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EFFECT OF WING LOADING, ASPECT RATIO, AND SPAN LOADING ON FLIGHT PERFORMANCES

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EFFECT OF WING LOADING, ASPECT RATIO, AND SPAN LOADING ON FLIGHT PERFORMANCES*

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SUMMARY

An investigation is made of the possible improvement in maximum, cruising, and climbing speeds attainable through increase in the wing loading. The decrease in wing area was considered for the two cases of constant aspect ratio and constant span loading. For a definite flight condition, an investigation is made to determine what loss in flight performance must be sustained if, for given reasons, certain wing loadings are not to be exceeded. With the aid of these general investigations, the trend with respect to wing loading is indicated and the requirements to be imposed on the landing aids are discussed.**

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*"Einfluss von Flächenbelastung, Flügelstreckung und Spannweitenbelastung auf die Flugleistungen." Luftfahrtforschung, vol. 16, no. 5, May 20, 1939, pp. 229-246. (From Thesis D83, accepted by the Technical High School of Berlin)

**In the course of the revision of this report dating from 1936, a number of foreign papers have been published which similarly take up the question of the increase in wing loading for constant aspect ratio (references 1 to 5). In these reports the equations for the optimum wing loadings for high speed are derived, also in general form, and a discussion is given of the difficulties in realizing these high wing loadings. The present report is concerned not only with the optimum wing loading for high-speed flight but also with the loss in flight performance conditioned by the unavoidable deviations from the optimum wing loading, as, for example, when for take-off and landing reasons certain wing loadings are not to be exceeded.

A. SYMBOLS USED

G, gross weight of airplane.

G', gross weight for reference condition $G/F = 100$ kg/m² and $\Lambda = 5$.

G_F, weight of wings.

N, engine power.

F, wing area.

$f_{ws} = c_{ws}\ F$, frontal drag flat plate area.

$f_{ws}' = f_{ws} - c_{wp}\ F = (c_{ws} - c_{wp})\ F$

$= c_{ws}'\ F$, parasite drag flat plate area.

ρ, air density.

η, propeller efficiency.

N/G, reciprocal of power loading.

$\eta f_{ws}'/G$, parasite drag loading.

$\Lambda = b^2/F$, wing aspect ratio.

G/b^2, span loading.

v, velocity in horizontal direction.

w, rate of climb.

$\Omega_\rho = \dfrac{\rho}{\rho_{4000}}$; $\Omega_\Lambda = \dfrac{\Lambda}{\Lambda = 5}$ etc.

$'$ denotes reference condition for $G/F = 100$ kg/m² and $\Lambda = 5$.

$*$ denotes condition at optimum wing loading.

B. PRELIMINARY REMARKS

I. Statement of the Problem

From the power distribution curves of a present-day high-speed airplane (fig. 1)* with a wing loading of 120 kg/m², it is found that at high speed the wing contributes about 50 percent of the total drag of the airplane. Since this wing drag consists for the most part of pure frictional drag, any decrease in its value can be obtained, except for smoothing of the surfaces, only by a reduction in the areas exposed to the air: i.e., for a given weight by an increase in the wing loading. In throttled engine flight, or what amounts to the same thing, in high-speed flight at high altitudes for which the proportion of the wing profile drag decreases as a result of the increase in the induced drag, the gain in flight performance resulting from the wing-area reduction will be small because while the part of the drag due to the wing reduction will be smaller the induced drag will increase with reduction in wing size.

In present-day airplanes, therefore, an increase in the maximum velocity through increase in the wing loading is apparently attainable, the amount of increase depending on the design data of the airplane. On the other hand, the increase in the wing loading above the usual present-day values will unfavorably affect the climb performance and the ceiling so that an optimum, compromise solution will have to be found between the contradicting requirements of maximum speed and climbing ability, depending on the purpose of the airplane in question. The object of the present investigation is to establish for what types of airplanes an increase in the wing loading is of particular advantage and up to what values this increase may be carried while still maintaining high speed and cruising flight with sufficient climb performance.

Further limits to the increase in the wing loading lie in the take-off and landing requirements of the airplane. In recent years various methods have been tested for take-off, so that a sufficiently short take-off run could be attained also for extremely high loading, for example, with the aid of catapults, cables, short-time boost power, etc. Assuming take-off aids with satisfactory characteristics will be developed in the future for still higher wing loadings of about 250 kg/m², the problem is

*Fig. 1 was taken from reference 6.

shifted to that of the safe control of the landing. Here the favorable circumstance enters: namely, that, due to the elimination of the fuel load and also of the useful load, wing loadings in landing are lower than in take-off. On the other hand, the external aids are no longer applicable in landing, so that the only possibility for shortening the landing run are the lowering of the landing speed through increase in the maximum lift of the wing and effective braking. Except for stationary curved flight, for which the diameter of the narrowest curve naturally increases with increasing wing loading, an increase in the wing loading for equal aspect ratio will react favorably on the maneuverability. According to Lachmann (reference 7), for equal flight speed and equal wing rolling moment coefficient, with decreasing span there is a decrease in the time required for carrying out a complete turn because the damping by the surfaces is reduced more rapidly than the accelerating rolling moments. This improvement in the handling qualities will be of advantage to military airplanes. In the case of commercial airplanes, however, which do not require any great maneuverability in flight, the reduction of the wing damping will have an unfavorable effect, particulary on the approach for a landing.

In the present investigation, we shall not for the moment consider the limits set on the wing-loading increase by take-off and landing characteristics and the changes in maneuverability brought about by an increase in the wing loading will not be taken into account, only those wing loadings being considered which are the optimum with regard to level and climbing flight.

II. Assumptions Made for the Computation

The wing loading of a given type of airplane may be increased by decreasing the wing area at constant wing aspect ratio or at constant span: i.e., equal span loading G/b^2, if the weight is kept constant (fig. 2). Constant span loading has the advantage that for equal airplane weight the induced drag is a function only of the flight speed. The limit for the wing loading increase is here set, however, not only by considerations of take-off and landing but also by the strength conditions of the very slender wings. It thus appeared to be more advantageous to consider the wing area reduction at constant aspect ratio and to introduce the latter as an independent variable. The designer is then immediately familiar with a design constant with which he is enabled to estimate rapidly the required strength of the wing. In order, however, to see what results are obtained with constant span loading, there is also briefly investigated the effect of increasing the wing loading at constant span. In changing the wing loadings, it may be assumed in the computation that either the tail surface area remains constant or that the tail surface, and hence the tail drag, varies in the same ratio as the wing area. In the latter case, the tail drag can be taken into account simply by a correspondingly proportional increase in the profile drag coefficient. In the following investigation in which only the profile drag coefficient c_{wp} is introduced, the latter may be considered as including the tail drag.

It is assumed in changing the wing and tail areas that no further drag and weight changes arise as a result of the change in wing loading: i.e., that the fuselage and nacelle drags, for example, are not affected by changes in the wing area. This assumption holds true with sufficient accuracy since the volume of the fuselage in the region of the maximum thickness is determined by the required loads and only by changes in the fuselage length or the interference effect between fuselage and wings and by changes in the parts of the nacelles projecting from the wings are deviations in the drag possible. The latter depend, however, to a large extent on the purpose of the airplane and are very difficult to take into account in a general way.* They are generally small in comparison with the total drag and moreover partly of opposite sign so that the error in the final result may be insignificant.

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*It is not entirely clear, for example, whether a given engine nacelle produces a greater drag when mounted on a deep or a narrow wing. The surface friction coefficient of the engine nacelle will be smaller in the case of the deep wing because the nacelle disappears farther into the wing. This frictional drag, however, with the present-day nacelles constitutes only the smaller portion, about 30 percent, of the total drag whereas the remaining principal portion is due to the pressure drag, particularly by the disturbance of the airfoil flow. How this pressure drag changes with variation of the wing chord is difficult to determine because, whereas in the case of the deep wing the disturbing nacelle parts projecting from the wing are smaller, the wing surface exposed to the flow is larger.

It was furthermore assumed that the profile-drag coefficient was the same, independent of the magnitude of the wing loading, so that no additional losses arise by the mounting of landing aids, as compared with the smooth wing. With clean streamlining, particularly with flaps having favorable hinge positions, as, for example, split flaps and Fowler wings, this required condition should readily be realized.

For the determination of the flight performance, particularly that of maximum rate of climb, and those magnitudes depending on it, as time of climb and ceiling, it is of considerable importance to know the changes in the gross weight of the airplane as a result of a change in the wing loading and wing aspect ratio. In order to determine these changes in weight, a detailed computation was carried out which is presented in the supplement. At this point there will only be pointed out the results of the computation which are represented in figure 3. This chart shows the relative change of the gross weight G'/G as a function of the wing loading, aspect ratio, and span loading, G' being the gross weight for an arbitrarily chosen reference state (wing loading $G/F = 100$ kg/m^2 and wing aspect ratio $b^2/F = 5$). Various degrees of fineness of wing structure and various sizes of the airplane were taken into account by the common parameter G_F'/G' which, for the reference state, denotes the ratio of weight of wing to weight of airplane. The curves of figure 3 thus show at a glance the effect of the weight of the wing for various wing sizes.

C. CHANGE IN WING LOADING FOR DEFINITE ASPECT RATIOS

I. Maximum and Cruising Speed

1. Example

The effect of a change in wing loading on the maximum speed will now be considered with the aid of an example of a typical airplane of 8,000 kg gross weight, engine power $2 \times 1{,}000$ hp., and having a maximum speed of about 450 km/h at 4 km altitude (fig. 4). If, for equal aspect ratio $\Lambda = 8$, the wing loading is increased beyond that of the initial loading of $G/F = 140$ kg/m^2, the maximum velocity at first increases appreciably, then at a slower rate, and finally at about 400 kg/m^2 the speed attains a maximum of about 480 km/h. On increasing the wing loading beyond this limiting value, the maximum speed again begins to drop.

This variation in the maximum speed with change in wing loading becomes understandable from a consideration of the distribution of the drag at the various wing loadings. Since with the initial loading of 140 kg/m², the induced drag forms only a very small portion of the entire drag, a decrease in the wing area is at first followed by a strong decrease in the profile drag whereas the increase in the induced drag is of secondary importance. As the wing loading is further increased, the induced drag approaches in value that of the profile drag until, finally, at the optimum condition the induced drag is equal to the profile drag. From this point on, further reduction in the wing causes the induced drag to exceed the profile drag, so that the maximum speed again drops. This result was considered first for constant gross airplane weight and secondly with account taken of the change in weight by the various wing sizes (fig. 4). It is found that the trend of the curve is essentially the same, so that in the consideration of the maximum speed an approximate computation with constant gross weight is sufficiently accurate for obtaining the effect of the wing loading.

In the case of the airplane considered above, the optimum of the wing loading is so flat that an increase in the wing loading up to the optimum value is not of advantage. An increase, for example, in the wing loading from 100 to 200 kg/m² gives a maximum speed increase of 10 percent; a further increase from 200 to 300 kg/m² gives a speed increase of only 3 percent and increasing the wing loading beyond 300 kg/m² to the optimum value of 400 kg/m² results in only a 0.6 percent further increase in the maximum speed.

For the airplane considered, it thus appears advantageous to develop take-off and landing aids that permit a wing loading of 200 to 250 kg/m². The gain in maximum speed for equal propulsive power as compared with the initial loading of 140 kg/m² amounts to about 27 km/h, corresponding to a 6 percent increase over the initial value.

2. Optimum Wing Loading for High-Speed and Cruising Flight

The problem is now to apply the results found in the previous section for a particular case to arbitrary airplanes.

The above considerations hold naturally not only for the flight at maximum speed but are also applicable to cruising speeds. In considering cruising flight, we may substitute the range ratio in place of the speed ratio, since for equal throttle setting of the engine an increase in the speed by a definite amount results in a proportionate increase in the range. If, for example, there is to be determined the optimum wing loading for cruising flight, it is necessary merely to substitute in the computation the engine power corresponding to the percent power rating used for cruising and the mean weight in flight.

From the power equation for level flight: Useful propulsive power = power of parasite drag and profile drag plus power of induced drag:

$$\frac{\eta\ N}{G} = \frac{\rho}{2}\ v_{max}{}^3 \left[\frac{f_{ws}'}{G} + \frac{c_{wp}}{G/F}\right] + \frac{2}{\pi \cdot \rho}\left(\frac{F}{b^2}\right)\frac{G}{F}\ \frac{1}{v_{max}} \qquad (1)$$

it follows that the principal variables are the power loading $\eta\ N/G$ and the parasite drag loading f_{ws}'/G if the altitude, wing aspect ratio $\Lambda = b^2/F$, and profile drag coefficient c_{wp} are considered as previously assigned constants. Differentiating the power equation with respect to G/F, there is obtained for constant gross weight, the condition for optimum wing loading with $\frac{\partial\ v_{max}}{\partial\ G/F} = 0$:

$$\frac{G/F^*}{\rho/2\ v^{*2}_{max}} = c_a^* = \sqrt{\pi\ c_{wp}\ \Lambda} \qquad (2)$$

which is the lift coefficient of the wing at optimum G/F. (Magnitudes denoted by * refer to the condition with optimum wing loading.)

This equation states that the wing loading in the most favorable case may be increased until the airplane flies at the maximum speed with the indicated lift coefficient c_a^*. The latter depends only on the profile-drag coefficient and on the wing aspect ratio. It corresponds to the condition of best L/D ratio as may be shown by a simple computation. Thus, in the polar diagram, it corresponds to the point of contact of the tangent to the wing polar (fig. 5). For this flight condition it is known that the induced drag is equal to the profile drag of the wing.

The state at the optimum wing loading is thus not identical with the state of flattest glide of the airplane because the lift coefficient for the optimum glide lies always considerably higher than the value corresponding to the maximum speed. This means that if, for example, an airplane at any altitude flies with the best L/D ratio, that is the state at which the maximum economy is attained for the constant wing size, this state for the flight speed under consideration is not the most economical if the wing size for equal aspect ratio may be considered as variable. It would be possible to attain the same flight speed with less propulsive power, hence greater economy if the wing area were increased to the extent indicated above. It may be shown that the ratio of the propulsive power at the best gliding angle $N_{\epsilon_{best}}$ to the propulsive power at the optimum wing loading and equal flight speed N_{G/F^*} is given by

$$\frac{N_{G/F^*}}{N_{\epsilon_{best}}} = \frac{c_{ws}'/c_{wp} + 2\sqrt{1 + c_{ws}'/c_{wp}}}{2(1 + c_{ws}'/c_{wp})}; \quad c_{ws}' = \frac{f_{ws}'/G}{G/F\epsilon_{best}} \qquad (3)$$

that is, at a ratio $c_{ws}'/c_{wp} = 2$, corresponding approximately to a high-speed airplane, the propulsive power for equal flight speed could be reduced by about 10 percent and thus the range increased by the same amount.

The result may also be expressed in a general form, as follows: For a given flight speed, the minimum power requirement is possessed by that airplane whose wing loading has the value obtained from equation (2)

$$G/F^* = \frac{\rho}{2} v^{*2} \sqrt{\pi \; c_{wp} \; \Lambda}$$

It is immaterial whether the airplane in question is of aerodynamically high quality with small propulsive power or an aerodynamically poor airplane with correspondingly higher propulsive power. For an airplane with 300 km/h maximum speed at 4 km altitude with a wing aspect ratio of $\Lambda = 8$, an optimum wing loading will therefore be obtained of 145 kg/m², and on doubling the speed at the same altitude a wing loading of 580 kg/m². An increase in the wing loading is therefore of particular advantage for high speed airplanes while for slow airplanes no appreciable gain is to be expected.

The above equation shows the effect of the density, aspect ratio, and profile drag on the optimum wing loading. For a practical application, however, it is of advantage to know the effect of the fundamental magnitudes of the airplane design as power loading and drag ratio. Figure 6 shows the results of such a computation, the optimum wing loading G/F^* being plotted as a function of the power loading $\eta\, N/G$ and the parasite drag to airplane weight ratio f_{ws}'/G. The chart was drawn for the altitude $H = 4$ km, the profile drag coefficient $c_{wp} = 0.01$, and the aspect ratio $\Lambda = 5$. In order to be able to use the diagram, however, for arbitrary conditions, the power equation was transformed with the aid of coefficients which were defined as follows:

$$\Omega_\rho = \frac{\rho}{\rho_{4000}};\quad \Omega_\Lambda = \frac{\Lambda}{\Lambda = 5};\quad \Omega_{c_{wp}} = \frac{c_{wp}}{c_{wp} = 0.01}$$

The transformed power equation then reads

$$\frac{\eta\, N}{G}\,\frac{\Omega_\rho^{1/2}\,\Omega_\Lambda^{3/4}}{\Omega_{c_{wp}}^{1/4}} = \frac{\rho_{4000}}{2}\left(v_{max}\,\Omega_\rho^{1/2}\,\Omega_\Lambda^{1/4}\,\Omega_{c_{wp}}^{1/4}\right)^3$$

$$\times\left[\frac{f_{ws}'}{G}\,\frac{1}{\Omega_{c_{wp}}} + \frac{c_{wp} = 0.01}{G/F}\right]$$

$$+\frac{2}{\pi\,\rho_{4000}}\,\frac{1}{\Lambda = 5}\,\frac{G}{F}\,\frac{1}{v_{max}\,\Omega_\rho^{1/2}\,\Omega_\Lambda^{1/4}\,\Omega_{c_{wp}}^{1/4}}$$

Written in the above manner, it may be seen that the chart may be made generally applicable if the following fictitious constants are introduced:

Fictitious power loading $\dfrac{\eta\, N}{G}\,\dfrac{\Omega_\rho^{1/2}\,\Omega_\Lambda^{3/4}}{\Omega_{c_{wp}}^{1/4}}$

Fictitious parasite drag ratio $f_{ws}'/G \times 1/\Omega_{c_{wp}}$

Fictitious maximum speed $v_{max}\,\Omega_\rho^{1/2}\,\Omega_\Lambda^{1/4}\,\Omega_{c_{wp}}^{1/4}$

From the diagram on figure 6, it is therefore possible to read off for any airplane the optimum wing loading under the flight conditions under consideration. It is seen that the optimum wing loading is greater the greater the ratio of propulsive power to airplane weight for equal f_{ws}'/G or the greater the aerodynamic efficiency of the airplane for equal $\eta\ N/G$, i.e., the smaller f_{ws}'/G.

On the same diagram, drawn to reduced scale in figure 7, are indicated for a number of recent airplanes the power loadings and drag ratios for full power flight at critical altitude, the aspect ratios of the various types being held constant. For these airplanes, the optimum wing loading increases up to values of about 400 kg/m² and tends to still larger values with further aerodynamic refinement. Since wing loadings of this order of magnitude show small promise of realization in the near future and moreover the optimum of the maximum speed is extremely flat (see sample computation, fig. 4), the next step is the investigation of the problem of how much may the wing loading be reduced with respect to the optimum value and still not permit the loss in speed to exceed 1 or 2 percent.

3. Wing Loading Allowing for a Definite Loss in Speed

Let the power equations for a definite airplane corresponding to an arbitrary wing loading G/F and to the optimum wing loading G/F^* be subtracted, keeping the values $\eta\ N/G$ and f_{ws}'/G unchanged.

There is then obtained the relation

$$G/F^* = \frac{\rho}{2}\, v^{*2} \sqrt{\pi \Lambda\ c_{wp}}$$

which, after some transformations, becomes

$$\frac{G/F}{G/F^*} = \frac{v_{max}}{v_{max}^*} \left\{ 1 + \frac{K}{2}\left(1 - \frac{v_{max}^3}{v_{max}^{*3}}\right) \pm \sqrt{\left[1 + \frac{K}{2}\left(1 - \frac{v_{max}^3}{v_{max}^{*3}}\right)\right]^2 - \frac{v_{max}}{v_{max}^*}} \right\} \qquad (4a)$$

where K is the ratio of the parasite drag to the profile drag at constant wing loading:

$$K = c_{ws}'/c_{wp} = f_{ws}'/G \; G/F^* \; 1/c_{wp} \qquad (4b)$$

The parameter K is easily determined for each airplane since the optimum wing loading is to be considered as known from figure 6.

The above relation between the decrease in wing loading and decrease in speed is plotted in figure 8. The sensitivity of each of the types with respect to deviations from the optimum wing loading may be seen to vary greatly. It is greater the smaller the value of the parameter $F_{ws}'/G \; G/F^* \; 1/c_{wp}$. How will this sensitivity to the proper choice of the wing loading change with the further development of airplanes? To answer this question, it is convenient to transform this parameter somewhat. With the relation $G/F^* = \frac{\rho}{2} v^{*2}_{max} \sqrt{\pi \, c_{wp} \, \Lambda}$ and the fact that at the optimum wing loading the induced drag is equal to the profile drag of the wing (see sec. CI2) the expression for the parameter K, making use of the power equation, may be transformed into the following:

$$K = \frac{f_{ws}'}{G} \frac{G}{F^*} \frac{1}{c_{wp}} = \frac{f_{ws}'}{G} v^{*2}_{max} \frac{\rho}{2} \times \sqrt{\frac{\pi \Lambda}{c_{wp}}} = \frac{\eta \; N/G}{v_{max}^* \sqrt{\frac{\pi \Lambda}{c_{wp}}}} - 2 \qquad (5)$$

If the maximum speed therefore, for equal aerodynamic efficiency and equal altitude, is increased by increasing the propulsive power, the value of K increases, i.e., the effect of the optimum choice of wing loading becomes less. If, however, the maximum speed for equal propulsive power is increased through improvement in the aerodynamic efficiency or through increasing the altitude, the value of K becomes less, so that the sensitivity of the airplane with respect to the proper choice of the wing loading becomes greater. In future development, however, we may expect a further refinement in the aerodynamic design as well as an increase in the flight altitude so that the point of view of suitable choice of wing loading will gain in importance.

For the airplanes of 1935 to 1937, the possible gain in maximum speed by increase in the wing loading amounts, according to figure 9, to as much as 10 percent. For the types Fw 200 and He 70, an increase in the wing loading thus appears to be of particular advantage, whereas for the types Ju 86, Do 17, and Ha 139 (twin-float seaplane) any further increase in the wing loading above the present values at constant aspect ratio promises no appreciable gain. Summarizing, we may therefore say: Through an increase in the wing loading above the present values in the case of high-speed airplanes, particularly airplanes of high aerodynamic efficiency, a considerable gain in speed is attainable. In the case of low-speed airplanes, however, the possible gain in speed through an increase in wing loading is small.

4. Effect of the Aspect Ratio

The above considerations hold for the particular case where the wing aspect ratio remains constant as the wing loading is increased. Since with increased wing loading, however, the induced drag becomes of increasingly greater importance and the optimum is finally determined by the interrelation between the induced and profile drags, it appears advantageous to increase the aspect ratio with increasing wing loading. As follows, however, from considerations of the optimum lift coefficient, the optimum wing loading, with increasing aspect ratio, is shifted toward higher values so that it becomes increasingly difficult to realize the optimum wing loading in practice. On the other hand, for the same area, the weight of the wing increases with increasing aspect ratio, so that the possible gain is again reduced.

In order to bring out the relations more clearly, there has been plotted on figure 10 the ratio of the maximum speed at each optimum wing loading for various aspect ratios. The change in the weight of the wing was estimated from the approximate relations of figure 3 first for a wing weight of 14 percent of the total at the initial condition. The results were then applied to different wing weights in the following manner:

Figure 3 may be applied to arbitrary wing weights by a simple shifting of the reference point. Denoting the wing loading at $G_F'/G' = 0.14$ by $(G/F)_o$ and the corresponding flight weight ratio by $(G'/G)_o$, then for any ratio $k_2(G/F)_o$ there corresponds the flight weight ratio $k_1(G'/G)_o$ where the factors k_1 and k_2 are constants for each wing weight, the values of which may be taken from figure 3. The power equation then reads

$$\frac{\eta N}{G'} k_1 \left(\frac{G'}{G}\right)_o = \frac{\rho}{2} v_{max}^3 \left[\frac{f_{ws}'}{G'} k_1 \left(\frac{G'}{G}\right)_o + \frac{c_{wp}}{k_2 (G/F)_o}\right] + \frac{2}{\pi \rho} \frac{1}{\Lambda} k_2 \left(\frac{G}{F}\right)_o \frac{1}{v_{max}}$$

or

$$\left(\frac{\eta N}{G} \frac{k_1}{\sqrt{k_2}}\right) \left(\frac{G'}{G}\right)_o = \frac{\rho}{2} \left(\frac{v_{max}}{\sqrt{k_2}}\right)^3 \times \left[\left(\frac{f_{ws}'}{G'} k_1 k_2\right) \left(\frac{G'}{G}\right)_o + \frac{c_{wp}}{(G/F)_o}\right] + \frac{2}{\pi \rho} \frac{1}{\Lambda} \frac{\sqrt{k_2}}{v_{max}} \left(\frac{G}{F}\right)_o$$

The application to different wing weights can therefore be made by introducing the following fictitious characteristics:

Power loading $\eta N k_1 / \sqrt{k_2}$

Parasite drag ratio $f_{ws}'/G' k_1 k_2$

Maximum speed $v_{max}/\sqrt{k_2}$

It is to be noted in conclusion that the results according to the above equation are first obtained as a function of the wing loading $(G/F)_o$, so that the computed wing loadings $(G/F)_o$ must be converted into the actual values with the aid of the relation $G/F = k_2 (G/F)_o$. The transformations described above can be carried out in a simple manner by a change in scale as shown, for example, in figure 10.

It may be seen that the airplanes with large propulsive power, that is, for example, racing planes, pursuit planes, high-speed planes of small useful load and range, with a power loading of more than 0.2 to 0.4 hp./kg are very unsensitive with respect to a change in the choice of aspect ratio. In the case of low-powered airplanes, however, as, for example, the recent high-speed transport planes of the type Fw 200 or the Douglas DC4 with a value of $\eta N/G$ of about 0.1 hp./kg and less in cruising flight, or in case of long-range airplanes with power loadings as low as 0.05 hp/kg, a considerable gain is attainable through increase in the aspect ratio which continues even beyond aspect ratios of 15. The increase in speed thus gained is the more marked the greater the aerodynamic efficiency of the airplane.

On consideration of figure 10, it is to be observed, however, that to a high aspect ratio there necessarily corresponds a high wing loading. It will therefore become increasingly difficult at high aspect ratios actually to attain the above speed gain. These relations will be brought out in the following for two airplane types, a moderate range high-speed airplane and a long-range high-speed airplane.

a) Moderate range airplane.- On figure 11 is shown the effect of aspect ratio and wing loading on the maximum speed for an airplane of 8,000 kg gross weight with two engines of 1,000 hp. developing at 4 km altitude a maximum speed of about 450 km/h. With the present wing loadings of about 150 kg/m², it is quite immaterial within wide limits what aspect ratio is chosen. Only with further increase in the wing loading to 200 and 300 kg/m² is there a slight displacement of the optimum aspect ratio toward values of $\Lambda = 10$. For the airplane under consideration, it is therefore of advantage to strive for wing loadings of the order of magnitude of 200 to 300 kg/m² at an aspect ratio of 9.

b) Long-range high-speed airplanes.- For a definitely long-range airplane of about 20,000 kg gross weight with 4 engines of 720 hp. each, which at 6 km altitude at 60 percent rated power develop a cruising speed of 360 km/h (maximum speed at 6 km altitude 430 km/h), figure 12 shows the corresponding relations. For this airplane, the attainable wing aspect ratio is of prime importance. In order to be able to utilize wing loadings of the order of magnitude of 200 kg/m², which still gives a considerable gain in the maximum speed, the wing aspect ratio must be raised at least to the value 12.

To attain the maximum economy in cruising flight for this airplane, wing loadings of 200 kg/m² with aspect ratios of about 12 should be striven for.

5. Effect of Profile-Drag Coefficient

In the relations given above, various profile-drag coefficients are included in the coefficients $\Omega_{c_{wp}}$. In order to bring out this effect more clearly, there was investigated for a typical medium-range high-speed airplane the dependence of the optimum wing loading on the profile-drag coefficient. It was found that the gain through increase in the wing loading was smaller the lower the drag coefficient of the wing. On improving the profile drag coefficient, for example, from 0.01 to 0.006 corresponding to the pure frictional drag of aerodynamically smooth surfaces, the wing loadings may be made about 18 percent lower with equal approximation to the optimum value.

At unusually high drag coefficients of, for example, 0.15, such as occur for suction wings with large thickness ratios, a further increase in the wing loading by 18 percent over the values at $c_{wp} = 0.01$ is still of advantage.

6. Effect of Altitude

From the equation for the optimum wing loading

$$G/F^* = \frac{\rho}{2} v_{max}^{*2} \sqrt{\pi\ c_{wp}\ \Lambda}$$

it follows immediately that for equal maximum speed the loading varies in proportion to the air density. Thus, on increasing the altitude, for example, from 3 to 12 km, the optimum wing loading drops to about one-third of the original value.

The above relation is shown graphically on figure 13 for a typical medium-range airplane with aspect ratio $\Lambda = 8$. With increase in the altitude, it has been assumed for simplicity that the weight of the power plant unit and the frontal drag do not increase with increasing altitude as is actually the case on passing to very high altitudes. Through the neglect of these changes, the speeds of the high altitude airplanes are overestimated as compared with the low-flying airplanes, so that the decrease in the optimum wing loading with altitude is actually even stronger. On increasing the altitude of this airplane from ground level to 8 km, the optimum wing loading drops to 70 percent and at 16 km altitude which corresponds approximately to the limiting critical altitude attainable with exhaust turbine drive, the wing loading drops to even 40 percent of the ground level value. With this type of airplane (medium-range high-speed airplane), it is therefore desirable at altitudes of about 4 km to have wing loadings of about 200 kg/m², since a further increase gives only very slight improvement in the performance. At altitudes of about 16 km there is hardly any justification for carrying the wing loading beyond 120 kg/m². Even with a simultaneous increase in the aspect ratio from 8 to 12, it would be of no advantage above 16 km altitude to go beyond wing loadings of 150 kg/m².

In order to be able to estimate from this figure the possibilities for development by refinement in the aerodynamic design of the airplane, another computation was carried out in which the parasite drag f_{ws}' was reduced to half the value and the profile drag coefficient to the value $c_{wp} = 0.06$ corresponding approximately to the lower limiting values for the case of completely smooth surfaces with freedom from flow separation. It may be seen that by this aerodynamic refinement the entire diagram is shifted to loadings of about 20 percent higher values. From this plot it clearly appears that at high altitudes the airplane is considerably more sensitive to the optimum choice of the wing loading than at low altitudes as was already concluded from the considerations in section CI3 for low-powered airplanes.

II. Rate of Climb

The maximum rate of climb was within restricted limits investigated in the same manner as the maximum speed, since the climb rate of the airplane is also of importance and moreover may be considered as a measure for the length of take-off run and ceiling altitude.

1. Example

Figure 14 shows the maximum rate of climb at 4 km altitude plotted as a function of the wing loading at equal aspect ratio for the same airplane used in investigating the effect of the wing loading on the maximum speed. The maximum rate of climb was taken to be that which for the best glide angle is obtained as the difference of the thrust horizontal speed w_h and the sinking speed w_s. If the change in the gross weight with change in wing loading is at first neglected (dotted curves) the rate of climb decreases continuously with increasing wing loading up to $w = 0$. On taking the change in weight into account, however, the curve obtained is fundamentally different. The combination of the effects of increased weight and decreased sinking velocity leads to a flat maximum at about $G/F = 100\ kg/m^2$.

2. Optimum Wing Loading for Climbing Flight

For the determination of the optimum wing loading in climb, it is not permissible, according to the above sample computation, to make the simplifying assumption, as was done in investigating the maximum speed, that the gross weight to a first approximation may be considered constant. By taking account of the complicated relations between wing weight and wing loading and aspect ratio, the solution of the problem can be found only graphically.

In agreement with the performance computation of Schrenk (reference 8), it was found that the maximum climb speed is that evaluated at the optimum L/D of the airplane.

Splitting the frontal drag coefficient c_{ws} into the two components of profile drag coefficient c_{wp} and parasite drag coefficient c_{ws}', there is obtained for the sinking speed w at the best L/D ratio

$$w_{s_{best}} = 2\left(\frac{1}{\pi\Lambda}\right)^{3/4}\left(\frac{G/F}{\rho/2}\right)\left(\frac{f_{ws}'}{G'}\frac{G'}{G}\frac{G}{F} + c_{wp}\right)^{1/4}$$

With the horizontal propulsive velocity $w_h = \eta\ N/G = \eta\ N/G'\ G'/G$, the climb speed at best glide angle is then

$$w = \frac{\eta\ N}{G'}\frac{G'}{G} - 2\left(\frac{1}{\pi\Lambda}\right)^{3/4}\left(\frac{G/F}{\rho/2}\right)^{1/2}\left(\frac{f_{ws}'}{G'}\frac{G'}{G}\frac{G}{F} + c_{wp}\right)^{1/4} \qquad (6)$$

The above equation is now evaluated for various wing loadings and the optimum value G/F^* of the wing loading determined by graphical methods. As the reference gross weight G', there was here taken the weight at the aspect ratio $\Lambda = 5$ and wing loading $G/F' = 100\ kg/m^2$. Figure 15 gives the results of the graphical computations for the optimum wing loadings G/F^* and the optimum climb speeds

w_{max}^* for the aspect ratio $\Lambda = 5$ as a function of the two principal parameters $\eta N/G'$ and f_{ws}'/G'. The altitude was taken as 4,000 meters and the profile drag coefficient of the wing as $c_{wp} = 0.012$ corresponding to the average values in climbing flight.

In order to make the above diagram applicable also to arbitrary altitudes and profile drag coefficients, equation (6) was transformed with the aid of the coefficients $\Omega_\rho = \frac{\rho}{\rho_{4000}}$ and $\Omega_{c_{wp}} = \frac{c_{wp}}{c_{wp} = 0.012}$ into the following form

$$w \frac{\Omega_\rho^{1/2}}{\Omega_{c_{wp}}^{1/4}} = \frac{\eta N}{G'} \frac{\Omega_\rho^{1/2}}{\Omega_{c_{wp}}^{1/4}} \frac{G'}{G} - 2\left(\frac{1}{\pi \Lambda}\right)^{3/4}$$

$$\times \left(\frac{G/F}{\rho_{4000}/2}\right)^{1/2} \left(\frac{f_{ws}'}{G'} \frac{1}{\Omega_{c_{wp}}} \frac{G'}{G} \frac{G}{F} + c_{wp_{0.012}}\right)^{1/4}$$

Written in the above manner, it may be seen that the curves may be immediately made generally applicable on introducing the following fictitious coefficients:

Power loading $\frac{\eta N}{G'} \frac{\Omega_\rho^{1/2}}{\Omega_{c_{wp}}^{1/4}}$

Parasite drag loading $f_{ws}'/G' \; 1/\Omega_{c_{wp}}$

Maximum rate of climb $w^* \frac{\Omega_\rho^{1/2}}{\Omega_{c_{wp}}^{1/4}}$

By a simple change of scale different ratios of wing weight to total weight at the initial condition ($\Lambda' = 5$, $G/F' = 100$ kg/m^2) may be obtained from the relations of figure 3. In agreement with the results of the relations for high speed, the optimum wing loadings corresponding to climb increase with increase in the power loading $\eta N/G'$ and aerodynamic efficiency, i.e., the smaller f_{ws}'/G' increase in profile drag coefficient c_{wp} decrease in altitude and increase in ratio of wing weight to total weight. It is to be noted that the optimum wing loadings for climb are in general only a fraction of the optimum wing loadings for high speed flight.

3. Effect of Aspect Ratio

Figure 15 for the determination of the optimum wing loading is valid only for the aspect ratio $\Lambda = 5$. Computation for arbitrary wing aspect ratios in the range between 3 and 15 gave the result that to an accuracy of 2 percent the following rule may be applied for conversion to any aspect ratios

$$G/F_{\Lambda}{}^{*} = \Omega_{\Lambda}\, G/F^{*}{}_{\Lambda=5}\,;\quad \Omega_{\Lambda} = \frac{\Lambda}{\Lambda = 5}$$

that is, an increase in the aspect ratio to double the value results in an increase in the optimum wing loading to double the original value. The corresponding displacements of the maximum rate of climb at the optimum wing loadings are plotted in figure 16 as a function of the aspect ratio. The differences at optimum wing loading are only small, being of the order of magnitude of 0.5 m/s. With respect to the rate of climb, it is therefore practically immaterial what aspect ratio is chosen, provided that care is taken to see that the optimum wing loading is realized with this aspect ratio. It may be remarked further that the optimum of the rate of climb is not obtained at about the same span loading G/b^2 but with increasing power loading tends toward smaller values of the span loading.

The effect of a change in aspect ratio on the rate of climb is thus mainly to shift the optimum of the rate of climb toward the region of wing loadings which had been found to be favorable for the optimum maximum speed, i.e., in general toward higher values.

4. Losses in Climb Performance through Deviations from the Optimum Wing Loading

Since it will not be possible in practice generally to attain the optimum wing loadings with respect to climb, the question is here investigated: namely, what losses in the rate of climb will be incurred by definite deviations from the optimum wing loading. Since it is usually sufficient to be able to estimate the losses in rate of climb only approximately, generally applicable mean curves were worked out for the variation of rate of climb with wing loading. These mean curves (fig. 17) apply accurately to parasite drag ratios of $f_{ws}/G' = 0.1 \times 10^{-3}$ m²/kg, aspect ratios of $\Lambda = 8$ and wing weight ratios of $G_F'/G' = 0.14$. For airplanes with considerably deviating characteristics, a maximum error of ±4 percent in the rate of climb was permitted through this simplification for the case that the wing loading was changed to half or double the value of the optimum wing loading.

Parallel with the considerations on the maximum speed, the rate of climb is found to be the more sensitive to the proper choice of the wing loading the smaller η N/G'.

5. Comparison of the Optimum with the Designed Wing Loadings of Present-Day Airplanes

In present-day airplanes, optimum wing loadings for climb, according to figure 15, are of the order of magnitude of 60 to 80 kg/m², so that in general the airplanes have already exceeded the wing loading optimum and with further increase in the wing loading the rate of climb and therefore also the take-off and ceiling will necessarily be impaired. This tendency is still more clearly indicated in figure 17 which shows the ratio of the rate of climb to the optimum obtainable as a function of the wing loading and holds for all aspect ratios. With the exception of the Fw 200, which, on account of its high aspect ratio, deviates from the other types, all the airplanes shown have already exceeded the optimum wing loading up to about 1.6 times the amount.

D. CHANGES IN WING LOADING AT DEFINITE SPAN LOADINGS

Having investigated the effect of the wing loading at equal aspect ratio, let us now briefly consider the results obtained if the assumption of constant span instead of constant aspect ratio is made. By introduction of this parameter, the results already obtained on the assumption of constant aspect ratio are not naturally extended in any direction but a somewhat different presentation is obtained of the same relations.

When the wing area is reduced, with the span constant, the profile drag on the one hand decreases while the induced drag for constant flight speed remains the same if the changes in weight resulting from the change in wing dimensions are neglected. The optimum wing loading for this case is therefore obtained for an infinitely large loading: i.e., when the wing at constant span shrinks to a lifting line.

If the changes in weight are taken into account, however, the optimum wing loading is shifted toward the region of finite wing loading. From the sample computation on figures 4 and 14, it can be concluded that the essential character of the curve of maximum speed is only slightly affected by the changes in weight, so that, in agreement with the previous investigations of the effect of the wing loading on the maximum speed, the computation based on the assumption of constant gross weight of airplane is sufficient.

I. Maximum and Cruising Speeds

1. Limiting Speed for Given Span Loading

The power equation (1) is transformed so that, instead of the aspect ratio, the span loading G/b^2 appears, as follows:

$$\frac{\eta N}{G} = \left(\frac{f_{ws}'}{G} + \frac{c_{wp}}{G/F}\right)\frac{\rho}{2} v_{max}^{3} + \frac{2}{\pi \rho}\left(\frac{G}{b^2}\right)\frac{1}{v_{max}}$$

The maximum speed v_{max}^* of an airplane with given wing loading therefore occurs for $G/F \longrightarrow \infty$ and is determined by the following equation:

$$\frac{\eta N}{G} = \frac{\rho}{2} v_{max}^{*3} \frac{f_{ws}'}{G} + \frac{2}{\pi \rho} \frac{G}{b^2} \frac{1}{v_{max}^*}$$

The above equation was first solved for the altitude $H = 4$ km and a certain initial span loading $G/b^2 = 20$ kg/m² and the results generalized with aid of the coefficients

$\Omega_\rho = \dfrac{\rho}{\rho_{4000}}$ and $\Omega_{G/b^2} = \dfrac{G/b^2}{(G/b^2)_{20}}$. In a general form,

the power equation may be written as follows:

$$\frac{\eta\ N}{G}\ \frac{\Omega_\rho^{1/2}}{\Omega_{G/b^2}^{1/2}} = \frac{\rho_{4000}}{2}\ \frac{f_{ws}'}{G}\ \Omega_{G/b^2}\ \left(v_{max}^*\ \frac{\Omega_\rho^{1/2}}{\Omega_{G/b^2}^{1/2}}\right)^3$$

$$+ \frac{2}{\pi\ \rho_{4000}} \left(\frac{G}{b^2}\right)_{20}\ \frac{1}{v_{max}^*\ \Omega_\rho^{1/2}/\Omega_{G/b^2}^{1/4}} \qquad (7)$$

The results of this computation are presented in figure 18, which shows the limiting speed v_{max}^* as a function of the power loading $\eta\ N/G$ and parasite drag ratio f_{ws}'/G at infinitely large wing loading. This limiting speed is naturally greater the greater the power loading and the smaller the parasite drag of the airplane.

2. Wing Loadings with Certain Losses in Speed

The maximum speed v_{max}^* at infinitely large wing loading is in general of slight significance. Much more important is the question: Up to what values is it necessary to increase the wing loading to obtain a given percent of the optimum speed, say, for example, 98, keeping the span constant. Assuming for this purpose that the wing loading is to be determined for a given degree of approximation to the optimum value v_{max}/v_{max}^*, the power equation for this condition becomes

$$\frac{\eta\ N}{G} = \frac{\rho}{2}\ \frac{f_{ws}'}{G}\ v_{max}^{*3} \left(\frac{v_{max}}{v_{max}^*}\right)^3$$

$$\times \frac{f_{ws}'/G + c_{wp}/G/F}{f_{ws}'/G} + \frac{2}{\pi\ \rho}\ \frac{G}{b^2}\ \frac{1}{v_{max}^*}\ \frac{v_{max}^*}{v_{max}}$$

$$= W_s^*\ \frac{f_{ws}'/G + c_{wp}/G/F}{f_{ws}'/G} \left(\frac{v_{max}}{v_{max}^*}\right)^3 + W_i^*\ \frac{v_{max}^*}{v_{max}}$$

where

W_s^* is frontal drag for infinitely large wing loading.

W_i^*, induced drag for infinitely large wing loading.

Subtracting from the above equation the corresponding relation for infinitely large wing loading $\eta N/G = W_s^* + W_i^*$, we obtain, after some transformation;

$$G/F \frac{f_{ws}'/G}{c_{wp}} = \frac{(v_{max}/v_{max}^*)^3}{w_i^*/w_s^* \, (1 - v_{max}^*/v_{max}) - (v_{max}/v_{max}^*)^3 + 1} \quad (8)$$

The parameter W_i^*/W_s^* in the above equation, i.e., the ratio of the induced drag to the frontal drag at given span loading and infinite wing loading is known for a definite airplane from equation (7) and for rapid computation has been plotted in figure 18 as a function of the power loading and parasite drag ratio.

The evaluation of equation (8) is shown in figure 19. The deviation of the curves from one another are extremely small within the practical range. Assuming $W_i^*/W_s^* = 0.02$ as lower limit and 0.4 as upper limit, a mean curve may be used with sufficient accuracy. From this a simple relation is obtained, which shows directly the loss in maximum speed with respect to the maximum value v_{max}^* as a function of the wing loading (fig. 20).

There are thus confirmed the results obtained in section CI2 and CI3: namely, that high wing loadings must be sought particularly for aerodynamically high quality airplanes, whereas for aerodynamically less efficient airplanes the attainable gain through increase in the wing loading at constant span remains small. These curves are with good approximation practically independent of the span loading chosen so that in the simplest manner they quickly show the possible gain through increase in the wing loading.

With the exception of this latter (fig. 20) the foregoing derived relations hold only for the case where the span loading remains constant. The question then arises whether this accidental span loading is also the optimum for the state of flight considered. This question corresponds, however, to that with regard to the optimum aspect ratio, discussed in detail in section CI4, to which the reader may therefore be referred.

D. APPLICATION OF THE RELATIONS DERIVED TO VARIOUS AIRPLANES

The above considerations have shown that the maximum speed on the one hand and the rate of climb, ceiling, and take-off on the other impose entirely different requirements on the wing size and shape. For any particular case therefore, an optimum compromise solution among the contradicting requirements must be found. In what follows the combined effects of wing loading and aspect ratio on the maximum and cruising speeds and rate of climb will be brought out for several typical airplanes: namely, a long-range high-speed airplane, a medium-range high-speed airplane, and a short-range high-speed airplane. For a complete discussion, it would naturally be necessary to carry out the investigation for different altitudes. For simplicity, however, this was not done and the relations only investigated for the most important operating altitudes.

I. Determination of the Characteristics of Typical Airplanes

Figure 21 shows the most important design magnitudes: namely, the power loading η N/G, and parasite drag ratio f_{ws}'/G for a few types of the years 1935 to 1937. It may be remarked that in spite of the strongly varied power loading the lower limit of the parasite drag ratio fluctuates about the value $0.1 \times 10^{-3} m^2/kg$. A further investigation with regard to the drag distribution of the most important airplanes considered gave the result that only about 50 percent of the entire drag is due to surface friction on smooth surfaces while the rest is to be ascribed to disturbances in the flow by separation, interference, and roughness. It may therefore be concluded that with increasing refinement in aerodynamical shape the parasite drag ratio of airplanes will tend toward a value of about 0.07×10^{-3} m^2/kg. This value was made the basis for the wing-loading investigations for all the types of airplanes investigated.

The power loadings were taken to be the following:

Long-range high-speed airplanes:

η N/G = 0.1 hp./kg or G/N = 8 kg/hp.

Medium-range airplanes:

= 0.18 hp./kg or G/N = 4.5 kg/hp.

Short-range airplane:

= 0.35 hp./kg or G/N = 2.3 kg/hp.

The relations between the wing loading and the aspect ratio, on the one hand, and the flight performances were obtained from figures 3, 6, 8, 15, 16, and 17.

II. Long-Range Airplane

(η N/G = 0.1 hp /kg, $f_{ws}'/G = 0.07 \times 10^{-3}$ m²/kg cruising flight with 60 percent throttling of the engines and mean weight in flight $G_m = 0.8$ G)

The airplane under consideration has a weight of 32 tons, a rated engine power of 4 x 1,000 hp. and develops at 6 km altitude with $G/F = 140$ kg/m² and $\Lambda = 8$ at 60 percent of the rated power of the engine in cruising flight a cruising speed of 320 km/h (maximum speed at 6 km altitude 400 km/h). Assuming a fuel load of 40 percent of the total weight, we may consider the flying weight to be 0.8 times the take-off weight.

Figure 22 shows for this airplane, the mean cruising speed (at 0.8 take-off weight, and 0.6 rated power) and the climbing speed at zero altitude with full power in take-off as a function of the take-off wing loading and the aspect ratio. If the take-off is not taken into consideration, since with these airplanes take-off aids may always be assumed, it may be seen from figure 22 that the wing should be given an aspect ratio of 12 and a take-off wing loading of 200 kg/m². As compared with the wing loading of 140 kg/m² with optimum aspect ratio, there would thereby be obtained a cruising-speed gain and hence an increase in range of about 5 to 6 percent.

Since, in landing, a decrease to about 60 percent of the take-off weight may be assumed, that is, for landing wing loadings of only 120 kg/m² are encountered, no particular problem is offered in landing with normal wing flaps. In order to assure the low wing loading of 120 kg/m² also in forced landing, the airplanes are to be provided with quick-release appliances for the fuel.

III. Medium-Range Airplane

(η N/G = 0.18 hp./kg, $f_{ws}'/G = 0.07 \times 10^{-3}$ m²/kg. Cruising with 80 percent rated power and mean flying weight $G_m = 0.85$ G)

The airplane under consideration has, for example, a flying weight of 9,000 kg, a rated engine power of 2 x 1,000 hp., and develops at 6 km altitude, with G/F = 140 kg/m² and $\Lambda = 7$, a maximum speed of 500 km/h. For this airplane type, there was assumed a mean gross weight 85 percent of the take-off weight and a power of 80 percent corresponding to the mean of high speed and cruising flight. The rate of climb was computed for the ground with full throttle at take-off weight so as to obtain a measure of the take-off ability (fig. 23). According to the latter, it is of advantage with respect to high speed and cruising flight to obtain wing loadings of 250 kg/m² at aspect ratios of 8 to 10. The gain in speed as compared with the wing loading of 140 kg/m² and optimum aspect ratio would then amount to about 8 percent.

Assuming for the landing, conservatively estimated, a decrease in speed of 20 percent, the wing loading in landing would be about 200 kg/m². At a landing speed of about 120 km/h, which may be looked upon at the present time as a reasonable value, the lift coefficient of the wing would then be 2.85. Maximum lift coefficients of this order of magnitude hardly offer any difficulties on the aerodynamic side.

Since it must also be required of airplanes of this application group that they have a sufficiently short take-off on their own power, an approximate computation was made for the take-off run. At a take-off power boost of the engines by about 10 to 20 percent take-off runs of the order of 600 m to 20 m altitude are obtainable with flaps. Care must be taken to see, however, that the flaps are set so that the small profile drag coefficients are obtained in spite of high lift coefficients of about 2.6.

IV. Short-Range High-Speed Airplanes

($\eta N/G$ = 0.35 hp./kg, $f_{ws}'/G = 0.07 \times 10^{-3}$ m²/kg, full-power flight with maximum gross weight)

As typical short-range high-speed airplane, there was taken an airplane of 2,300 kg gross weight and rated power of 1,000 hp. The maximum speed and the rate of climb to 5 km altitude were considered for full-power flight at maximum weight, since the change in weight during flight is in general very small.

Figure 24 shows that a considerable increase in the maximum speed is attainable through an increase in the wing loading up to values of 300 kg/m², the aspect ratio being only of secondary importance. The maximum rate of climb for the aspect ratios investigated is about the same; after exceeding the maximum value the rate of climb rapidly decreases.

An increase in the wing loading up to 250 to 300 kg/m^2 should therefore be accompanied by a simultaneous increase in the aspect ratio in order that a favorable compromise solution may be possible between the maximum speed and the climbing ability. A desirable wing size should therefore be that for a wing loading of 250 to 300 kg/m^2 at an aspect ratio of about 10. The gain in maximum speed as compared with $G/F = 140$ kg/m^2 amounts to from 8 to 10 percent for about the same rate of climb of the airplane.

For the landing of these airplanes, only a very small decrease in gross weight of about 10 percent of the take-off weight may be assumed, so that the problem of safe landing must be fully confronted. At a landing speed of 120 km/h, there must thus be available maximum lift coefficients of 3.2 to 3.9.

The take-off run of these highly loaded airplanes, on account of the very large power excess, is still of the order of 600 meters to attain 20 meters altitude, so that from this aspect a sufficient take-off is assured also without particular take-off aids.

V. General Viewpoints for Wing Loading Increases for Airplane Types Considered

In increasing the wing loading on the airplanes considered the general characteristic appeared that the wing loading increases are of advantage only with a simultaneous increase in the aspect ratio, both as regards increasing the maximum speed and obtaining a favorable compromise solution between high speed and climbing speed. This tendency is also favored by the fact that the sinking velocity in landing may be kept within normal limits at high aspect ratios.

In general, it will be difficult in such greatly reduced wings to carry loads such as, for example, fuel, retractable landing gear for the single seater, etc., which generally may be carried in the wing. For constructional reasons of this kind, the performance gains discussed above cannot always be realized. To determine the limits that must apply would take us beyond the scope of this paper, which has for its object only the investigation of the gain in flight performance by reduction in the wing size without considering the constructional side of the problem.

By increasing the wing loadings beyond the value 140 kg/m², which corresponds approximately to the present state of development, speed gains of 5 to 10 percent, according to the design data of the airplane, are thus obtainable. These increases in the loading are not in themselves of very great magnitude but they are an important step toward further refinement of the airplane design which without increase in the propulsive power leads to an increase in the flight performance.

F. CONCLUSIONS

I. High Speed

1. The optimum wing loading, G/F^*, for which the maximum speed at given aspect ratio under certain simplifying assumptions attains its greatest value is given by the following relation:

$$G/F^* = \frac{\rho}{2} v_{max}{}^2 \sqrt{\pi \Lambda c_{wp}}$$

i.e., an increase in the wing loading is of particular advantage for high-speed airplanes at low altitude, whereas for slow-speed airplanes and high altitudes no appreciable gain is to be expected. The most favorable wing loadings, other conditions being equal, are higher the greater the aspect ratio and the profile drag coefficient and the lower the altitude.

2. Because of the flatness of the speed optimum with change in wing loading, it is of advantage and most often even necessary to remain considerably below the optimum wing loading. The resulting loss in maximum speed is greater the greater the flight speed and the aerodynamic efficiency of the airplane; i.e., for the future airplane development in the direction of high speed and aerodynamically still more efficient airplanes, the sensitivity to the optimum choice of wing loading will increase.

3. In the case of high-powered airplanes with high-power engines, the wing aspect ratio has only a secondary importance. In the case of airplanes with low-power engines (long-range airplane) the increase in the wing loading offers, however, considerable gains in speed which are available up to aspect ratios of 15.

II. Climbing Flight

1. In agreement with the relations holding for the maximum speed, the optimum wing loadings for climbing flight are higher the more powerful the engine, the greater the aerodynamic efficiency, the higher the aspect ratios and the profile drag coefficients, and the lower the altitude. For the present-day airplanes the optimum wing loadings are generally at smaller than the designed values, so that an increase in the wing loading for equal aspect ratio necessarily leads to impairment in the climb ability. These losses can be balanced, however, for the greater part if on increasing the wing loading the aspect ratio is simultaneously increased.

2. The sensitivity of the airplanes to deviations from the optimum wing loading is greater the smaller the ratio η N/G.

3. The aspect ratio at optimum wing loading has only a small effect on the rate of climb. A change in the aspect ratio is therefore generally of advantage only if the losses in rate of climb through high wing loadings are to be kept small.

APPENDIX

CHANGE IN WEIGHT OF AIRPLANE DUE TO CHANGE IN WING DIMENSIONS

I. General Remark

For the determination of the flight performances, particularly the maximum rate of climb and the magnitudes like climbing time and ceiling altitude depending on it, it is of great importance to know the change in weight of the airplane as a result of the change in the wing loading and aspect ratio.

Since the weight of the wings is mainly affected by a change in wing loading and aspect ratio and to a smaller extent by the weight of the fuselage, tail, and landing gear, it will be assumed in this approximate computation that the latter three structures do not undergo any changes in weight. The error arising from this assumption may usually be expected to be very small because small changes in the weights of these parts, which changes may even compensate each other*, affect the over-all weight inappreciably. If these changes are neglected, the problem may be restricted to that of finding a reliable relation between the weight of the wing on the one hand and the wing loading and aspect ratio on the other. It will further be assumed that the wings are cantilever monoplane which maintain their thickness ratio and taper ratios with change in dimensions and have similar structure to the original wing.

II. Unit Weight of Wing

Although a considerable portion of the weight of a wing is made up of additional weights, for example, as a result of cut-outs, overdimensioning of parts, etc., it is nevertheless to be expected that the total weight is mainly affected by that portion which is required for taking up the external wing stresses. For this reason, the weight of the wing, following an unpublished work of Bock (reference 9) is divided into the following main groups:

1. Portion of the weight that is proportional to the

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*On reducing the wing loading, the weight of the fuselage and tail will, in general, be increased whereas that of the landing gear is decreased.

flange of the longerons (for taking up the bending moments).

2. Portion of the weight proportional to the web of the longerons (for taking up the shear forces).

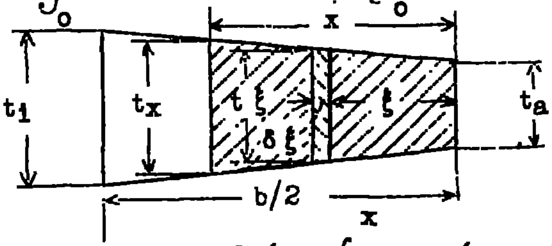
3. Portion of the weight proportional to the weight of the covering (for taking up the wing torsion).

4. Portion of the weight proportional to the wing area to cover the additional weights, as by overdimensioned plates, etc.

For simplification, the effect of the stiffness requirements was not taken into consideration, since these requirements would lead to a very complicated computation.

1. Flange weight.- Assuming that in taking up the bending moments the effective distance between the longeron flanges is equal to the maximum height of the wing profile, i.e., if the decrease of the effective distance by the finite thickness of the flanges is neglected, there is obtained from the bending moments for the flange weight of a half wing:

$$G_G = \gamma \int_0^{b/2} F_{G_x}\, dx = 2\,\frac{\gamma}{\sigma_G} \int_0^{b/2} \frac{M_x}{h_x}\, dx$$



$$G_G = 2\,\frac{\gamma}{\sigma_G}\, n_A\, G/F \int_0^{b/2} \frac{\int_0^x t_\xi\, (x - \xi)\, \delta_\xi}{h_x}\, dx$$

where

F_{G_x} is cross section of flange at position x;

h_x, maximum profile height at position x;

M_x, bending moment at x;

γ, specific weight of the material (duralumin);

σ_G, mean stress in flange of longeron;

n_A, breaking load factor of the wing in case A.

Assuming tapered wings with a taper ratio t_i/t_a and thickness ratio which decreases linearly from d_i/t_i at the wing root to d_a/t_a at the wing tip, we obtain, after solving the integral and transforming:

$$G_G/F = \frac{1}{2} \frac{\gamma}{\sigma_G} n_A \; G/F \; \frac{b}{2} \Lambda \frac{1}{d_i/t_i} f_G \qquad (1a)$$

In the above equation, the factor f_G takes care of the effect of the taper ratio t_i/t_a and the effect of the decrease of the wing thickness ratio toward the wing tip and is defined by the following relation

$$f_G = \int_0^{x=b/2} \left(\frac{x}{b/2}\right)^2 \frac{\left[1 + \frac{1}{3} \frac{x}{b/2}\left(\frac{t_i}{t_a} - 1\right)\right] d\left(\frac{x}{b/2}\right)}{\left[1 + \frac{x}{b/2}\left(\frac{t_i}{t_a} - 1\right)\right]\left[\frac{d_a/t_a}{d_i/t_a} + \frac{x}{b/2}\left(1 - \frac{d_a/t_a}{d_i/t_i}\right)\right]} \qquad (1b)$$

For equal taper ratio and equal variation of the thickness ratio, the factor f_G is thus a constant.

2. Weight of web.- If the transverse forces of the wing Q_x are assumed to be taken up by a web of shearing strength τ_{St}, the weight of the web is

$$G_{St} = \gamma \int_0^{b/2} F_{St}\, dx = \frac{\gamma}{\tau_{St}} \int_0^{b/2} Q_x\, dx$$

$$= \frac{\gamma}{\tau_{St}} n_A \; G/F \int_0^{b/2} \left[\int_0^{x} t\xi \cdot d\xi \right] dx$$

Again, if a tapered wing with the taper ratio t_i/t_a is assumed, there is obtained for the weight of the web referred to the wing area

$$G_{St}/F = \frac{\gamma}{\tau_{St}} n_A \; G/F \; \frac{b}{2} f_{St} \qquad (2a)$$

where the factor f_{St} again takes care of the effect of the wing shape and is determined by the relation

$$f_{St} = \int_0^{b/2} \frac{2\ x/b/2}{1 + t_i/t_a} \left[1 + \frac{1}{2}\frac{x}{b/2}\left(\frac{t_i}{t_a} - 1\right)\right] d\left(\frac{x}{b/2}\right) \quad (2c)$$

3. Weight of covering.- Considering the wing torsion in diving flight with dynamic pressure q_c and the moment coefficient c_{m_0} as determining the dimension of the covering, we find in the same way as for determining the weights of the flanges and web

$$G_B/F = \frac{1}{6}\frac{\gamma}{\tau_B} c_{m_0}\ q_c\ \frac{b}{2}\left(\frac{U_{pr_i}\ t_i}{F_{pr_i}}\right) f_B$$

From a statistic consideration of ordinary wing sections, it was found that with good accuracy, we may set $\frac{U_{pr}\ t}{F_{pr}} = \frac{3}{d/t}$. With this relation, there is obtained

$$G_B/F = \frac{1}{2}\frac{\gamma}{\tau_B} c_{m_0}\ q_c\ \frac{b}{2}\frac{1}{d_i/t_i} f_B \quad (3a)$$

where the factor f_B is defined by

$$f_B = \int_0^{b/2} \frac{x}{b/2}\frac{2}{1 + t_i/t_a}\frac{d_i/t_i/d_a/t_a}{1 + \frac{x}{b/2}\left(\frac{d_i/t_i}{d_a/t_a} - 1\right)} \times \frac{\left(\frac{x}{b/2}\right)^2\left(\frac{t_i}{t_a} - 1\right)^2 + 3\frac{x}{b/2}\left(\frac{t_i}{t_a} - 1\right) + 3}{1 + \frac{x}{b/2}\left(\frac{t_i}{t_a} - 1\right)} d\left(\frac{x}{b/2}\right) \quad (3b)$$

In this formula for G_B/F it is inconvenient that not the load factor n_A but the diving flight dynamic pressure q_c must be introduced for estimating the external load. According to the airplane strength specifications of December

1936, a safe dynamic pressure is assumed for the majority of airplanes $q_{c_{safe}} = 2.25\ q_h$ where q_h is the maximum dynamic pressure in unaccelerated level-flight. For a simple estimate, we may introduce, in agreement with the load assumptions of January 1935, the following relation between the maximum dynamic pressure q_h and the load factor n_A, which relation also well agrees with the load specifications of December 1936 in the practically important range of $\frac{q_h}{G/F} = 5$ to 9 (reference 10):

$$n_{A_{safe}} = \sim 0.7\ \frac{q_h}{G/F} = \frac{1}{2}\ n_A$$

With this relation, equation (3c) assumes the form

$$G_B/F = 1.6\ c_{n_0}\ \frac{\gamma}{\tau_B}\ G/F\ \frac{b}{2}\ n_A\ \frac{1}{d_i/t_i}\ f_B \qquad (3c)$$

4. Total weight of wing unit.- With the partial weights derived above, there is obtained for the entire unit weight of a wing the following relation

$$G_F/F = n_A\ G/F\ \frac{b}{2}\left[\frac{1}{2}\ \frac{\gamma}{\sigma_G}\ \frac{1}{d_i/t_i}\ \Lambda\ f_G + 2\ \frac{\gamma}{\tau_{St}}\ f_{St} + 1.6\ c_{n_0}\ \frac{\gamma}{\tau_B}\ \frac{1}{d_i/t_i}\ f_B\right] + k_Z \qquad (4)$$

This unit weight formula is not to be used for computing the weight of a given wing but only for the estimate of the change in wing weight in passing from a given wing size or aspect ratio to different dimensions, keeping the same wing structure, thickness ratio, taper ratio, etc. It is sufficiently accurate to introduce mean values for the form parameters f_G, f_{St}, and F_B, and mean fictitious stresses into the computation. Under these simplifying assumptions, the wing weight equation (4) takes the following form:

$$G_F/F = (k_1 + k_2\ \Lambda)\ n_A\ b\ G/F + k_Z \qquad (5)$$

By comparison with statistically determined weights of wings, the following values were found for the constants k_1, k_2, and k_Z:

$$\left.\begin{array}{l} k_1 = 0.58 \times 10^{-3}\ m^{-1} \\ k_2 = 0.041 \times 10^{-3}\ m^{-1} \\ k_Z = 3.5\ kg/m^2 \end{array}\right\} \text{State corresponding to the years 1934 to 1936*}$$

The unit weights of the wing determined in the above manner sufficiently well agree with most of the wing weights of present-day design. Since the above computation is intended essentially for future airplanes for which, corresponding to the continued progress in airplane design, smaller weights may be expected, the wing weight corresponding to the present state of development was reduced by about 15 percent: i.e., the constants of the unit weight formulas (equation 5) were taken to be the following:

---

*Assuming in the mean a thickness ratio at the wing root $d_i/t_i = 0.17$, decrease of the thickness ratio toward the wing tip $d_a/t_a = 0.6\ d_i/t_i$ and taper ratio $t_a/t_i = 0.5$, we obtain the following wing shape parameters:

$$f_G = 0.26; \qquad f_{St} = 0.44; \qquad f_B = 1.2$$

With these values, the mean fictitious rupture stresses of the wing and the effective wing moment coefficient are found to be the following:

$$\sigma_G = 25\ kg/mm^2;\ \tau_{St} = 3.5\ kg/mm^2;$$

$$\tau_B/c_{m_0} = 3.0/0.07 = 43\ kg/mm^2$$

For comparison, there was determined, according to Heck and Ebner (reference 11) the critical buckling stress for a covering of a wing with fixed ends (30 cm long, 15 cm wide, 1 mm thick) as 3.0 kg/mm² and for the web plate corresponding to the better support of the web and greater sheet thicknesses 3.5 kg/mm². Taking account of the usual additional weights by cut-outs, overdimensioning, etc., the mean fictitious stresses assumed compare reasonably with the critical buckling stresses of several typical structural members and also with the mean tensile and rupture stress of duralumin.

$k_1 = 0.50 \times 10^{-3}\ m^{-1}$

$k_2 = 0.035\ 10^{-3}\ m^{-1}$

$k_Z = 3.0\ kg/m^2$

Figure 25 shows the wing unit weights for various aspect ratios obtained with these values as a function of the principal parameter $n_A\ G/F\ b$. Figure 26 shows for comparison the wing weights of airplanes of the years 1934 to 1936 computed with the aid of figure 25 for the same aspect ratio $\Lambda = 5$ by the addition of the weight increment $\Delta G/F = G_F/F_{\Lambda=5} - G_F/F_\Lambda$. The mean curve lies in the more favorable half of the range of weights and agrees well with the statistical values.

III. Change in Weight of Airplane through Change in the Wing Dimensions

For use in later computation, it is of advantage to determine the change in the weight of the airplane resulting from a change in the wing area and aspect ratio by means of ratios which refer to a definite initial state to be more accurately defined later. All values which refer to the initial state will be denoted in the computation by primes, F', n_A', etc., and the ratios of the changed magnitudes by Ω: for example, $\Omega_F = F/F'$, $\Omega_{n_A} = n_A/n_A'$.

If we assume furthermore that the ratio of the wing weight to the total weight in the initial state is known, for example, $G_F' = (1 - \varphi')G'$, then from equation (5) the relation may be determined as follows:

$$n_A'\ b' = \frac{1 - \varphi' - k_Z/G/F'}{k_1 + k_2\ \Lambda'}$$

Substituting the above relation and the notations referred to the initial state in equation (5), we obtain

$$\frac{G_F/F}{G/F} = \sqrt{\frac{\Omega_G}{\Omega_{G/F}}}\left[k_1\ \Omega_\Lambda^{1/2}\ \frac{1 - \varphi' - \frac{k_Z}{G/F'}}{k_1 + k_2\ \Lambda'} + k_2\ \Omega_\Lambda^{3/2}\ \Lambda'\ \frac{1 - \varphi' - \frac{k_Z}{G/F'}}{k_1 + k_2\ \Lambda'}\right] + \frac{1}{\Omega_{G/F}}\ \frac{k_Z}{G/F'}$$

or

$$G_F/G = \sqrt{\frac{\Omega_G}{\Omega_{G/F}}}\left[k_1{}^* \, \Omega_\Lambda^{1/2} + k_2{}^* \, \Omega_\Lambda^{3/2}\right] + k_Z{}^* \frac{1}{\Omega_{G/F}}$$

Setting up the equation for the gross weight of the airplane under the assumption that only the weight of the wing undergoes change, we obtain

$$1 = \varphi' \frac{1}{\Omega_G} + \sqrt{\Omega_G} \, \frac{1}{\sqrt{\Omega_{G/F}}} \times \left[k_1{}^* \, \Omega_\Lambda^{1/2} + k_2{}^* \, \Omega_\Lambda^{3/2}\right] + k_Z{}^* \frac{1}{\Omega_{G/F}} \qquad (6)$$

From the above relation the airplane weight ratio Ω_G can be computed for any wing load ratio $\Omega_{G/F}$ and aspect ratio Ω_Λ. Further consideration shows that the above is a cubic equation for $\Omega_G{}^{1/2}$ (irreducible case) of whose three real solutions only one lies within the practical range $\varphi' < \Omega_G < +\infty$. For this solution, there is obtained

$$\Omega_G{}^{-1/2} = (G/G')^{-1/2}$$

$$= -2\sqrt{\frac{1}{3\,\varphi'}\left(1 - k_Z{}^* \frac{1}{\Omega_{G/F}}\right)} \cos\left(60^\circ - \frac{\alpha}{3}\right) \qquad (7a)$$

$$\cos\alpha = \frac{\frac{1}{2\,\varphi'}\left(k_1{}^* \, \Omega_\Lambda{}^{1/2} + k_2{}^* \, \Omega_\Lambda{}^{3/2}\right)\sqrt{\frac{1}{\Omega_{G/F}}}}{\left[\frac{1}{3\,\varphi'}\left(1 - k_Z{}^* \frac{1}{\Omega_{G/F}}\right)\right]^{3/2}} \qquad (7b)$$

To evaluate this equation, the following constants were chosen as initial values:

Wing loading $G/F' = 100$ kg/m²,

Wing aspect ratio $\Lambda' = 5$,

Ratio of wing weight to total weight $G_F'/G' = 1 - \varphi' = 0.14$ (will be extended later to a greater range).

The assumed wing weight proportion of 14 percent corresponds to favorable mean relations for airplanes of the years 1935 to 1937. Figure 27 shows the numerical evaluation of equation (6) for the above initial values.

IV. Sensitivity to the Change in the Weight of Wing

The relations derived above for the change in the airplane weight through a various choice of the wing loading and aspect ratio depends on numerous assumptions, so that in conclusion, it seems useful to investigate the effect on the curves of making changes in the assumptions. The most important assumptions are the unit wing weight curve of figure 26 and the assumed ratio of the wing weight to total weight of 14 percent.

In order to investigate first the effect of the unit wing weight curve, the weights were first increased by 30 percent and then lowered by the same percentage and with those changed values there was again determined the wing weight to total weight ratio for the aspect ratio 8. Since the flight ratio weight in spite of the various assumptions for the weight of the wing tends toward the limiting value $G'/G = 1/\varphi' = 1.163$ at infinitely high wing loading the deviations of the actual values from the mean values remain extremely small also in the range of high wing loadings. Numerical evaluation gave only slight deviations of less than 1 percent of the mean values. In contrast to this, larger deviations were obtained if for the same unit wing weight curve the ratio of wing weight to total weight was varied. Figure 27 shows the change in weight of airplane for assumed wing weight proportions of 10, 14, and 18 percent of the total weight at the initial state ($G/F' = 100$ kg/m², $\Lambda' = 5$). Since with increasing wing loading the curves approach various limiting values asymptotically, the differences increase considerably with increasing wing loading.

In order to be able to estimate also the relations for changed wing to airplane weight, for example, for $G_F' = (1 - \varphi_2') G'$, may be used with sufficient accuracy, the airplane weight scale being multiplied by φ'/φ_2' and the wing loading scale by

$$\frac{G/F'}{G/F_{\Lambda=5} \text{ (for } G'/G\ \varphi'/\varphi_2' = 1.0)},$$

i.e., on the diagram that point is sought on the G'/G curve for which the weight of the wing at the aspect ratio $\Lambda' = 5$, has the desired value $(1 = \varphi_2') G'$ instead of $(1 - \varphi') G'$ and the entire diagram is referred to this point as initial point (see approximate points, fig. 27). The corresponding new scales for the wing weight proportions of 10, 14, and 18 percent at the aspect ratio 5 and the wing loading $G/F' = 100$ kg/m² are given in figure 27 of the main report, so that this diagram may be made applicable for a wide range of wing weight ratios, thus providing a means for taking account of the effect of the wing weight on the airplane performance.

Translation by S. Reiss,
National Advisory Committee
for Aeronautics.

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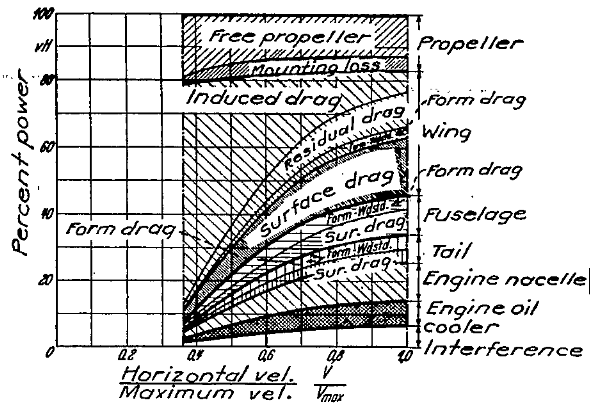


Figure 1.- Power balance of a two-engined high-speed airplane.
$G/F = 120\ kg/m^2 \quad G/N = 4.5\ kg/hp$

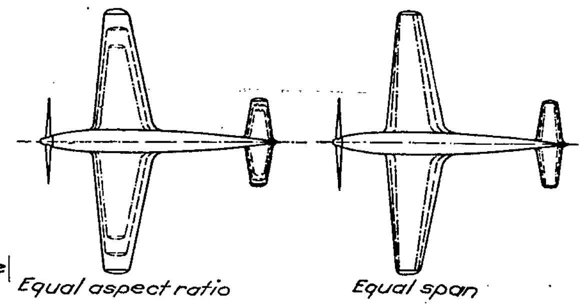


Figure 2.- Airplanes with various reduced wings.

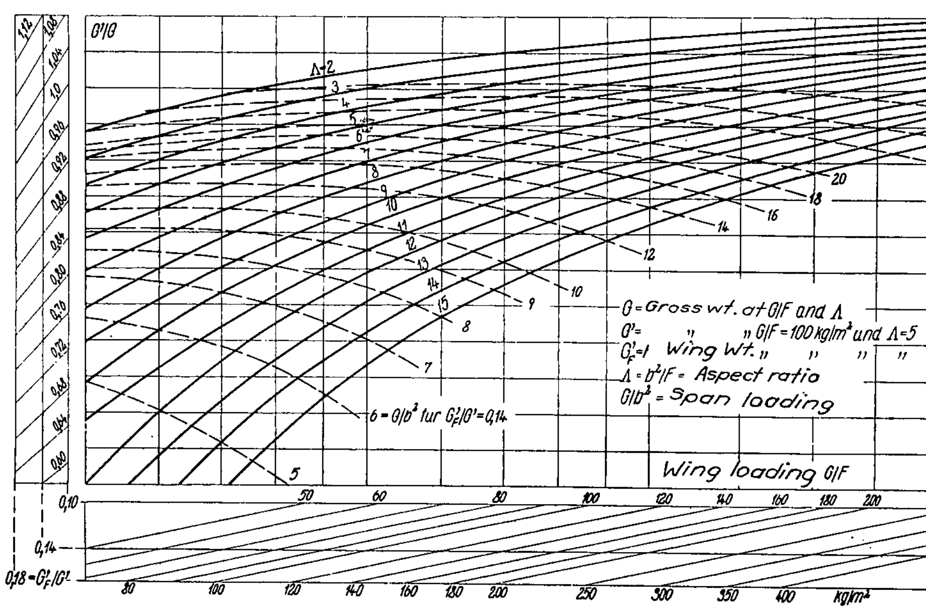


Figure 3.- Gross weight ratio G'/G as a function of the wing loading and aspect ratio.

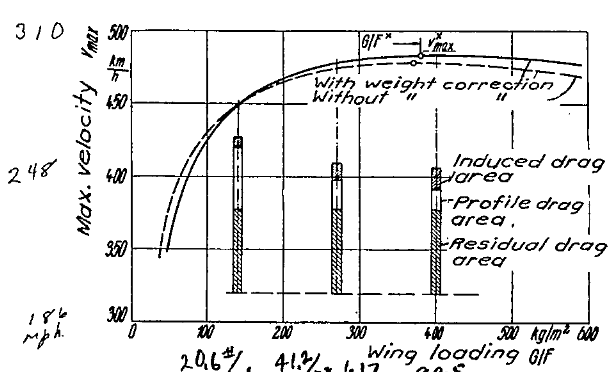


Figure 4.- Maximum speed of a high-speed airplane at various wing loadings.
Power loading $\eta N/G' = 0.2$ hp/kg
Parasite drag ratio $f'_{ws}/G' = 0.1 \times 10^{-3} m^2/kg$
Altitude = 4 km, aspect ratio $\Lambda = 8$
(ratio of wing weight to total weight $G_F'/G' = 0.14$)

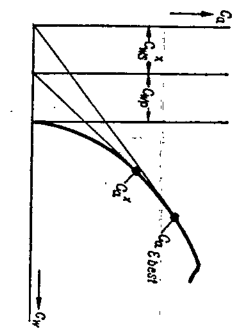


Figure 5.- Airplane polars with operating points for flight with optimum G/F and flight with flattest glide angle.

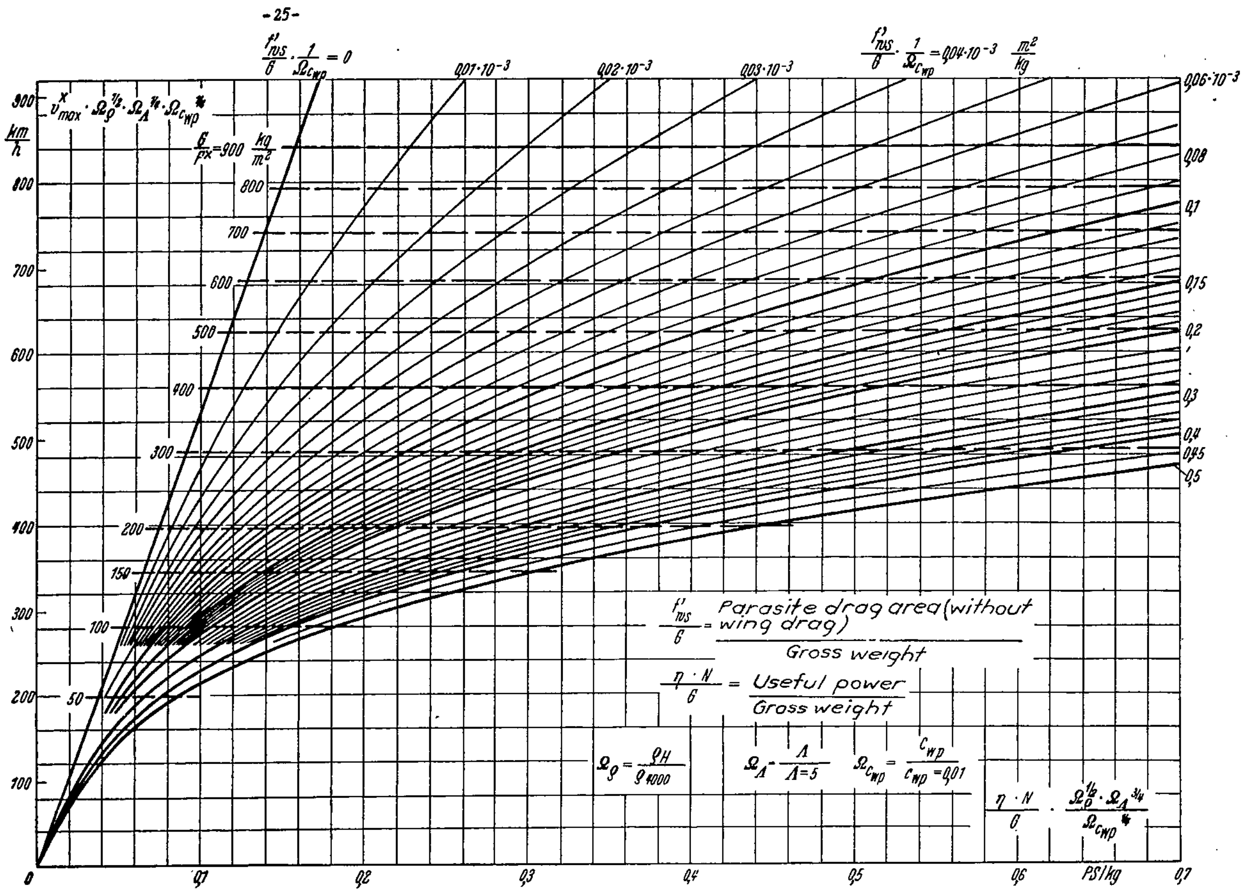


Figure 6.- Maximum speed v_{max}* at optimum wing loading G/F* for equal aspect ratio Λ (without weight correction).

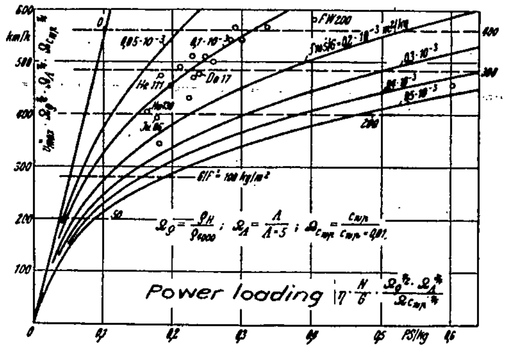


Figure 7.- Optimum wing loading G/F* and corresponding maximum speed V_{max}* for recent airplanes (without weight correction).

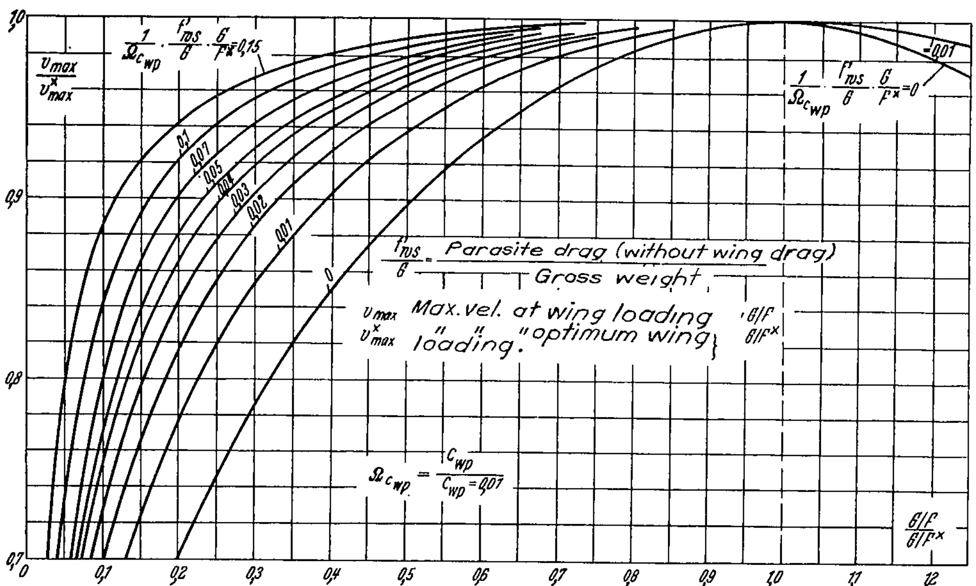
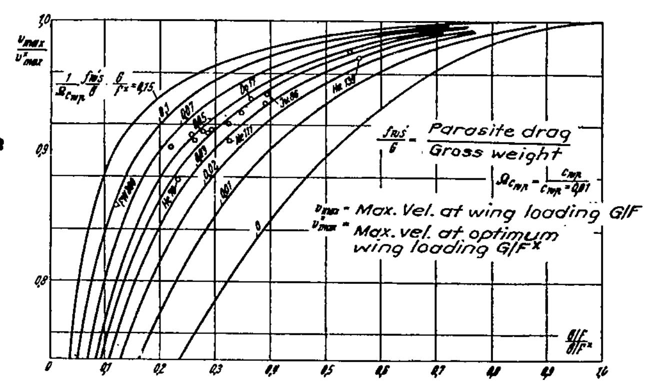
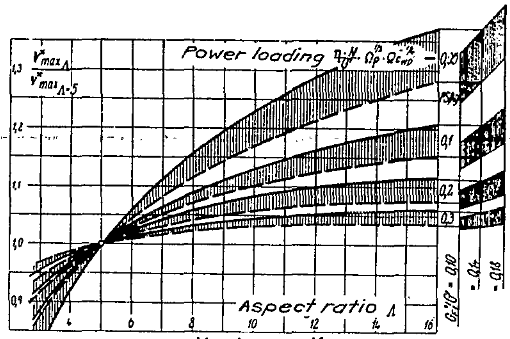


Figure 8.- Dependence of the maximum speed on the wing loading for equal aspect ratio (without weight correction).

Figure 9.- Dependence of the maximum speed on the wing loading for recent airplanes (without weight correction).





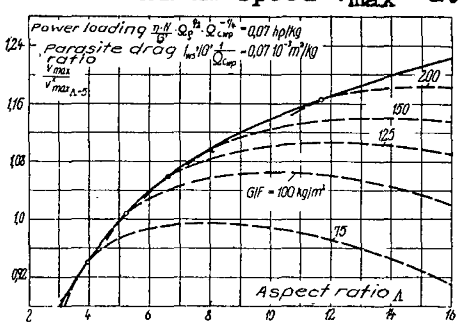
G'_F/G'	0,10	0,14	0,18	
$f'_{ws}/G' \frac{1}{\Omega_{cwp}} 10^3$ m²/kg	0,093 0,37	0,05 0,2	0,033 0,131	—— - - -

G'=airplane weight at G/F =100 kg/m² and $\Lambda=5$

G'_F=weight of wing at G/F =100 kg/m² and $\Lambda=5$

$\Omega_\rho=\rho/\rho_{4000}$; $\Omega_{cwp}=\frac{c_{wp}}{c_{wp}=.01}$

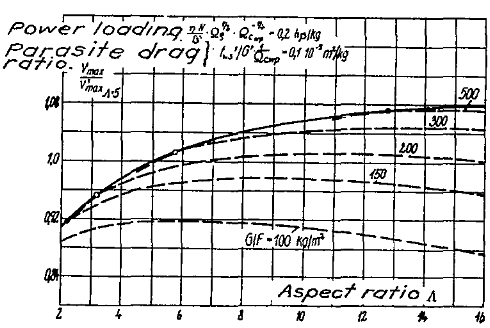
Figure 10.- Effect of the aspect ratio on the maximum speed v_{max}* at optimum wing loading (with weight correction).

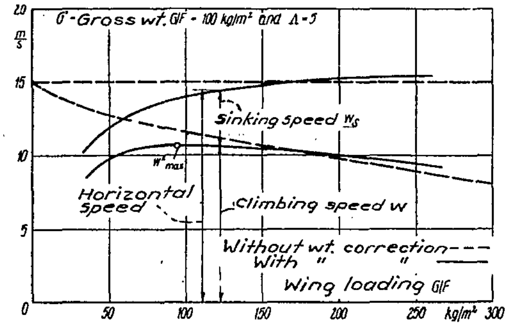


$v_{max}{}^*{}_{\Lambda=5}$=maximum speed at optimum G/F and $\Lambda=5$.

G' weight of airplane at G/F =100 kg/m² and $\Lambda=5$.

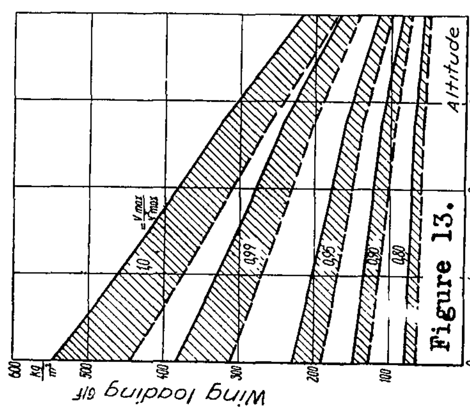
Figure 12.- Effect of the wing aspect ratio on the maximum speed for a long range airplane (with weight correction $G'_F/G'=0.14$).





Power loading: $\eta N/G'$ 0.2 hp/kg
Parasite drag ratio: f'_{ws}/G' = 0.1×10^{-3} m²/kg.
Altitude =4 km,aspect ratio Λ=8.
(ratio of wing weight to total weight $G'_F/G'=0.14$)

Figure 14.- Rate of climb of a high speed airplane at various wing loadings.



Power loading $\eta N/G'=0.2$ hp/kg
Parasite drag ratio: $f'_{ws}/G'=0.05\times10^{-3}$ m²/kg ($c_{wp}=0.006$) ————
$f'_{ws}/G'=0.1\times10^{-3}$ m²/kg ($c_{wp}=0.01$) - - - - - - -
Aspect ratio Λ=8. Maximum speed at optimum wing loading $=v_{max}$*.

Figure 13.- Effect of altitude on the choice of wing loading(with weight correction $G'_F/G'=0.14$).

$v_{max}{}^*{}_{\Lambda=5}$ = maximum speed at optimum G/F and $\Lambda=5$.

G'=weight of airplane at G/F=100 kg/m² and $\Lambda=5$.

Figure 11.- Effect of the aspect ratio on the maximum speed for a high speed airplane (with weight correction, $G'_F/G'=0.14$).

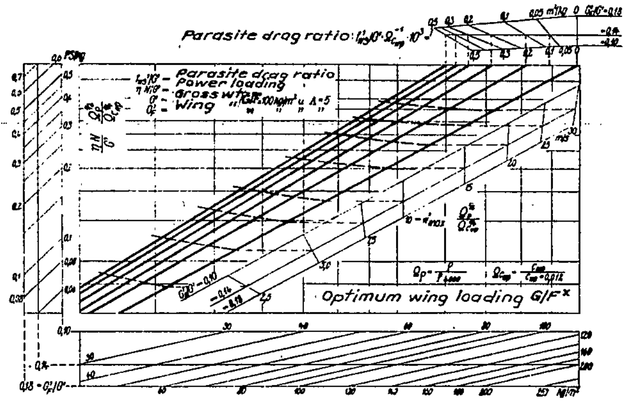


Figure 15.- Optimum wing loading G/F* for climb for aspect ratio $\Lambda = 5$ and corresponding rate of climb w_{max}*(with weight correction).

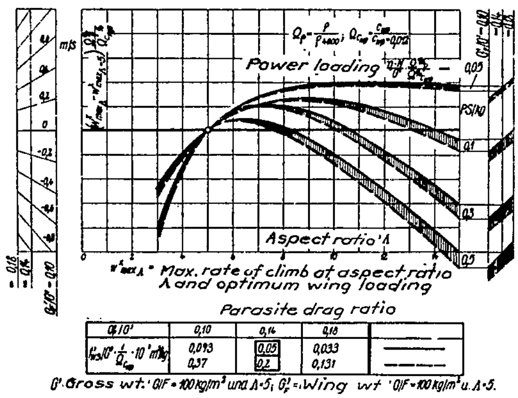
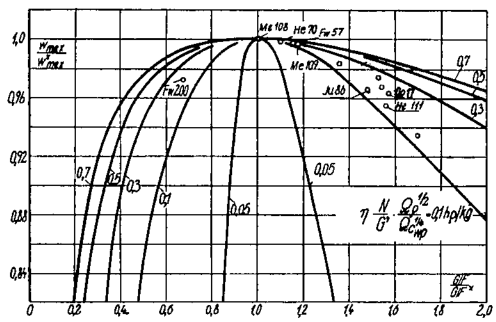


Figure 16.- Effect of the aspect ratio on the rate of climb at optimum wing loading (with weight correction).



w_{max}*= rate of climb at optimum wing loading G/F*.

$\eta N/G'$ = power loading;

$\Omega_p = \rho/\rho_{4000}$;

$$\Omega_{c_{wp}} = \frac{c_{wp}}{c_{wp} = 0.012}$$

w_{max} rate of climb at wing loading G/F.

Figure 17.- Dependence of the rate of climb on the wing loading for equal aspect ratio (with weight correction).

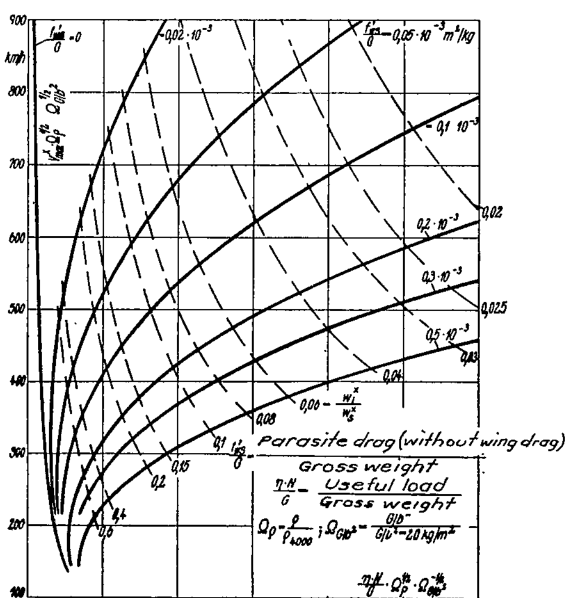


Figure 18.- Maximum speed v_{max}* at infinite power loading and equal span loading (without weight correction).

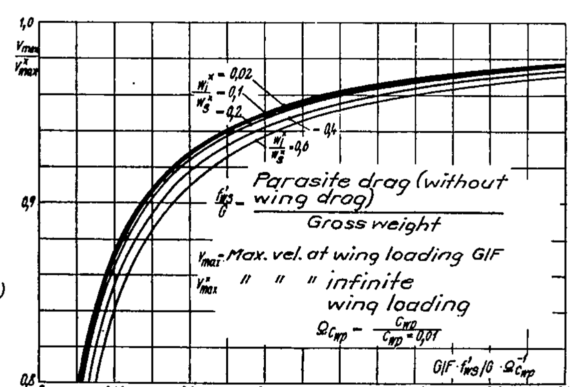


Figure 19.- Dependence of maximum speed on the wing loading at equal span loading G/b^2 (without weight correction).

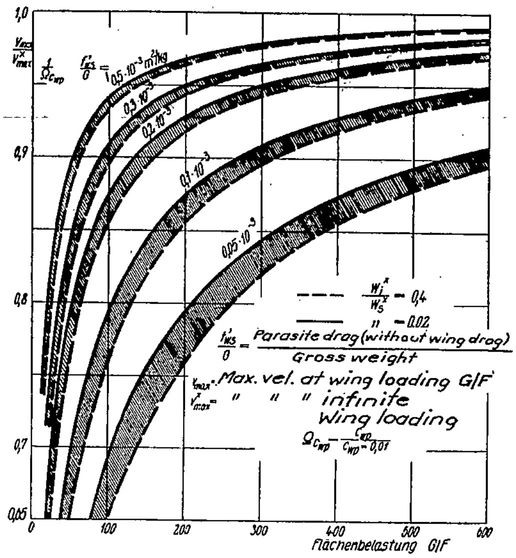


Figure 20.- Dependence of the maximum speed on the wing loading at equal span loading G/b^2 (without weight correction).

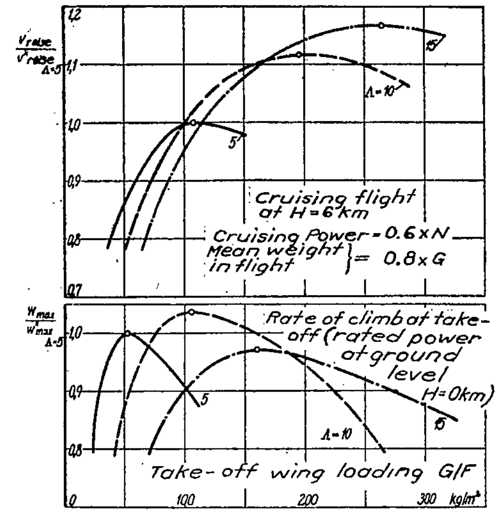


Figure 22.- Long range airplane (with weight correction; $G_F'/G' = 0.14$). $G = 32000$ kg; $N = 4 \times 1000$ hp, $v_{max_6} = 400$ km/h; ($v_{cruis}*_{\Lambda=5} = 295$ km/h)

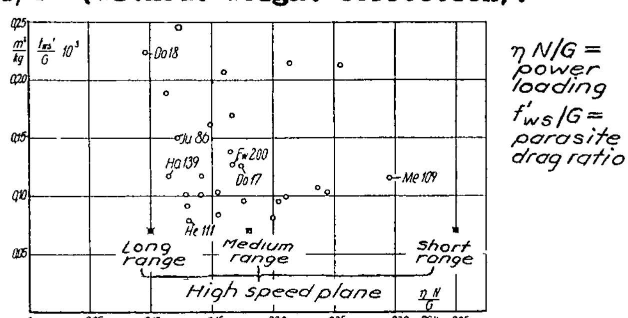


Figure 21.- Design magnitudes of airplanes of the years 1935 to 1937.

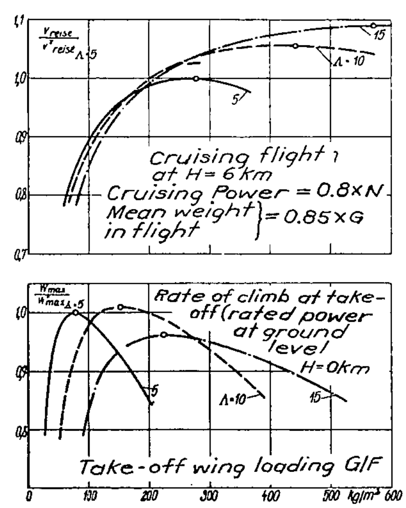


Figure 23.- Medium range airplane (with weight correction ;$G'_F/G' = 0.14$) $G = 9000$ kg; $N = 2 \times 1000$ hp, $v_{max_6} = 500$ km/h ($v_{cruis}*_{\Lambda=5} = 480$ km/h).

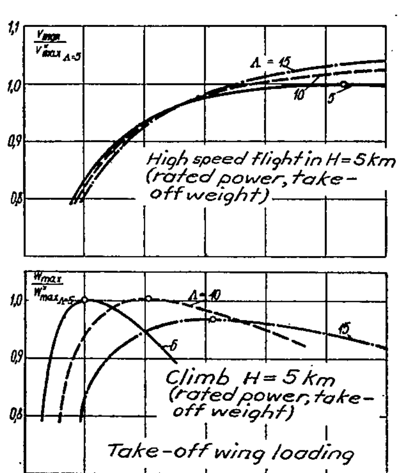


Figure 24.- Short range airplane (with weight correction; $G'_F/G' = 0.14$) $G = 2300$ kg; $N = 1000$ hp, $v_{max_5} = 600$ km/h ($v_{cruis}*_{\Lambda=5} = 680$ km/h).

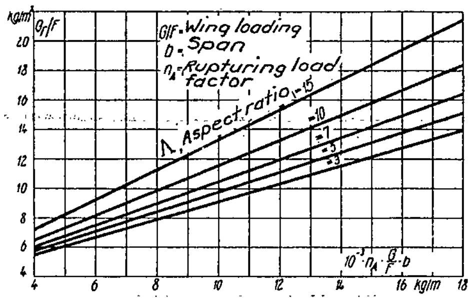


Figure 25.- Weight of unit wing G_F/F for cantilever monoplane wing of all metal construction.

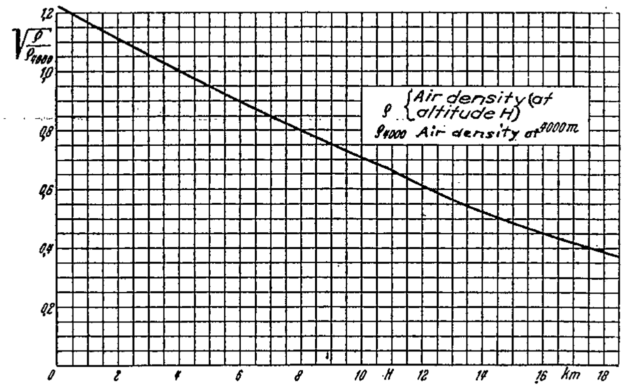


Figure 28.- Graph for the computation of the air density ratio $\sqrt{\rho/\rho 4000} = \Omega_\rho^{1/2}$.

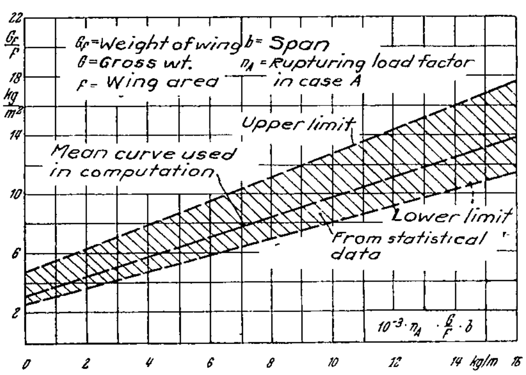


Figure 26.- Comparison of computed weights of unit wing with statistics. Unit weights improved by 15 percent and computed with aid of fig. 1 for aspect ratio A = 5.

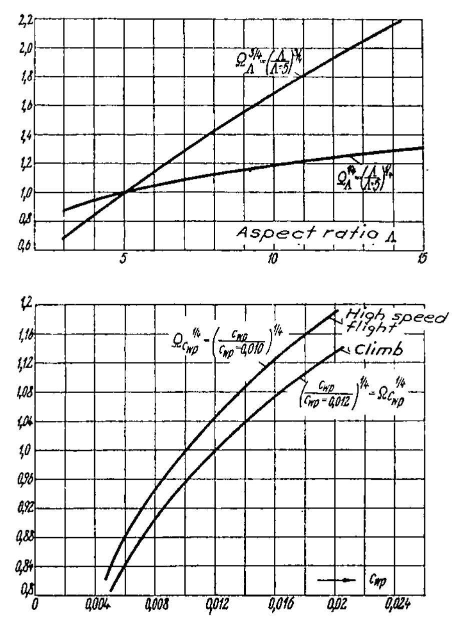


Figure 29.- Graphs for the determination of the coefficients Ω_Λ and $\Omega_{c_{wp}}$.

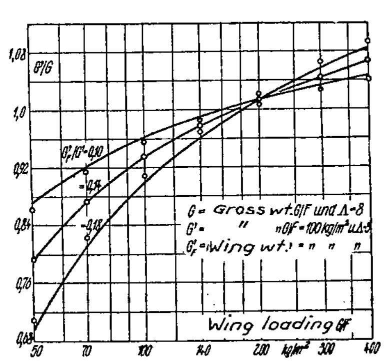


Figure 27.- Airplane weight ratios for various proportions of the wing weight. Continuous curves; accurate computation. Points determined from fig. 3 of the body of the report by scale shifting.