T_a}$$

$-\eta = c \int (\varphi + \xi) dt$; on the other hand, the system is equally susceptible to increasing oscillations, equation (17). This ties in with the fact that a system susceptible to vibrations receives energy through an exciting force with a phase lag of $\pi/2$ behind the deflection. Here the regulating system is susceptible to superperiodic vibrations (equation (3)); the exciting force with $\pi/2$ lag is given through the control-link action which, for the governing principle, $-\eta = c \int (\varphi + \xi) dt$ lags in phase by $\pi/2$ behind the state φ of the system. There is therefore a limit for the magnitude of displacement effect c , at the transgression of which the regulated system executes increasing oscillations. This limit follows from equation (17) at:*

$$c = \frac{1}{T_V} + \frac{1}{T_A}$$

The oscillation frequency ω existing at this limit is:

$$\omega = \sqrt{\frac{c}{T_A + T_V}} = \frac{1}{\sqrt{T_V T_A}}$$

It is seen therefrom that the displacement effect c de-

*With the formula $\varphi = \varphi_0 \sin \omega t$ of an undamped oscillation, equation (17) gives: $-\omega^3 T_V T_A \varphi_0 \cos \omega t - (T_V + T_A) \omega^2 \varphi_0 \sin \omega t + \omega \varphi_0 \cos \omega t + c \varphi_0 \sin \omega t = \xi + T_V \dot{\xi} - c \xi$; hence transformed:

$$(-\omega^3 T_V T_A + \omega) \cos \omega t + (-(T_V + T_A) \omega^2 + c) \sin \omega t = \frac{\ddot{\xi} + T_V \dot{\xi} - c \xi}{\varphi_0}$$

This equation must be applicable for every moment, which can be the case only if both parentheses - each for itself - becomes zero. Then,

$$-\omega^3 T_V T_A + \omega = 0$$

$$-(T_V + T_A) \omega^2 + c = 0$$

hence

$$\omega^2 = \frac{1}{T_A T_V} = \frac{c}{T_V + T_A}$$

and

$$c = \frac{T_V + T_A}{T_V T_A} = \frac{1}{T_A} + \frac{1}{T_V}$$

creases with increasing starting time T_a , as well as with increasing deceleration time T_v . The increase of these two time constants implies a growth of the accumulator masses, hence a lowering of frequency ω , and conse-

quently, an increase of the undamping $c \int \varphi dt$, each semi-oscillation. But even by infinite time constants, the displacement effect need not fall short of the value $1/T_a$ or $1/T_v$ in order to assure damping, because the inherent resetting ability of the regulating system supports a damped state.

The use of principle $-\eta = a (\varphi + \xi) + c \int (\varphi + \xi) dt$,

also produces here the best conditions. Admittedly, the possibility of increasing oscillations remains, but then the damping limit lies at:

$$c = \left(\frac{1}{T_a} + \frac{1}{T_v} \right) (1 + a)$$

$$\omega = \sqrt{\frac{1 + a}{T_v T_a}} = \sqrt{\frac{c}{T_a + T_v}}$$

The damping limit is postponed by the action of the resetting effect (because of higher frequencies and hence, a proportionally reduced action of term

$$c \int (\varphi + \xi) dt$$

A permanent disturbance of the equilibrium (for instance, opened oven door) by conjugate setting indicates a new state of equilibrium that departs from the theoretical state. This departure is:

$$\Delta\varphi = \frac{\Delta\xi}{1 + a}$$

for it requires the existence of just such a departure $\Delta\varphi$ to assure the setting of the control link needed for the new state of equilibrium. Here also this departure is so much less as the control action (its resetting effect a) is greater. By conjugate running speed or $-\eta = a (\varphi + \xi)$

$+ c \int (\varphi + \xi) dt$, a temporary departure occurs, as on the pressure and rpm control.

A directional control of the regulator process secured by a change of quantity ξ , alters - by conjugate setting - the equilibrium condition by

$$\Delta\varphi = \frac{a}{1+a} \Delta\xi$$

This change is not proportionate, since the regulating system has an inherent resetting ability which the directional control must overcome also. On the running speed regulator or $-\eta = a(\varphi + \xi) + c \int (\varphi + \xi) dt$, the change

$\Delta\varphi$ of the theoretical state is proportional to the change of the control quantity, because the natural resetting ability of the regulating system is neutralized by the action of the displacement effect c .

8) Automatic Directional Control

The relative equations are as follows:

Governing Principle

Regulated System

$$-\eta = a(\varphi + \xi) \quad \ddot{\varphi} + M\dot{\varphi} + aN\varphi = \xi - aN\xi \quad (18)$$

$$-\eta = a(\varphi + \xi) + c \int (\varphi + \xi) dt \quad \ddot{\varphi} + M\dot{\varphi} + aN\dot{\varphi} + cN\varphi = \dot{\xi} - cN\xi - aN\dot{\xi} \quad (19)$$

$$-\eta = a \int (\varphi + \xi) dt \quad \ddot{\varphi} + M\dot{\varphi} + cN\varphi = \dot{\xi} - cN\varphi \quad (20)$$

The regulator with conjugate setting, $-\eta = a(\varphi + \xi)$, changes the vehicle to a system susceptible to vibration. A mechanical oscillation circuit is formed between the craft's inertia moment and the resetting action of the regulator. The damping follows as damping of the vehicle in its surrounding medium; it is always positive. The characteristic quantities of the natural oscillation of the system follow from equation (18) at:

$$D = \frac{1}{2} \frac{M}{\sqrt{aN}}$$

$$\omega_e = \sqrt{aN - \frac{1}{4} M^2}$$

The natural frequency rises with increasing resetting effect a , and increasing rudder effectiveness N ; the

damping decreases, as on every oscillation circuit.

The use of a regulator with conjugate running speed is out of order here, since the system executes only increasing oscillations, as seen from equation (20). With

$$\varphi = e^{Kt} \sin \omega_e t$$

follows

$$\begin{aligned} & (K^3 + M K^2 - 3 \omega_e^2 K - M \omega_e^2 + c N) e^{Kt} \sin \omega_e t \\ & + (3 \omega_e K^2 + 2 M \omega_e K - \omega_e^3) e^{Kt} \cos \omega_e t \\ & = \dot{\xi} - c N \xi = \text{const} \end{aligned}$$

Since this equation must be satisfied at every instant, the factors before the terms with sin and cos must each be zero, which gives the equations:

$$K^3 + M K^2 - 3 \omega_e^2 K - M \omega_e^2 + c N = 0$$

$$3 \omega_e K^2 + 2 M \omega_e K - \omega_e^3 = 0$$

The relation (see footnote**, p. 14):

$$D = \frac{K}{\omega_e} \frac{1}{\sqrt{1 + K^2/\omega_e^2}}$$

holds by damping as by increasing oscillations; hence, after minor changes, the formula for the increase factor (-D) and the displacement effect c of the regulator reads:

$$\frac{cN}{4M^3} = \frac{16}{\left(\frac{1}{D^2} - 4\right)^3} + \frac{8}{\left(\frac{1}{D^2} - 4\right)^2} + \frac{1}{\left(\frac{1}{D^2} - 4\right)}$$

This equation indicates that the increase continues by increasing displacement effect c, and that it becomes greater as the damping capacity M of the vehicle is less. However, this increase does not exceed the factor D = 0.5, reached by the vehicle without damping.

Even for $\ddot{\eta} = a(\varphi + \xi) + c \int (\varphi + \xi) dt$, the possi-

bility of increasing oscillations exists by considerable application of displacement effect c . The damping limit follows from equation (19) at:*

$$c = a M$$

$$\omega = \sqrt{a N}$$

The displacement effect c can be so much greater as the resetting effect a is greater (because at the then-existing higher natural frequency, smaller undamping displacement quantities $c \int \varphi dt$ prevail).

Preservation of equilibrium by an external permanent disturbing moment $\Delta \xi$ (for instance, through failure of a lateral power plant in a ship) - by conjugate setting - postulates a departure $\Delta \varphi$ from the theoretical state

$$\Delta \varphi = \frac{\Delta \xi}{a}$$

This departure is, of course, so much less as the regulator is more effective (as its resetting effect a is greater).

By $-\ddot{\eta} = a(\varphi + \xi) + c \int (\varphi + \xi) dt$, such departure is eliminated in the course of time as action of c .

A directional control of the regulator process by way of control link ξ induces a proportionate change $\Delta \varphi$ of the theoretical state by conjugate setting:

$$\Delta \varphi = \Delta \xi$$

inasmuch as the regulating system here has no resetting ability of its own, like the speed regulator, hence has no preferred status φ . The same holds true by conjugate running speed.

*Equation (19), treated exactly as (17) in footnote, p. 24, gives:

$$-\omega^3 + \omega a N = 0$$

$$-\omega^2 M + c N = 0$$

hence,

$$\omega^2 = a N = \frac{cN}{M}$$

and

$$c = a M$$

The problem of automatic directional control has still another aspect: the effect of the control link - here, of the rudder - is not constant but dependent upon the dynamic pressure. In consequence, natural frequency, damping, and damping limit become dependent on the speed of travel, and on airplanes; and in addition, on the flying height (reference 4). The natural frequency rises with increasing speed, the damping decreases, and its limit is shifted to greater values of displacement effect c . In airplane directional control the damping limit computed here is not the decisive one. Path oscillations of the center of gravity are apt to form by existing displacement effect c through the lateral motions of the airplane center of gravity, the damping limit of which is reached sooner than the limit of the rotary oscillations about the airplane c.g. in question.

A comparison of the results obtained on the different regulating systems discloses an increasing difficulty of the regulating problem from pressure control to rpm, and temperature control to automatic directional control. The results are appended in table II. On the regulator with conjugate setting, both the pressure control and the rpm control show aperiodic processes; temperature regulator and automatic directional control disclose damped oscillations.

By $-\ddot{\eta} = a(\varphi + \xi) + c \int (\varphi + \xi) dt$, the processes on the pressure and rpm regulator already follow damped oscillations; temperature regulator and automatic control show potential increasing oscillations. The control with conjugate running speed, lastly, shows damped oscillations only as pressure regulator; the rpm regulator already executes undamped oscillations, the temperature regulator manifests potential increasing oscillations, and the oscillations on the automatic directional control are continuously increasing. Hence, the justification of the enumeration in the order chosen.

The rpm regulator and automatic directional control stand out as systems without inherent resetting ability; that is, without preferred status φ . By directional control to another state they follow willingly and come to rest in the purposed new state, even on the regulator with conjugate setting. Pressure regulator and temperature regulator dispose of their own resetting ability; after a directional control they do not come to rest on the desired new state

by conjugate setting. They require a departure $\Delta\varphi$, to equalize their different internal resetting force in the new state. It requires the action of a displacement effect c , which produces such an equalization after a certain time, automatically, to make them seek the desired position. The inherent resetting ability of pressure and temperature regulating systems is also apparent in the plots of table II; both also show by vanishing resetting effect a of the regulator, finite values of time constants K , damping D , or damping limit; while on the rpm regulator and the automatic directional control, the corresponding values are, respectively, zero and infinite.

The regulator with $-\eta = a(\varphi + \xi) + c \int (\varphi + \xi) dt$,

shows up to the best advantage, according to table II, since a very energetic application of the regulator on the regulating system is desired in order to achieve a minimum of departure from the theoretical state. This is impossible on the running-speed regulator because of its damping limit. In fact, only the pressure regulator manifests here consistently, damped behavior in this respect, and even there the damping declines with increasing control action. By

$-\eta = a(\varphi + \xi) + c \int (\varphi + \xi) dt$, on the other hand, damp-

ing can always be maintained by adequate resetting effect a ; the displacement effect can be chosen smaller than for the running-speed regulator, since now it is no longer the sole and true regulating effect, its sole purpose being rather to remove remaining departures. The damping values prevalent by $-\eta = a(\varphi + \xi)$, naturally cannot be exceeded.

However, in order to achieve still greater damping values, as is desirable for specific control problems, the regulator process is given an effect corresponding to the time derivative $\dot{\varphi}$ of the state. Then the regulator opposes a change of rate of the state. This rate of change $\dot{\varphi}$ can be measured direct (for instance, as rate of rotation of the craft by means of fettered gyroscopes on the automatic control, as rotary acceleration of the shaft by the inertia governor on the rpm control), or derived by instrumentative differentiation from the measured state (for instance, by methods which measure the difference between the state φ and a sluggish subsequent comparative state). If this "damping effect b " is combined with a setting regulator, it follows the principle:

$$- \ddot{\eta} = a (\varphi + \xi) + b \dot{\varphi} \quad (8a)$$

for which the modified formulas are as follows:

Pressure control:

$$(T_a + b) \dot{\varphi} + (z + a) \varphi = \xi - a \xi$$

$$K = \frac{T_a + b}{z + a}$$

rpm control:

$$(T_a + b) \dot{\varphi} + a \varphi = \xi - a \xi$$

$$K = \frac{T_a + b}{a}$$

Temperature control:

$$T_v T_a \ddot{\varphi} + (T_v + T_a + b) \dot{\varphi} + (1+a) \varphi = \xi + T_v \dot{\xi} - a \xi$$

$$D = \frac{T_v + T_a + b}{2} \frac{1}{\sqrt{T_v T_a (1+a)}}$$

$$\omega_e = \sqrt{\frac{1+a}{T_v T_a} - \frac{(T_v + T_a + b)^2}{4 T_v^2 T_a^2}}$$

Automatic directional control:

$$\ddot{\varphi} + (M + bN) \dot{\varphi} + aN \varphi = \xi - aN \xi$$

$$D = \frac{M + bN}{2 \sqrt{aN}}$$

$$\omega_e = \sqrt{aN - \frac{(M + bN)^2}{4}}$$

There is evident in all cases a rise in damping D and time constant as a result of the damping effect b of the regulator.

To better visualize the different control systems, the use of mechanical substitutes of the regulator processes is helpful. The linear, homogeneous, differential equations of the 1st, 2d, and 3d orders, given in the description of the regulating processes, appear in the same manner in simple mechanical vibration patterns; hence, manifest the same behavior as the regulating systems while being at the same time easier to grasp, table III.

The state ϕ of a regulating system is represented by the position (emphasized in table III) of a linearly moving reference point; unless otherwise stated, the systems illustrated in table III are frictionless, weightless, and massless. The analogy is most easily apparent on the automatic directional control; the mechanical oscillation circuit is simply transferred. The inertia moment of the vehicle is represented by a linearly moving mass, the vehicle damping by a brake cylinder. This system, like the vehicle, discloses no resetting ability of its own, while the temperature-regulating system has one such and in addition, a vibration susceptible attitude (equation. (3)), which appears in the substitute picture as a mechanical oscillation circuit with mass, spring, and damping. The rpm regulating system, which only opposes a change of its state, appears as a simple brake cylinder. The pressure-regulating system, on the other hand, with resetting forces of its own, manifests also a reset spring. Since the inherent resetting forces of this system grow with increasing load z , this spring should be stiffer if greater loads are involved. Following a disturbance of its state of equilibrium, such as giving it a push by hand, they act as described in table I.

The effects of mounting a regulator with conjugate setting on the regulating system, are proportional to the departure ϕ from the theoretical state. Such effects can, however, in the substitute patterns, be exerted through a spring which is exactly untense in the theoretical state. Then it is seen how the automatically steered vehicle has now become susceptible to vibrations, how the natural frequency of the temperature regulator has now increased and its damping decreased. It is readily apparent how the rpm controlled system receives a resetting ability over the regulator, to the theoretical position, how the natural resetting ability on the pressure-regulated system is increased by the regulator, and so on. It is also quite plain that a permanent outside disturbance can be equalized only by a departure from the theoret-

ical state, because then only a corresponding spring tension appears. The damping action of a supplementary damping influence b on the regulator process is represented by another brake cylinder. The action of a displacement effect c is represented by a device which a flowing medium integrates. To illustrate: Sand pours out of a pipe on a scale which, on departure from the theoretical state, is asymmetrically loaded (the medium being, of course, subject to gravity). It takes the action of a constant interference force on the reference point to force it out of its theoretical position, but the scale accumulates asymmetric weight thereby until equilibrium is restored. The substitute system therefore acts exactly like the actual regulated system. Even the necessity for representing the displacement effect by an auxiliary force corresponds to the actual conditions, for without energy input no undamping action can develop. The conditions by directional control also are correctly and plainly represented. By conjugate setting the directional control is accomplished by shifting of the hinge point of the spring representing the regulator resetting effect; on pressure and temperature regulators the spring - not adjusted with it, and representing the natural resetting ability of the system - acts against such an adjustment. This condition does not exist on the rpm regulator or on the automatic directional control; it is absent also in the presence of displacement effect, since by directional control the adjustment of the position of the integration device itself effects a directional control.

b) Actual Regulator

The actual regulator, contrary to the faultproof regulator, is subject to faults which appear as departures from the governing principle in consequence of manufacturing defects and type characteristics. These departures or "errors," as they are termed hereafter, fall into the following groups:

The sluggish regulator: Its control link does not assume the faultproof setting η , but a setting η_f^* , linked to the theoretical setting η , through

$$\dot{\eta}_f + \eta_f = \eta \quad (21)$$

* In equations (1) to (4) of the regulating system, η_f then, of course, substitutes for η .

The control moves sluggishly toward its theoretical setting η , the rate of lag depending upon the difference between what the control-link setting is and what it should be; the deciding factor δ is termed the "inertia δ " of the regulator. After intentional removal of the control link out of its theoretical setting by arrested regulating system (impossible to achieve on the faultproof regulator), the control link reaches its theoretical setting, according to an e-function. Such behavior, for instance, systematically occurs on the amplifier with conjugate setting by return motion; it also occurs on the amplifier with conjugate running speed, encumbered with masses.

The regulator susceptible to vibration follows the equation:

$$\ddot{\eta}_f + 2 D_R \omega_R \dot{\eta}_f + \omega_R^2 \eta_f = \omega_R^2 \eta \quad (22)$$

If this regulator is considered by itself, a removal of the control link from its theoretical setting returns it again after a damped oscillation. The characteristic quantities of this oscillation are ω_R and D_R . Equation (20) represents simply the known form of the oscillation equation; the factor ω_R before η must be present in order that $\eta_f = \eta$ in the swing-in state. This attitude is particularly manifested by almost all detectors of rpm regulators and, to a lesser extent, by those of pressure regulators. This principle (22) finds considerable use on transmission links of indirect regulating systems, and especially on amplifiers operating with elastic auxiliary forces (compressed air, for instance).

The control with friction requires first a certain minimum impulse before it responds. Its behavior is not immediately amenable to practical mathematical treatment. Regarding the action of an adjustment of effects a , b , and of the regulator on the regulator process, the friction on the detector side of the adjustment point of these influences makes itself felt differently than the friction on the control-link side. The study of the friction is therefore divided into: friction in the detector system and friction in the control link. They differ to the extent that the effect of friction of the detector system on the control link changes along with a change in regulator adjustment while the friction in the control link remains constant. Inasmuch as it is impossible to produce bearing points without friction, every regulating system disposes of a certain measure of friction.

The effect of these errors on the various regulating systems is now analyzed on the basis of the following restrictions: There is no outside disturbance f of the equilibrium; no directional control ξ of the process is required. It is further assumed that at the moment only one type of error occurs, which brings out its effect unrestrictedly. For the rest, the characteristic effects of the individual groups of errors overlap fairly undisturbedly.

The running-speed regulator is no longer discussed as rpm regulator and automatic directional control, after having proved its impracticability on the faultproof regulator. The results are illustrated in tables. The discussion of the results is less with respect to the effect of the constants T_a , T_v , M , N , z of the regulating system - whose effect on the regulator process was essentially disclosed by the faultproof regulator - than with a view to the modification of this behavior by the errors of the actual regulator. For identification of the governing principle, the faultproof principle is employed; in the study of tables IV to VII, for instance, it is to be noted that the faultproof principle, as a result of the error of the actual regulator (equations (21), (22), (39)), is now, naturally, no longer the real one, but merely the desired correlation between phase quantities and rudder setting.

a) The Sluggish Regulator

The sluggish regulator shows a lag in the control link relative to its theoretical setting. Thus it undamps systems susceptible to vibration and makes aperiodic, systems susceptible to vibration (because when the system passes through neutral the control link - because of inertia ϕ - still has a deflection that drives the system to the other side beyond the neutral position). During an oscillation, the sluggish regulator evinces a lag in phase shift α , and a reduction in amplitude for the control-link motion. With

$$\eta = \eta_0 \sin \omega t$$

$$\eta_f = \eta_{f0} \sin (\omega t - \alpha)$$

equation (21) gives the phase displacement α at

$$\cot \alpha = \frac{1}{\delta \omega}$$

and the reduction in control-link amplitude at

$$\eta_{f_0} = \eta_0 \cos \alpha$$

From this it is apparent that with rising frequency ω , and inertia δ , the phase angle continues to increase and finally reaches the value $\pi/2$. The amplitude η_f continues to decrease at the same time and reaches zero when the phase angle has reached the value $\pi/2$.

The behavior of the sluggish regulator is appended in table IV.

Pressure regulator: The combined equations (1) and (21) and principles (5) to (7) yield the equations:

Governing Principle

Regulated System

$$-\eta = a\varphi \quad \delta T_a \ddot{\varphi} + (\delta z + T_a) \dot{\varphi} + (z + a)\varphi = 0 \quad (23)$$

$$-\eta = a\varphi + c \int \varphi dt \quad \delta T_a \ddot{\varphi} + (\delta z + T_a) \dot{\varphi} + (z + a)\varphi + c\varphi = 0 \quad (24)$$

$$-\eta = c \int \varphi dt \quad \delta T_a \ddot{\varphi} + (\delta z + T_a) \dot{\varphi} + z\dot{\varphi} + c\varphi = 0 \quad (25)$$

On a regulator with conjugate setting, $-\eta = a\varphi$, the pressure regulated system is now susceptible to vibrations (equation (23)). The characteristic quantities of this vibration are:*

*

Equation (23), transformed, gives:

$$\ddot{\varphi} + \frac{\delta z + T_a}{\delta T_a} \dot{\varphi} + \frac{z + a}{\delta T_a} \varphi = 0$$

whence

$$2D \omega_0 = \frac{\delta z + T_a}{\delta T_a}$$

and

$$\omega_0^2 = \frac{z + a}{\delta T_a}$$

$$D = \frac{1}{2} \frac{x\delta + T_a}{\sqrt{(x+a)\delta T_a}}$$

$$\omega_e = \sqrt{\frac{x+a}{\delta T_a} - \frac{(x\delta + T_a)^2}{4\delta^2 T_a^2}}$$

At first the damping decreases as the inertia δ increases (as a result of the undamping effect of the lagging phase of the "sluggish" control link), then increases (since, by further increasing δ the amplitude of the control-link motion decreases consistently and so lowers the undamping effect again despite the great phase displacement). Under high load the damping is great, since then the natural (not phase-shifted) resetting ability of the system is great.

On the regulator with conjugate running speed, $-\eta = c \int \varphi dt$, the system previously susceptible to vibrations, now displays increasing possibilities. The damping limit follows from equation (25) at*

$$c = \frac{x}{\delta} + \frac{x^2}{T_a}$$

$$\omega = \sqrt{\frac{x}{\delta T_a}} = \sqrt{\frac{c}{\delta x + T_a}}$$

With increasing regulator inertia δ , its action (c) on the regulating system must be continuously reduced, in order to avoid increasing oscillations. However, a finite amount x^2/T_a of displacement effect c can be left, even by infinitely great inertia because, while at very great inertia the phase difference is close to $\pi/2$, the deflection of the control link itself has abated considerably. Here also this limit lies by great load at greater control effects; by vanishing load x , damping is wholly impossible.

* With $\varphi = \varphi_0 \sin \omega t$ from equation (25), the equations are:

$$-\omega^3 \delta T_a + \omega x = 0$$

$$-\omega^2 (\delta x + T_a) + c = 0$$

Hence the values given in the text.

For $\eta = a \varphi + c \int \varphi dt$, there appears a mixture of the above phenomena. The regulated system can execute increasing oscillations; the damping limit lies, according to (24), at:

$$\delta = \frac{(z + a) T_a}{c T_a - z^2 - a z}$$

$$\omega = \sqrt{\frac{z + a}{\delta T_a}} = \sqrt{\frac{c}{\delta z + T_a}}$$

By increasing inertia δ , either the displacement effect c , must be lowered again or the resetting effect a increased (because its effect approaches the action of the system with pure setting control, $\eta = a \varphi$, where continuous damping prevails). Since the pressure-regulating system disposes of its own resetting ability, a certain amount of inertia δ is permissible even by vanishing resetting effect a . But when the natural resetting ability becomes zero by vanishing load z , a certain resetting effect a of the regulator, must be available. If load z exceeds the value,

$$z = -\frac{a}{2} + \sqrt{\frac{a^2}{4} + c T_a}$$

which is equivalent to a resetting effect a greater than

$$a = \frac{c T_a}{z} - z$$

then damping is always present, even by infinitely great inertia δ . Then the undamping effect of inertia δ , which becomes smaller again as δ increases, is no longer sufficient with respect to the (not phase-changed) natural resetting force of the regulating system to produce increasing oscillations.

IDE regulator: The pertinent equations are:

Governing Principle

Regulated System

$$\eta = a \varphi \quad \delta T_a \ddot{\varphi} + T_a \dot{\varphi} + a \varphi = 0 \quad (26)$$

$$\eta = a \varphi + c \int \varphi dt \quad \delta T_a \ddot{\varphi} + T_a \dot{\varphi} + a \varphi + c \varphi = 0 \quad (27)$$

$$\eta = c \int \varphi dt \quad \delta T_a \ddot{\varphi} + T_a \dot{\varphi} + c \varphi = 0 \quad (28)$$

The sluggish regulator with conjugate setting makes the erstwhile aperiodic system susceptible to vibrations by its own inertia (equation (26)). The characteristic quantities are:

$$D = \frac{1}{2} \sqrt{\frac{T_a}{a\delta}}$$

$$\omega_e = \sqrt{\frac{a}{\delta T_a} - \frac{1}{4\delta^2}}$$

As δ increases, the damping of the regulator process increases and does not rise again as on the pressure regulator. It lacks the inherent resetting ability of the regulating system; the regulator resetting effect cannot replace it since it is phase-shifted, and so appears undamping. Greater resetting effect therefore actually reduces the damping. By increasing starting time T_a , the damping increases, as on the pressure regulator, for by great T_a the control processes are slowed down, hence the inertia effect is less.

The sluggish rpm regulator induces the previously undamped oscillating system through its phase difference of the control link (equation (28)) to continuously increasing oscillations (also treatment of (20)). The sluggish regulator with conjugate setting makes the system, already susceptible to vibrations, amenable to increased vibrations (equation (29)), with a damping limit at:*

$$a = \frac{M^2}{N} + \frac{M}{\delta N}$$

$$\omega = \sqrt{\frac{a N}{\delta M + 1}} = \sqrt{\frac{M}{\delta}}$$

By increasing inertia δ of the regulator, its effect (a) must, on account of the undamping action, be continuously reduced, in order to avoid increasing vibrations. However,

*With $\varphi = \varphi_0 \sin \omega t$, equation (29) gives:

$$- \omega^3 \delta + \omega M = 0$$

$$- \omega^2 (\delta M + 1) + a N = 0$$

giving the relations in the text.

the value need not fall below $a = M^2/N$, even by the greatest δ , since the effect of the control itself diminishes as a result of the lowered control-link amplitude perceptible at great δ . By great damping capacity M of the vehicle (or by great damping effect b of the control), the resetting effect can again be chosen greater.

With $-\eta = a\phi + c \int \phi dt$, the previously susceptible system has become capable of increasing vibrations. The damping limit follows from equation (27) at:

$$a = c \delta$$

$$\omega = \sqrt{\frac{c}{T_a}} = \sqrt{\frac{a}{\delta T_a}}$$

As on the pressure regulator, the displacement effect c must be reduced, or the resetting effect a increased when δ increases. Here the damping limit is unaffected by the regulating system (quantity T_a is nonexistent). The reason for this is that at short starting periods T_a , high frequencies ω , and hence large undamping phase angles occur on the control link because of the inertia δ ; by long starting periods T_a , the frequencies ω are low, but the undamping phase angles are large as a result of the then-great displacement effect c .

Temperature regulator.— No analysis is needed. Its processes take place in time intervals for which the time lag, even by awkward control arrangement, plays no part because of inertia δ .

Automatic directional control.— The pertinent equations are:

Governing Principle

Regulated System

$$-\eta = a\phi \quad \delta \ddot{\phi} + (\delta M + 1)\ddot{\phi} + M\dot{\phi} + aN\phi = 0 \quad (29)$$

$$-\eta = a\phi + c \int \phi dt \quad \delta \phi^{(IV)} + (\delta M + 1)\ddot{\phi} + M\dot{\phi} + aN\phi + cN\phi = 0 \quad (30)$$

$$-\eta = c \int \phi dt \quad \delta \phi^{(IV)} + (\delta M + 1)\ddot{\phi} + M\dot{\phi} + cN\phi = 0 \quad (31)$$

By conjugate running speed, previously increasing oscillations are now naturally much amplified.

With $-\eta = a\phi + c \int \phi dt$, the possibility of increasing oscillations remains. The damping limit follows from equation (30) at:

$$a = \left(M + \frac{1}{\phi} \right) \left[\frac{M}{2N} \pm \sqrt{\frac{1}{4} \left(\frac{M}{N} \right)^2 - \frac{c\phi}{N}} \right]$$

This implies a certain zone for the resetting effect a , which, if exceeded or fallen short of, shifts the control process into the zone of increasing vibrations. The resetting effect a may not exceed a certain limit; otherwise, its undamping, phase-shifted effect on the control link brings the system to increasing oscillations. But another limit must not be fallen short of, since by too little resetting effect a , the undamping action of the displacement effect c , predominates. Hence, with increasing c , as with increasing ϕ , the useful range of a is curtailed.

The behavior of the sluggish regulator in table IV again shows the correctness of previously chosen sequence: From pressure control to automatic directional control, and from setting control to running-speed control, the regulator process becomes consistently more unfavorable.

β) The Regulator Susceptible to Vibration

The natural oscillations of the regulator are imparted to the regulated system. This reacts back by way of the detector on the regulator, through which frequency and damping alone change with respect to the values of the regulator. Since everyone of the regulating systems requires a certain time to become fully effective (equations (1) to (4), and table I), the damping on the regulated system is impaired or even annulled. If the regulated system by faultproof regulator was already susceptible to vibrations, this vibration form is retained, and a second form of vibration arising from the natural vibration of the regulator is superimposed. At very high natural damping D_R of the regulator, its vibration attitude is almost completely suppressed; it changes to the sluggish regulator. There usually is a certain damping D_R zone which may not be exceeded nor fallen short of. In the latter case, the energy withdrawal through insufficient D_R is too little; if exceeded, the undamping through the great phase difference is excessive. Since the phase difference grows as

the displacement effect c increases, the zone shrinks; starting from a certain displacement effect, the damped zone disappears completely, leaving only increasing motions. The transition to infinitely high natural regulator frequency ω_R leads to the faultproof regulator, since by infinitely high natural frequency ω_R , a "true" representation of the orders follows. Table V shows the control effect (a and c) plotted against damping D_R for different natural frequencies ω_R ; the possibility of increasing oscillations exists in every case, the graphs therefore represent a damping limit.

Pressure regulator.— The relative equations are:

Governing Principle

Regulated System

$$- \eta = a \varphi \quad T_a \ddot{\varphi} + (z + 2D_R \omega_R T_a) \dot{\varphi} + (2D_R \omega_R z + \omega_R^2 T_a) \varphi + \omega_R^2 (z+a) \varphi = 0 \quad (32)$$

$$- \eta = a\varphi + c \int \varphi dt \quad T_a \varphi^{(IV)} + (z + 2D_R \omega_R T_a) \ddot{\varphi} + (2D_R \omega_R z + \omega_R^2 T_a) \dot{\varphi} + \omega_R^2 (z+a) \dot{\varphi} + \omega_R^2 c \varphi = 0 \quad (33)$$

$$- \eta = c \int \varphi dt \quad T_a \varphi^{(IV)} + (z + 2D_R \omega_R T_a) \ddot{\varphi} + (2D_R \omega_R z + \omega_R^2 T_a) \dot{\varphi} + \omega_R^2 z \dot{\varphi} + \omega_R^2 c \varphi = 0 \quad (34)$$

The damping limit for the regulator with conjugate settings follows from equation (32) at:*

*With $\varphi = \varphi_0 \sin \omega t$, equation (32) gives:

$$- \omega^3 T_a + \omega (2 D_R \omega_R z + \omega_R^2 T_a) = 0$$

$$- \omega^2 (z + 2 D_R \omega_R T_a) + \omega_R^2 (z + a) = 0$$

hence,

$$\omega^2 = \frac{2 D_R \omega_R z + \omega_R^2 T_a}{T_a} = \frac{z + a}{z + 2 D_R \omega_R T_a} \omega_R^2$$

A second transformation gives:

$$z + a = \frac{2 D_R \omega_R z + \omega_R^2 T_a}{T_a \omega_R^2} (z + 2 D_R \omega_R T_a)$$

and

(Continued on p. 43)

$$a = 2 D_R \left(\frac{z^2}{T_a \omega_R} + 2 D_R z + T_a \omega_R \right)$$

$$\omega = \sqrt{\frac{\omega_R^2 (z+a)}{z + 2 D_R \omega_R T_a}} = \sqrt{\omega_R^2 + \frac{2 D_R \omega_R z}{T_a}}$$

From this it is apparent that with increasing D_R , the action of the regulator on the regulating system (its resetting effect a) can also be increased (since by great D_R , much energy is withdrawn from the back-and-forth swinging regulator). Likewise, the resetting effect a can be chosen large by high natural frequency ω_R of the vibration-susceptible regulator (because then the regulator follows its orders quickly and readily). But the same is possible also by very low ω_R ; then the regulator swings so slowly that the regulating system has time to follow its oscillations fairly true and without appreciably undamping them. Because of the then abnormally slow regulator response, the latter setting is, of course, practically useless. Under smaller load z , the resetting effect a must be chosen smaller (since the system - because of the then-lower resetting force - follows the faulty one of the regulator now more readily). The same influence as the natural frequency ω_R is shown by the starting time T_a , and for the very same reasons.

For the running-speed regulator, equation (34) gives the damping limit at:*

(Continued from p. 42)

$$z + a = \left(2 \frac{D_R z}{T_a \omega_R} + 1 \right) (z + 2 D_R \omega_R T_a)$$

whence

$$z + a = 2 \frac{D_R z^2}{T_a \omega_R} + z + 4 D_R^2 z + 2 D_R \omega_R T_a$$

and finally, the value shown in the text.

* Assuming an undamped oscillation as basis, equation (34) gives:

$$\begin{aligned} \omega^4 T_a - \omega^2 (2 D_R \omega_R z + \omega_R^2 T_a) + \omega_R^2 c &= 0 \\ - \omega^3 (z + 2 D_R \omega_R T_a) + \omega \omega_R^2 z &= 0 \end{aligned}$$

(Continued on p. 44)

$$c = \frac{\omega_R^2 z}{z + 2 D_R \omega_R T_a} \left[\frac{2 D_R z}{\omega_R} + T_a - \frac{T_a z}{z + 2 D_R \omega_R T_a} \right]$$

$$\omega = \sqrt{\frac{\omega_R^2 z}{z + 2 D_R \omega_R T_a}}$$

$$= \sqrt{\frac{2 D_R \omega_R z + \omega_R^2 T_a}{2 T_a}} = \sqrt{\frac{(2 D_R \omega_R z + \omega_R^2 T_a)^2}{4 T_a^2} - \frac{c \omega_R^2}{T_a}}$$

It is seen that the possible displacement effect c increases at first with increasing damping (because of the increased energy extraction), but then drops again to the value z^2/T_a of the sluggish regulator (since ultimately the undamping action of the consistently increasing phase difference lag of the control link outweighs the energy removal). Therefore, a given displacement effect c , if damping is at all possible, results in a certain damping zone D_R , which may not be abandoned. This zone increases with the natural frequency ω_R ; it is also more favorable by great load. This is associated with the appearance of two natural oscillation forms - one due to the regulator, the other to the system.

For $\ddot{\eta} = a \varphi + c \int \varphi dt$, a damping limit follows at:*

(Continued from p. 43)

From the first, c follows at:

$$c = -\omega^4 \frac{T_a}{\omega_R^2} + \omega^2 \left(\frac{2 D_R z}{\omega_R} + T_a \right)$$

from the second ω^2 follows at:

$$\omega^2 = \frac{\omega_R^2}{z + 2 D_R \omega_R T_a}$$

which, inserted for ω^2 , gives:

$$c = \frac{\omega_R^2 z}{z + 2 D_R \omega_R T_a} \left(\frac{2 D_R z}{\omega_R} + T_a \right) - \frac{T_a}{\omega_R^2} \frac{\omega_R^4 z^2}{(z + 2 D_R \omega_R T_a)^2}$$

*With equation (33), the formula $\varphi = \varphi_0 \sin \omega t$ yields:

$$\omega^4 T_a - \omega^2 (\omega_R^2 T_a + 2 D_R \omega_R z) + \omega_R^2 c = 0$$

$$- \omega^3 (z + 2 D_R \omega_R T_a) + \omega \omega_R^2 (z + a) = 0$$

whence

$$a = \omega^2 \frac{z + 2 D_R \omega_R T_a}{\omega_R^2} \quad (\text{Cont. on p. 45})$$

$$a = \frac{(z + 2 D_R \omega_R T_a) (\omega_R^2 T_a + 2 D_R \omega_R z)}{2 \omega_R^2 T_a} - z$$

$$\pm \sqrt{\frac{(z + 2 D_R \omega_R T_a)^2 (\omega_R^2 T_a + 2 D_R \omega_R z)^2}{4 \omega_R^4 T_a^2} - \frac{c (z + 2 D_R \omega_R T_a)^2}{\omega^2}}$$

$$\omega = \sqrt{\frac{\omega_R^2 (z + a)}{z + 2 D_R \omega_R T_a}} =$$

$$= \sqrt{\frac{\omega_R^2 T_a + 2 D_R \omega_R z}{2 T_a}} \pm \sqrt{\frac{(\omega_R^2 T_a + 2 D_R \omega_R z)^2}{4 T_a} - \frac{c}{T_a} \omega_R^2}$$

The result is a limited zone for the possible resetting effect a of the regulator, due to the existence of two forms of oscillations in the regulated system itself; one, which had already appeared on the faultproof regulator, the other having originated from the natural oscillation of the regulator. Choosing the resetting effect a too high, causes the natural oscillation mode of the regulator to increase (because of the increasing frequency of the regulating process and hence, of the phase change of the control link). If chosen too small, the oscillation of the system increases (because the undamping displacement effect becomes too active). This permissible zone of a resetting effect a becomes wider as the displacement effect c decreases (ultimately passing into the zone of the purely conjugate setting), but it also widens out by increasing ω_R . It shifts to higher values as D_R increases; for in that case both limits increase (the oscil-

(Continued from p. 44)

The frequency is:

$$\omega^2 = \frac{\omega_R^2 T_a + 2 D_R \omega_R z}{2 T_a} \pm \sqrt{\frac{(\omega_R^2 T_a + 2 D_R \omega_R z)^2}{4 T_a^2} - \frac{c \omega_R^2}{T_a}}$$

whence

$$a = \frac{(\omega_R^2 T_a + 2 D_R \omega_R z)}{2 T_a} \frac{(z + 2 D_R \omega_R T_a)}{\omega_R^2} - z$$

$$\pm \sqrt{\left[\frac{(\omega_R^2 T_a + 2 D_R \omega_R z)(z + 2 D_R \omega_R T_a)}{2 T_a \omega_R^2} \right]^2 - \frac{c (z + 2 D_R \omega_R T_a)^2}{T_a \omega_R^2}}$$

lations of the system because of the growing phase change, the other - more rapid regulator oscillation - because of the accelerated energy removal). The range shrinks to zero when

$$c > \frac{(\omega_R^2 T_a + 2 D_R \omega_R z)^2}{4 T_a \omega_R^2}$$

The rpm regulator.- The equations are:

Governing Principle

Regulated System

$$- \eta = a \varphi \quad T_a \ddot{\varphi} + 2T_a D_R \omega_R \dot{\varphi} + T_a \omega_R^2 \varphi + a \omega_R^2 \varphi = 0 \quad (35)$$

$$- \eta = a \varphi + c \int \varphi dt \quad T_a \varphi^{(IV)} + 2T_a D_R \omega_R \ddot{\varphi} + T_a \omega_R^2 \dot{\varphi} + a \omega_R^2 \varphi + c \omega_R^2 \varphi = 0 \quad (36)$$

$$- \eta = c \int \varphi dt \quad (\text{impractical, since it puts the rpm control under continually increasing amplitude})$$

The rpm regulator shows, consistent with theory, the behavior of the pressure regulator by zero load. The setting regulator shows a damping limit at:

$$a = 2 D_R \omega_R T_a$$

$$\omega = \sqrt{\frac{a \omega_R}{2 D_R T_a}} = \omega_R$$

The permissible resetting effect a increases with increasing frequency ω_R and increased damping D_R . It must be kept small by low frequency ω_R ; the only resetting force is that from the regulator, and it shifted considerably in phase at low ω_R .

The principle $-\eta = a \varphi + c \int \varphi dt$ shows, as on the pressure regulator and for the same reasons, a gone for the resetting effect a .*

*The relations for the rpm regulator can, as always, be obtained from those of the pressure regulator by putting the load $z = 0$.

$$a = D_R \omega_R \left(T_a \pm \sqrt{T_a^2 - \frac{4c T_a}{\omega_R^2}} \right)$$

$$\omega = \sqrt{\frac{a \omega_R}{2D_R T_a}} = \sqrt{\frac{\omega_R^2}{2} \pm \omega_R \sqrt{\frac{1}{4} - \frac{c}{T_a}}}$$

Here the zone tapers toward the zero point, since the rpm regulating system, having no resetting ability of its own, cannot function without regulator resetting effect a . An analysis of the running-speed regulator is omitted for the same reason. The damped zone shrinks to zero when

$$c > \frac{\omega_R^2 T_a}{4}$$

Temperature regulator.— An analysis of the temperature-regulating system with regulator susceptible to vibrations, is superfluous. Relative to the time intervals within which the regulating processes transpire, all natural frequencies of the regulator are so rapid, that they are negligible.

Automatic directional control.— The pertinent equations are:

Governing Principle

Regulated System

$$- \eta = a\varphi \quad \varphi^{(VI)} + (M + 2D_R \omega_R) \ddot{\varphi} + (2D_R \omega_R M + \omega_R^2) \ddot{\varphi} + \omega_R^2 M \dot{\varphi} + \omega_R^2 a N \varphi = 0 \quad (37)$$

$$- \eta = a\varphi + c \int \varphi dt \quad \varphi^{(V)} + (M + 2D_R \omega_R) \varphi^{(IV)} + (2D_R \omega_R M + \omega_R^2) \ddot{\varphi} + \omega_R^2 M \dot{\varphi} + \omega_R^2 a N \dot{\varphi} + \omega_R^2 c N = 0 \quad (38)$$

- $\eta = c \int \varphi dt$ (impractical, since it puts the automatic control under continually increasing amplitude)

The regulator with conjugate settings shows a damping limit at:*

* See footnote on p. 48.

$$\begin{aligned}
 aN &= \frac{1}{4} (2D_R M + \omega_R)^2 - \left(\frac{\omega_R M}{M + 2D_R \omega_R} - \frac{2D_R M + \omega_R}{2} \right)^2 \\
 &= \sqrt{\frac{M \omega_R^2}{M + 2D_R \omega_R}} \\
 &= \sqrt{\frac{2D_R \omega_R M + \omega_R^2}{2}} \pm \sqrt{\frac{1}{4} (2D_R \omega_R M + \omega_R^2)^2 - aN M \omega_R^2}
 \end{aligned}$$

The resetting effect a of the regulator can therefore be chosen so much greater as its natural frequency ω_R is higher. The damping D_R must remain within a certain zone. If it falls short of this zone, the energy removal is no longer sufficient to damp the rapid natural oscillation form (of the regulator). If the zone is exceeded the slow oscillation form of the system is accelerated by the great phase lag of the control link. By infinitely great damping D_R , the process changes to that of the sluggish regulator, with a damping limit at

$$a < \frac{M^2}{N}$$

(Footnote from p. 47)

Equation (37), together with $\varphi = \varphi_0 \sin \omega t$, gives:

$$\begin{aligned}
 \omega^4 - \omega^2 (2D_R \omega_R M + \omega_R^2) + \omega_R^2 aN &= 0 \\
 - \omega^3 (M + 2D_R \omega_R) + \omega \omega_R^2 M &= 0
 \end{aligned}$$

whence

$$aN = \frac{-\omega^4}{\omega_R^2} + \omega^2 \left(\frac{2D_R M}{\omega_R} + 1 \right)$$

and

$$\omega^2 = \frac{\omega_R^2 M}{M + 2D_R \omega_R}$$

which, inserted, leaves:

$$aN = - \frac{\omega_R^2 M^2}{(M + 2D_R \omega_R)^2} + \frac{2D_R \omega_R M^2 + \omega_R^2 M}{M + 2D_R \omega_R}$$

For $\ddot{\eta} = a + c \int \varphi dt$, the damping limit lies at*

$$\begin{aligned} aN = \frac{1}{4} (2D_R M + \omega_R)^2 - \left[\frac{1}{2} \frac{\omega_R M}{M + 2D_R \omega_R} - \frac{2D_R M + \omega_R}{2} \right. \\ \left. \pm \sqrt{\frac{1}{4} \left(\frac{\omega_R M}{M + 2D_R \omega_R} \right)^2 - \frac{cN}{M + 2D_R \omega_R}} \right]^2 \end{aligned}$$

The behavior is similar to that of the setting regulator. But now the permissible range for the resetting effect a is curtailed, even for small values. As on the faultproof regulator, a must not fall below a certain amount, or the undamping displacement effect c becomes predominant. The range within which a must remain, becomes continuously smaller as c increases.

Y) Regulator with Friction

Here we differentiate between friction in the detector system and friction in the control link. Friction in the regulator means that its control link does not assume an exactly defined setting but can fluctuate within a certain range around the theoretical setting. By changing theoretical setting, the control link lags to the amount of this zone, so that the actions of the regulator with

* Equation (36) gives:

$$\omega^5 - (\omega_R^2 + 2D_R \omega_R M) \omega^3 + aN \omega_R^2 \omega = 0$$

$$\omega^4 (M + 2D_R \omega_R) - \omega_R^2 M \omega^2 + \omega_R^2 cN = 0$$

whence

$$\omega^2 = \frac{2D_R \omega_R M + \omega_R^2}{2} \pm \sqrt{\frac{1}{4} (2D_R \omega_R M + \omega_R^2)^2 - aN \omega_R^2}$$

$$\omega^2 = \frac{1}{2} \frac{\omega_R^2 M}{M + 2D_R \omega_R} \pm \sqrt{\frac{1}{4} \left(\frac{\omega_R^2 M}{M + 2D_R \omega_R} \right) - \frac{cN \omega_R^2}{M + 2D_R \omega_R}}$$

Equating these two values for ω^2 and squaring, gives:

$$\begin{aligned} \frac{1}{4} (2D_R \omega_R M + \omega_R^2)^2 - aN \omega_R^2 = \\ = \left[\frac{1}{2} \frac{\omega_R^2 M}{M + 2D_R \omega_R} - \frac{2D_R \omega_R M + \omega_R^2}{2} \right. \\ \left. \pm \sqrt{\frac{1}{4} \left(\frac{\omega_R^2 M}{M + 2D_R \omega_R} \right)^2 - \frac{cN \omega_R^2}{M + 2D_R \omega_R}} \right]^2 \end{aligned}$$

friction are likely to be the same as on the sluggish regulator (which lags also). In fact, the ensuing calculation shows that oscillations of the faultproof regulated system are undamped by the regulator friction and aperiodic motions are made susceptible to vibration. Since the amount of lag due to friction is constant, its effect at great departures is small and becomes perceptible only by small departures; by further increasing deflections φ from the theoretical state the system changes to the faultproof regulator. This means that (after a disturbance, for instance) the deflection process is initially the same as on the faultproof regulator and evinces departures from it only by small deflections which, however, may then be so severe that the regulated system remains in permanent oscillations altogether.

For oscillations the conditions accompanying friction can be readily approximated. The course of a command φ from the detector altered by friction effects, can be represented by two sine curves φ_f (fig. 3) modified in phase and amplitude with respect to the theoretical curve: One, the first approximation, represents a value that is too high; the other, second approximation, represents a value that is too low - placing the true conditions by friction between both and so forming a limit.

Preserving the assumption of a periodic solution, the two approximate curves can be looked upon as a solution of a differential equation of the form:

$$d\dot{\varphi}_f + \varphi_f = e\varphi \quad (39)$$

where quantities d and e must satisfy a special relationship. For harmonic motions:

$$\varphi = \varphi_0 \sin \omega t$$

$$\varphi_f = \varphi_{f_0} \sin (\omega t - \alpha)$$

it yields:

Approximation 1.-

$$d\omega \varphi_{f_0} \cos (\omega t - \alpha) + \varphi_{f_0} \sin (\omega t - \alpha) = e \varphi_0 \sin \omega t$$

hence,

$$\sin \alpha = \frac{d\omega \varphi_{f_0}}{e\varphi_0}$$

$$\cos \alpha = \varphi_{f_0}/e\varphi_0$$

On the other hand, the aspect of the curve (fig. 3) indicates

$$\sin \alpha = \frac{\varphi_r}{\varphi_0}; \text{ hence } \cos \alpha = \sqrt{1 - \left(\frac{\varphi_r}{\varphi_0}\right)^2}$$

leaving for d the value:

$$d = \left(\frac{\varphi_r}{\varphi_0}\right) \frac{1}{\omega \sqrt{1 - \left(\frac{\varphi_r}{\varphi_0}\right)^2}}$$

with φ_r denoting the amount of change of state necessary to make the friction-encumbered regulator respond and serve as indication of the amount of friction. In addition, the approximation curve φ_f in the first approximation is to be so placed that it intersects the theoretical curve φ in its maximum amplitude; thereby it probably indicates greater deflections than the curve distorted by friction. Thus, we obtain:

$$\varphi_{f_0} = e\varphi_0 \cos \alpha = \varphi_0 \sin \left(\frac{\pi}{2} + \alpha\right)$$

hence, $e = 1$.

For periodic solutions of the regulator process, according to approximation 1, the behavior of the friction should be approximated by the differential equation (whereby, for reasons of simplicity, φ_r/φ replaces φ_r/φ_0 , and φ represents an indication for the departure from the theoretical state instead of a time function).

$$\frac{\varphi_r}{\varphi} \frac{1}{\omega \sqrt{1 - \left(\frac{\varphi_r}{\varphi}\right)^2}} \dot{\varphi}_f + \varphi_f = \varphi \quad (40)$$

Approximation 2. - Here the equation reads:

$$d\omega \varphi_{f_0} \cos(\omega t - \alpha) + \varphi_{f_0} \sin(\omega t - \alpha) = e\varphi_0 \sin \omega t$$

With the phase difference α again at (fig. 3)

$$\sin \alpha = \varphi_r / \varphi_0$$

the value d amounts to

$$d = \frac{\varphi_r}{\varphi_0} \frac{1}{\omega \sqrt{1 - \left(\frac{\varphi_r}{\varphi_0}\right)^2}}$$

But with the maximum deflection φ_{r_0} chosen at $\varphi_0 - \varphi_r$, we have:

$$\varphi_{r_0} = e\varphi_0 \cos \alpha = \varphi_0 - \varphi_r$$

hence

$$e = \left(1 - \frac{\varphi_r}{\varphi_0}\right) \frac{1}{\sqrt{1 - \left(\frac{\varphi_r}{\varphi_0}\right)^2}}$$

For periodic solutions of the control process by the second approximation, the attitude of the regulator with friction is approximated by the differential equation:

$$\frac{\varphi_r}{\varphi} \frac{1}{\omega \sqrt{1 - \left(\frac{\varphi_r}{\varphi}\right)^2}} \dot{\varphi}_f + \varphi_f = \left(1 - \frac{\varphi_r}{\varphi}\right) \frac{1}{\sqrt{1 - \left(\frac{\varphi_r}{\varphi}\right)^2}} \varphi \quad (41)$$

The final results of the analytical calculation of the regulator with friction are appended in tables VI and VII. The damping by damped processes and the damping limit of ampliative processes are shown plotted against the ratio φ/φ_r of momentary departure φ of the state to friction zone $\pm \varphi_r$. The space between the two curves of first and second approximations, within which the actual curve follows, is indicated by crosshatching. The study was made only for the setting regulator and the running-speed regulator; it did not include the principle $\ddot{\eta} = a\varphi + c \int \varphi dt$, the behavior of which lies between the other two.

If, in the treatment here, a damping limit is reached, then both approximations show the same frequency curve. In that case the phase shift α for both approximations is the same and the vector related to the permanent oscillation closes momentarily only by a phase difference. As to the frequency curve $\omega = f(\varphi/\varphi_r)$, it should be noted

that the frequency ω is valid for the damping limit, and therefore can be traversed only by a change of φ/φ_r concurrently with a change of a, c, T_a , etc.

Pressure regulator.— For the pressure regulator with conjugate settings, the equations:

$$T_a \dot{\varphi} + z\varphi = T_f \quad (1)$$

$$- T_f = a\varphi_f \quad (5)$$

$$a\dot{\varphi}_f + \varphi_f = e\varphi \quad (39)$$

give the equation of the friction-decelerated pressure regulated system at:

Approximation 1:

$$\frac{\varphi_r}{\varphi} \frac{1}{\omega} \frac{T_a}{\sqrt{1 - (\varphi_r/\varphi)^2}} \dot{\varphi} + \left(\frac{\varphi_r}{\varphi} \frac{z}{\omega} \frac{1}{\sqrt{1 - (\varphi_r/\varphi)^2}} + T_a \right) \dot{\varphi} + (z + a) \varphi = 0 \quad (42a)$$

Approximation 2:

$$\frac{\varphi_r}{\varphi} \frac{1}{\omega} \frac{T_a}{\sqrt{1 - (\varphi_r/\varphi)^2}} \ddot{\varphi} + \left(\frac{\varphi_r}{\varphi} \frac{z}{\omega} \frac{1}{\sqrt{1 - (\varphi_r/\varphi)^2}} + T_a \right) \dot{\varphi} + \left(z + \frac{a \left(1 - \frac{\varphi_r}{\varphi} \right)}{\sqrt{1 - (\varphi_r/\varphi)^2}} \right) \varphi = 0 \quad (42b)$$

Equation (42) states that the regulated system is susceptible to vibration, hence the use of the approximate method is justified. The characteristic quantities of this natural oscillation are:*

* Equation (42a) gives for approximation 1:

$$\omega_o^2 = \frac{z + a}{\frac{\varphi_r}{\varphi} T_a} \omega_e \sqrt{1 - \left(\frac{\varphi_r}{\varphi} \right)^2}$$

$$D = \frac{1}{2} \left(\frac{\frac{\varphi_r}{\varphi} z}{\omega_e \sqrt{1 - \left(\frac{\varphi_r}{\varphi} \right)^2}} + T_a \right) \frac{\omega_e}{\frac{\varphi_r}{\varphi} T_a} \sqrt{1 - \left(\frac{\varphi_r}{\varphi} \right)^2}$$

(Cont. on p. 54)

Approximation 1:

$$\left(\frac{\varphi}{\varphi_r}\right)^2 = \left[\frac{D}{\sqrt{1-D^2}} \pm \sqrt{\frac{D^2}{1-D^2} - \frac{z}{z+a} \frac{1}{(1-D^2)}} \right]^2 + 1$$

$$\omega_e = \frac{z+a}{T_a} \sqrt{\left(\frac{\varphi}{\varphi_r}\right)^2 - 1 \times (1-D^2)}$$

The evaluation of the above formulas (table VI) shows that with decreasing φ/φ_r the damping D itself decreases

(Footnote continued from p. 53)

hence for the frequency:

$$\frac{\omega_e}{\omega_0} = \sqrt{1-D^2}$$

$$\omega_e = \frac{z+a}{T_a} \sqrt{\left(\frac{\varphi}{\varphi_r}\right)^2 - 1 \times (1-D^2)}$$

On the other hand, the introduction of the same relations and of this formula for ω_e in the equation for D give:

$$D = \frac{1}{2} \frac{z}{T_a} \sqrt{1-D^2} \frac{T_a}{z+a} \frac{1}{1-D^2} - \frac{1}{\sqrt{\left(\frac{\varphi}{\varphi_r}\right)^2 - 1}} + \frac{\sqrt{1-D^2}}{2} \sqrt{\left(\frac{\varphi}{\varphi_r}\right)^2 - 1}$$

and after a few changes:

$$\left(\left(\frac{\varphi}{\varphi_r}\right)^2 - 1\right)(1-D^2) - \sqrt{\left(\left(\frac{\varphi}{\varphi_r}\right)^2 - 1\right)(1-D^2)} \times 2D + \frac{z}{z+a} = 0$$

hence

$$\sqrt{\left(\left(\frac{\varphi}{\varphi_r}\right)^2 - 1\right)(1-D^2)} = D \pm \sqrt{D^2 - \frac{z}{z+a}}$$

After dividing by $\sqrt{1-D^2}$ and rearranging, we find the value for φ/φ_r as given in the text.

For approximation 2, the same calculation process gives the frequency ω_e at:

$$\omega_e = \frac{1-D^2}{T_a} \left[z \sqrt{\left(\frac{\varphi}{\varphi_r}\right)^2 - 1} + a \left(\frac{\varphi}{\varphi_r} - 1\right) \right]$$

The calculation for damping D affords: (Cont. on p. 55)

(since by decreasing φ/φ_r the undamping phase shift α increases). By greater departures from the theoretical state the process is aperiodic (since the effect of the phase displacement α due to friction is small by great departures; $\sin \alpha = \varphi_r/\varphi$). By very small departures φ/φ_r from the theoretical state, damping D is greater again (because by small deflections the control-link amplitude, while being materially distorted, is considerably weakened and can no longer bring the regulating system with its own resetting power to overswinging; ultimately the process terminates in a dead zone $\varphi = \pm \varphi_r$. With increasing load z , damping D increases because of the then-increasing natural resetting ability; D likewise increases by decreasing resetting effect a , where the very distortion caused by friction makes the system susceptible to vibration.

For the running speed regulator, equations:

$$T_a \dot{\varphi} + z\varphi = \eta_f \tag{1}$$

$$- \eta_f = c \int \varphi_f dt \tag{7}$$

$$d\dot{\varphi}_f + \varphi_f = e\varphi \tag{39}$$

give the formula of the regulated system at:

Approximation 1:

$$\frac{\varphi_r}{\varphi} \frac{T_a}{\omega \sqrt{1 - \left(\frac{\varphi_r}{\varphi}\right)^2}} \dot{\varphi} + \left(\frac{\frac{\varphi_r}{\varphi} z}{\omega \sqrt{1 - \left(\frac{\varphi_r}{\varphi}\right)^2}} + T_a \right) \ddot{\varphi} + \dot{\varphi} + c\varphi = 0 \tag{43a}$$

(Continuation of footnote from p. 54)

$$\left(\left(\frac{\varphi}{\varphi_r}\right)^2 - 1 \right) (1-D^2) - \sqrt{\left(\left(\frac{\varphi}{\varphi_r}\right)^2 - 1 \right) (1-D^2)} \times 2D + \frac{z}{z + a \frac{\left(\frac{\varphi}{\varphi_r}\right)^2 - 1}{\sqrt{\left(\frac{\varphi}{\varphi_r}\right)^2 - 1}}} = 0$$

which show the values of damping D to be greater than with approximation 1, but otherwise with the same curve.

Approximation 2:

$$\frac{\varphi_r}{\omega} \frac{T_a}{\sqrt{1 - \left(\frac{\varphi_r}{\varphi}\right)^2}} \dot{\varphi} + \left(\frac{\frac{\varphi_r}{\varphi} z}{\omega \sqrt{1 - \left(\frac{\varphi_r}{\varphi}\right)^2}} + T_a \right) \ddot{\varphi} + z \dot{\varphi} + \frac{c \left(1 - \frac{\varphi_r}{\varphi}\right)}{\sqrt{1 - \left(\frac{\varphi_r}{\varphi}\right)^2}} \varphi = 0 \quad (43b)$$

These formulas state that the already-susceptible system regulated as faultproof, now discloses the possibility of increasing oscillations as a result of further undamping. The damping limit follows from equation (43) at:*

*With $\varphi = \varphi_0 \sin \omega t$, equation (43) gives:

Approximation 1:

$$- \omega^3 \frac{\varphi_r}{\varphi} \frac{T_a}{\omega \sqrt{1 - \left(\frac{\varphi_r}{\varphi}\right)^2}} + \omega z = 0$$

$$- \omega^2 \left(\frac{\varphi_r}{\varphi} \frac{z}{\omega \sqrt{1 - \left(\frac{\varphi_r}{\varphi}\right)^2}} + T_a \right) + c = 0$$

hence

$$\omega = \frac{z}{T_a} \sqrt{\left(\frac{\varphi_r}{\varphi}\right)^2 - 1}$$

$$c = \frac{z}{T_a} \sqrt{\left(\frac{\varphi_r}{\varphi}\right)^2 - 1} \frac{\frac{\varphi_r}{\varphi} z}{\sqrt{1 - \left(\frac{\varphi_r}{\varphi}\right)^2}} + \frac{T_a z^2}{T_a} \left(\left(\frac{\varphi_r}{\varphi}\right)^2 - 1 \right)$$

Approximation 2:

$$- \omega^3 \frac{\varphi_r}{\varphi} \frac{T_a}{\omega \sqrt{1 - \left(\frac{\varphi_r}{\varphi}\right)^2}} + \omega z = 0$$

$$- \omega^2 \left(\frac{\varphi_r}{\varphi} \frac{z}{\omega \sqrt{1 - \left(\frac{\varphi_r}{\varphi}\right)^2}} + T_a \right) + c \frac{1 - \frac{\varphi_r}{\varphi}}{\sqrt{1 - \left(\frac{\varphi_r}{\varphi}\right)^2}} = 0$$

(Cont. on p. 57)

Approximation 1:

$$c = \frac{\kappa}{E_1} \frac{B}{a} \left(\frac{\phi}{\phi_r} \right)^B$$

$$\omega = \frac{\kappa}{E_1} \frac{B}{a} \sqrt{\left(\frac{\phi}{\phi_r} \right)^B}$$

Approximation 2:

$$c = \frac{\kappa}{E_1} \frac{B}{a} \left(\frac{\phi}{\phi_r} \right)^B \frac{\sqrt{\left(\frac{\phi}{\phi_r} \right)^B - 1}}{\frac{\phi}{\phi_r} - 1}$$

$$\omega = \frac{\kappa}{E_1} \frac{B}{a} \sqrt{\left(\frac{\phi}{\phi_r} \right)^B - 1}$$

The evaluation of these formulas (table VI) shows that damping always exists

$$c < \frac{\kappa}{E_1} \frac{B}{a}$$

(Continuation of footnote from p. 56)

hence

$$\omega = \frac{\kappa}{E_1} \frac{B}{a} \sqrt{\left(\frac{\phi}{\phi_r} \right)^B - 1}$$

inserted

$$c = \frac{\kappa}{E_1} \frac{B}{a} \sqrt{\left(\frac{\phi}{\phi_r} \right)^B - 1} \frac{\frac{\phi}{\phi_r} \kappa}{\sqrt{1 - \left(\frac{\phi}{\phi_r} \right)^B}} \frac{\sqrt{1 - \left(\frac{\phi}{\phi_r} \right)^B}}{1 - \frac{\phi}{\phi_r}} + \frac{\kappa}{E_1} \frac{B}{a} \left(\left(\frac{\phi}{\phi_r} \right)^B - 1 \right) \frac{\sqrt{1 - \left(\frac{\phi}{\phi_r} \right)^B}}{1 - \frac{\phi}{\phi_r}}$$

which, abbreviated, give the values shown in the text.

At substantially greater c values, there is some damping by great φ (since then the proportionate effect of friction φ_r is quite small); but the deflection process does not terminate in the dead zone but rather is maintained as permanent oscillation (since by the then-small deflections φ , the phase-shifting effect of the friction is proportionally strong enough to maintain the system in permanent oscillations). This danger of permanent oscillation increases, of course, by decreasing load x (because the natural resetting ability of the system diminishes as the load decreases).

The rpm regulator.— In this case the equations of the regulated system read as follows:

Approximation 1:

$$\frac{\varphi_r}{\varphi} \frac{T_a}{\omega \sqrt{1 - \left(\frac{\varphi_r}{\varphi}\right)^2}} \ddot{\varphi} + T_a \dot{\varphi} + x\varphi = 0 \quad (44a)$$

Approximation 2:

$$\frac{\varphi_r}{\varphi} \frac{T_a}{\omega \sqrt{1 - \left(\frac{\varphi_r}{\varphi}\right)^2}} \ddot{\varphi} + T_a \dot{\varphi} + x \frac{1 - \frac{\varphi_r}{\varphi}}{\sqrt{1 - \left(\frac{\varphi_r}{\varphi}\right)^2}} \varphi = 0 \quad (44b)$$

In this case also the regulated system is found to be susceptible to vibration, hence the application of the approximate method is justified. The characteristic quantities of the oscillation are:

Approximation 1:

$$D = \frac{\sqrt{\left(\frac{\varphi_r}{\varphi}\right)^2 - 1}}{\left(\frac{\varphi_r}{\varphi}\right)^2 + 3}$$

$$\omega_0 = \frac{x}{T_a} \sqrt{\left(\frac{\varphi_r}{\varphi}\right)^2 - 1} \times (1 - D^2)$$

Approximation 2:

$$D = \sqrt{\frac{(\varphi/\varphi_r)^2 - 1}{(\varphi/\varphi_r)^2 + 3}}$$

$$\omega_0 = \frac{a}{T_a} \left(\frac{\varphi}{\varphi_r} - 1 \right) \times (1 - D^2)$$

No great damping D exists now by small deflections φ/φ_r . (because the inherent resetting effect of the regulating system is lacking, which on the pressure regulator effectuated at this point a second rise in damping). By great φ/φ_r the effect of friction φ_r is proportionately so small that the control process becomes aperiodic. The damping (for which both approximations yield the same formula) continues to decrease with decreasing deflections φ/φ_r , since the total resetting effect of the regulated system traces back to the consistently distorted resetting effect of the regulator. Increasing oscillations, however, are not possible.

Temperature regulator.— Equations for the regulator with conjugate setting follow from (3), (5), and (39) at:

Approximation 1:

$$\begin{aligned} \frac{\varphi_r}{\omega} \frac{T_v T_a}{\sqrt{1 - \left(\frac{\varphi_r}{\varphi}\right)^2}} \ddot{\varphi} + \left(\frac{\frac{\varphi_r}{\varphi} (T_v + T_a)}{\omega \sqrt{1 - \left(\frac{\varphi_r}{\varphi}\right)^2}} + T_v T_a \right) \dot{\varphi} \\ + \left(\frac{\frac{\varphi_r}{\varphi}}{\omega \sqrt{1 - \left(\frac{\varphi_r}{\varphi}\right)^2}} + T_v + T_a \right) \varphi + (1+a) \varphi = 0 \end{aligned} \quad (45a)$$

Approximation 2:

$$\begin{aligned} \frac{\varphi_r}{\omega} \frac{T_v T_a}{\sqrt{1 - \left(\frac{\varphi_r}{\varphi}\right)^2}} \ddot{\varphi} + \left(\frac{\frac{\varphi_r}{\varphi} (T_v + T_a)}{\omega \sqrt{1 - \left(\frac{\varphi_r}{\varphi}\right)^2}} + T_v T_a \right) \dot{\varphi} \\ + \left(\frac{\frac{\varphi_r}{\varphi}}{\omega \sqrt{1 - \left(\frac{\varphi_r}{\varphi}\right)^2}} + T_v + T_a \right) \varphi + \left(1 + a \frac{1 - \frac{\varphi_r}{\varphi}}{\sqrt{1 - \left(\frac{\varphi_r}{\varphi}\right)^2}} \right) \varphi = 0 \end{aligned} \quad (45b)$$

Both show that the vibration-susceptible system with fault-proof regulator now manifests potential increasing oscillations as a result of the friction-induced lag of the control link. The related damping limit follows from equation (45) at:

Approximation 1:

$$a = \frac{T_v + T_a}{\left(\frac{\varphi}{\varphi_r}\right)^2} \left[\frac{1}{2} \frac{T_v + T_a}{T_v T_a} + \sqrt{\left(\frac{1}{2} \frac{T_v + T_a}{T_v T_a}\right)^2 + \frac{1}{\left(\left(\frac{\varphi}{\varphi_r}\right)^2 - 1\right) T_v T_a}} \right]$$

Approximation 2:

$$a = \frac{\left(\frac{\varphi}{\varphi_r}\right)^2}{\frac{\varphi}{\varphi_r} - 1} (T_v + T_a) \sqrt{\left(\frac{\varphi}{\varphi_r}\right)^2 - 1} \left[\frac{1}{2} \frac{T_v + T_a}{T_v T_a} + \sqrt{\left(\frac{1}{2} \frac{T_v + T_a}{T_v T_a}\right)^2 + \frac{1}{\left(\left(\frac{\varphi}{\varphi_r}\right)^2 - 1\right) T_v T_a}} \right]$$

The evaluation of these formulas (table VI) shows that the resetting effect a may not exceed a certain value if the deflection process is to terminate in the dead zone. If it exceeds this limit, a deflection process ends in a permanent oscillation by $\varphi > \varphi_r$. Admittedly, by the inherent resetting ability of the regulated system, a damped zone can still develop a little outside of $\varphi = \varphi_r$, but this cannot be reached by a deflection from greater changes of state because a zone of increasing oscillations lies between. In any case, there is always damping so long as

$$a < \frac{(T_v + T_a)^2}{T_v T_a}$$

is applicable.

For the regulator with conjugate running speed, the formulas read:

Approximation 1:

$$\begin{aligned} & \frac{\frac{\phi}{\phi_R} T_V T_A}{\omega \sqrt{1 - \left(\frac{\phi}{\phi_R}\right)^2}} \phi^{(IV)} + \left(\frac{\frac{\phi}{\phi_R} (T_V + T_A)}{\omega \sqrt{1 - \left(\frac{\phi}{\phi_R}\right)^2}} + T_V T_A \right) \ddot{\phi} \\ & + \left(\frac{\frac{\phi}{\phi_R}}{\omega \sqrt{1 - \left(\frac{\phi}{\phi_R}\right)^2}} + T_V + T_A \right) \dot{\phi} + \phi + c\phi = 0 \end{aligned} \quad (46a)$$

Approximation 2:

$$\begin{aligned} & \frac{\frac{\phi}{\phi_R} T_V T_A}{\omega \sqrt{1 - \left(\frac{\phi}{\phi_R}\right)^2}} \phi^{(IV)} + \left(\frac{\frac{\phi}{\phi_R} (T_V + T_A)}{\omega \sqrt{1 - \left(\frac{\phi}{\phi_R}\right)^2}} + T_V T_A \right) \ddot{\phi} \\ & + \left(\frac{\frac{\phi}{\phi_R}}{\omega \sqrt{1 - \left(\frac{\phi}{\phi_R}\right)^2}} + T_V + T_A \right) \dot{\phi} + \phi + c \frac{\left(\frac{\phi}{\phi_R}\right) - 1}{\sqrt{\left(\frac{\phi}{\phi_R}\right)^2 - 1}} \phi = 0 \end{aligned} \quad (46b)$$

Again it is manifest that the regulator process has a damping limit which for the values:*

*With $\phi = \phi_0 \sin \omega t$, equation (46) gives:

$$\begin{aligned} & \omega^4 \frac{\phi}{\phi_R} \frac{T_V T_A}{\omega \sqrt{1 - \left(\frac{\phi}{\phi_R}\right)^2}} - \omega^2 \left(\frac{\phi}{\phi_R} \frac{1}{\omega \sqrt{1 - \left(\frac{\phi}{\phi_R}\right)^2}} + T_V + T_A \right) + c = 0 \\ & - \omega^3 \left(\frac{\phi}{\phi_R} \frac{T_V + T_A}{\omega \sqrt{1 - \left(\frac{\phi}{\phi_R}\right)^2}} + T_V T_A \right) + \omega = 0 \end{aligned}$$

hence

$$c = \omega^2 (T_V + T_A) + \frac{\omega \frac{\phi}{\phi_R}}{\sqrt{1 - \left(\frac{\phi}{\phi_R}\right)^2}} - \omega^3 \frac{T_V T_A}{\sqrt{1 - \left(\frac{\phi}{\phi_R}\right)^2}} \frac{\phi}{\phi_R}$$

and

(Cont. on p. 62)

Approximation 1:

$$c = \frac{1}{\sqrt{\left(\frac{\phi}{\phi_r}\right)^2 - 1}} (\omega - \omega^3 T_V T_A) + \omega^2 (T_V + T_A)$$

$$\omega = -\frac{1}{2} \frac{\phi_r}{\phi} \frac{T_V + T_A}{T_V T_A \sqrt{1 - \left(\frac{\phi_r}{\phi}\right)^2}} \pm \sqrt{\frac{1}{4} \left(\frac{\phi_r}{\phi}\right)^2 \frac{(T_V + T_A)^2}{T_V^2 T_A^2 (1 - \left(\frac{\phi_r}{\phi}\right)^2)} + \frac{1}{T_V T_A}}$$

Approximation 2:

$$c = \frac{1}{\frac{\phi}{\phi_r} - 1} (\omega - \omega^3 T_V T_A) + \frac{\sqrt{\left(\frac{\phi}{\phi_r}\right)^2 - 1}}{\frac{\phi}{\phi_r} - 1} \omega^2 (T_V + T_A)$$

ω is the same as by approximation 1.

(Continued from footnote on p. 61)

$$\omega = -\frac{1}{2} \frac{\phi_r}{\phi} \frac{T_V + T_A}{T_V T_A} \frac{1}{\sqrt{1 - \left(\frac{\phi_r}{\phi}\right)^2}} \pm \sqrt{\frac{1}{4} \left(\frac{\phi_r}{\phi}\right)^2 \frac{(T_V + T_A)^2}{T_V^2 T_A^2 (1 - \left(\frac{\phi_r}{\phi}\right)^2)} + \frac{1}{T_V T_A}}$$

In the numerical calculation, ω is computed and written in the formula for c . The values for approximation 2 follow from approximation 1 when putting:

$$c \left(\frac{\phi}{\phi_r} - 1\right) / \sqrt{\left(\frac{\phi}{\phi_r}\right)^2 - 1}$$

instead of c .

Evaluation of these formulas (table VI) discloses for great departure φ from the theoretical state, the same limiting conditions as the faultproof regulator. On approaching the theoretical state, however, during the deflection process, the displacement effect c of the regulator can be chosen greater than for the faultproof regulator; for the phase shift of the control link relative to the factor of the state already amounts to $\pi/2$ on the faultproof running-speed regulator. Now it becomes even greater, which means that its undamping proportion becomes less again.

Automatic directional control.— For the setting regulator alone involved here, equation (47) gives:

Approximation 1:

$$\frac{\frac{\varphi_r}{\varphi}}{\omega \sqrt{1 - \left(\frac{\varphi_r}{\varphi}\right)^2}} \ddot{\varphi} + \left(\frac{\frac{\varphi_r}{\varphi} M}{\omega \sqrt{1 - \left(\frac{\varphi_r}{\varphi}\right)^2}} + 1 \right) \dot{\varphi} + M\dot{\varphi} + aN\varphi = 0 \quad (47a)$$

Approximation 2:

$$\frac{\frac{\varphi_r}{\varphi}}{\varphi \sqrt{1 - \left(\frac{\varphi_r}{\varphi}\right)^2}} \ddot{\varphi} + \left(\frac{\frac{\varphi_r}{\varphi} M}{\omega \sqrt{1 - \left(\frac{\varphi_r}{\varphi}\right)^2}} + 1 \right) \dot{\varphi} + M\dot{\varphi} + aN \frac{1 - \frac{\varphi_r}{\varphi}}{\sqrt{1 - \left(\frac{\varphi_r}{\varphi}\right)^2}} \varphi = 0 \quad (47b)$$

There is a possibility of increasing oscillations as a result of the phase shift of the control link due to the friction. The related damping limit is:

Approximation 1:

$$a = \frac{M^2}{N} \left(\frac{\varphi}{\varphi_r} \right)^2$$

$$\omega = N \sqrt{\left(\frac{\varphi}{\varphi_r} \right)^2 - 1}$$

Approximation 2:

$$a = \frac{M^2}{N} \left(\frac{\varphi}{\varphi_r} \right)^2 \frac{\sqrt{\left(\frac{\varphi}{\varphi_r} \right)^2 - 1}}{\frac{\varphi}{\varphi_r} - 1}$$

$$\omega = M \sqrt{\left(\frac{\varphi}{\varphi_r} \right)^2 - 1}$$

In any case, damping continues to exist so long as

$$a < \frac{M^2}{N}$$

At greater values a zone is formed in which increasing oscillations occur which prevent the deflection process (for instance, after a disturbance) from running in the dead zone. The deflection process terminates in a permanent oscillation with so much greater deflection as the chosen resetting effect a is greater (because with increasing a - hence increased frequency - the natural damping of the system diminishes).

The regulator with friction in the control link. - If the friction is on the control-link side, a change in adjustment (a , b , c) of the regulator changes the proportional effect of this friction as well. By decreasing regulator effect, the proportional amount of friction increases materially, for by small regulating effect small control-link motions occur which the friction in the control link distorts much more than the greater motions by greater regulating effect. Hence it is to be expected that the results by friction in the detector system are modified to the extent that they are shifted toward greater φ by small regulator effects. The proportionate amount of friction is inversely proportional to effects a and c of the regulator, so that the earlier results are applicable if values $\frac{\varphi}{\eta_r/a}$ and $\frac{\varphi}{\eta_r/c}$, respectively, are used instead of φ/φ_r . Here η_r indicates the amount by which the contact switch must be adjusted in order to induce motion on the control link. The results obtained with these new values are shown in table VII. There is,

indeed, a postponement of the damping limits toward greater deflections as the regulating effect diminishes; the dead zone within which the regulating process comes to rest is now bounded by a parabola $\varphi/\eta_r = 1/a$. (By small effects of a or c , a great deflection must already exist in order to overcome the friction in the control link.)

Translation by J. Vanier,
National Advisory Committee
for Aeronautics.

REFERENCES

1. Wunsch, G.: Regler für Druck und Menge, München und Berlin, 1930.
2. Lehr, E.: Schwingungstechnik, II, Berlin, 1934, p. 319.
3. Lang, H.: Theorie des Heizungsreglers. Gesundh.-Ing. 56, 1933, p. 529.
4. Oppelt, W.: Die Flugzeugkurssteuerung im Geradeausflug. Luftfahrtforschung, Bd. 14, 1937, p. 270.

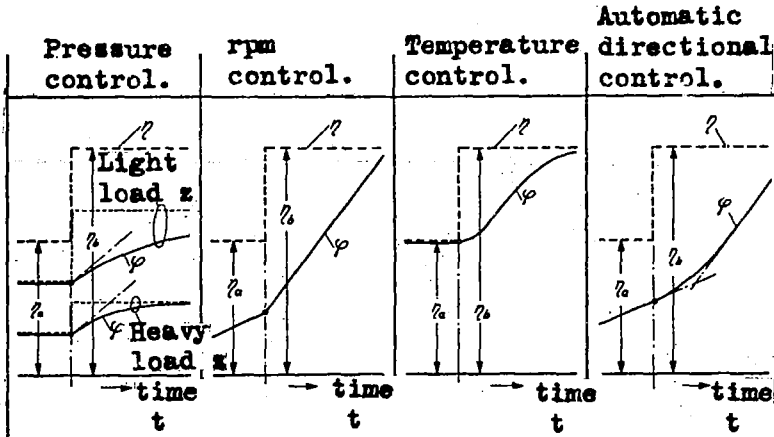
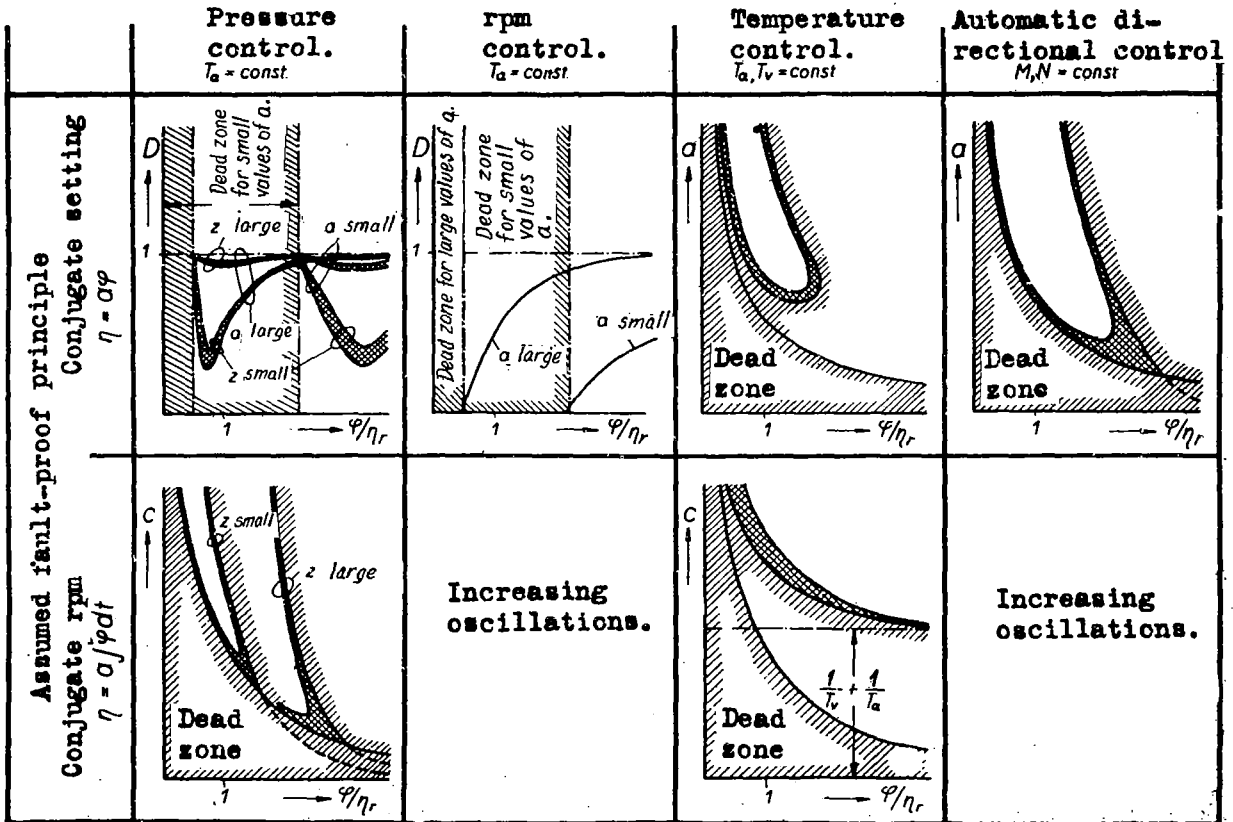


Table I.- The regulating system after abrupt adjustment of control link from η_a to η_b .

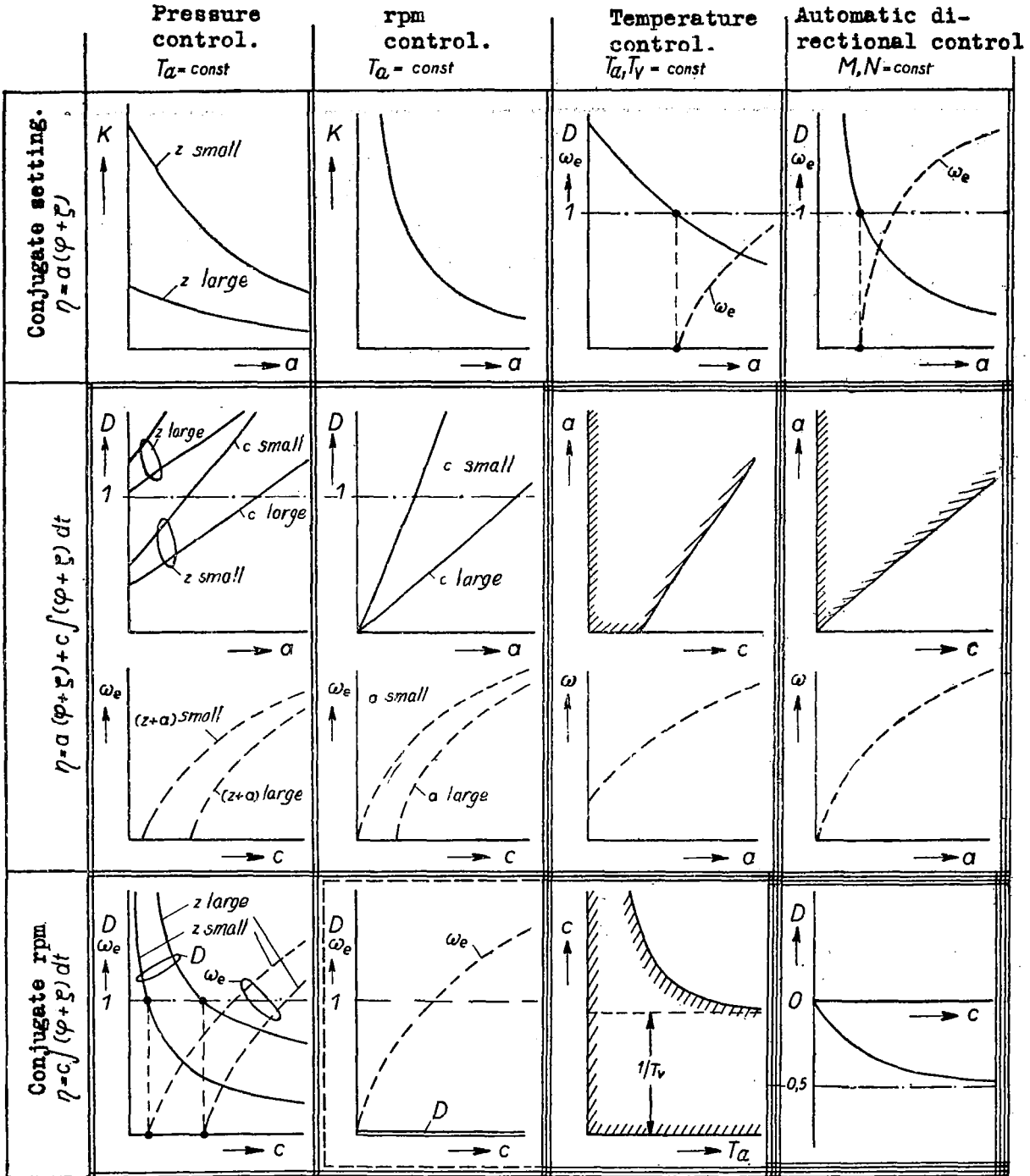
ϕ = Departure from theoretical state.
 η = Control link deflection.

Table VII.-- Control with control-link friction.



Cross hatched zone denotes zone between the two approximations.

Table II.- Fault-proof regulators.



Hatched zone = zone where damping exists.

- = Denotes aperiodic process.
- — — = Damped periodic process.
- — — — = Undamped periodic process.
- — — — — = Periodic processes with possible increase.
- — — — — — = Increasing periodic processes.

Table III.- Mechanical substitutes explaining the control processes.

	Pressure cotrol heavy loadz	light loadz	rpm control	Temperature control	Automatic di- rectional control.
Regulating system.					
$\eta = a(\phi + \xi)$					
$\eta = a(\phi + \xi) + b\phi$					
$\eta = a(\phi + \xi) + c \int (\phi + \xi) dt$					
$\eta = c \int (\phi + \xi) dt$					

Table IV. * Sluggish control.

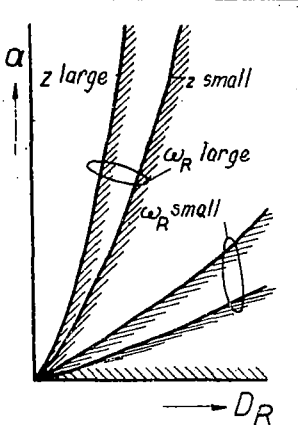
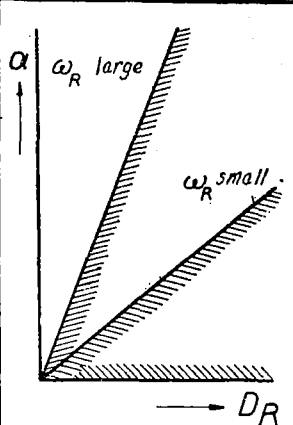
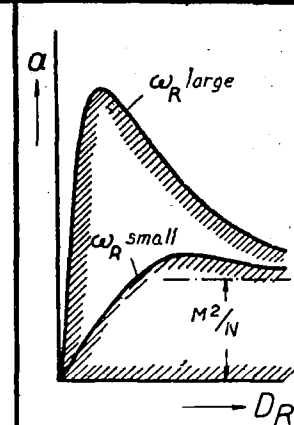
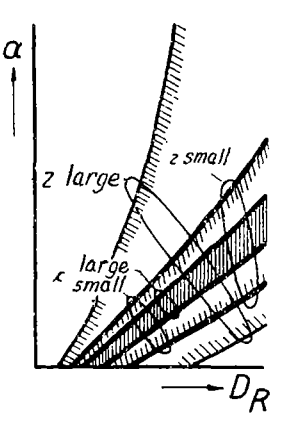
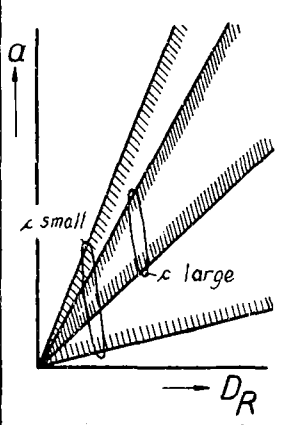
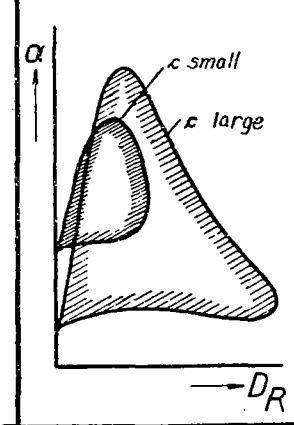
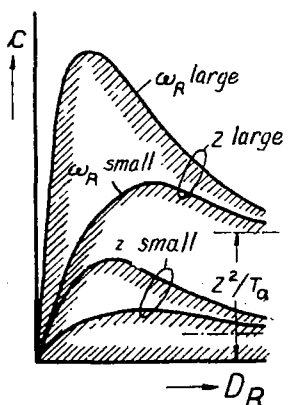
$$\partial \eta_f + \eta_f = \eta.$$

	Pressure control. $T_a = \text{const}$	rpm control. $T_a = \text{const}$	Temperature control	Automatic directional control $M, N = \text{const}$
Conjugate setting $\eta = a\varphi$				
Assumed fault-proof principle $\eta = a\varphi + c/\varphi dt$			Temperature regulating processes are so slow that the control inertia ∂ is negligible	
Conjugate rpm $\eta = c/\varphi dt$		Increasing oscillations.		Increasing oscillations.

Shaded range: range of damping.

Table V.- Regulator susceptible to vibration.

$$\eta_f'' + 2 D_R \omega_R \eta_f' + \omega_R^2 \eta_f = \eta.$$

	Pressure control. $T_\alpha = \text{const}$	rpm control. $T_\alpha = \text{const}$	Temperature control.	Automatic directional control $M, N = \text{const}$
Conjugate setting. $\eta = a\phi$			Temperature control processes slow enough to be negligible with respect to oscillation susceptibility of control.	
Assumed fault-proof principle $\eta = a\phi + c \int \phi dt$				
Conjugate rpm $\eta = c \int \phi dt$		Increasing oscillations.		Increasing oscillations.

Shaded range: range of damping.

Table VI.- Regulator with friction in detector system.

	Pressure control. $T_a, a \text{ const}$	rpm control. $T_a, a \text{ const.}$	Temperature control. $T_a, T_v \text{ const.}$	Automatic directional control $M, N \text{ const.}$
Conjugate setting $\eta = a \phi$				
	Assumed fault-proof principle Conjugate rpm $\eta = c \int \phi dt$		Increasing oscillations.	

Shaded range: range of damping.

Cross hatched zone denotes zone between the two approximations.

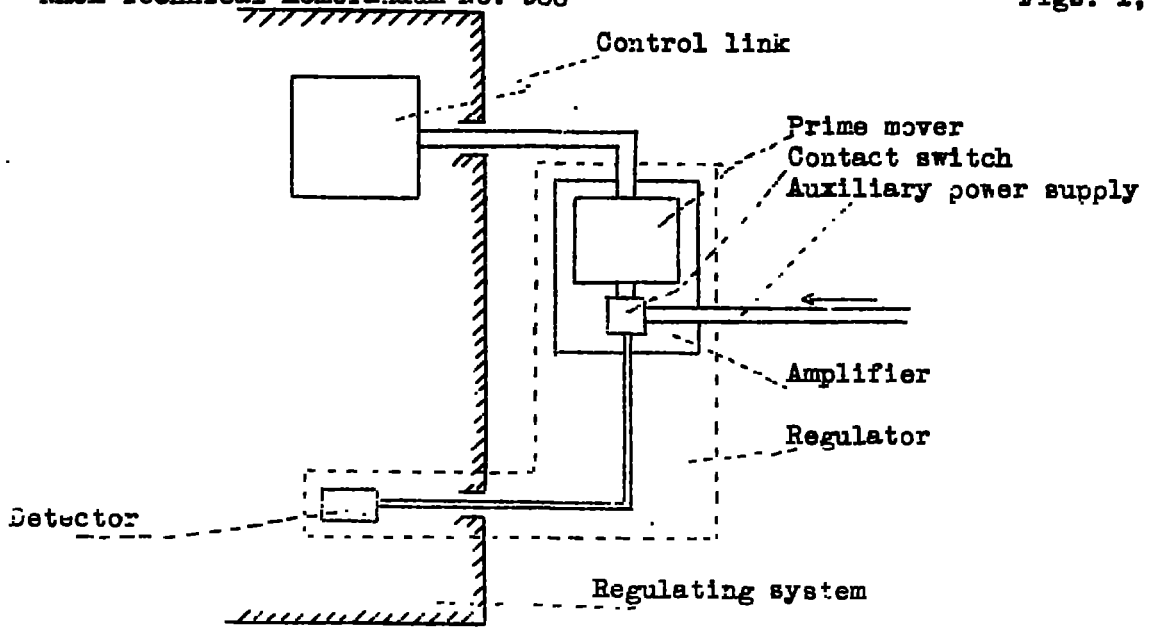


Figure 1.-Diagrammatic sketch of a regulated system

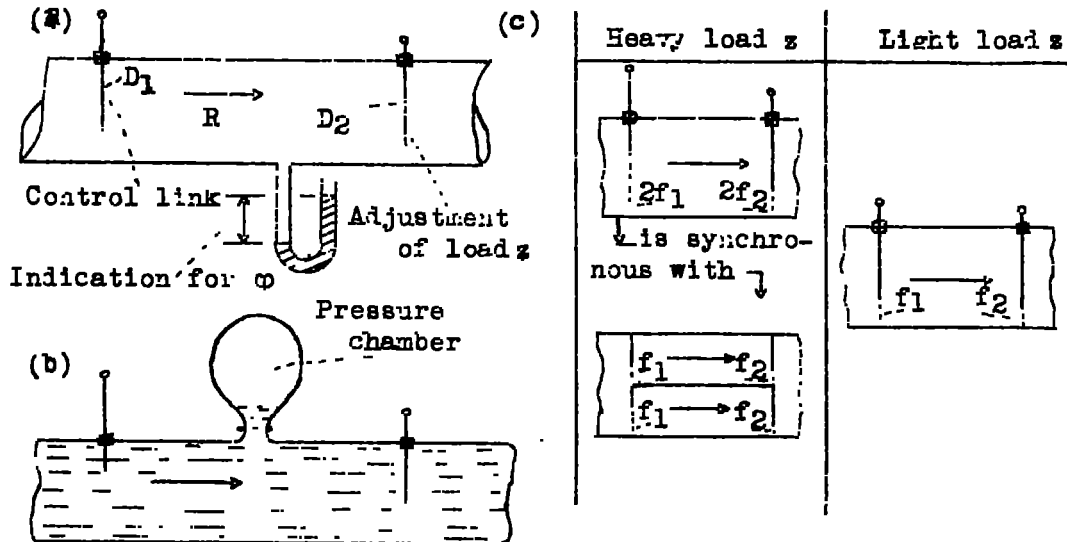


Figure 2.-Definition of pressure regulating process

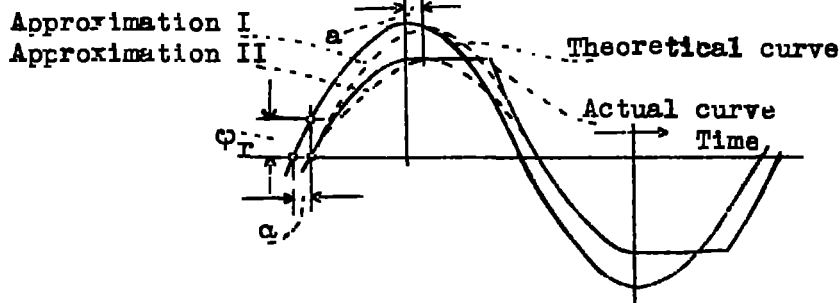


Figure 3.-Explanation of conditions on regulator with friction

NASA Technical Library



3 1176 01440 7168