

COPIES
10

CASE FILE
COPY



TECHNICAL MEMORANDUMS

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

Original No. 774

No. 774

TENSION FIELDS IN ORIGINALLY CURVED, THIN SHEETS
DURING SHEARING STRESSES

By H. Wagner and W. Ballerstedt

Luftfahrtforschung
Vol. XII, No. 2, May 16, 1935
Verlag von R. Oldenbourg, München und Berlin

REPRODUCED BY
NATIONAL TECHNICAL
INFORMATION SERVICE
U. S. DEPARTMENT OF COMMERCE
SPRINGFIELD, VA. 22161

Washington
August 1935

TECHNICAL MEMORANDUM NO. 774

TENSION FIELDS IN ORIGINALLY CURVED, THIN SHEETS
DURING SHEARING STRESSES

By H. Wagner and W. Ballerstedt

SUMMARY

So-called shell (or stressed skin) bodies or wings consist of outer skin and stiffening sections, the latter running in part longitudinally (longitudinal stiffeners) and partly transversely (transverse stiffeners, rings).

The curved skin is usually so thin that it buckles long before the structure has reached the ultimate load. Buckling is followed by the appearance of oblique wrinkles in the sheet.

The analysis of the stresses in the sheet and stiffeners is predicated upon the direction of the wrinkles, particularly the tensile stresses (principal stresses). This analysis and the calculation of the stresses after buckling form the subject of the present article. It includes:

1. Metal cylinders with closely spaced longitudinal stiffeners.
2. Metal cylinders with closely spaced transverse rings.

As concerns the longitudinal stiffeners, the report forms, aside from minor modifications, a repetition of the theoretical part of an experimental report by H. Wagner at the Rohrbach Metal Airplane Company in 1927, and whose publication has been permitted by Dr. Rohrbach.

*"Über Zugfelder in ursprünglich gekrümmten, dünnen Blechen bei Beanspruchung durch Schubkräfte." Luftfahrtforschung, May 16, 1935, pp. 70-74.



The literature index refers to H. Wagner's report on flat sheet metal girders with very thin metal web (reference 1).

NOTATION

- s, sheet thickness.
- r, radius of cylinder.
- x, coordinate in axial (longitudinal) direction.
- y, coordinate in peripheral direction.
- F_x , sectional surface of a longitudinal stiffener.
- F_y , sectional surface of a transverse ring.
- t_x , distance between two longitudinal stiffeners.
- t_y , distance between two transverse rings.
- X, compressive force in a longitudinal stiffener.
- Y, compressive force in a transverse ring.
- σ_x , tensile strain in a longitudinal stiffener.
- σ_y , tensile strain in a transverse ring.
- f, deflection of a longitudinal stiffener.
- τ , shearing stress due to given external load.
- τ_0 , buckling stress in shear of sheet.
- σ , tension in sheet after formation of tension field.
- α , direction between principal tensile stress σ ; particularly, elongation and axis x.
- E, modulus of elasticity.
- $\left. \begin{array}{l} \epsilon \\ \epsilon_x \\ \epsilon_y \end{array} \right\}$ principal elongation and elongation in the tension field in direction of x and y.
- γ , shear strain.

THE STRESS IN SHEET AND STIFFENERS

The premises of the calculation are that the dimensioning of sheet and stiffeners and the stress in shear and tension are uniform within the entire range.

In a suitably designed flat sheet wall the buckling load of the sheet versus shear is usually so low compared with the permissible stress in the tension field that the influence of the compressive stiffness of the sheet may be disregarded. Now, in curved sheets the buckling stress is usually substantially higher on the one hand, and on the other, no such high stresses are permissible once buckling has taken place; otherwise, the wrinkling in the curved sheet would induce permanent deflections. Thus the analysis of the diagonal tension fields of curved sheets must allow for the buckling stiffness of the sheet.

The following considerations are based upon the argument that the sheet, even after buckling, continues to transmit the proportion of the total shearing stress τ corresponding to the buckling stress in shear τ_0 , and only the proportion $\tau - \tau_0$ extending beyond the buckling load, in tension. From this follows: the stiffeners which, admittedly, are not stressed prior to the buckling, undergo a stress only to the amount of the excess $\tau - \tau_0$.

Figure 1 illustrates a piece of the shell. From the equilibrium of the forces in section AA in the y direction, and section BB in the x direction, the proportion $\tau - \tau_0$ gives the forces (compression) in the stiffeners (equation (10), N.A.C.A. Technical Memorandum No. 604, reference 1):

$$X = - (\tau - \tau_0) s t_x \cot \alpha \quad (1a)$$

$$Y = - (\tau - \tau_0) s t_y \tan \alpha \quad (1b)$$

Thus the tension in longitudinal and transverse stiffeners becomes:

$$\sigma_x = - \xi \tau \cot \alpha \quad (2a)$$

$$\sigma_y = - \eta \tau \tan \alpha \quad (2b)$$

where, for the sake of brevity:

$$\zeta = \frac{\tau - \tau_0}{\tau} \frac{s t_x}{E_x} \quad (3a)$$

$$\eta = \frac{\tau - \tau_0}{\tau} \frac{s t_y}{E_y} \quad (3b)$$

By the same argument the stress of the sheet is the resultant stress of buckling stress in shear τ_0 and a tensile stress $\frac{\tau - \tau_0}{\sin \alpha \cos \alpha}$. However, the tensile stress decisive for the dimensioning of the sheet is simply put at

$$\sigma = \frac{\tau}{\sin \alpha \cos \alpha} \quad (4)$$

and the principal elongation of the sheet at

$$\epsilon = \frac{\tau}{E \sin \alpha \cos \alpha} \quad (5)$$

The minor inaccuracy introduced with this simplification is well within the degree of accuracy of the assumption that the stress after buckling in shear, is τ_0 .

DIRECTION OF WRINKLES IN SHEETS WITH CLOSELY SPACED LONGITUDINALS

It is presumed in the following derivation that the spacing of the transverse rings is greater than that of the longitudinal stiffeners - by at least twice as much. The spacing of the longitudinal stiffeners is presumed to be small relative to the radius. For example,

$$t_x \leq \frac{1}{3} r$$

In order to compute the direction of the wrinkles in the tension field, we first consider the elongations ϵ_x and ϵ_y of the sheet in x and y directions. The elongation ϵ_x is given through the force in the longitudinal

stiffener

$$\epsilon_x = \frac{\sigma_x}{E} = -\frac{\tau}{E} \zeta \cot \alpha \quad (6)$$

As concerns the determination of the elongation ϵ_y of the sheet panel in y direction, we refer to figure 2. Under shear the originally unstressed curved sheet is stretched straight while forming tension wrinkles. The small distance $-t_x \epsilon_y$, for which the straight connection of points O' and U' is shorter after loading than the curved connection of points O and U before loading, is now calculated. The shortening due to the straight stretching alone is ($\varphi = t_x/r$),

$$r \varphi - 2r \sin \frac{\varphi}{2} = r \frac{\varphi^3}{24} = \frac{1}{24} \frac{t_x^3}{r^2}.$$

Besides, the two longitudinal stiffeners O and U approach each other to the measure of

$$- \frac{\sigma_y}{E} t_x$$

as a result of the compressive stress σ_y in the cross members. The desired shortening of \widehat{OU} to $\widehat{O'U'}$ becomes

$$- t_x \epsilon_y = + \frac{1}{24} \frac{t_x^3}{r^2} - \frac{\sigma_y}{E} t_x$$

Consequently, the elongation in transverse direction ϵ_y is (see (2b)):

$$\epsilon_y = - \frac{1}{24} \frac{t_x^3}{r^2} - \frac{\tau}{E} \eta \tan \alpha \quad (7)$$

The elongation ϵ of the sheet itself being given in (5), the three quantities ϵ , ϵ_y , ϵ_x defining the tension field are known also. Three such quantities, however, establish a plain (flat) elongation condition and consequently, also the elongation attitude of the tension field and the direction angle α of the principal elongation ϵ , particularly, the wrinkles.

$$\tan^2 \alpha = \frac{\epsilon - \epsilon_x}{\epsilon - \epsilon_y} \quad (8)$$

ϵ , ϵ_x , ϵ_y themselves are dependent on α according to (5), (6), and (7). The insertion of the elongations conformably to these three equations finally gives

$$\frac{t_x}{r} \sqrt{\frac{E}{\tau}} = \sqrt{24 \tan \alpha \left(\frac{\cos 2 \alpha}{\sin^4 \alpha} + \zeta \cot^4 \alpha - \eta \right)} \quad (9)$$

This equation is resolved according to $\frac{t_x}{r}$, rather than α , which is not explicitly possible.

Now we effect a correction on ϵ_y . When establishing the equation for ϵ_y , we allowed for the straight stretching of the sheet and the approach of O and U induced by the compressive stress σ_y in the transverse stiffeners. Through the deflection of the skin tension the longitudinal stiffeners are loaded with a force directed toward the axis of the cylinder

$$s t_y (\tau - \tau_0) \frac{t_x}{r} \tan \alpha$$

which is approximately evenly distributed over length t_y of the stiffener, and slightly bend inward for a readily calculable degree f . This deflection f modifies ϵ_y for an amount ϵ_f ; that is (see (7)),

$$\epsilon_y = - \frac{1}{24} \frac{t_x^2}{r^2} + \frac{\sigma_y}{E} + \epsilon_f \quad (10)$$

To illustrate: If several adjacent longitudinals bend for the same amount of f (fig. 3), then ϵ_f has the value $\frac{f}{r}$; but as the deflection of these longitudinals is zero at the points connecting the transverse stiffeners, we may write approximately:

$$\epsilon_f = - \frac{1}{2} \frac{f}{r} \quad (11)$$

Thus with ϵ_y written in (8) conformable to (10) rather than to (7), it gives

$$\frac{t_x}{r} \sqrt{\frac{E}{\tau}} \sqrt{1 + \frac{12 f r}{t_x^2}} = \sqrt{24 \tan \alpha \left(\frac{\cos 2 \alpha}{\sin^4 \alpha} + a \right)} \quad (12)$$

whereby
$$a = \zeta \cot^4 \alpha - \eta \quad (13a)$$

or
$$a = \frac{\sigma_x}{\tau} \cot^3 \alpha - \eta \quad (13b)$$

Now follows the discussion of equations (9) to (13). In any case, only one of the two equations (13) is to be used. In the absence of external longitudinal forces other than the shearing forces (torque), (13a) is suitable, whereas (13b) is preferable when the permissible compression σ_x of the longitudinals is given from the start; that is, chiefly, when the shell is to be dimensioned for concurrent action of transverse and longitudinal forces. If accompanied by simultaneous bending of the longitudinal stiffeners, then σ_x denotes the tension of the fiber of the longitudinal lying on the skin of the sheet; that is, the mean tension of this fiber in longitudinal direction.

In a design $\frac{t}{r}$ will probably be chosen beforehand, and the loading τ will also be given. The deflection f of the longitudinal stiffeners - usually of little influence - may be estimated first, at (say $f = 0.3$ cm) or put at zero. This affords the left-hand side of (12). This is followed by assuming arbitrary values for a (say 10, 20, 30), and reading on figure 4 the corresponding values of $\cot \alpha$ conformable to the known coordinate (left side of (12)) and the axis of the abscissa. Compute $\cot \alpha$ according to the chosen a values from (13a) or (13b); that is, from either

$$\cot \alpha = \sqrt[4]{\frac{a + \eta}{\zeta}} = \sqrt[4]{\frac{F_x}{s t_x} \left(\frac{\tau}{\tau - \tau_0} a + \frac{s t_y}{F_y} \right)} \quad (14a)$$

or

$$\cot \alpha = \sqrt[3]{\frac{\tau}{\sigma_x} (a + \eta)} = \sqrt[3]{\frac{\tau}{\sigma_x} \left(a + \frac{s t_y}{F_y} \frac{\tau - \tau_0}{\tau} \right)} \quad (14b)$$

The plotting of the two unlike $\cot \alpha$ against a , once from figure 4, then from one of (14) gives an intersection point which denotes the correct value of $\cot \alpha$

Lastly, the angle of the tension stress is:

$$\gamma = 2 \cot \alpha (\epsilon - \epsilon_x) \quad (15)$$

EXAMPLE

Given in kg, cm

$$\begin{array}{ll}
 \tau = 400 & E = 700,000 \\
 s = .06 & r = 50 \\
 t_x = 18 & F_x = 1 \\
 t_y = 40 & F_y = 1.2 \\
 f = 0.4 & \text{(estimated)}
 \end{array}$$

The buckling stress in shear is:

$$\tau_0 = 0.1 E \frac{s}{r} + 5.3 \left(\frac{s}{t} \right)^2 E = 125 \frac{\text{kg}}{\text{cm}^2}$$

for which the left side of (12) gives 19.9.

Now choose three values for a : say, 10, 15, 20, and read, once from figure 4, with an ordinate of 19.9 on the abscissa, the three values: $\cot \alpha = 2.3, 2.1, 1.8$.

No external axial forces being present, compute from (14a) corresponding to the three chosen a values: $\cot \alpha = 1.98, 2.17, 2.30$. Then plot both $\cot \alpha$ curves against a (fig. 5) and find that both values agree at $\cot \alpha = 2.14$; consequently, $\cot \alpha = 2.14, \alpha = 25.1^\circ$.

DIRECTION OF WRINKLES IN SHEETS WITH
CLOSELY SPACED TRANSVERSE STIFFENERS (RINGS)

For the following, it is presumed that the spacing of the transverse stiffeners (rings) is, at the most, about half as great as that of the longitudinal stiffeners, in addition to being small compared with the radius of the cylinder, say $t_y \leq \frac{1}{3} r$. Such an experimentally obtained tension field is shown in figure 6.

The analysis of the elongation attitude preferably proceeds from constant principal stress σ , particularly

principal elongation ϵ within the whole tension field range and equal direction angle α . The elongation ϵ_y in y direction (peripheral direction) at each point of the tension field, is readily ascertainable.

Figure 7 shows two views of a metal cylinder between two transverse stiffeners (rings). Because of the flat stretching of the sheet fibers, each fiber appears as chord of the base circle. The mid-ordinate p of the segment is

$$p = \frac{t_y^2}{8r} \tan^2 \alpha \quad (16)$$

This mid-ordinate represents the contraction of the cylinder in the center between the rings. The periphery of the circle being proportional to the radius, the elongation of the field peripherally in the center between the rings due to the contraction, is

$$\epsilon_k = -\frac{p}{r} \quad (17)$$

As a result of the flat stretching of the fibers, the tension field lies on a hyperboloid of rotation; hence contraction and elongation in y direction are unlike at unlike points x of the height. Assume all variable quantities over x to be denoted by $'$ such as p' , ϵ_k' . The contour of the hyperboloid of rotation is a parabola within the degree of accuracy of our analysis; that is, p' and ϵ_k' are parabolic over the height of the cylinder.

$$\frac{p'}{p} = \frac{\epsilon_k'}{\epsilon_k} = 1 - \left(\frac{x}{\frac{1}{2} t_y} \right)^2 \quad (18)$$

The total, probably always negative elongation ϵ_y' in peripheral direction then is the sum of ϵ_k' and elongation $\frac{\sigma_y}{E}$ of the rings:

$$\epsilon_y' = \epsilon_k' + \frac{\sigma_y}{E} \quad (19)$$

On the other hand, the quantities ϵ , α , and ϵ_y' defining the elongation field being known at every point of

the field of the sheet, the elongation attitude which at every point of the hyperboloid of rotation must be considered perpendicular to the surface, is definitely established at each point. All other quantities of the elongation field can be expressed through

$$\gamma' = 2 \tan \alpha (\epsilon - \epsilon_{y'}) \quad (20)$$

$$\epsilon_{x'} = \epsilon (1 - \tan^2 \alpha) + \epsilon_{y'} \tan^2 \alpha \quad (21)$$

$\epsilon_{y'}$ being variable along the height of the cylinder, these two elongation quantities are also variable. Now the whole peripheral displacement of the upper ring relative to the lower, is denoted by γt_y , and the change in distance of the two rings, by $\epsilon_x t_y$. These quantities are obtained by integrating γ' and $\epsilon_{x'}$ over the length t_y of the cylinder (over x). The resulting shear strain is

$$\gamma = 2 \tan \alpha \left(\epsilon - \frac{\sigma_y}{E} - \frac{2}{3} \epsilon_k \right) \quad (22)$$

In the calculation of the changes in $\epsilon_x t_y$ it must, in addition to $\epsilon_{x'}$, be observed that the surface elements of the tension field lying on the hyperboloid of rotation, slope at $\frac{dp'}{ax}$ relative to the cylinder axis; that is, that the ring spacing is less than the length of the (approximately parabolic) meridian of the surface of the tension field:

$$\epsilon_x t_y = \int_{-\frac{t_y}{2}}^{+\frac{t_y}{2}} \left[\epsilon_{x'} - \frac{1}{2} \left(\frac{dp'}{dx} \right)^2 \right] dx$$

With due regard to (21) and (16), (17), (18), and (19), the calculation finally gives the specific approach ϵ_x of both rings at

$$\epsilon_x = \epsilon (1 - \tan^2 \alpha) + \tan^2 \alpha \left(\frac{\sigma_y}{E} + \epsilon_k \right) \quad (23)$$

or, in different form:

$$\tan^2 \alpha = \frac{\epsilon - \epsilon_x}{\epsilon - \left(\frac{\sigma_y}{E} + \epsilon_k \right)} \quad (24)$$

For computing the direction of the wrinkles with given size of sheet and stiffeners, we write the values of (5), (23), (2b), and (17) in equation (24) for ϵ , ϵ_x , σ_y , and ϵ_k . It gives

$$\frac{t_y}{r} \sqrt{\frac{E}{T}} = \sqrt{8 \cot \alpha \left(\frac{\cos 2 \alpha}{\sin^4 \alpha} + a \right)} \quad (25)$$

whereby a has the significance denoted in (13a) or (13b). Figure 8 was plotted conformably to equation (25).

The manipulation of these equations is precisely as for closely spaced, longitudinal stiffeners.

Translation by J. Vanier,
National Advisory Committee
for Aeronautics.

REFERENCE

1. Wagner, Herbert: Flat Sheet Metal Girders with Very Thin Metal Web.
 - Part I - General Theories and Assumptions. T.M. No. 604, N.A.C.A., 1931.
 - Part II - Sheet Metal Girders with Spars Resistant to Bending - Oblique Uprights - Stiffness. T.M. No. 605, N.A.C.A., 1931.
 - Part III - Sheet Metal Girders with Spars Resistant to Bending - The Stress in Uprights - Diagonal Tension Fields. T.M. No. 606, N.A.C.A., 1931.

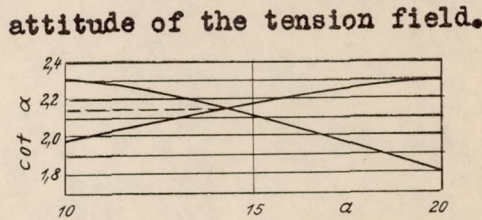
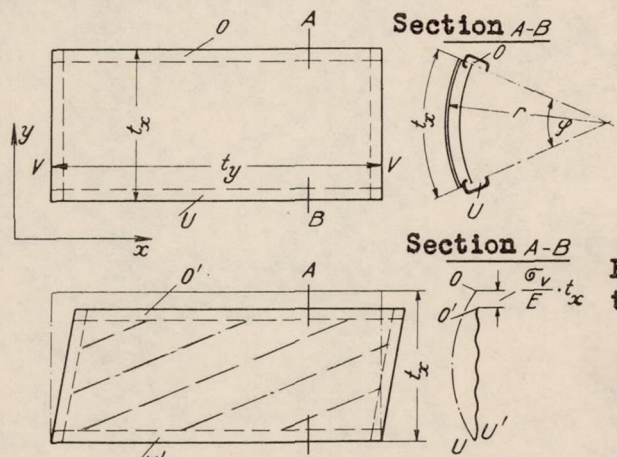
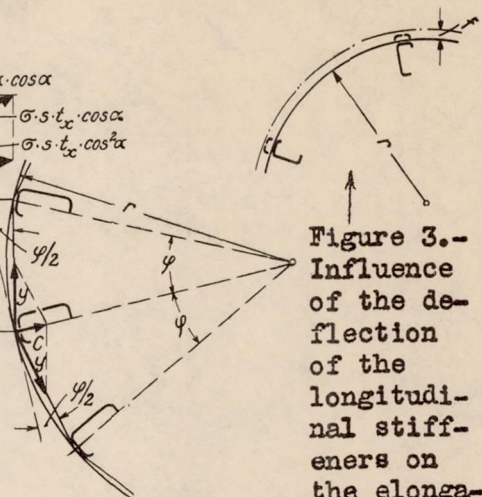
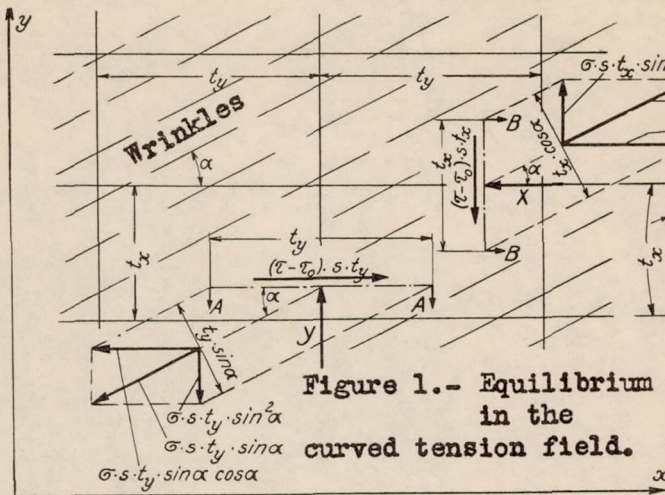


Figure 2.- The tension field in a curved piece of the shell is comparable to that in a flat tension field accompanied by an additional approach of both flanges.

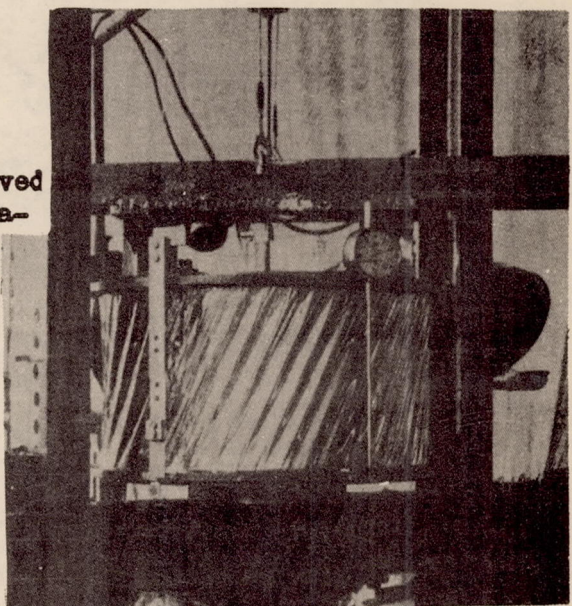
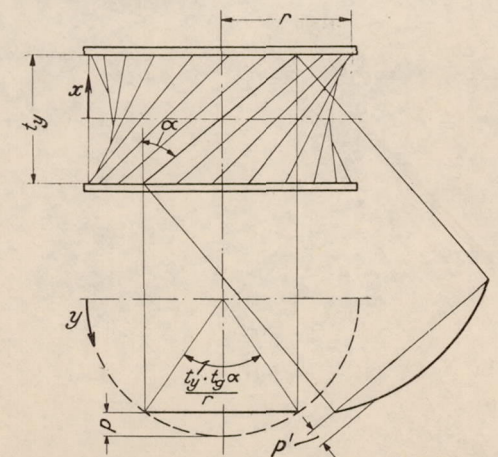


Figure 6.- Wrinkling of a metal cylinder under shear; experiment.

Figure 7.- Tension field closely stiffened in peripheral direction; effect of straight stretching of fibers on the elongation attitude of the tension-field components.

Figure 7.- Tension field closely stiffened in peripheral direction; effect of straight stretching of fibers on the elongation attitude of the tension-field components.

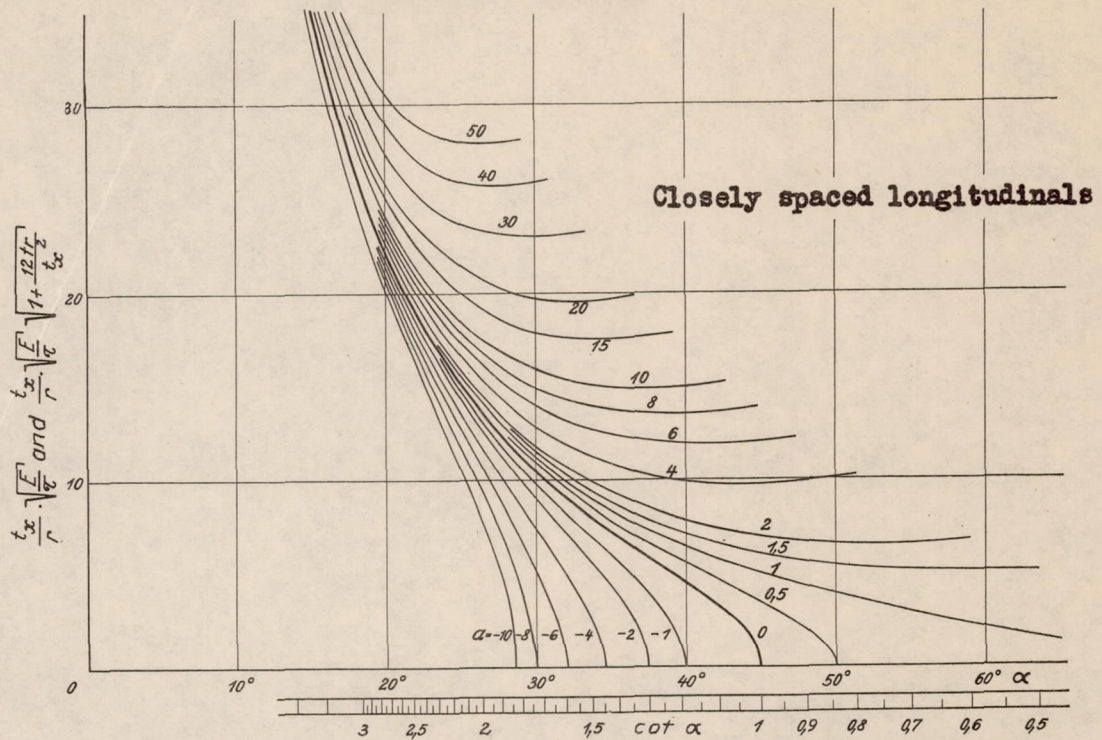


Figure 4.- Cylinder with closely spaced longitudinal stiffeners; relation between distance and size of stiffeners, shear load τ and direction of wrinkles.

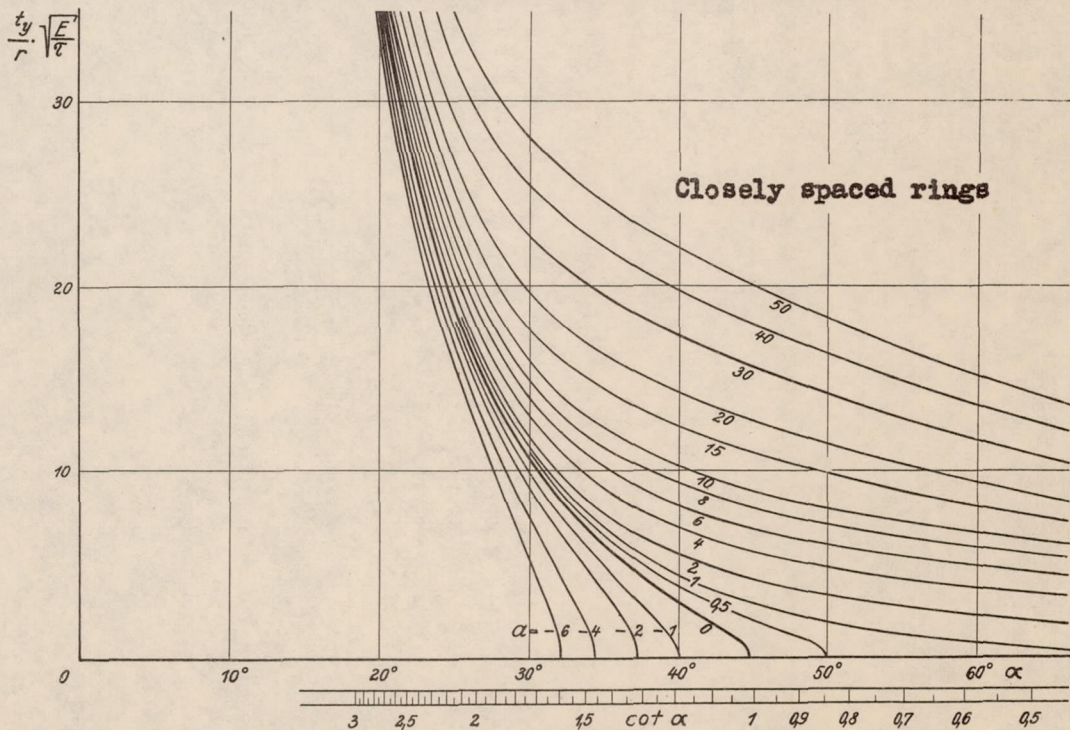


Figure 8.- Metal cylinder with closely spaced circumferential stiffeners; relation between distance and size of stiffening sections, shear τ and direction of wrinkles.