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STRESSES IN REINFORCING RINGS DUE TO AXIAL FORCES IN CYLINDRICAL AND CONICAL STRESSED SKINS

By K. Drescher and H. Gropler

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STRESSES IN REINFORCING RINGS DUE TO AXIAL FORCES IN CYLINDRICAL AND CONICAL STRESSED SKINS*

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At the ends of a monocoque fuselage concentrated axial forces in the skin must generally be taken up. Such axial forces must also be taken up in the case of other members where axial forces from the neighboring stressed skin construction must be considered. In order to take up these axial forces two bulkheads or reinforcing frames may be arranged at the positions where the forces are applied. If these bulkheads are in the form of rings, bending moments are set up in them. In the present paper computations are performed for obtaining the value of those bending moments. It is assumed that the stressed skin is cylindrical or conical and that its cross section is circular or elliptical. (See in this connection, H. Wagner (reference 1).)

I. INTRODUCTION

When it is required to take up axial loads in a stressed skin structure - for example, a monocoque fuselage - two reinforcing frames are commonly attached at the positions where those loads are to be taken up. How the loads and the resulting stresses are determined, is well known although the actual carrying out of a complete computation is quite tedious. This is especially true where the frames are to be in the form of rings, in which case the computation is statically indeterminate. In order to lighten this task for the practicing designer, the bending moments set up in such reinforcing rings are computed and the results presented in the form of charts (figs. 8 to 13).

In order to reduce the scope of the computation only, the following monocoque shapes were considered:

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*"Über die bei Einleitung von Längskräften in Zylinder- und Kegelschalen auftretende Beanspruchung von Ringspanten. Luftfahrtforschung, vol. 14, no. 2, February 20, 1937, pp. 63-70.

Cylindrical and conical shapes.

Cross section, circular or elliptical (ratio of semiaxes 2/3).

Rings of constant bending stiffness along their circumferences.

Some of the most usual symmetrical and "antisymmetrical" (i.e., one force tensile, the other compressive) loading conditions were considered from which, by superposition, different loading conditions could be obtained.

In order to extend the range of applicability of the results, they were presented in such a form as to enable at least an estimate of the stresses to be obtained with stressed skin constructions of other cross sections. An estimate of this sort should be sufficient since a knowledge of the accurate values of the moments in the rings is generally not required.

In order to be able to make an intelligent application of the results, the general principles underlying the computations will be reviewed below. The computation procedure itself, however, will be omitted. Only for the case of the circle will the formulas used for constructing the charts be given.

II. UNDERLYING PRINCIPLES

Figure 1 shows a thin-walled sheet-metal tube fixed at its right-hand end. Let a concentrated force be applied in any direction at the left-hand end. The stress at a great distance away from the point of application of the force may be computed from the relations for an infinitely long prismatical beam as given in textbooks on the strength of materials (linear distribution of axial stresses, etc., fig. 1).

The stresses set up in the region where the forces are taken up, depend on the type of construction of the end of the tube. In the case of very thick-walled tubes, the bending stiffness of the sheet metal is considered sufficient to take up the stresses. Such cases will not be considered here. In the case of the very thin-walled monocoque structures that we shall consider below, we shall assume that the bending and twisting strength of the sheet metal is negligible. Stiffener frames are then required that are strong enough to take up the stresses acting in their planes, such as, for example, latticework or reinforcing rings that resist bending.

The object of the present paper is to compute the bending stresses for such ring stiffeners; in particular, for the case where axial forces are to be taken up.

1. Reinforcing against a Transverse Force

Where a transverse force is to be taken up in a thin-walled tube, only a single reinforcing ring lying in the plane of the transverse force is required. The stress distribution up to the reinforcing frame is that corresponding to the theory of infinitely long thin tubes. The stress in the ring itself is determined from the equilibrium between the outer transverse force and the shear stresses transmitted from the sheet metal to the frame and corresponding to the axial stress distribution (fig. 2). The formulas for the bending stresses occurring with this type of loading for the case of the circular reinforcing ring have been given by Professor Pohl (reference 2).

2. Reinforcing against an Axial Force

Where an axial force is to be taken up, two reinforcing frames at a sufficient distance from each other are required at the end of the tube. Furthermore, it is necessary to have a longitudinal member extending from the point of application of the axial force to the second reinforcing ring and which may be riveted, for example, to the sheet metal (fig. 3, top), To the right of the second ring, the stress distribution is the linear one corresponding to the infinitely long rod. To obtain the stress at the end of the tube, we consider the equilibrium of the portion of the tube cut off to the right of the second ring (fig. 3, below). At the right-hand end the external force is that corresponding to the stress distribution of the infinitely long tube, and this force must be in equilibrium with the external force acting at the left-hand end of the tube. This structure, consisting of two reinforcing frames, the longitudinal rod, and the stressed skin is a statically determinate structure. If the two frames are considered as flat disks, then the space enclosed may be considered as simply connected.

For the stressed skin the "shear flow" q, that is, the shear stress times the wall thickness, is constant along the generating line of the tube as may readily be seen by neglecting the bending strength of the metal. From this it follows further that the axial force decreases linearly along the tube from the value P at the left-hand end to zero at the right.

The "shear flow" q may be found from the following considerations of equilibrium: Imagine the cylindrical tube, which is loaded by the axial force P, cut along some generating line as shown in figure 4. There will then act along the edge A-A a constant shear flow q whose value is to be determined (fig. 4, center). The value of q along any other line X-X (fig. 4, below) is obtained from the equilibrium of the axial forces acting upon the portion AX of the tube:

$$h\ q = h\ q_A + \int_0^u p\ du + \Sigma\ P$$

In order to determine q_A we make a cut through the cylindrical shell near the end ring parallel to the surface of the latter. In this surface (for example, in the riveting between the tube and the ring), the shear flow q is transmitted from the tube to the reinforcing ring. Since it is assumed that there are no external forces acting in the ring surface, the forces due to the shear flow q must be in equilibrium. In particular, the moment M_q about an axis perpendicular to the ring area must be zero. From this condition q_A is determined.*

In the particular case of symmetrical loading of a symmetrical shell this method of computing q_A is not necessary, q_A then being determined from considerations of symmetry. The value of the shear flow q transmitted from the tube to the ring is now known at all positions about the ring circumference, and the ring load consists

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*There is first determined the moment M_{Δ_q} due to the increment of the shear $\Delta q = q - q_A$. The moment $2F\ q_A$ due to the increment of shear $\Delta\ q_A$ must then be equal and opposite to the moment M_{Δ_q} (formula of Bredt). From this it follows that $q_A = - M_{\Delta_q}/2F$ where F is the area enclosed by the cylindrical shell.

of these shear forces held in equilibrium. The second reinforcing ring experiences an equal and opposite force.

Figure 5 shows, for example, a circular tube in equilibrium under the action of an axial force at one end and the corresponding linear stress distribution at the other end and the shear load transmitted from the tube to the reinforcing ring. If these rings are built up in the form of frames, the bending moments occurring in them are obtained from the usual computation for statically indeterminate frames with a degree of redundancy of three. These moments depend on the bending strength along the ring circumference.

For monocoque conical tubes the following may be shown to be true. The bending moments that arise in both reinforcing rings in taking up the forces P acting along the generating line (fig. 6) are equal and opposite in the two rings and have the same value as the bending moment for the case of a cylindrical tube of equal loading P and whose reinforcing rings are the same distance apart and of the same shape, and whose linear cross-sectional dimensions are equal to the geometric mean of the corresponding cross-sectional dimensions of the two rings of the conical tube.*

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*The proof is as follows: Let an axial force be applied in the direction of the generating line at the left end of the portion of the conical tube between the two rings (fig. 7). At the other end, in the direction of the generating lines, there is a load distribution whose axial components in the same manner as for the cylindrical tube may be assumed as linear. The conical tube portion is in equilibrium under these forces as may readily be shown (all the forces pass through the cone vertex). If we now imagine the circumference of the right reinforcing ring to be divided into a definite number of parts, then the axial component of the force applied to each part is independent of the size of the ring. Thus the axial component does not change if the cone is converted into a cylinder. The bending moments in the rings at the ends of the conical tube portion included between the two rings are now computed in the general form. From this computation the result is obtained that these bending moments for a given form of cross section of the tube and given loading (and thus also for given values of the axial components) are proportional to the product of the linear dimensions of the two rings. For a conical tube these bending moments are therefore of the same magnitude as for a cylindrical tube of equal form of cross section, equal length, and equal loading when these products are also the same, as was to be proved.

Having thus carried out the computation of the ring moments for cylindrical tubes, these moments may at once be given for the conical tubes.

III. RESULTS

Following the method indicated above, the bending moments arising in rings used for reinforcing against longitudinal forces in conical tubes were computed for several particular shapes, and the computations enable the designer to obtain an estimate of the moments also for other similar shapes. The particular shapes considered were:

Bulkhead rings of constant bending strength along their circumference, circular rings, and elliptical rings with semiaxis ratio 3:2.

1. Notation

b_1, length of semiaxis of elliptical end ring representing symmetrical of "antisymmetrical" loading condition. For the circle b becomes equal to the radius a.

a_1, length of the other semiaxes of the elliptical end ring or the radius of the circle.

b_2, a_2, lengths of the corresponding semiaxis of the second reinforcing ring.

h, distance apart of the two reinforcing rings.

M_b, bending moment in the rings. M_b is considered positive when the outside fiber of the end ring is under tension, and in the second ring under pressure.

v/u, nondimensional coordinate of the point at each ring where the bending moment M_b acts; v denoting the length of arc measured along the ring circumference from the point B on the axis of symmetry, and u the semicircumference of the ring. (See sketch, fig. 8.)

Notation (Cont.)

φ, the angular coordinate corresponding to v/u for the circle.

P, the force applied at the end ring in the direction og the generatrix. Tensions are considered as positive.

x/a_1, y/b_1 nondimensional coordinates of the point of application of the force. For the circle and the ellipse both coordinates are connected by the relation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

φ_o, in the case of the circle the angular distance of the point of application of the force from the axis of symmetry or antisymmetry.

m, m', the moment coefficient corresponding to the moment M_b of the symmetrical forces P.

M, the external moment about the axis b_1 due to the antisymmetric forces P.

n, n', the moment coefficient corresponding to the moment M of the nonsymmetric forces P at the end ring.

2. Results for Symmetrical Loading

The symmetrical load chosen consisted of two equally large tensile forces P. As a particular case, the two forces P coincide to produce a single force of magnitude 2P.

The results were presented nondimensionally in the form of a moment coefficient corresponding to the bending moment M_b in the ring. For the circle and for the two ellipses this coefficient is plotted as ordinate on the figures (8, 9, 10) against the abscissa v/u, at which the bending moment M_b is applied. The equal and opposite bending moments in the two rings are

$$M_b = m\,P\,\frac{a_1\,b_2}{h} = -\,m\,P\,\frac{a_2\,b_1}{h}$$

The different m curves apply to the different points of application of the load P, the value x/a_1 being chosen as the parameter of the system. The heavy drawn curves correspond to positive values of y/b_1 while the lightly drawn curves correspond to negative values. For the circle the mathematical relation holds:

$$M_b = \frac{P\,a^2}{\pi\,h}\left[\cos\varphi\left\{\frac{5}{2}\cos\varphi_0 - (\pi - \varphi_0)\sin\varphi_0\right\} + \varphi\sin\varphi\cos\varphi_0 - \frac{\varphi^2}{2} - \frac{\varphi_0{}^2}{2} + \pi\,\varphi_0 - \frac{\pi^2}{3} + 1\right]$$

applicable to the range $0 < \varphi < \varphi_0$. For $\varphi > \varphi_0$ the relation is:

$$M_b = \frac{P\,a^2}{\pi\,h}\left[\cos\varphi\left(\frac{5}{2}\cos\varphi_0 + \varphi_0\sin\varphi_0\right) - (\pi-\varphi)\sin\varphi\cos\varphi_0 - \frac{\varphi^2}{2} - \frac{\varphi_0^2}{2} + \pi\,\varphi - \frac{\pi^2}{3} + 1\right]$$

<u>Example</u>: Let an axial force of 1,000 kilograms act on a cylindrical tube of radius $a_1 = b_1 = a_2 = b_2 = 60$ cm with $h = 80$ cm. The bending moment in the ring with $2P = 1{,}000$ kg, $P = 500$ kg is:

$$M_b = \frac{P\,a_1\,b_1}{h}\,m$$

where m has the value given by the curve $\frac{x}{a_1} = 0$ in figure 8. The maximum bending moment corresponds to the maximum value of m and therefore occurs for the position $\frac{v}{u} = 0$; that is, at the point of application of the force, and amounts to

$$M_b = 500 \times \frac{60^2}{80} \times 0.067 = 1{,}500 \text{ kg cm}$$

If four equal forces P are applied and two of these symmetrically lying forces are directed opposite to the other two, then this load represents an external bending moment, for which the ring moment M_b is again given by the figure (8). We have, namely,

$$M_b = \frac{P\ a_1\ b_1}{h}\ \Delta\ m$$

where Δ m is the difference in the ordinates of the two m curves, corresponding to the two points of application of the forces.

The formula for the circle for the region $0 < \varphi < \varphi_o$ is

$$M_b = \frac{P\ a^2}{\pi\ h}\left[\cos\varphi\left\{5\cos\varphi_o - (\pi-2\varphi_o)\sin\varphi_o\right\} + 2\varphi\sin\varphi\cos\varphi_o - \frac{\pi^2}{2} + \pi\,\varphi_o\right]$$

and for $\varphi > \varphi_o$

$$M_b = \frac{P\ a^2}{\pi\ h}\left[\cos\varphi\left\{5\cos\varphi_o + 2\varphi_o\sin\varphi_o\right\} - (\pi-2\varphi)\sin\varphi\cos\varphi_o - \frac{\pi^2}{2} + \pi\,\varphi\right]$$

The particular case is to be noted where $\varphi_o = \frac{\pi}{2}$, corresponding to a load of two pure bending moments. If the magnitude of both moments is 2M, then

$$M_b = \frac{M\ a}{\pi\ h}\ [3\cos\varphi + 2\varphi\sin\varphi - \pi]$$

<u>Example</u>: A bending moment is to be transmitted in an elliptical conical shell ($b_1 = 60$, $a_1 = 40$, $b_2 = 48$, $a_2 = 32$, and $h = 80$ cm). Four forces $P = 1{,}000$ kg act symmetrically with respect to the major axis b of the ellipse. For two of the forces $x = 28$ cm, and for the other two $x = 38.8$, ring I.

a) For all of the four forces acting on the same side of the minor axis, the heavy curves $x/a_1 = 0.7$ and $x/a_1 = 0.97$ are to be considered. The maximum difference in the m values between these two curves, according to figure 9, occurs for $v/u = 0.48$ and amounts to 0.047.

The maximum moment is therefore

$$M_{b_{max}} = 0.047 \frac{1000 \times 40 \times 48}{80} = 1130 \text{ cm kg}$$

b) If the two pairs of forces act on different sides of the minor semiaxis, the heavily drawn curve ($x/a_1 = 0.7$) and the lightly drawn curve ($x/a_1 = 0.97$), are the ones considered. The maximum difference now obtained for the m values for $v/u = 0$ is 0.057. Thus, the maximum moment now is

$$M_{b_{max}} = 0.057 \frac{1000 \times 40 \times 48}{80} = 1370 \text{ cm kg}$$

3. Results for "Antisymmetrical" Loading

As an antisymmetrical loading there are chosen a tensile force P and a compressive force of equal magnitude $-P$, which together exert an external moment M about the axis b_1.

The results are presented in nondimensional form in terms of a moment coefficient n. This value is computed for the circle and plotted on figure 11 as ordinate. The bending moment in the ring is then

$$M_b = \frac{M\,a_1}{h}\,n$$

The formula for a couple P, $-P$ valid for the range $0 < \varphi < \varphi_o$ is

$$M_b = \frac{P\,a^2}{\pi\,h}\left[\sin\varphi\left\{\frac{5}{2}\sin\varphi_o + (\pi-\varphi_o)\cos\varphi_o\right\} - \varphi\cos\varphi\sin\varphi_o - \varphi\,(\pi-\varphi_o)\right]$$

and for $\varphi > \varphi_o$

$$M_b = \frac{P\,a^2}{\pi\,h}\left[\sin\varphi\left\{\frac{5}{2}\sin\varphi_o - \varphi_o\cos\varphi_o\right\} + (\pi-\varphi)\cos\varphi\sin\varphi_o - \varphi_o\,(\pi-\varphi)\right]$$

Here too there is a limiting case for $\varphi_0 = 0$, corresponding to a pure moment M at the position $\varphi_0 = 0$. The formula obtained is

$$M_b = \frac{M\ a}{2\pi\ h}\left[\frac{3}{2}\ \sin\varphi - (\pi-\varphi)\ (1-\cos\varphi)\right]$$

On superimposing another couple P, $-P$ in analogy to the previous case considered, we have for $0 < \varphi < \varphi_0$

$$M_b = \frac{P\ a^2}{\pi\ h}\left[\sin\varphi\ \left\{5\ \sin\varphi_0 + (\pi-2\varphi_0)\ \cos\varphi_0\right\} - 2\varphi\ \cos\varphi\ \sin\varphi_0 - \pi\ \varphi\right]$$

This case corresponds to the one previously considered, rotated by $\pi/2$, and the formula may in fact be obtained from the other by a transformation of coordinates.

Example: Let two equal and oppositely directed forces of 1,000 kilograms each be applied at the end of a circular conical shell ($a_1 = b_1 = 60$, $a_2 = b_2 = 50$, and $h = 80$ cm). Let the angle subtended at the center be 100°. We first compute the moment M of the two forces about the axis b_1. We have:

$$x = a_1\ \sin 50^\circ = 60 \times 0.766 = 46\ \text{cm}$$

$$M = P \times 2x = 1{,}000 \times 92 = 92{,}000\ \text{cm kg}$$

The bending moment in the reinforcing ring, therefore, is:

$$M_b = M\ \frac{a_1\ a_2}{h}\ n$$

where for $x/a_1 = 0.766$, n is to be obtained from figure 11 by interpolation. The maximum value of n occurs for $v/u = 0.23$ and is equal to 0.040. The maximum moment in each ring is, therefore:

$$M_{b_{max}} = 92{,}000 \times \frac{50 \times 60}{80} \times 0.040 = 2{,}520\ \text{cm kg}$$

Example: Let four forces be applied at the end of the cylindrical shell ($R = 60$ cm, $h = 80$ cm) that have a resultant zero. One pair of oppositely directed forces is applied at $\varphi_1 = 30^\circ$ and possesses the moment $M_0 =$

100,000 cm kg. The other oppositely directed pair act at $\varphi_2 = 135^\circ$ and exert an equal and opposite moment. We have:

$$M_b = M_o \frac{R}{h} \left[(n)_{\substack{\frac{x}{a} = 0.5 \\ \frac{y}{a} = \text{positive}}} - (n)_{\substack{\frac{x}{a} = 0.707 \\ \frac{y}{a} = \text{negative}}} \right]$$

The maximum difference between the values in the brackets occurs, according to figure 11, for $\varphi = 41^\circ$ and amounts to $n_1 - n_2 = 0.087$. Thus

$$M_{b_{max}} = 100,000 \times \frac{60}{80} \times 0.087 = 6,530 \text{ cm kg}$$

Figure 13 shows the particular case for two pairs of forces corresponding to a pure moment. This figure is used to particular advantage when the maximum moment in the ring and not the point applied, is desired. The graph clearly shows the large oscillations undergone by the maximum stress depending on the position of the points of application of the forces.

IV. EXTENSION OF APPLICABILITY OF THE RESULTS

1. Other Forms of Reinforcing Frames

In order to extend the range of applicability of the computations which have been carried out for only three particular forms of reinforcing rings, it was attempted to find such nondimensional values for the abscissa, ordinate, and parameter of the point of application of the forces, as to cover the curve families for all three forms as far as possible. This point of view led to the choice of the variables m, n, v/u, x/a, and y/b as given above. The comparison made in figure 12 of the symmetrical loading for several points of application shows quite good agreement with the groups of curves. In particular, if the comparison is made, for definite points of application, between the maximum values of m for the ellipse with those for the circle, then the difference amounts to - at most - 20 percent.

From the above agreement, the following rule may be set up for estimating an upper limit for the maximum bending moments for ellipses with other semiaxis ratios and for other elliptical-shaped frames; for example, those put together out of four circular arcs:

For the given end frame with semiaxes a_1 and b_1, the ratios x/a_1 and y/b_1 corresponding to the given points of load application, are determined. The maximum value of m or n is taken for the less favorable of these two ratios from figures 8, 11, or 13 applicable to circular forms. The maximum bending moment for the given form of reinforcing ring is then approximately

$$M_b \approx P \frac{a_1 \ b_2}{h} m$$

or

$$M_b \approx M \frac{b_2}{h} n$$

The value obtained is then multiplied by the larger of the two values $\sqrt{a/b}$ and $\sqrt{b/a}$.

<u>Example</u>: Given a cylinder having a cross section similar to an ellipse with semiaxes $a = 45$, $b = 75$, and distance $h = 80$ cm. Four forces, each of 1,000 kilograms, are applied at the four points with coordinates $x/a = \pm 0.65$ and $y/b = \pm 0.8$ and produce a pure moment about the axis b, The coordinates x, y do not satisfy the ellipse equation to the value $y/b = 0.8$, corresponding to a value $x/a = 0.60$ on the ellipse. According to figure 13, there corresponds, for the circle, to the value $x/a = 0.60$ a value $n = 0.0043$; to $x/a = 0.65$ a value $n = 0.0039$. The larger value is the one used. Hence,

$$M_b \approx P \frac{a_1 \ b_2}{h} n = 1{,}000 \times \frac{45 \times 75}{80} \times 0.0043 = 181 \text{ cm kg}$$

Multiplying by the correction factor $\sqrt{b/a}$ the final result becomes

$$M_b \approx 181 \sqrt{\frac{75}{45}} = 234 \text{ cm kg}$$

2. Reinforcing Frames with Variable Strength in Bending

For those frames whose strength in bending varies along the circumference, it generally will not be difficult to estimate the maximum values of the bending moments on the basis of the above-computed results. If the frame is built so as to have a constant moment of inertia and is reinforced only at the places of maximum bending moments, then the strain energy in bending - that is, the mean value of the moments squared - decreases at the nonreinforced portion of the member. It will always be on the safe side therefore to dimension this nonreinforced portion in accordance with the maximum moment which would occur at this portion if the bending strength were constant all about the circumference.

V. SUMMARY

In taking up the axial forces in monocoque structures, bending moments are set up in the reinforcing rings. For the case where two reinforcing rings are provided to take up the forces, the bending moments are determined for several loading conditions and plotted on charts, thus removing the burden of involved computation from the designer.

Translation by S. Reiss,
National Advisory Committee
for Aeronautics.

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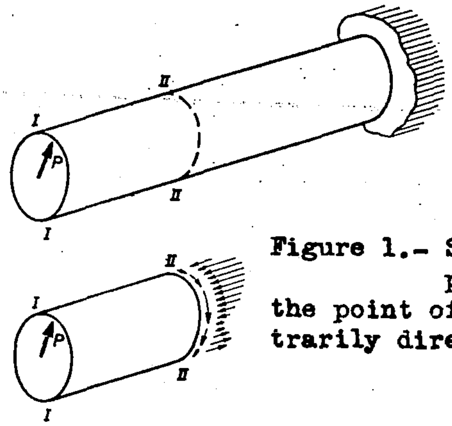

Figure 1.- Stress distribution at a position far removed from the point of application of an arbitrarily directed force.

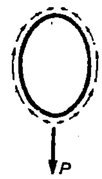

Figure 2.- Ring loading due to a transverse force.

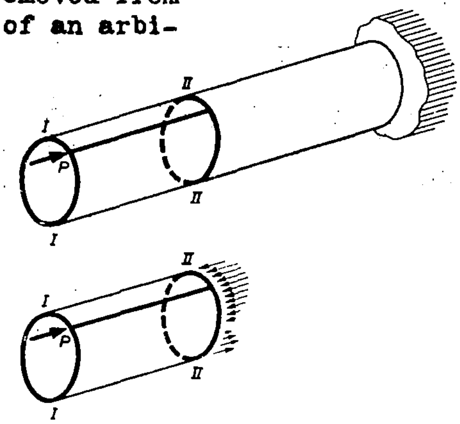

Figure 3.- Transmission of an axial force in a cylindrical shell.

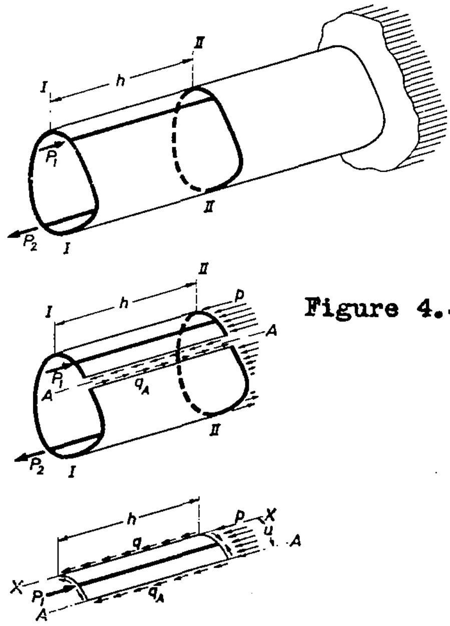

Figure 4.- Axial force distribution on cylindrical shell.

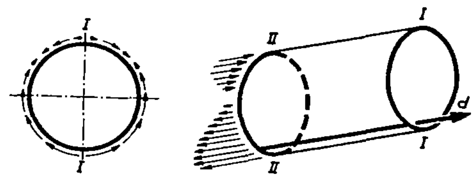

Figure 5.- Frame loading of a circular cylindrical shell.

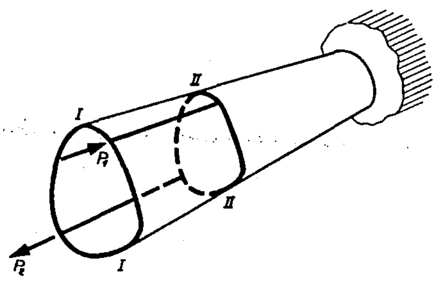

Figure 6.- Taking up of axial forces in a conical shell.

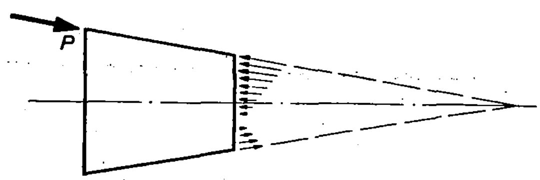

Figure 7.- Equilibrium of a truncated conical shell.

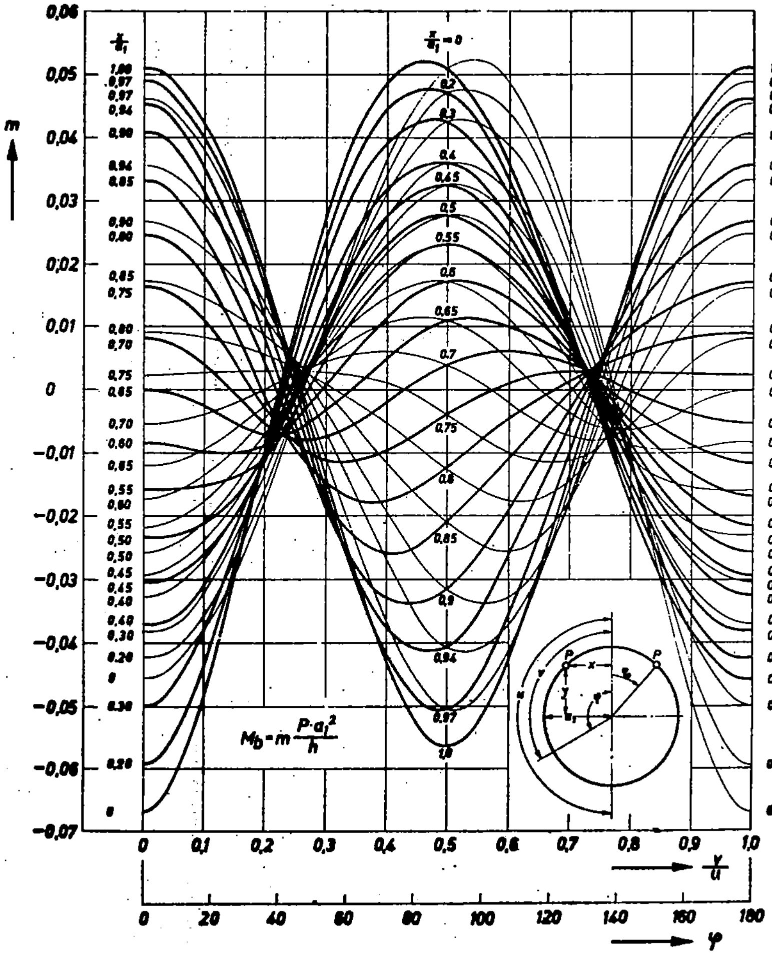

Figure 8.- Moment coefficient for symmetrically loaded circular ring.

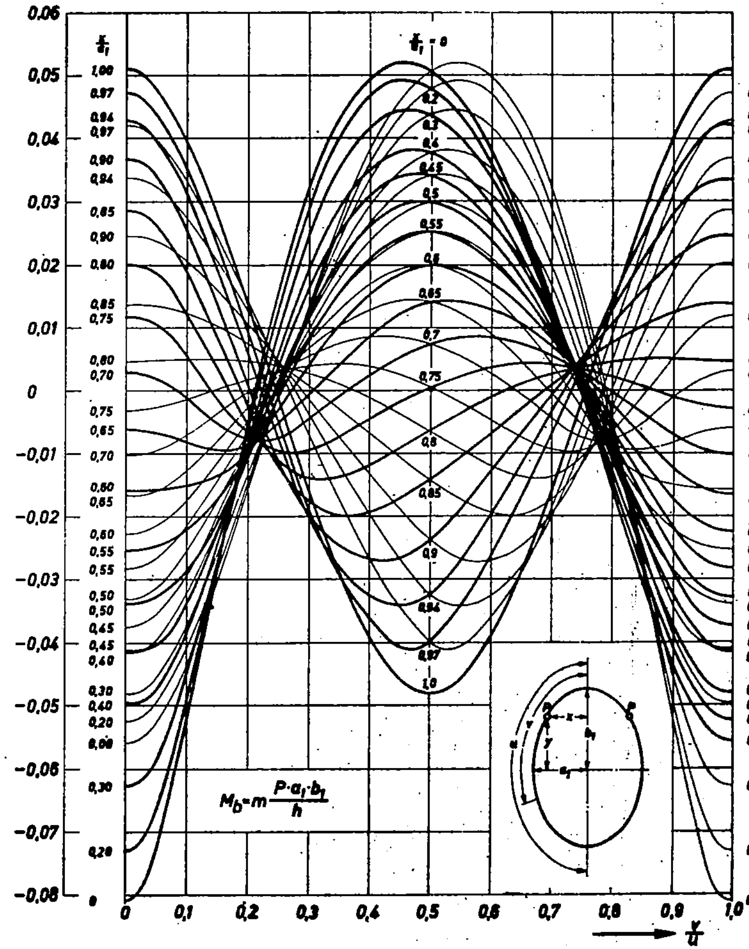

Figure 9.- Moment coefficient for symmetrically loaded elliptical ring with semi-axes ratio $a_1/b_1=2/3$.

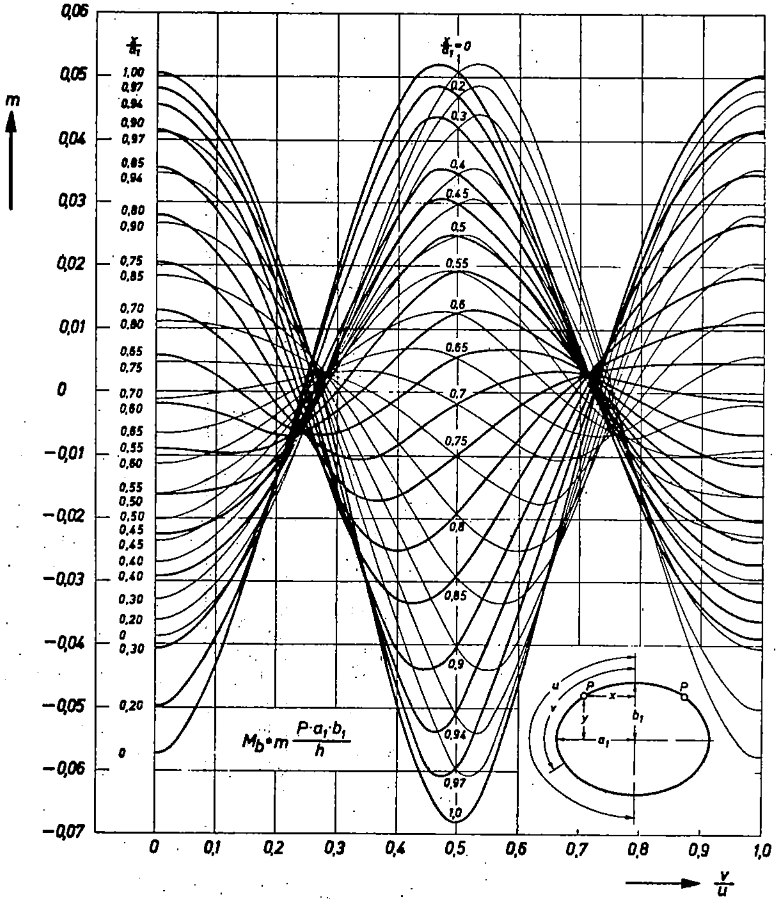

Figure 10.- Moment coefficient for symmetrically loaded elliptical ring with semi-axes ratio $a_1/b_1=3/2$

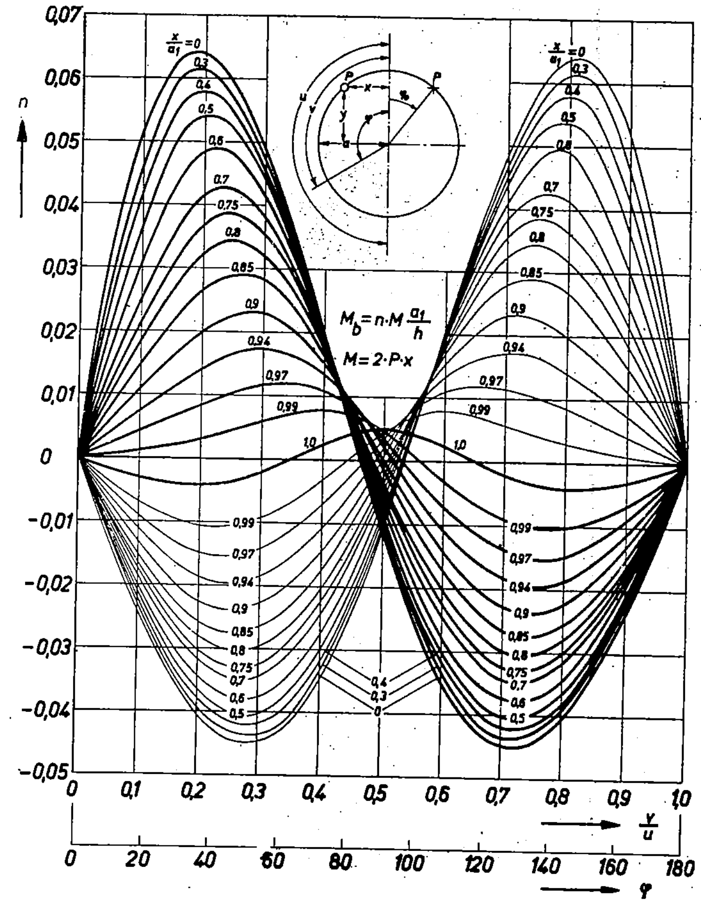

Figure 11.- Moment coefficient for"anti-symmetrically"loaded circular ring.

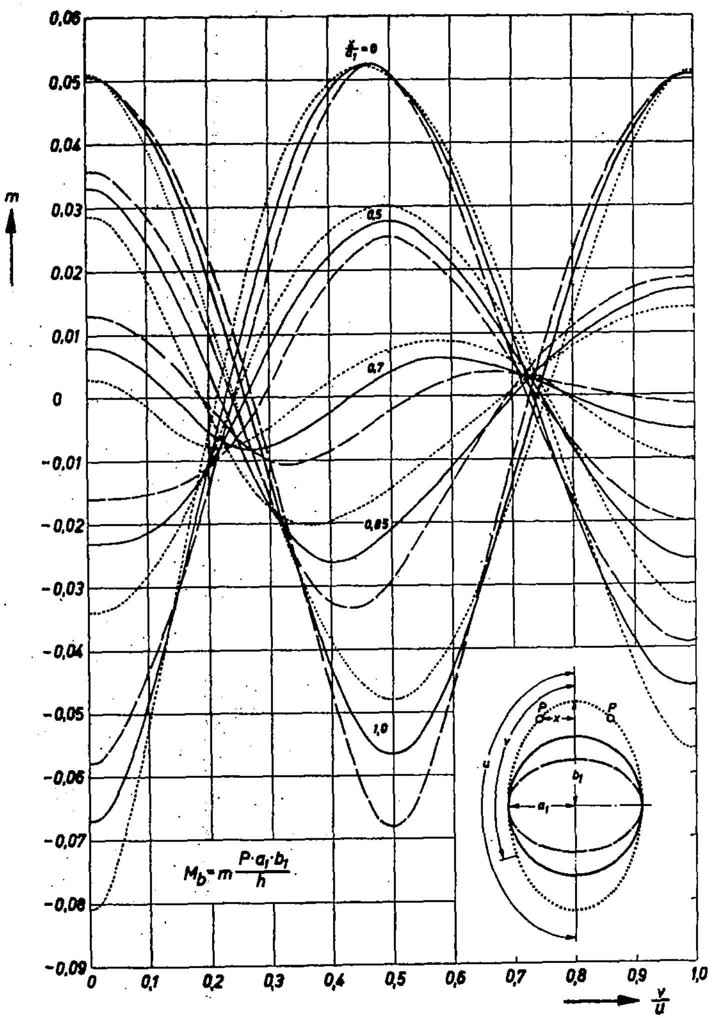

Figure 12.- Comparison of moment coefficient for various elliptical rings (semi-axes ratio a/b=2/3,1/1,3/2).

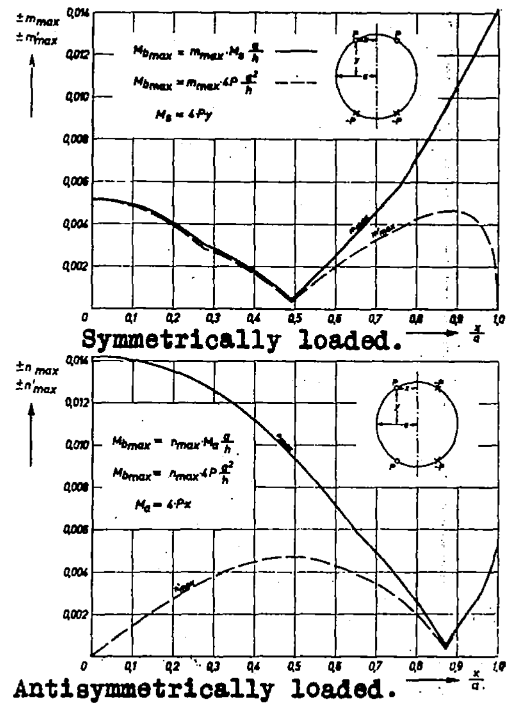

Figure 13.- Maximum moment coefficients for circular rings.
1, depending on the moment M_a of the load ———
2, depending on the magnitude of the concentrated force P — — — —