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THE DANGEROUS FLAT SPIN AND THE FACTORS AFFECTING IT

By Richard Fuchs and Wilhelm Schmidt

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1. Notation

a) Axes

All the axes pass through the center of gravity (C.G.) of the airplane, their positive direction being as indicated by the arrows in Figure 1.

Air axes (fixed with respect to flight path):

X, path axis tangent to path of C.G. of airplane;
Z, horizontal axis perpendicular to X;
Y, axis perpendicular to both X and Z.

Body axes (fixed with respect to airplane):

x, fuselage axis (longitudinal axis);
y, strut axis (normal axis);
z, spar axis (lateral axis).

Other axes:

y_1, axis of lift in plane of symmetry perpendicular to X;
z_1, axis perpendicular to X and y_1;
x_1, axis perpendicular to y and z_1.

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*"Der gefährlicliche flache Trudelflug und seine Beeinflüssung," from Zeitschrift für Flugtechnik und Motorluftschiffahrt, July 14 (p. 325), and July 28 (p. 359), 1930, publishedby R. Oldenbourg, Münich und Berlin.

b) Determination of Path of C.G. of Airplane

v (m/s), velocity along flight path (path velocity);

$\dot{v}$ (m/s²), change in path velocity with time;

φ (deg.), angle of glide, between axis X and its projection on horizontal plane; positive, when airplane climbs;

$\dot{\varphi}$ (1/s), change in gliding angle with time, accompanied by change in angular velocity about Z; positive, when acting clockwise about the corresponding axis as viewed in the positive direction of the latter;

ω (1/s), angular velocity about the vertical, positive upward; positive, when acting clockwise about the corresponding axis as viewed in the positive direction of the latter.

c) Position of Airplane with Respect to Path

α (deg.), angle of attack
μ (deg.), angle of bank
τ (deg.), lateral angle (angle of yaw)

} See fig. 1;

α_H (deg.), angle of attack of horizontal empennage;

α' (deg.), angle of attack of vertical empennage;

$\dot{\alpha}$ (1/s), change in angle of attack of airplane with time, accompanied by change in angular velocity about z_1;

$\dot{\mu}$ (1/s), change in angle of bank with time, accompanied by change in angular velocity about X;

$\dot{\tau}$ (1/s), change in angle of yaw with time, accompanied by change in angular velocity about y.

All angular velocities are considered positive, when acting clockwise about the corresponding axis as viewed in the positive direction of the axis.

d) Rotation of Airplane in Space

$\Omega (1/s)$, total rotational velocity about an axis fixed in space; also vectorial sum of rotational velocities $\dot{\omega}$, $\dot{\varphi}$, $\dot{\alpha}$, $\dot{\mu}$ and $\dot{\tau}$.

$$\Omega_X = \omega \sin \varphi + \mu - \dot{\tau} \sin \alpha$$

$$\Omega_{y_1} = \omega \cos \varphi \cos \mu + \dot{\varphi} \sin \mu + \dot{\tau} \cos \alpha$$

$$\Omega_{z_1} = - \omega \cos \varphi \sin \mu + \dot{\varphi} \cos \mu + \dot{\alpha}$$

$$\Omega_x = [(\omega \cos\varphi \cos\mu + \varphi \sin\mu) \sin\alpha + (\omega \sin\varphi + \dot{\mu}) \cos\alpha] \cos\tau - [- \omega \cos\varphi \sin\mu + \dot{\varphi} \cos\mu + \dot{\alpha}] \sin \tau$$

$$\Omega_y = (\omega \cos\varphi \cos\mu + \varphi \sin\mu) \cos\alpha - (\omega \sin\varphi + \dot{\mu}) \sin\alpha + \dot{\tau}$$

$$\Omega_z = [(\omega \cos\varphi \cos\mu + \dot{\varphi} \sin\mu) \sin\alpha + (\omega \sin\varphi + \dot{\mu}) \cos\alpha] \sin\tau + [- \omega \cos\varphi \sin\mu + \dot{\varphi} \cos\mu + \dot{\alpha}] \cos \tau.$$

<u>Components of Ω about the corresponding axes.</u>- All rotations are positive when acting clockwise about the corresponding axis as viewed in the positive direction of the axis.

$\dot{\Omega}_x (1/s^2)$, $\dot{\Omega}_{y_1} (1/s^2)$, $\dot{\Omega}_{z_1} (1/s^2)$: Change of angular velocity, with time about the corresponding body axis.

e) Local Constants

$g\,(m/s^2)$, acceleration due to gravity,

$\gamma\,(kg/m^3)$, air density.

In this treatise $\frac{\gamma}{2g} = \frac{1}{20}$, corresponding to an altitude of about 2300 m (7546 ft.).

$q = \frac{\gamma}{2g} v^2$ (kg/m^2), dynamic pressure,

q_H (kg/m^2), dynamic pressure on horizontal empennage,

q' (kg/m^2), dynamic pressure on vertical empennage.

f) Characteristics of a Junkers A 35 Low-Wing Monoplane (Figs. 2-4)

G = weight of airplane = 1600 kg,

F = wing area = 29.76 m^2,

b = span = 15.94 m,

t = wing chord (m),

t_1 = " " in middle = 2.2 m,

t_2 = " " at tips = 1.6 m,

r = distance of C.G. back of leading edge of wing = 0.80 m,

h = distance of C.G. above wing chord = 0.42 (Fig. 4),

J_x = inertia moment of airplane about axis x = 300 $mkgs^2$,

J_y = inertia moment of airplane about axis y = 550 $mkgs^2$,

J_z = inertia moment of airplane about axis z = 290 $mkgs^2$,

F_H = area of horizontal empennage = 4.80 m²,

l_H = distance of c.p. (center of pressure) of horizontal empennage from axis z = 5.27 m,

F_s = area of vertical empennage = 1.79 m²,

l_s = distance of c.p. of vertical empennage from axis y = 5.48 m,

$F'(m^2)$, effective damping area of fuselage tip,

$F''(m^2)$, " " " " vertical empennage,

$l'(m)$, distance of c.p. of vertical empennage from axis y,

β_H(deg.), elevator deflection when generating a positive moment about axis z (elevator down).

g) Air-Force Coefficients, Air-Force Moments and Gyroscopic Moments.

All coefficients have been obtained by dividing the measured forces in kilograms by qF and the measured moments in mkg by qFt_1.

c_a, lift; positive in positive direction of axis y_1,

c_w, drag; " " negative " " " X,

c_q, cross-wind force; positive in negative direction of axis z_1,

$c_n = c_a \cos \alpha + c_w \sin \alpha$, normal force; positive in positive direction of axis y,

$c_t = c_w \cos \alpha - c_a \sin \alpha$, tangential force at zero angle of yaw; positive in negative direction of axis x.

When the above coefficients belong to the wing alone, it is indicated by the subscript F.

c_{n_H}, normal force of horizontal empennage,

c_n', normal force of vertical empennage,

M_L, aerodynamic moment of whole airplane about axis z,

M_F, " " " wing alone " " z,

M_H, " " " horizontal tail " " z.

When the subscript 0 is used, it indicates that the above moments are about the leading edge of the wing.

K_F, aerodynamic moment of wing alone about axis x,

L_F, " " " " " " " y,

L', " " " vertical tail about axis y,

K_L, " " " whole airplane " " x,

L_L, " " " " " " " y.

M_K, gyroscopic moment about spar axis z,

K_K, " " " fuselage axis x,

L_K, " " " strut axis y.

All moments are positive when acting counterclockwise about the corresponding axis, as viewed in the positive direction of the axis.

2. Purpose and Scope of this Investigation

It is known that the operation of the control surfaces has hardly any effect in a flat spin, so that the airplane can recover from it only after a long time and often not at all. A flat spin must therefore be considered extremely dangerous, so long as no way is known for restoring the normal effect of the controls.

The article on the combined lateral and longitudinal motion of airplanes (reference 1) shows what steady motions can be produced by a given angle of deflection of a control surface. Several examples are given of how a spin can be developed.

The purpose of the present investigation is first to determine all the conditions under which a flat spin is possible; and then to determine the forces which cause a perceptible disturbance of the equilibrium of all the forces and moments existing in a flat spin, especially the forces tending to reduce the angle of attack of the airplane; furthermore to compare the efficacy of the available means and, for certain given cases, to find a way by which an airplane can be quickly and safely brought out of a dangerous flat spin and restored to a small angle of attack, i.e., to normal flight.

A knowledge of the article mentioned in Reference 2, at the end of this paper, is assumed. It is there shown how to calculate, by a comparatively simple method, the values of the variables belonging to a steady spin by starting with the assumption that the angle of yaw and the cross-wind force are small and that the lift and drag, as well as the aerodynamic moment about the spar axis z, are independent of the total rotation. In the course of the following investigation it will be shown that the angle of yaw must be small in a flat spin, thus justifying the above assumption. It will also be shown that the total rotation is very large and that the lift and drag, as well as the moment about the axis z, can no longer be regarded as independent of the total rotation. Nevertheless, the results of the investigation in Reference 2 were hardly changed, even for the case in which consideration was given to the effect of the total rotation on the lift, drag and aerodynamic moment. Hence, we must differentiate between two kinds of spins. In both kinds the airplane has a very large angle of glide and falls almost vertically at an angle of attack exceeding that of maximum lift. It is spoken of as a steep or flat spin, according to whether the angle of attack is very large or relatively small. Figure 5, taken from an American publication (reference 3) shows an airplane in a flat spin.

The aerodynamic forces and moments acting on an airplane in a flat spin have never been determined in a wind tunnel. A mathematical determination of the aerodynamic forces and moments acting on an airplane in the most common case, even in a side wind and for any total rotation, is not yet possible, because of the lack of the requisite wind-tunnel data. Fortunately, it is known that, in a flat spin, the angle of yaw must be small and the total rotation consists essentially of a rotation about the path axis X. For this case a mathematical determination of the aerodynamic forces and moments is possible, provided the lift and drag, as well as the moments about the spar axis z are first measured in a wind tunnel for angles of attack up to about 90°. Such measurements were made on a Junkers A 35 low-wing monoplane. The results of the wind-tunnel tests with this airplane, as well as the calculated aerodynamic forces and moments acting on the flat-spinning airplane, are given in Section 3 of the present report. The fundamental equations for balancing all the forces and moments acting on the airplane are also given in the same section.

The very large angle of attack and angle of glide, as well as the relatively large total rotation belonging to a flat spin, depend primarily on the mass distribution and on the vertical empennage and tail end of the fuselage. The importance of the mass distribution for initiating a flat spin is explained in Reference 4, while the effect of the shape of the vertical empennage and fuselage tip is shown in References 1 and 2. In Section 4 we shall find that the position of an airplane entering a flat spin can be quite accurately determined and that the corresponding angle of yaw must be small. The assumption that the angle of yaw must be small, which was taken as the basis of all previous investigations, proves therefore to be correct for the flat spin.

The effect of disturbances in a flat spin can be determined mathematically. It has been shown that the fundamental equations can be greatly simplified and solved in such a way as to indicate which quantities are affected by a change in the angle of attack. It is thus possible to compare these quantities and to determine which are the most effective. This subject will be considered in Section 5.

It will be shown that, in agreement with reality, the control surfaces have hardly any effect and that there is practically but one way to reduce the angle of attack very much in a flat spin. This is to suddenly increase the upward slope of the curve of the coefficient of the aerodynamic moment about the spar axis z as plotted against the angle of attack, which is equivalent to a sudden enlargement of the horizontal empennage during flight.

The solution of the fundamental equations is obviously correct only in so far as the basic assumptions are themselves correct. This is the case only just after the disturbance of the equilibrium. The effect of a disturbance can be followed longer only with the aid of a numerical integration. This is done in Section 6 for the case of the sudden doubling of the horizontal empennage during flight. This is the only way to judge regarding the possibility of recovery from a flat spin. It will be shown that, while the angle of attack decreases greatly, the angle of yaw increases considerably, at least in the beginning. Therefore the accurate mathematical determination of all the aerodynamic forces and moments acting on the airplane becomes impossible, due to the lack of the requisite wind-tunnel data. Nevertheless the approximate calculation of these forces and moments, as here made, without regard to the side wind, may at least be regarded as giving correct qualitative results, all the more because the airplane, due to the sudden enlargement of the horizontal empennage during flight, tips forward and passes into a vertical dive without rotation or side wind. Hence the results of the calculation are also physically instructive.

Lastly, the questions of especial interest to airplane designers will be considered in Section 7. It will be shown, by way of example, how the horizontal empennage might be constructed, so as to enable a sudden enlargement of its area during flight, i.e., a quick and safe recovery even from the hitherto justly feared flat spin. It will also be shown how the tail end of the fuselage and the different tail surfaces could be designed so that a flat spin would be impossible.

3. Introduction of the Mathematical Data Required for This Investigation

a) The Airplane Investigated

The investigation was conducted with a Junkers A 35 low-wing monoplane (Figs. 2-4), whose dimensions were given in Section 1,f. The inertia moments were determined mathematically.

b) Available Wind-Tunnel Data

All the measurements were made in the Göttingen wind tunnel on a rigidly mounted model of the above low-wing monoplane, both on the whole airplane and on the wing alone. The angle of attack α was varied between $-20°$ and $+90°$. The lateral angle τ was kept at $0°$. In general the elevator and rudder remained in the neutral position, measurements being made with an elevator displacement β_H of $\pm 10°$ only at small angles of attack.

Measurements were made of the lift and drag and of the moment about the leading edge of the wing. The lift and drag were divided by qF and the leading-edge moment by qFt_1, thus obtaining the respective absolute or nondimensional coefficients. In Figure 6 the lift and drag coefficients are plotted against the angle of attack, both for the whole airplane and also for the wing alone. In Figure 7 the coefficient of the leading-edge moment is plotted against the angle of attack with the elevator displacement as parameter.

c) Aerodynamic Forces and Moments Acting on an Airplane in a Flat Spin

All the forces and moments depend essentially on the angle of attack α, the lateral angle (angle of yaw) τ and on the total rotation Ω, whose components about the air axes are represented by the equations:

Path axis X,

$$\Omega_X = \omega \sin \varphi + \dot{\mu} - \dot{\tau} \sin \alpha \tag{1}$$

Lift axis y_1,

$$\Omega_{y_1} = \omega \cos \varphi \cos \mu + \dot{\varphi} \sin \mu + \dot{\tau} \cos \alpha \tag{2}$$

Axis $z_1 + X$ and y_1,

$$\Omega_{z_1} = - \omega \cos \varphi \sin \mu + \dot{\varphi} \cos \mu + \dot{\alpha} \tag{3}$$

Any accurate determination of the aerodynamic forces and moments acting on an airplane with a side wind and a given rotation is generally impossible, due to the lack of the requisite wind-tunnel data, but is quite possible in a flat spin.

A "flat spin" is a very steep, nearly steady spiral flight in which the fuselage is almost horizontal. For such a flight case the angle of attack is accordingly very very large and the angle of glide is approximately $-90°$. (Fig. 5.)

Since, according to Section 4, the angle of yaw must be small in a flat spin and can therefore exert hardly any influence on the aerodynamic forces and moments at the corresponding large angles of attack, and since, moreover, according to equations (1) to (3), the components Ω_{y_1} and Ω_{z_1} of the resultant rotation Ω are very small in comparison with the component Ω_X, all the aerodynamic forces and moments acting on the airplane depend chiefly on the angle of attack α and on the rotation Ω_X about the path axis. They can be calculated when they have not been determined by wind-tunnel tests.

As mentioned above (3,b) the results of the wind-tunnel tests cover only the coefficients of lift, drag and leading-edge moment in terms of the angle of attack. The corresponding values arising from the rotation about the path axis, as well as the newly added cross-wind force and the moments about the fuselage and strut axes (x and y), must be calculated.

In a flat spin the cross-wind force is negligible in comparison with the lift and drag. By cross-wind force with the coefficient c_q is meant a force in the direction of the axis z_1 perpendicular to the path axis X and to the lift axis y_1. In a flat spin it is produced chiefly by the fuselage and vertical empennage which, due to the rotation Ω_X, are exposed to a lateral air current.

Assuming the forces acting on the fuselage and vertical empennage to be combined into a single force and designating the coefficient of their components in the direction of the spar axis z by c_n'', the corresponding dynamic pressure by q' and the corresponding area by F', we then have, according to Figure 8,

$$c_q \approx \frac{F'}{F}\frac{q'}{q} c_n' \cos \tau$$

Since F'/F is always small and q'/q never exceeds 2, it follows that c_q is negligible in comparison with the lift and drag, even for large values of c_n' and zero angle of yaw.

The lift and drag produced by the other parts of the airplane are hardly affected by the rotation Ω_X about the path axis. For any given rotation Ω_X the corresponding values of the wing alone are:

$$c_{a_F} = \frac{1}{F} \int_{z=-\frac{b}{2}}^{+\frac{b}{2}} [c_{a_F} (\alpha + \Delta\alpha) \cos \Delta\alpha + \\ + c_{w_F} (\alpha + \Delta\alpha) \sin \Delta\alpha] \frac{1}{\cos^2 \Delta\alpha} t\, d\, z$$

$$c_{w_F} = \frac{1}{F} \int_{z=-\frac{b}{2}}^{+\frac{b}{2}} [c_{w_F} (\alpha + \Delta\alpha) \cos \Delta\alpha - \\ - c_{a_F} (\alpha + \Delta\alpha) \sin \Delta\alpha] \frac{1}{\cos^2 \Delta\alpha} t\, d\, z$$

where $\Delta\alpha = 57.3 \text{ arc tan } \frac{z\,\Omega_X}{v}$. The values c_{a_F} and c_{w_F}, applicable only to the wing at rest, were derived from Figure 6.

In Figure 9, c_{a_F} and c_{w_F} are plotted against α and $\frac{b\Omega_X}{2\,v}$. At the large values of $\frac{b\Omega_X}{2\,v}$ for the flat spin, the corresponding lift and drag coefficients for the stationary wing are considerably altered. (Reference 5.)

If the coefficients of lift and drag for the wing alone are subtracted from the corresponding coefficients for the whole airplane at the same angle of attack (fig. 6) and the resulting values are added to those plotted in Figure 9, the lift and drag coefficients are obtained for the whole airplane, as plotted in Figure 10 against α and $\frac{b\Omega_X}{2\,v}$.

A knowledge of the moments about the spar axis z is necessary for spin investigations. These moments are therefore determined from the corresponding measured moments about the leading edge of the wing as follows.

Let M_{F_0} be the coefficient of the aerodynamic moment about the leading edge for the wing alone and M_{L_0} the corresponding coefficient for the whole airplane. Then $M_{L_0} - M_{F_0}$ yields the coefficient M_{H_0} of the moment about the leading edge of the wing produced chiefly by the horizontal empennage.

If we draw a vertical line from the C.G. of the airplane to the plane of the wing chord and designate the distance of its bottom point from the leading edge of the wing and from the c.p. of the horizontal empennage by r and l_H, respectively, we obtain for the coefficient M_H of the moment about the spar axis, as produced by the horizontal empennage,

$$M_H = \frac{l_H}{r + l_H} (M_{L_0} - M_{F_0})$$

In Figure 11, M_H is plotted against the angle of attack α.

The moment about the spar axis z is derived from the coefficient M_{F_0} of the corresponding moment about the leading edge due to the wing alone by the formula

$$M_F = M_{F_0} - \frac{r}{t_1} c_{n_F} + \frac{h}{t_1} c_{t_F},$$

h being the vertical distance from the C.G. of the airplane to the plane of the wing chord and c_{n_F} and c_{t_F} the respective coefficients of the normal and tangential forces of the wing alone. In Figure 12 these coefficients are plotted against the angle of attack.

The addition of M_F and M_H yields the coefficient M_L of the aerodynamic moment about the spar axis for the whole airplane. These coefficients are plotted against the angle of attack in Figure 11. So long as the distance r is approximately $0.36t_1$, as for the airplane investigated, the moment about the spar axis produced by the wing alone is small in comparison with the corresponding moment of the horizontal empennage at the large angles of attack belonging to the flat spin.

At large angles of attack of the flat spin, the moment produced by the wing alone about the spar axis z, due to a rotation Ω_X about the path axis X, is hardly changed.

Designating the coefficient of the normal component of the total aerodynamic force acting on the horizontal empennage by c_{n_H}, the corresponding dynamic pressure by q_H, the area of the horizontal empennage by F_H and the distance between the c.p. of the aerodynamic force and the spar axis by l_H, the coefficient M_H of the moment about the spar axis, due to the horizontal empennage, becomes

$$M_H = \frac{c_{n_H}\, q_H\, F_H\, l_H}{q\, F\, t_1} \tag{4}$$

The coefficient c_{n_H} is hardly affected, even by the large rotations about the path axis occurring in a flat spin, so that M_H depends chiefly on the dynamic pressure q_H, for which we have the formula

$$q_H = q\left[1 + \frac{4\, l_H^2 \sin^2 \alpha}{b^2}\left(\frac{b\, \Omega_X}{2\, v}\right)^2\right]$$

Accordingly the moment about the spar axis for any rotation Ω_X about the path axis becomes

$$M_H\left[1 + \frac{4\, l_H^2 \sin^2 \alpha}{b^2}\left(\frac{b\, \Omega_X}{2\, v}\right)^2\right]$$

The addition of the coefficient M_F of the moment about the spar axis due to the wing alone yields the coefficient of the moment about the spar axis due to the whole airplane. The latter is plotted against α and $\frac{b\Omega_X}{2\, v}$ in Figure 13. At the large values of α and $\frac{b\Omega_X}{2\, v}$ for a flat spin, the moment about the spar axis for the stationary model is considerably changed.

For a rotation Ω_X about the path axis, the wing alone produces moments about the fuselage and strut axes, whose respective coefficients K_F and L_F can be calculated as follows:

$$K_F = \frac{1}{F\ t_1} \int_{z=-\frac{b}{2}}^{+\frac{b}{2}} c_{n_F}\ (\alpha + \Delta\alpha)\ \frac{1}{\cos^2 \Delta\alpha}\ t\ z\ d\ z$$

$$L_F = \frac{1}{F\ t_1} \int_{z=-\frac{b}{2}}^{+\frac{b}{2}} c_{t_F}\ (\alpha + \Delta\alpha)\ \frac{1}{\cos^2 \Delta\alpha}\ t\ z\ d\ z$$

where $\Delta\alpha = \text{arc tan} \frac{z\Omega_X}{v}$. The respective coefficients c_{n_F} and c_{t_F} of the normal and tangential forces for the wing alone are taken from Figure 12. In Figures 14 and 15 K_F and L_F are plotted against α and $\frac{b\Omega_X}{2\ v}$, respectively.

In addition to the above moment about the strut axis y produced by the wing alone, due to the rotation Ω_X, there is another very important moment about the same axis, produced principally by the vertical empennage and the tail end of the fuselage, whose coefficient is designated by L'.

If we imagine all the forces acting on the vertical empennage and the tail end of the fuselage combined into a single force and denote the coefficient of the component acting in the direction of the spar axis z by c_n', the corresponding dynamic pressure by q', the effective area of the vertical empennage and fuselage end by F'' and the distance between the strut axis y and the c.p. of the aerodynamic force by l' we then have

$$L' = \frac{c_n'\ q'\ F''\ l'}{q\ F\ t_1} \tag{5}$$

On the airplane in question the greater part of the vertical empennage lies above the fuselage and the horizontal empennage. When the angle of attack is small, the whole area of the vertical empennage is exposed to the air flow. In a flat spin, however, the angle of attack is very large, so that almost all the vertical empennage above the fuselage and the horizontal empennage is blanketed. (Fig. 16.) Hence the effective area F'' is considerably smaller in a flat spin than in normal flight and has approximately the following value:

$$F'' = F' - C \sin \alpha, \tag{6}$$

in which C is the blanketed portion at $\alpha = 90^\circ$.

The coefficient c_n' depends principally on the angle α', at which the effective area F'' is struck by the air flow. In the absence of experimental data, we are using the normal-force coefficient as plotted against α in Figure 17.

The dynamic pressure q' is represented by the formula

$$q' = q\left[1 + \frac{4\ l'^2 \sin^2 \alpha}{b^2} \left(\frac{b\ \Omega_X}{2\ v}\right)^2\right]$$

Consequently the coefficient L' of the moment about the strut axis y, principally produced by the vertical empennage and the tail end of the fuselage due to a rotation Ω_X about the path axis, becomes

$$L' = \frac{(F' - C \sin \alpha)\ l'}{F\ t_1} \left[1 + \frac{4\ l'^2 \sin^2 \alpha}{b^2} \left(\frac{b\ \Omega_X}{2\ v}\right)^2\right] c_n' .$$

The coefficient c_n' depends on the angle α', as represented by the formula

$$\alpha' = \operatorname{arc\ tan} \frac{2\ l'}{b} \frac{b\ \Omega_X}{2\ v} \sin \alpha.$$

In Figure 18, L' is plotted against the angle of attack α with the parameter $\frac{b\Omega_X}{2\ v}$.

d) Fundamental Equations

Equilibrium of the forces in the direction of the air axes:

Path axis X,

$$\frac{G}{g} \dot{v} = -G \sin \varphi + S \cos \tau \cos \alpha - c_w\ q\ F \tag{7}$$

Lift axis y_1,

$$0 = \frac{G}{g} v \; (\omega \cos \varphi \sin \mu - \dot{\varphi} \cos \mu) -$$
$$- G \cos\varphi \cos\mu + S \cos\tau \sin\alpha + c_a \, qF \tag{8}$$

Axis $z_1 + X$ and y_1,

$$0 = \frac{G}{g} v \; (\omega \cos \varphi \cos \mu + \dot{\varphi} \sin \mu) +$$
$$+ G \cos\varphi \sin\mu - S \sin\tau - c_q \, qF \tag{9}$$

Equilibrium of the moments about the body axes:

Fuselage axis x,

$$J_x \dot{\Omega}_x - (J_y - J_z) \, \Omega_y \, \Omega_z = - K_L \tag{10}$$

Strut axis y,

$$J_y \dot{\Omega}_y - (J_z - J_x) \, \Omega_z \, \Omega_x = - L_L \tag{11}$$

Spar axis z,

$$J_z \Omega_z - (J_x - J_y) \, \Omega_x \, \Omega_y = - M_L \tag{12}$$

The rotational velocities $(\Omega_x, \Omega_y$ and $\Omega_z)$ about the body axes are defined as follows:

$$\Omega_x = [(\omega \cos\varphi \cos\mu + \dot{\varphi} \sin\mu)\sin\alpha + (\omega \sin\varphi + \dot{\mu})\cos\alpha] \cos\tau$$
$$- [-\omega \cos\varphi \sin\mu + \dot{\varphi} \cos\mu + \dot{\alpha}] \sin\tau \tag{13}$$

$$\Omega_y = (\omega \cos\varphi \cos\mu + \dot{\varphi} \sin\mu)\cos\alpha - (\omega \sin\varphi + \dot{\mu})\sin\alpha + \dot{\tau} \tag{14}$$

$$\Omega_z = [(\omega \cos\varphi \cos\mu + \dot{\varphi} \sin\mu)\sin\alpha + (\omega \sin\varphi + \dot{\mu})\cos\alpha] \sin\tau$$
$$+ [-\omega\cos\varphi \sin\mu + \dot{\varphi} \cos\mu + \dot{\alpha}] \cos\tau \tag{15}$$

On the assumption that the propeller thrust S is zero, equations (7) to (15) may better be written as follows:

$$\dot{\alpha} = -\Omega_x \sin\tau + \Omega_z \cos\tau + g\,\frac{\cos\mu\cos\varphi}{v} - \frac{\gamma F}{2G}\,v\,c_a \qquad (16)$$

$$\dot{\mu} = \frac{\Omega_x \cos\tau + \Omega_z \sin\tau}{\cos\alpha} + g\,\frac{\tan\alpha\sin\mu\cos\varphi}{v}$$

$$+ \frac{\gamma F}{2G}\,v\,[c_a \sin\mu\tan\varphi - c_q(\cos\mu\tan\varphi + \tan\alpha)] \qquad (17)$$

$$\dot{\tau} = (\Omega_x \cos\tau + \Omega_z \sin\tau)\tan\alpha +$$

$$+ \Omega_y + g\,\frac{\sin\mu\cos\varphi}{v\cos\alpha} - \frac{\gamma F}{2G}\,\frac{v\,c_q}{\cos\alpha} \qquad (18)$$

$$\dot{\varphi} = -g\,\frac{\cos\varphi}{v} + \frac{\gamma F}{2G}\,v\,(c_a\cos\mu + c_q\sin\mu) \qquad (19)$$

$$\dot{v} = -g\sin\varphi - \frac{\gamma F}{2G}\,c_w\,v^2 \qquad (20)$$

$$\omega = -\frac{\gamma F}{2G}\,\frac{v}{\cos\varphi}\,(c_a\sin\mu - c_q\cos\mu) \qquad (21)$$

$$\dot{\Omega}_x = -\frac{\gamma F}{2g}\,\frac{t_1}{J_x}\,v^2 K_L + \frac{J_y - J_z}{J_x}\,\Omega_y\,\Omega_z \qquad (22)$$

$$\dot{\Omega}_y = -\frac{\gamma F}{2g}\,\frac{t_1}{J_y}\,v^2 K_L + \frac{J_z - J_x}{J_y}\,\Omega_z\,\Omega_x \qquad (23)$$

$$\dot{\Omega}_z = -\frac{\gamma F}{2g}\,\frac{t_1}{J_z}\,v^2 M_L + \frac{J_x - J_y}{J_z}\,\Omega_x\,\Omega_y \qquad (24)$$

4. Equilibrium Conditions of a Flat Spin

a) Equilibrium of the Forces

For the case when the propeller thrust S and the coefficient of the cross-wind force c_q, as well as all their derivatives, vanish, the fundamental equations (7) to (9) show that:

$$v = \sqrt{- \frac{G}{F} \frac{2\,g}{\gamma} \frac{\sin \varphi}{c_w}} \tag{25}$$

$$\omega = \sqrt{\frac{F^2\,\gamma^2}{4\,G^2} \frac{{c_a}^2\,v^2}{\cos^2 \varphi} - \frac{g^2}{v^2}} \tag{26}$$

$$\mu = -\,57.3 \text{ arc tan } \frac{v\,\omega}{g} \tag{27}$$

In these equations c_a and c_w are the respective coefficients of lift and drag for the whole airplane. These coefficients are nearly independent of the angle of yaw at the large angles of attack prevailing in a flat spin. On the other hand, according to Section 3,c, they are largely dependent on the total rotation Ω, which, in a flat spim, consists essentially of the rotation Ω_x about the path axis. This dependence is shown in Figure 10. Taking c_a and c_w from Figure 10, $\frac{b\Omega_x}{2\,v}$ can be calculated by equations (25) and (26) for any given angle of attack and angle of glide. In Figure 19, $\frac{b\Omega_x}{2\,v}$ is plotted against the angle of attack α with the angle of glide φ as parameter. It is obvious that the value corresponding to any given angle of glide is nearly constant for the angles of attack in a flat spin, so that the coefficients c_a and c_w for these angles of attack, corresponding to a constant value $\frac{b\Omega_x}{2\,v}$, can be taken from Figure 10. It is obvious that, for the airplane investigated, $\frac{b\Omega_x}{2\,v}$ cannot exceed a maximum value of about 1.5 corresponding to an angle of glide φ of about -87°. On the basis of these values, v, ω and μ are calculated by equations (25) and (27) and likewise plotted in Figure 19.

At the angles of attack and glide in a flat spin we have the following results according to Figure 19:

The path velocity v varies but little with the angle of attack, being about 25 m/s (82 ft./sec.) for the airplane investigated.

The rotational velocity ω diminishes with increasing angle of attack. For example, at an angle of attack of 60° and an angle of glide of -87°, ω has a value of about 5, that is, the airplane requires about 1.3 seconds for a complete revolution about the vertical axis. The angle of bank μ is about -85°.

b) Equilibrium of the Moments

The balancing of the moments about the strut axis y is expressed according to equation (11) by

$$J_z - J_x) \Omega_z \Omega_x = L$$

the negative gyroscopic moment $-L_K$ being equal to the aerodynamic moment L_L.

It is obvious that the factor $J_z - J_x$ is so small for airplanes of the ordinary type that no considerable gyroscopic moment can develop about the strut axis. Hence the balancing of the moments about the strut axis is restricted to the aerodynamic moments alone. These consist essentially of the moments produced by the wing alone and, above all, by the vertical empennage and the end of the fuselage, so that

$$L_L = L_F + L'.$$

can be written for the aerodynamic moment about the strut axis.

The moments about the strut axis are therefore balanced when $L_L = 0$, that is, when $L_F = - L'$. The moment produced by the wing alone, which accelerate the existing rotation, must therefore at least equal the damping moment due to the vertical empennage and the tail end of the fuselage. Both moments are nearly independent of the angle of yaw. Hence the latter has hardly any effect in the balancing of the moments about the strut axis. An accurate determination of both moments is therefore difficult. It may still be maintained that the damping moment produced by the fuselage tip and the vertical empennage continually increases with increasing $\frac{b\Omega_x}{2\,V}$, while the moment, which is produced by the wing alone and which accelerates the existing rotation, is nearly independent of $\frac{b\Omega_x}{2\ v}$ and remains small. For the investigated airplane, according to Figures 15 and 18, a comparison of these two moments is possible only when $\frac{b\Omega_x}{2\ v}$ is not greater than about 1.5 - i.e., when the angle of glide is not greater than about -87°. Hence the maximum value of $\frac{b\Omega_x}{2\ v}$, and consequently of φ, is determined by the balancing of the moments about the strut axis.

The moments about the fuselage axis are balanced according to equation (10)

$$(J_y - J_z)\ \Omega_y\ \Omega_z = K,$$

the negative gyroscopic moment $-K_K$ being equal to the aerodynamic moment K_L. The nondimensional coefficients K_L and $-K_K$ are obtained through division by qFt_1 and the introduction of equations (14) and (15). The aerodynamic moment consists essentially of the wing moment, so that we may put $K_L = K_F$. It is nearly independent of the angle of yaw and can be taken from Figure 14, corresponding to an angle of glide $\varphi = -87°$, i.e., to a value of about 1.5 for $\frac{b\Omega_x}{2\ v}$. In Figure 20, K_L and K_K are plotted against the angle of attack, with the angle of yaw τ as parameter, for a gliding angle of -87°. It follows that the gyroscopic moment about the fuselage axis is greatly affected by the angle of yaw. Even at an angle of yaw of ±20°, any balancing of the moments about the fuselage axis is no longer possible at the angles of attack prevailing in a flat spin.

The moments about the spar axis z are balanced according to equation (12)

$$(J_x - J_y)\ \Omega_x\ \Omega_y = M$$

the negative gyroscopic moment $-M_K$ being equal to the aerodynamic moment M_L. The nondimensional coefficients M_L and $-M_K$ are obtained through division by qFt_1 and the introduction of equations (13) and (14). The aerodynamic moment is nearly independent of the angle of yaw. In Figure 21, M_L and $-M_K$ are plotted against the angle of attack with the angle of yaw τ as parameter, the gliding φ being -87°. It is found that the angle of yaw has hardly any effect on the balancing of the moments about the spar axis, so long as it is less than about $\pm 20^\circ$, the angle of attack being about 60°.

In summarizing, it may be said: that the maximum value of $\frac{b\Omega_x}{2\,v}$ and the corresponding angle of glide φ are determined by balancing the moments about the strut axis y; that any balancing of the moments about the fuselage axis x is possible only at small angles of yaw τ; lastly, that the angle of attack α is determined by balancing the moments about the spar axis z. However, when the angle of glide φ, the lateral angle (angle of yaw) τ, the angle of attack α and the value of $\frac{b\Omega_x}{2\,v}$ are known, the velocity v, the rotation ω and the angle of bank μ can be calculated. Thus the fundamental equilibrium conditions of a flat spin are fully determined.

5. Effect of Disturbances in a Flat Spin

a) Simplification of the Fundamental Equations

On the assumption that the engine is stopped, the suitably altered fundamental equations (16) to (20) can be still further simplified for flat spins, as follows.

If the initial state of equilibrium of a flat spin be disturbed, a numerical integration (Section 6) shows that the path velocity v changes but little at first, so that v may be considered constant. In a flat spin a balancing of all the forces and moments acting on the airplane is possible, according to Section 4, only when the angle of yaw τ is small. In the following investigation of the initial position of equilibrium, it is assumed that τ_0 is so small that $\sin \tau_0$ is approximately zero and $\cos \tau_0$ is approximately 1.

According to Section 3,c the aerodynamic forces and moments acting on an airplane in a flat spin depend not only on the angle of attack but also on the total rotation Ω, which consists essentially of the rotation Ω_x about the path axis. If this initial position of equilibrium is disturbed, the numerical integration shows (fig. 24) that the only important effect at first is a change in the rotation Ω_z about the spar axis, while the components of the total rotation remain nearly constant. The cross-wind force (Section 3,c) is negligibly small in comparison with the lift and drag, so that its coefficient c_q may be considered zero.

The lift is produced principally by the wing and is hardly affected by the accompanying rotation Ω_z. It accordingly depends almost entirely on the angle of attack α and on the rotation Ω_x about the path axis and can be taken from Figure 10 to correspond to a given rotation Ω_x, that is, to a given value of $\frac{b\Omega_x}{2\,v}$ at constant v. The curve of the lift coefficient c_a, as plotted against the angle of attack α with $\frac{b\Omega_x}{2\,v}$ as parameter, can, according to Figure 10, be regarded as a straight line for the angles of attack in question, so that we can put $c_a = m_1\,\alpha + n_1$. No assumptions need to be made regarding the drag coefficient c_w.

The moment about the spar axis is affected by the accompanying rotation Ω_z in so far as an additional moment is produced, mainly by the horizontal empennage, which tends to damp the rotation Ω_z. This additional damping moment of the horizontal empennage is due mainly to a change in the angle α_H caused by the rotation Ω_z. On the other hand the accompanying dynamic pressure can at first be regarded as constant, because Ω_z is then small. The coefficient of this additional horizontal-empennage moment can accordingly be written $\frac{d\ M_H}{d\ \alpha_H}\,\Delta\,\alpha_H$, in which

$$\Delta\,\alpha_H = \text{arc tan}\,\frac{l_H\ \Omega_z}{v} \approx \frac{l_H}{v}\,\Omega_z$$

Since, according to Figure 11, the wing moment is nearly constant at the large angles of attack prevailing in a flat spin, we can put

$$\frac{d\ M_H}{d\ \alpha_H} \approx \frac{d\ M_L}{d\ \alpha_H}.$$

The curve of the coefficient M_L of the total moment about the spar axis, as plotted against the angle of attack α with the parameter $\frac{b\Omega_x}{2\,v}$, can, according to Figure 13, be regarded as a straight line $M_L = m_2\,\alpha + n_2$ for the angles of attack in question, so that

$\frac{d\,M_L}{d\,\alpha}$ is constant and we can put $\frac{d\,M_H}{d\,\alpha_H} \approx \frac{d\,M_L}{d\,\alpha} = n_2$.

Hence

$$M_L = n_2\,\alpha + n_2 + \frac{l_H}{v}\,m_2\,\Omega_z.$$

The moment about the fuselage axis is hardly affected by the operation of the ailerons at the large angles of attack prevailing in a flat spin. If any change in the shape of the wing during flight be disregarded (which will be shown to have hardly any effect on the alteration of the angle of attack), the moment about the fuselage axis is materially affected only by the angle of attack and by the rotation about the path axis. The curve of the corresponding coefficient in terms of the angle of attack can, according to Figure 14, be represented by a straight line $K_L = n_3\,\alpha + n_3$ for the parameter $\frac{b\Omega_X}{2\,v}$ belonging to a flat spin.

The airplane moment L_L about the strut axis consists essentially of the moment L_F produced by the wing and the moment L' produced by the tail end of the fuselage and the vertical empennage, as expressed by the equation $L_L = L_F + L'$. The moment L_F produced by the wing alone may be regarded as constant at first.

The coefficient of the moment L' produced by the fuselage end and the vertical empennage is represented, according to equation (5), by the formula

$$L' = \frac{c_n'\,q'\,F''\,l'}{q\,F\,t_1}$$

in which, according to equation (6), $F'' = F' - C\sin\alpha$ can be put for the effective area of the fuselage end and vertical empennage. In a flat spin the total rotation consists essentially of a rotation Ω_X about the path axis, which, at the large angles of attack, nearly equals the rotation Ω_y about the strut axis. The dynamic pressure q' may be regarded as constant at first and be calculated as follows:

$$q' = q\left[1 + \left(\frac{l'\,\Omega_{y_0}}{v}\right)^2\right].$$

On the contrary, the angle α', at which the fuselage end and the vertical empennage are struck by the air flow, and the coefficient c_n' of the corresponding normal force are quite sensitive to any change in the rotation Ω_y.

$$\alpha' = 57.3 \text{ arc tan } \frac{l'}{v} \Omega_y \approx 57.3 \frac{l'}{v} \Omega_y$$

In Figure 17 the coefficient c_n' is plotted against α' or (which amounts to nearly the same thing) against $57.3 \frac{l'}{v} \Omega_y$. This curve can be represented by a straight line, as follows:

$$c_n' = m_4' \alpha' = m_4 \Omega_y$$

in which m_4 approximates $57.3 \frac{l'}{v} m_4'$. Hence the coefficient of the total moment about the strut axis becomes

$$L_L = L_F + [1 + \left(\frac{l' \Omega y_0}{v}\right)^2] \frac{F''}{F} \frac{l'}{t_1} m_4 \Omega_y$$

Hence L_L depends principally on the rotation Ω_y about the strut axis.

Of the fundamental equations (16) to (24), serving for the determination of the nine variables,

$$\alpha, \mu, \tau; \quad v, \varphi, \omega; \quad \Omega_x, \Omega_y \text{ and } \Omega_z$$

equation (20) does not need to be considered, because of the assumption that the path velocity v is constant at first, which leaves only eight differential equations to be integrated. By simplifying these on the basis of the above assumptions, by developing the terms from the products of several variables into a Taylor series, only the first terms of which are considered and, lastly, by neglecting the terms which are small in comparison with the others, we obtained the following eight differential equations of the first order:

$$\dot{\alpha} = a_1 \alpha + b_1 \tau + c_1 \Omega_z + f_1 \tag{28}$$

$$\dot{\tau} = a_2 \alpha + d_2 \Omega_x + e_2 \Omega_y + f_2 \tag{29}$$

$$\dot{\Omega}_z = a_3 \alpha + c_3 \Omega_z + d_3 \Omega_x + e_3 \Omega_y + f_3 \tag{30}$$

$$\dot{\Omega}_x = a_4 \alpha + c_4 \Omega_z + e_4 \Omega_y + f_4 \tag{31}$$

$$\dot{\Omega}_y = e_5 \Omega_y + f_5 \tag{32}$$

$$\mu = a_6 \alpha + b_6 \tau + c_6 \Omega_z + d_6 \Omega_x + f_6 + g_6 \mu + h_6 \varphi \tag{33}$$

$$\dot{\varphi} = a_7 \alpha + f_7 + g_7 \mu + h_7 \varphi \tag{34}$$

$$\omega = a_8 \alpha + f_8 + g_8 \mu + h_8 \varphi \tag{35}$$

Equations (28) to (32) no longer contain the quantities μ, φ and ω, so that a separation of the variables is possible, and equations (33) to (35) no longer need to be considered in calculating the especially important change in the angle of attack α.

The coefficients in equations (28) to (32) have the following values:

$$a_1 = -\frac{\gamma}{2}\frac{F}{G}v\, m_{1_0}$$

$$b_1 = -\Omega_{x_0}$$

$$c_1 = 1$$

$$f_1 = -\frac{\gamma}{2}\frac{F}{G}v\, n_{1_0}$$

$$a_2 = \frac{\Omega_{x_0}}{\cos^2 \alpha_0}$$

$$d_2 = \tan \alpha_0$$

$$e_2 = 1$$

$$f_2 = -\frac{\Omega_{x_0} \alpha_0}{\cos^2 \alpha_0}$$

$$a_3 = -\frac{\gamma}{2}\frac{v^2}{g}\frac{F t_1}{J_z}\, m_{2_0}$$

$$c_3 = - \frac{\gamma\ F\ t_1\ l_H\ v}{2\ g\ J_z}\ m_{2_0}$$

$$d_3 = \frac{J_x - J_y}{J_z}\ \Omega_{y_0}$$

$$c_3 = \frac{J_x - J_y}{J_z}\ \Omega_{x_0}$$

$$f_3 = - \frac{\gamma\ v^2\ F\ t_1}{2\ g\ J_z}\ n_{2_0} - \frac{J_x - J_y}{J_z}\ \Omega_{x_0}\ \Omega_{x_0}$$

$$a_4 = - \frac{\gamma\ v^2\ F\ t_1}{2\ g\ J_x}\ m_{3_0}$$

$$c_4 = \frac{J_y - J_z}{J_x}\ \Omega_{y_0}$$

$$e_4 = \frac{J_y - J_z}{J_x}\ \Omega_{z_0}$$

$$f_4 = - \frac{J_y - J_z}{J_x}\ \Omega_{y_0}\ \Omega_{z_0} - \frac{\gamma\ v^2\ F\ t_1}{2\ g\ J_x}\ n_{3_0}$$

$$e_5 = - \frac{\gamma\ F''\ l'\ v^2}{2\ g\ J_y}\ [1 + \left(\frac{l'\ \Omega_{y_0}}{v}\right)^2]\ m_{4_0}$$

$$f_5 = - \frac{\gamma\ F\ t_1\ v^2}{2\ g\ J_y}\ L_{F_0}.$$

b) Solution of the Fundamental Equations

From equations (28) to (32) we can derive an equation of the following form:

$$\ddot{\alpha} - a_1\alpha - (a_2 b_1 + a_3 c_1)\alpha - (b_1 d_2 + c_1 d_3)\Omega_x - (b_1 e_2 + c_1 e_3)\Omega_y$$
$$- c_1 c_3\ \Omega_z - (b_1 f_2 + c_1 f_3) = 0 \qquad (37)$$

in which

$$a_1 = -\frac{\gamma}{2}\frac{F}{G}\frac{v}{}\, m_{1_0} \approx -0.1$$

$$a_2\, b_1 + a_3\, c_1 \approx -\omega_0{}^2 - \frac{\gamma}{2}\frac{v^2}{g}\frac{F\, t_1}{J_z}\, m_{2_0} \approx -18.5$$

$$b_1\, d_2 + c_1\, d_3 \approx -\omega_0 \sin\varphi_0 \sin\alpha_0 \left(1 + \frac{J_x - J_y}{J_z}\right) \approx +0.3$$

$$b_1\, e_2 + c_1\, e_3 \approx -\omega_0 \sin\varphi_0 \cos\alpha_0 \left(1 - \frac{J_x - J_y}{J_z}\right) \approx +3.2$$

$$c_1\, c_3 = -\frac{\gamma}{2}\frac{F}{g}\frac{t_1\, l_H\, v}{J_z}\, m_{2_0} \approx +1.4$$

$$b_1\, f_2 + c_1\, f_3 \approx \omega_0{}^2 \sin^2\varphi_0 \left(\alpha_0 + \frac{J_x - J_y}{2\, g\, J_z} \sin 2\,\alpha_0\right) -$$

$$-\frac{\gamma}{2}\frac{v^2}{g}\frac{F\, t_1}{J_z}\, n_{2_0} \approx +8.9$$

so that equation (37) may be expressed numerically as follows:

$$\ddot{\alpha} + 0.1\,\dot{\alpha} + 18.5\,\alpha - 0.3\,\Omega_x - 3.2\,\Omega_y - 1.4\,\Omega_z - 8.9 = 0$$

with $\Omega_{x_0} \approx -1.7$ and $\Omega_{z_0} = 0.2$

It is therefore obvious that, in normal construction, the terms with Ω_x and Ω_z are negligibly small in comparison with the other terms and can be omitted. The angle of attack is at first hardly affected by any change in the rotation Ω_x about the fuselage axis, which might, e.g., be forcibly produced by changing the shape of the wing during flight. According to equation (32), the rotation Ω_y can be expressed by the formula

$$\Omega_y = -\frac{f_5}{e_5} + \left(\Omega_{y_0} + \frac{f_5}{e_5}\right) e^{e_5 t} \approx \Omega_{y_0} + (\Omega_{y_0}\, e_5 - f_5)\ t,$$

so that, by using this value, the following nonhomogeneous differential equation of the second order is obtained for α.

$$\ddot{\alpha} - a_1 \dot{\alpha} + q \alpha = r t + s.$$

The damping is so slight that it does not need to be considered at the beginning. Hence oscillation sets in about a mean position which changes with time. At the beginning (as we shall see), this is in thoroughly satisfactory accord with a numerical integration carried out as accurately as possible. The solution is derived from the initial conditions and the constants q, r and s of the differential equation with the following values:

$$q = \left(\frac{\Omega_{x_0}}{\cos \alpha_0}\right)^2 + \frac{\gamma\ F\ t_1\ v^2}{2\ g\ J_z} \boxed{m_{2_0}} \tag{38}$$

$$r = \left(\frac{\gamma\ F\ t_1\ v^2}{2\ g\ J_y}\right)^2 \Omega_{x_0} \left(1 + \frac{J_x - J_y}{J_z}\right)$$

$$\left\{L_{F_0} + \frac{F''\ l'}{F\ t_1} \left[1 + \left(\frac{l'\ \Omega_{y_0}}{v}\right)^2\right] \Omega_{y_0} \boxed{m_{4_0}}\right\} \tag{39}$$

$$s = \left(\frac{\Omega_{x_0}}{\cos \alpha_0}\right)^2 \alpha_0 - \Omega_{x_0} \Omega_{y_0} - \frac{\gamma\ v^2\ F\ t_1}{2\ g\ J_z} \boxed{n_{2_0}} \tag{40}$$

This form of the differential equation and its solution is very essential, because it indicates the means by which the angle of attack can be sufficiently reduced to enable recovery from the dangerous condition. From this solution we can determine what quantities are really involved.

If we refrain from altering the wing and the inertia moments during the spin, then m_{2_0}, n_{2_0} and m_{4_0} are the only quantities remaining in equations (38) to (40), which enable the pilot to change the angle of attack.

6. Danger of the Flat Spin

The elimination of danger from a flat spin is synonymous with the determination of the effect of measures for disturbing it and for restoring the airplane to normal flight at a small angle of attack.

It has already been stated in Section 5 that the only practicable way to change the angle of attack is to alter the quantities m_{2_0}, n_{2_0} and m_{4_0}. The quantities m_{2_0} and n_{2_0} indicate only the slope or parallel displacement of the straight lines representing the curve of the coefficient corresponding to the moment about the spar axis, as plotted against the angle of attack according to Figure 13. Likewise, m_4 indicates only the slope of the straight lines which represent the curve of the normal force corresponding to the fuselage end and the vertical empennage as plotted against the angle of attack according to Figure 17. Figure 22 represents the beginning of the oscillation produced by doubling the values m_{2_0}, n_{2_0} and m_{4_0} corresponding to the state of equilibrium. The angle of attack is but slightly reduced by a parallel displacement of the moment line corresponding to the doubling of n_{2_0}, which might perhaps be attained by extreme pressures. Aileron and rudder deflections do not enter into equations (38) and (40) and consequently have nothing to do with the question of reducing the angle of attack. This again confirms the fact that these controls have almost no effect in a flat spin. Even the doubling of n_{4_0}, which, as shown by a more thorough investigation, might be attained by doubling the area of the vertical empennage, effects only a slight change in the angle of attack. The greatest change is effected, according to Figure 22, by doubling m_{2_0}. According to equation (4) the coefficient M_H of the moment of the horizontal empennage about the spar axis can be represented by the formula

$$M_H = \frac{c_{n_H}\, q_H\, F_H\, l_H}{q\, F\, t_1},$$

in which

$$q_H = q\left[1 + \frac{4\, l_H^2 \sin^2\alpha}{b^2}\left(\frac{b\,\Omega_X}{2\,v}\right)^2\right]$$

So long as the rotation Ω_x about the path axis does not change much, q_H remains nearly constant. The coefficient c_{n_H} of the normal force on the horizontal empennage depends not only on the angle of attack, but also on the plan form of the horizontal empennage F_H and on its profile. It remains constant for one and the same angle of attack, if the profile is not changed and the shape of the altered horizontal empennage is similar to the original shape. Under the conditions

$$M_H = C\, F_H\, l_H,$$

i.e., the coefficient of the moment of the horizontal empennage increases with its area F_H and with the distance l_H of the c.p. of the horizontal empennage from the spar axis. If, for example, the area of the horizontal empennage is doubled, the coefficient M_H of the moment of the horizontal empennage is also doubled, involving, however, no change in the wing moment. The curve $M_L = m_2\, \alpha + n_2$ of the coefficient of the aerodynamic moment about the spar axis, as plotted in Figure 13 against the angle of attack α, assumes therefore very nearly the desired doubled slope m_2. Accordingly, the most effective way to change the angle of attack is to increase the area of the horizontal empennage. This enlargement must be effected suddenly and during the spin, since it is only in this way that the large calculated change in the angle of attack due to the sudden doubling of the surface area can take place according to Figure 22. Section 8 will show how this requirement can be structurally fulfilled.

Of course the approximate solution of the fundamental equations is valid only so long as the fundamental assumptions are correct. Such is the case only for the time immediately after the disturbance. Further changes, especially in the angle of attack, can be determined only by numerical integration. This has been done for the case of a sudden doubling of the area of the horizontal empennage during a spin. In Figures 23 and 24, the nine variables v, φ, ω, α, μ, τ, Ω_x, Ω_y and Ω_z are plotted against the time. The assumptions underlying the approximate integration hold good when the path velocity v and the rotations Ω_x and Ω_y about the respective fuselage and strut axes vary but little, at least during the period immediately following the disturbance. Almost the whole change in the rotation Ω

then becomes evident through a change in the rotation Ω_z about the spar axis. In Figure 22 the corresponding change in the angle of attack, as determined by the numerical integration, is plotted alongside the exact solution. At the beginning, the approximate integration yields almost the same result as the accurate numerical integration.

However, as soon as the angle of attack assumes considerably smaller values, the numerical integration deviates from the approximate one. There is no oscillation about the mean position, as in the approximate integration, but the angle of attack drops to still smaller values, whereby the angle of yaw increases very rapidly, at least in the beginning. The airplane noses over and turns about the strut axis in such a way that the air flow strikes the fuselage almost perpendicularly. The resulting pressure against the vertical empennage and the tail end of the fuselage creates a moment about the strut axis, which turns the fuselage in the direction of fall, i.e., again reduces the angle of yaw. Since the total rotation also diminishes greatly with time, and the angle of glide approaches -90°, the airplane soon enters a dive which is free from rotation and side wind and from which it can level off. The pilot is therefore able to pull an airplane quickly out of a dangerous flat spin by suddenly enlarging the area of the horizontal empennage.

A diminution of the angle of attack, very similar to that produced by the sudden enlargement of the horizontal empennage, can also be brought about, without the action of the pilot, by the airplane suddenly encountering a strong ascending current, since it is entirely indifferent, as regards recovery, whether the quantity m_{z_0} or v^2 is doubled in equation (38). Doubling v^2 would correspond to increasing the sinking speed in a flat spin from about 30 m/s (100 ft./sec.) to about 42 m/s (138 ft./sec.). The ascending current must accordingly have a velocity of about 12 m/s (40 ft./sec.) to produce the same effect as doubling the area of the horizontal empennage. Such an up-current is conceivable, so that it is not impossible for an airplane to nose over without the aid of the pilot. On the other hand, the encountering of a down-current would greatly increase the difficulty of recovering from a flat spin.

It should also be mentioned that an airplane can also be brought out of a dangerous flat spin by the proper handling of the elevator. Figure 22 shows that a single strong push produces a periodic change in the angle of attack, which is practically synonymous with an oscillation of the airplane about the spar axis. It is obviously possible, by repeated pushes in time with the initiated oscillations, to increase their amplitude to whatever degree may be necessary for recovery from the spin. Americans have conducted a series of experiments on the recovery from a flat spin. They succeeded in leveling off the airplane by the above maneuver, but only after falling a long distance. The pertinent paragraph in the American report (reference 3) reads:

"Should this be ineffective after several additional turns an attempt should be made to rock the plane out, using the engine in conjunction each time the controls are moved for recovery. It is necessary, of course, to work with the natural period of the plane in attempting recovery by this means, the controls being operated very much in the same manner that a seaplane is rocked on the step for taking off."

7. Constructional Measures

a) For Recovering from a Flat Spin

The present theoretical investigation shows, in agreement with practical experience, that the operation of the controls causes only a very slight disturbance of the equilibrium of all the forces and moments acting on an airplane in a flat spin. Hence an airplane can be brought out of such a spin only very gradually and often not at all. The investigation shows, moreover, that a sudden increase in the moment of the horizontal empennage during a spin immediately and greatly reduces the angle of attack and causes the airplane to nose over strongly, so that it can be quickly and safely brought out of the flat spin, which has hitherto been justly feared.

This result is not at all surprising. It is obviously possible to recover from the dangerous flat spin if the airplane can be forced down suddenly to the small angle of attack at which all the controls become fully effective. Just the fact that the controls have practically no effect at the large angles of attack prevailing in a flat spin, is the real reason why the operation of the controls can bring an airplane out of such a spin but very gradually and often not at all.

On the other hand, enlarging the area of the horizontal empennage increases its moment just when it has a very large angle of attack, as in a flat spin. Constructionally there is no insurmountable obstacle to such an enlargement of the horizontal empennage. A possible construction is shown, e.g., in Figure 25. In ordinary flight the dotted movable portions are enclosed in the stabilizer. In case of need they can be projected so as to form a considerable enlargement of the horizontal empennage.

b) For Preventing the Possibility of a Flat Spin

The best way known to make a flat spin impossible, is to give the proper form to the vertical empennage and to the tail end of the fuselage. In reference 2 we called attention to the fact that the surfaces for damping the rotation about the strut axis (particularly the vertical empennage and the tail end of the fuselage) must be made as large as possible and so arranged that they are exposed to the air flow from all directions, especially from obliquely underneath.

Many airplane fuselages are so built that, as seen from the side, they taper greatly toward the tail, while, as seen from above, they are too broad even at the tail, so that the area of the lateral surfaces of the fuselage at the tail are small and the vertical empennage, situated chiefly above the fuselage tip, is largely shielded from the air flowing from obliquely underneath. The horizontal empennage produces a similar effect to a still greater degree, so that the vertical empennage is almost completely blanketed from air currents coming from obliquely underneath.

Due to the unfavorable shape of the fuselage and the shape and arrangement of the tail surfaces, the damping moments about the strut axis are frequently too small to enable the prevention of a flat spin. A flat spin cannot always be prevented by enlarging the vertical empennage, at least when the latter is situated above the fuselage and the horizontal empennage.

The fact that, despite the unfavorable shape and arrangement of the fuselage and vertical empennage, many of the airplanes thus constructed have not fallen into flat spins, does not prove that they are incapable of doing so. It has been found that airplanes supposed to be spinproof have nevertheless fallen into dangerous flat spins under special conditions.

The risk of falling into a flat spin can be greatly reduced by a suitable construction of the fuselage tip and of the vertical empennage. There are airplanes whose fuselage tips, as seen from above, are but little broader than the superimposed vertical empennage. If, moreover, spaces are left between the fuselage and the horizontal empennage, as shown in Figure 26, so that the vertical empennage is exposed to the air flow from all possible directions, including that from obliquely underneath, and if, furthermore, the lateral surfaces of the fuselage end and the vertical empennage are made as large as possible, the flat spin will then be impossible.

8. Summary

This report deals first with the fundamental data required for the investigation. These are chiefly the aerodynamic forces and moments acting on an airplane in a flat spin. It is shown that these forces and moments depend principally on the angle of attack and on the rotation about the path axis, and can therefore either be measured in a wind tunnel or calculated from wind-tunnel measurements of lift, drag and moment about the leading edge of the wing of an airplane model at rest. The lift, drag and moment about the spar axis are so greatly altered by the rapid rotation in a flat spin, that they can no longer, as in reference 2, be regarded as independent of the rotation. No substantial change in the angles of attack and glide occurring in a flat spin, as found in reference 2, is involved. The cross-wind force, as compared with the lift and drag, can be disregarded in a flat spin. Practically the only aerodynamic moment about the fuselage axis is that produced by the wing as a result of the rotation. In addition to the corresponding aerodynamic moment about the strut axis, as produced by the wing alone, there is another aerodynamic moment principally produced by the vertical empennage and the tail end of the fuselage. This moment is due to the rapid rotation, which it constantly tends to damp.

The initial conditions for a flat spin are as follows. The principal moment about the strut axis consists of the aerodynamic moment produced by the wing alone, which accelerates the existing rotation, and the damping moment produced by the vertical empennage and the tail end of the fuselage. These moments can be balanced only for a given maximum value $\frac{b\Omega x}{2\,v}$, corresponding to a definite maximum angle of glide φ. The lateral angle (angle of yaw) τ must be small, in order for it to be possible to balance the moments about the fuselage axis at a given angle of glide. Lastly, the angle of attack α is determined by balancing the moments about the spar axis, when the angle of glide φ and the lateral angle τ are known.

The characteristics of a flat spin are thus determined. For the airplane in question, they are:

Path velocity	v	=	25 m/s,
Angle of glide	φ	=	-87°,
Rotational velocity	ω	=	5(1/s),

i.e., the airplane requires about 1.3 s for one revolution about the vertical axis;

Angle of attack	α	=	60°,
Angle of bank	μ	=	-85°,
Lateral angle (angle of yaw)	τ		small.

The investigation of the effect of disturbances in a flat spin begins with the integration of the known fundamental equations pertaining to the equilibrium of the forces and moments acting on the airplane. These fundamental equations can be advantageously transformed and simplified, so as to enable the separation of the variables and especially the determination of the very important angle of attack, from which we can ascertain the requisite factors for recovery from the dangerous condition. It is shown that, in agreement with experience, the controls have hardly any effect and that the only remaining means is a sudden increase in the slope of the curve of the moment coefficient about the spar axis in terms of the angle of attack. The requisite increase in the slope of this curve can be effected by a sudden enlargement of the horizontal empennage during the spin.

The dangerousness of the flat spin can only be determined by following the effect of a disturbance on the variables over a considerable period of time. This is possible only by a numerical integration, which was made for the case of a sudden doubling of the horizontal empennage and led to the following at least qualitatively correct result.

Due to the sudden doubling of the area of the horizontal empennage during flight, the airplane quickly noses so far over that the angle of attack is reduced to that of normal flight. For a time the rotation of the airplane about the strut axis continues, so that the lateral angle is very large at first and the side of the fuselage is struck almost vertically by the air current. The fuselage end and the vertical empennage then damp the rotation about the strut axis so strongly, that the fuselage axis gradually approaches the direction of the path axis and the lateral angle diminishes. The rotation gradually diminishes, and the angle of glide approaches -90°. The airplane soon goes into a dive without rotation or side wind. The sudden enlargement of the horizontal empennage is therefore the most effective means for bringing an airplane quickly out of a dangerous flat spin.

The sudden encountering of an up-current by a flat-spinning airplane may cause it to nose over and recover from the spin without the aid of the pilot. The sudden encountering of a down-current would, however, greatly increase the difficulty of recovering from a flat spin. It is quite possible that an airplane which is often put into a steep spin without going into a dangerous flat spin may nevertheless fall into the latter under certain conditions. Hence it is very rash, simply on the basis of test flights, to claim that an airplane cannot flat-spin, until it has been satisfactorily demonstrated under all possible conditions of flight.

Another way to recover from a flat spin, although not so quickly, is by the correct operation of the elevator. A single strong push on the control stick causes a slight periodic variation in the angle of attack and starts an oscillation of the airplane about the spar axis. By alternate pushing and pulling in time with the original oscillations, their amplitude can be increased as much as may be necessary for recovery from the spin. American experiments with flat-spinning airplanes have demonstrated that an airplane can thus be brought out of a dangerous flat spin.

The sudden enlargement of the horizontal empennage during flight, as required for quick recovery from a flat spin, can be accomplished as shown, for example, in Figure 25. The stabilizer is provided with telescoping parts which are enclosed in the stabilizer during ordinary flight, but can be projected, in case of need, so as greatly to enlarge the area of the horizontal empennage.

If it be desired to avoid all possibility of a flat spin, the horizontal empennage may be so constructed as to leave spaces next to the fuselage (fig. 26), thus exposing the largely dimensioned vertical empennage and lateral surfaces of the end of the fuselage to air currents from all possible directions and especially from obliquely underneath.

9. References

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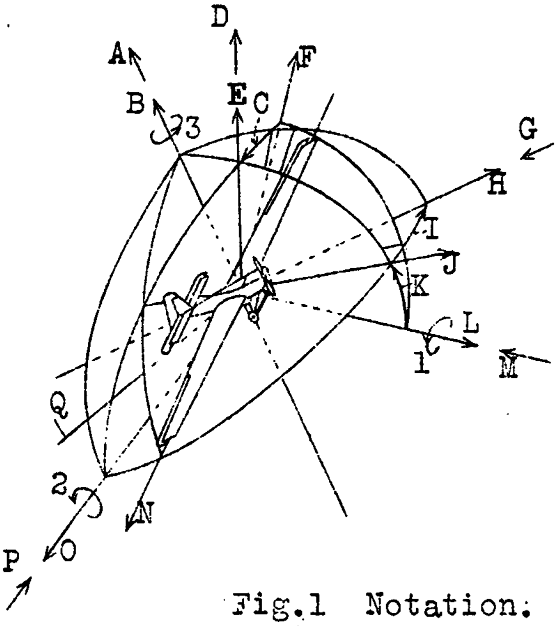
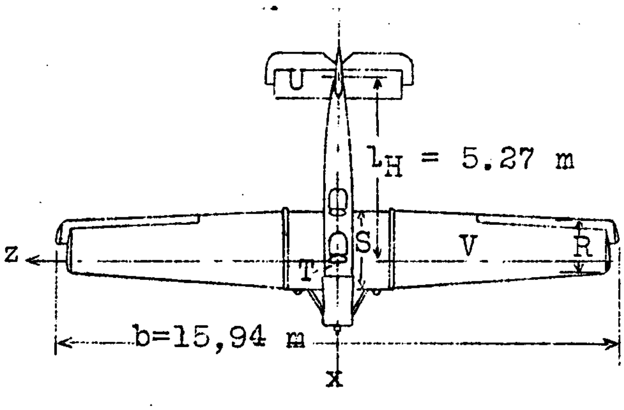


A, Normal force N
B, Strut axis y
C, Angle of wing setting μ
D, Lift A
E, Lift axis y_1
F, y axis $\perp$ to x and z
G, Tangential force T
H, Fuselage axis x
I, Lateral angle τ
J, x axis $\perp$ to y and z_1
K, Angle of attack α
L, Path axis x
M, Drag W
N, Spar axis z
O, z_1 axis $\perp$ to x and y_1
P, Cross-wind force Q
Q, Horizontal axis z $\perp$ to x

Fig.1 Notation.



R, t_2=1.60 m
S, t_1=2.20 m
T, G=1600 kg
U, F_H=4.80 m^2
V, F= 29.76 m^2

Fig.2

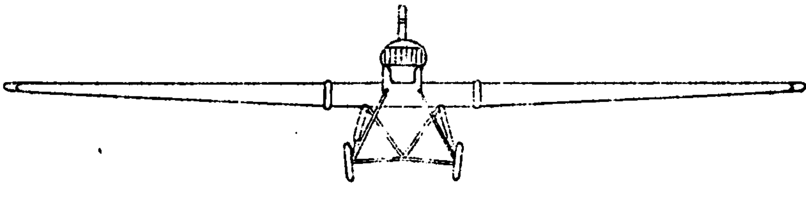


Fig.3

W , F_s=1.79 m^2
A', r=0.80 m
B', h=0.42 m

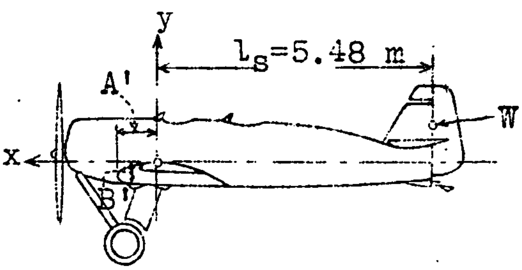


Fig.4

Figs.2,3,4 Plan, front and side views of Junkers A35 low-wing monoplane.

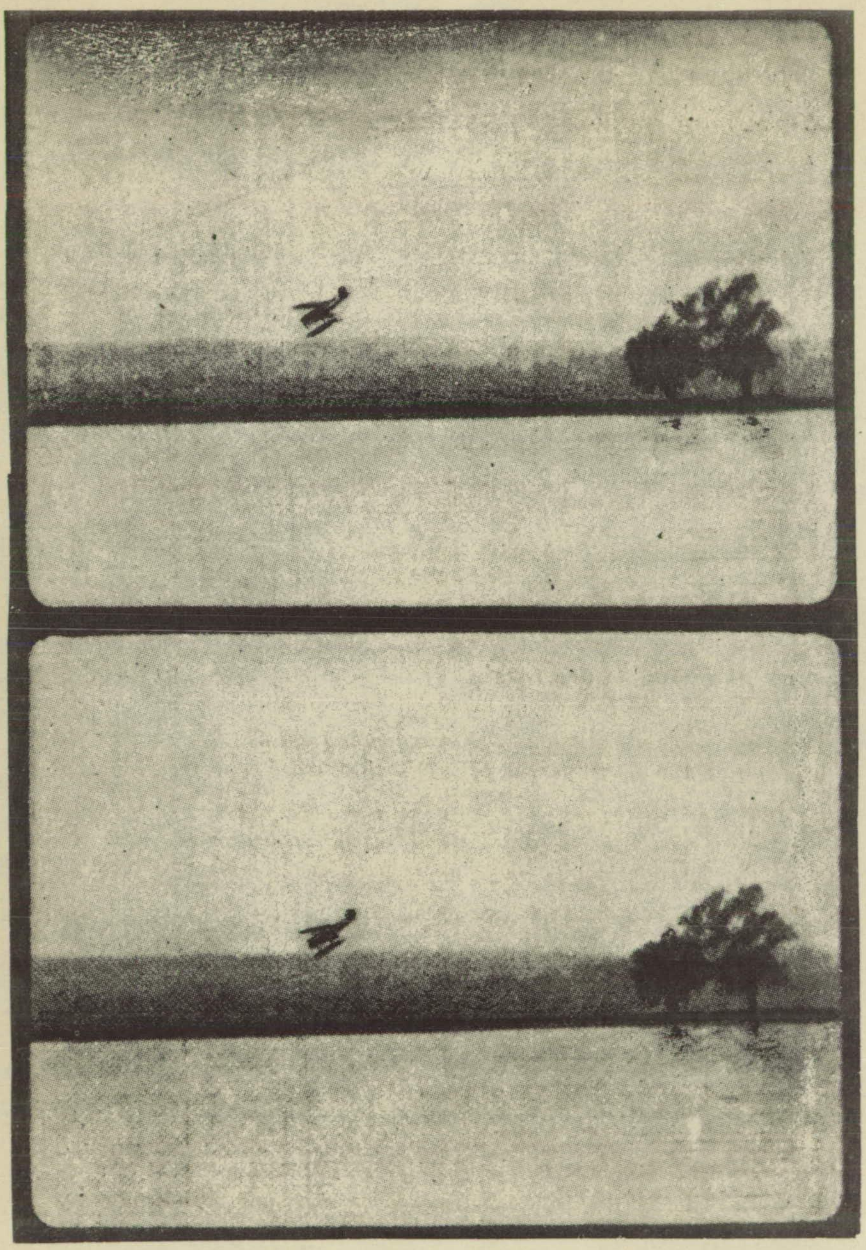


Fig. 5 Flat-spinning seaplane falling almost vertically at a midspan angle of attack of about 60°

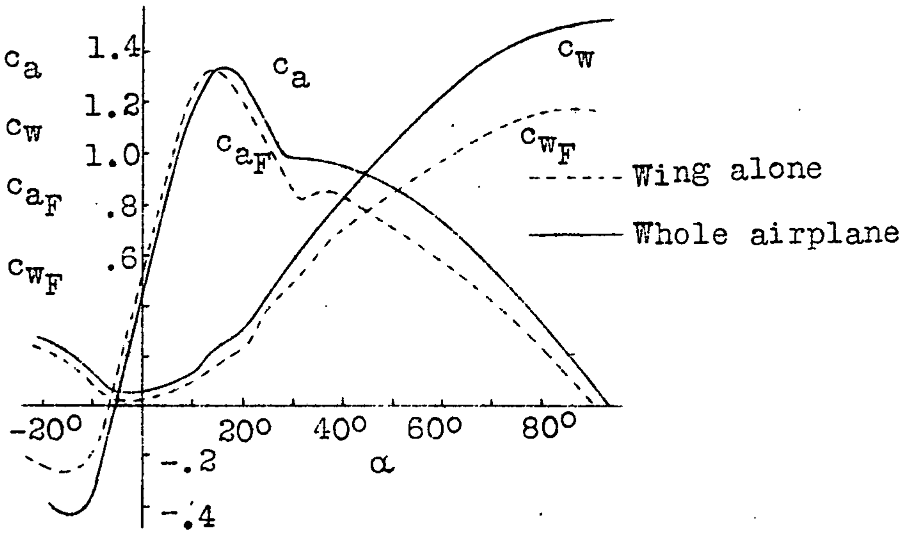


Fig.6 Coefficients of lift and drag plotted against angle of attack.

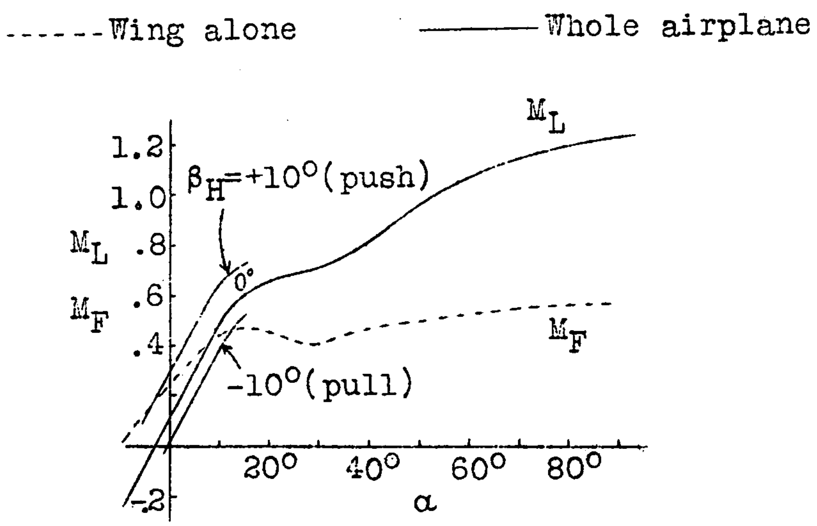


Fig.7 Coefficient of moment about leading edge of wing plotted against angle of attack with elevator angle as parameter.

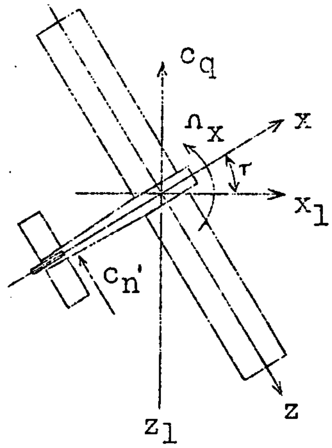


Fig.8 For calculating the cross-wind force.

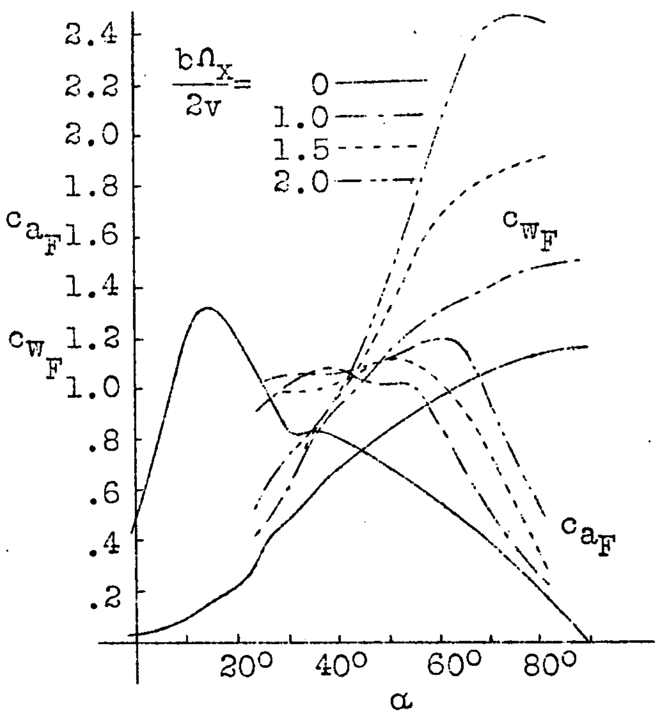


Fig.9 Coefficients of wing lift and drag plotted against angle of attack with parameter $b\Omega_x/2v$.

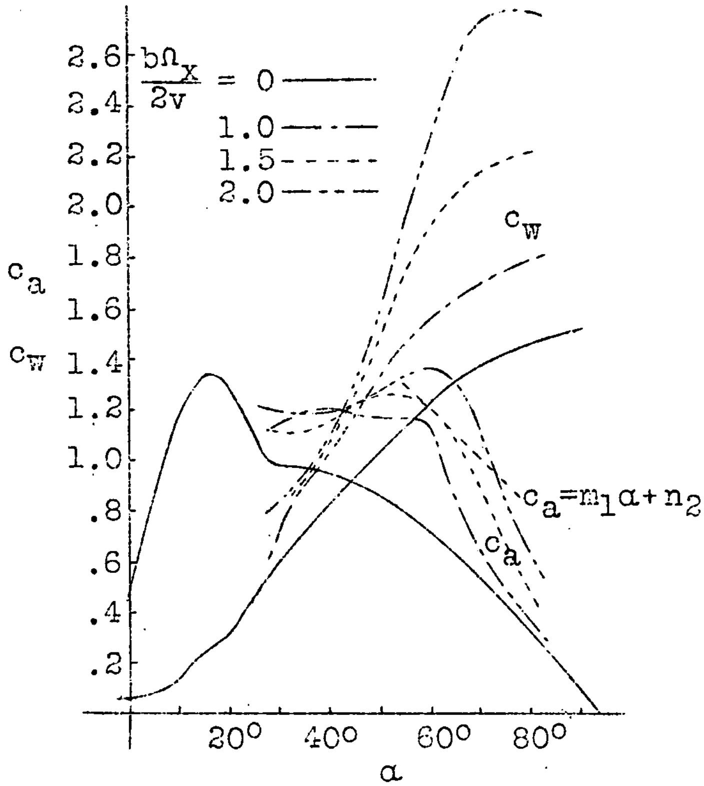


Fig.10 Coefficients of airplane lift and drag plotted against angle of attack with parameter $b\Omega_x/2v$ (n_1 instead of n_2).

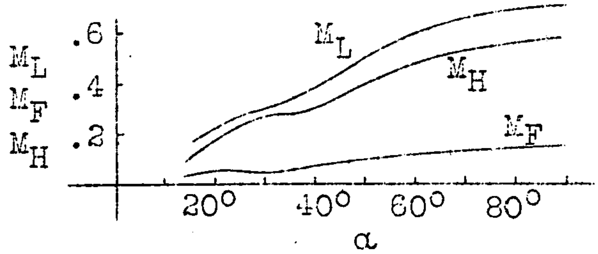


Fig.11 Coefficients of aerodynamic moments about spar axis plotted against angle of attack.

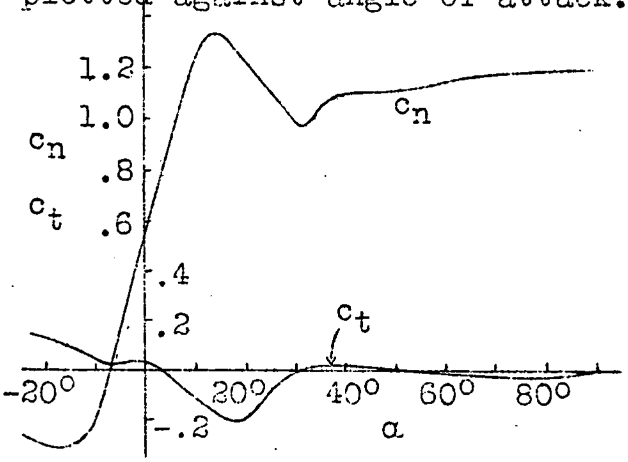


Fig.12 Coefficients of normal and tangential forces of wing alone plotted against angle of attack.

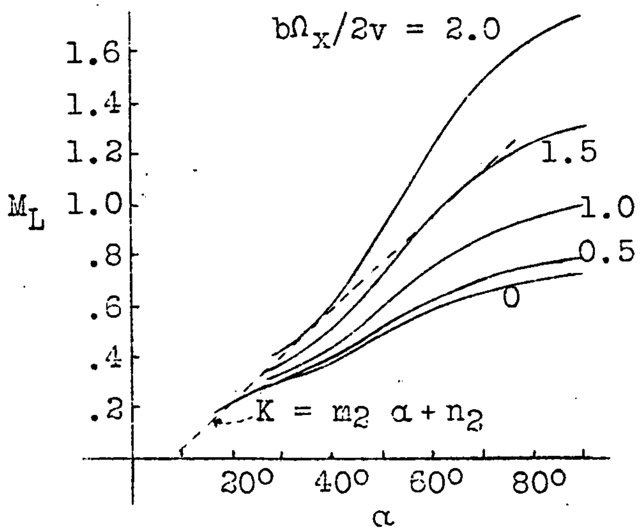


Fig.13 Coefficients of aerodynamic moments about spar axis plotted against angle of attack with parameter $b\Omega_x/2v$.

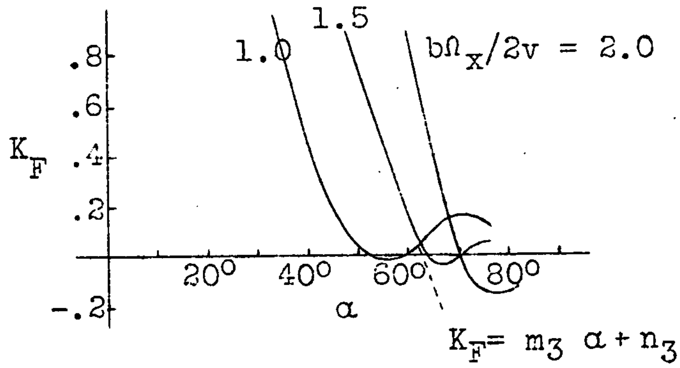


Fig.14 Coefficients of aerodynamic moments of wing alone about fuselage axis plotted against angle of attack with parameter $b\Omega_x/2v$.

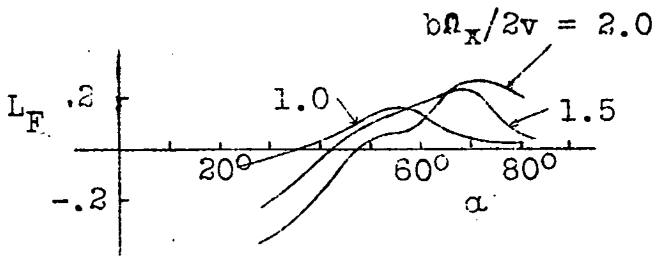


Fig.15 Coefficients of aerodynamic moments of wing alone about strut axis plotted against angle of attack with parameter $b\Omega_x/2v$.

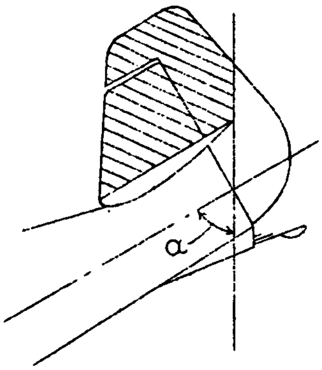


Fig.16 Blanketing of vertical empennage.

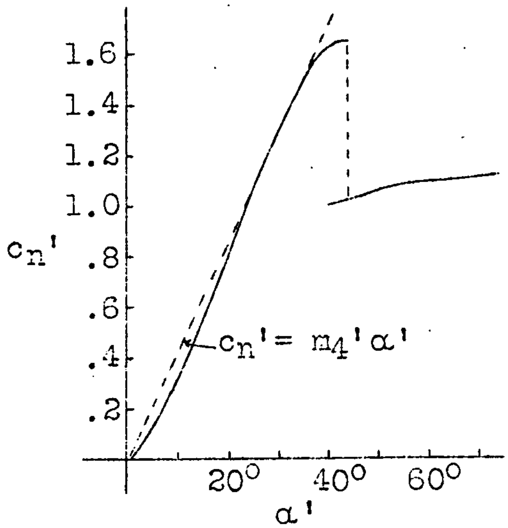


Fig.17 Coefficient of normal force of a square flat surface plotted against angle of attack.

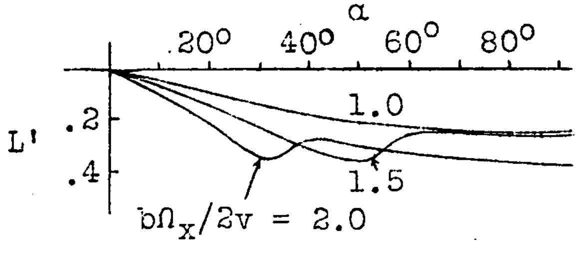


Fig.18 Coefficients of aerodynamic moments about strut axis, produced chiefly by fuselage end and vertical empennage, plotted against angle of attack with parameter $b\Omega_x/2v$.

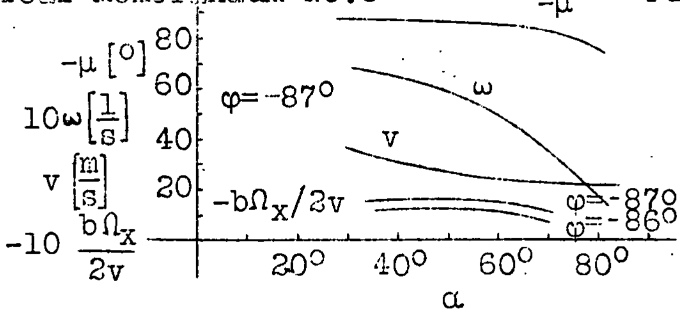


Fig.19 Path velocity, rotational velocity about vertical axis and angle of wing setting plotted against angle of attack for angle of glide -87°, as likewise $b\Omega_x/2v$ plotted against angle of attack with angle of glide as parameter.

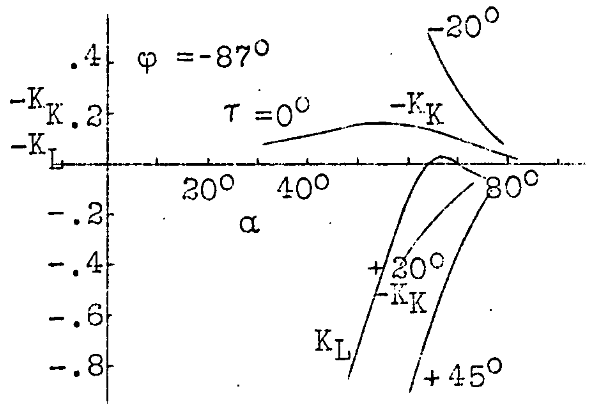


Fig.20 Balancing of moments about fuselage axis at angle of attack -87°. Coefficients of aerodynamic and gyroscopic moments plotted against angle of attack with lateral angle as parameter.

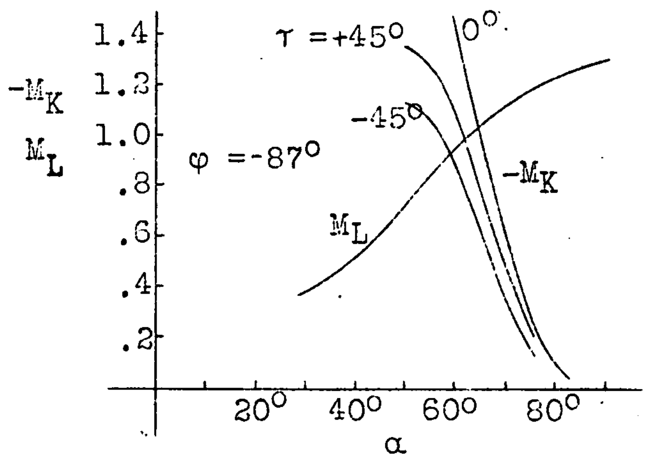


Fig.21 Balancing of moments about spar axis at angle of glide -87°. Coefficients of aerodynamic and gyroscopic moments plotted against angle of attack with lateral angle as parameter.

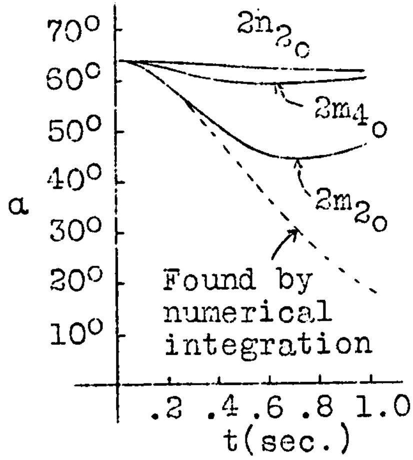


Fig.22 Angle of attack plotted against time.

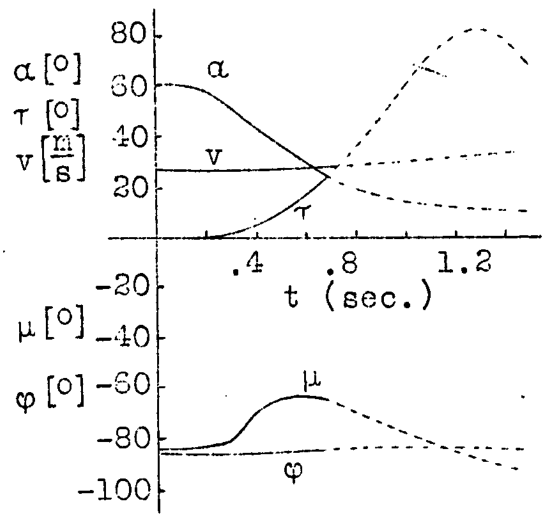


Fig.23

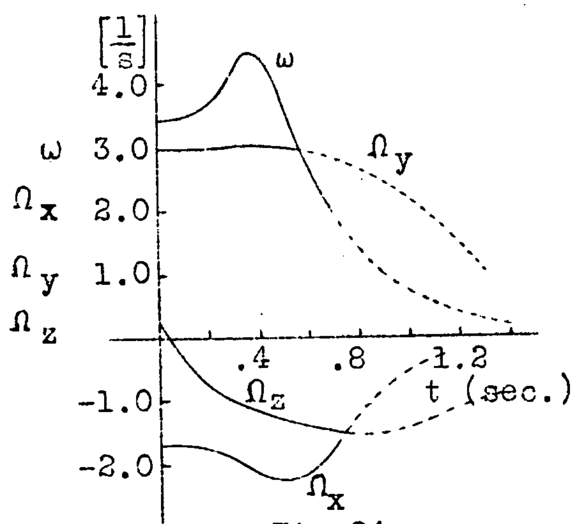


Fig.24

Figs.23,24 The nine variables plotted against time.

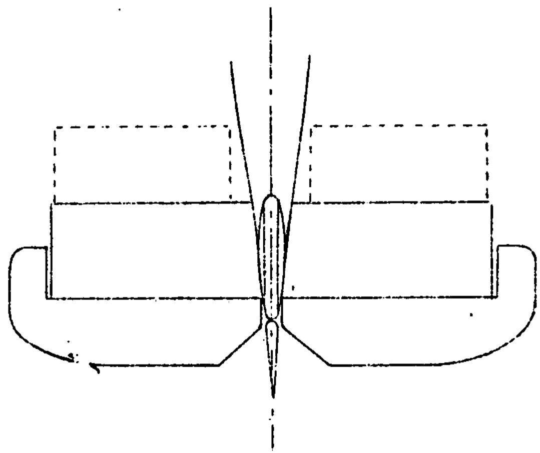


Fig.25 Horizontal empennage enlargable during flight.

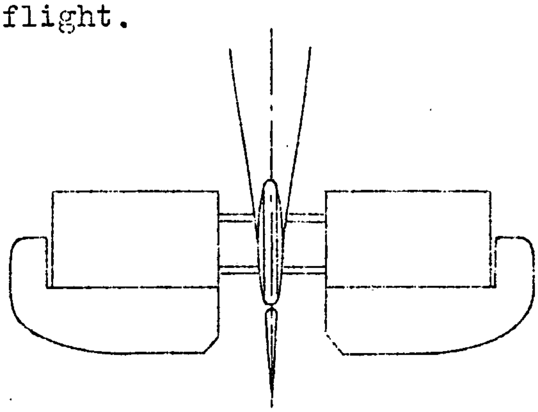


Fig.26 Horizontal empennage with open spaces next to fuselage.