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THE FRICTIONLESS FLOW IN THE REGION AROUND TWO CIRCLES

By M. Lagally

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TECHNICAL MEMORANDUM NO. 626

THE FRICTIONLESS FLOW IN THE REGION AROUND TWO CIRCLES*

By M. Lagally

1. Object of the investigation - Outlook.- Two nonintersecting circles are given in a plane; a frictionless flow is sought which fills the entire space A around both of the circles. The flow must be free of singularities in finite space; however, it may possess a velocity at infinity. Also if this is given, the flow is therewith nonuniquely determined; there results a doubly infinite number of solutions to the problem, the first of which will be unique provided that we write down a determined value for the circulation around each of the two circles. The solution of the problem is carried out with the aid of the conformal representation of the space A as a rectangle; this gives four simple flows from which the general flow appears to be linearly built up. Of particular interest is the determination of the flux that passes through the gap between the two circles, with which the flow coming from infinity and its dependence on both of the circulations and the velocity at infinity are interrelated.

The investigation is carried thus far. It attempts to surpass the boundaries of pure mathematical interest in that it possesses as an example, a flow investigation in a multiply-connected region. Then the results appear to be carried out by means of an appropriate conformal representation of the region around two chosen closed curves. Thus we have the basis of an exact plane theory of the biplane. Certainly, the known difficulties of the accomplishment of this representation depend upon the region around the given curves. It is easier to choose the transformation function so that we obtain the transformed curves which, as contours of the wings of a biplane are at least useful to a certain extent, and correspond to the Joukowski monoplane wing. Both wings have sharp trailing edges and, moreover, in order to obtain a smooth flow at the trailing edges, we have to locate a branch point of the transformation function which falls at a stagnation point of the flow around the circles. This application is only pointed out here and will not be entered into

*"Die reibungslose Strömung im Aussengebiet zweier Kreise."
Zeitschrift für angewandte Mathematik und Mechanik, Vol. 9, No. 4, August 1929, pp. 299-305.

in what follows.*

2. Bipolar coordinates and associated conformal representation.— We make two points $Q_1 (c, 0)$ and $Q_2 (-c, 0)$ on the x-axis in the z-plane, the origins of two polar coordinate systems r_1, ϑ_1 and r_2, ϑ_2 so that we may take the complex variable $z = x + iy$ for a point P (x, y) in both the forms:

$$z = c + r_1 e^{i\vartheta_1} \quad \text{and} \quad z = -c + r_2 e^{i\vartheta_2}. \quad (1)$$

Then the quotient

$$\frac{z + c}{z - c} = \frac{r_2}{r_1} e^{i(\vartheta_2 - \vartheta_1)} \quad (2)$$

contains in a known manner, the bipolar coordinates $\frac{r_2}{r_1}$ and $\vartheta_2 - \vartheta_1$. (Fig. 1a.)

The curves $\frac{r_2}{r_1} = \text{constant}$ are according to the theorem of Apollonius, the circles of a circular pencil which contains the points Q_1 and Q_2 as real zero circles. The curves $\vartheta_2 - \vartheta_1 = \text{constant}$ are, by means of the theorem on the peripheral angle, circles of a circular pencil which have the points Q_1 and Q_2 as real points of intersection; each of these circles is split into two arcs by Q_1 and Q_2 for which the corresponding peripheral angles $\vartheta_2 - \vartheta_1$ and $\vartheta_1 - \vartheta_2$ total 2π . Both the circular pencils are orthogonal.

We introduce a new complex variable $Z = X + iY$ by means of the equation

$$Z = \log_e \frac{z + c}{z - c} \quad (3)$$

which separates into

$$X = \log_e \frac{r_2}{r_1}, \quad Y = \vartheta_2 - \vartheta_1 \quad (4)$$

*Note by the Editor: Mr. Stephan Bergmann has called to my attention, that Julius Bonder has handled the same problem in his thesis (Proceedings and reports of the Warsaw Polytechnical Society, Vol. IV, 1, 1925). J. Bonder applies the method of images, however, he does not recognize that the series found by him are only partial fractional developments of elliptic functions. (See the report of St. Bergmann on "Nouevre polnische Arbeiten uber Mechanik," this journal, Vol. IX, 1929, pp. 241-247.) Furthermore, Dmitri Riabouchinsky has treated in his thesis (1922) in part 2, Chap. IV ("Mouvement cyclique plan d'un liquide autour d'un solide qui se meut parallelement a une paroi rectiligne) a somewhat special problem with the aid of Jacobi functions.

so that we obtain a conformal representation of the orthogonal circular pencils. (Fig. 1b.)

The circles $\frac{r_2}{r_1} = \text{constant}$ correspond in a unique manner to the straight lines $X = \text{constant}$; in particular, the zero points Q_2 and Q_1 to the lines $X = -\infty$ and $X = +\infty$. The circular arcs $\vartheta_2 - \vartheta_1 = \text{constant}$ between Q_1 and Q_2 correspond to the lines $Y = \text{constant}$ and indeed since the angles ϑ_1 and ϑ_2 are determined only up to multiple values of 2π , each arc $\vartheta_2 - \vartheta_1 = \text{constant}$, corresponds to infinitely many parallels $Y = \text{constant}$ of difference 2π .

The whole z -plane will thus have infinitely many values on the Z -plane, but a strip of width 2π bounded by two parallels to the X -axis is conformally represented in a unique manner. For the following investigation, the strip of the Z -plane lying between the lines $Y = -\pi$ and $Y = +\pi$ will be considered as the representation of the z -plane cut along the distance $Q_1 Q_2$. By this transformation the point $z = \infty$ corresponds to $Z = 0$.

3. The flow-field and its conformal representation.- (Fig. 2a, b.)- It will now be necessary to investigate the flow in the z -plane which occupies the region A enclosing two circles K_1 and K_2 of the circular pencil with the real zero circles Q_1 and Q_2 . Let K_1 and K_2 be determined by means of $\frac{r_2}{r_1} = \lambda$ and $\frac{r_2}{r_1} = \mu$, respectively; thus, if the zero circle Q_1 lies in the interior of K_1 and the zero circle Q_2 in the interior of K_2 , then $\lambda > 1$, $\mu < 1$. If to shorten, we put $\log_e \lambda = \alpha$, $\log_e \mu = -\beta$, then the circles K_1 and K_2 correspond to the conformal representation of the two lines $X = \alpha$ and $X = -\beta$.

The representation of the region A is thus a rectangle which is formed by the four lines $X = \alpha$, $X = -\beta$, $Y = \pi$, $Y = -\pi$. The two sides $Y = +\pi$, $Y = -\pi$ correspond to the lower and upper edges, respectively, of a cut along the x -axis which is drawn between the two circles. The rectangle contains the point $Z = 0$ as an inner point.

If the flow in the z -plane has the circulations Γ_1 and Γ_2 about the circles K_1 and K_2 , it thus keeps as its complex potential at infinity:

$$\Omega = -i \frac{\Gamma_1 + \Gamma_2}{2\pi} \log_e z. \quad (5)$$

$\Gamma_1 + \Gamma_2$ is the circulation around every closed curve surrounding K_1 and K_2 ; the infinitely distant point of the flow itself has the character of a point vortex of circulation

$$\Gamma = - (\Gamma_1 + \Gamma_2).. \quad (6)$$

By means of the conformal representation, there corresponds to it a point vortex with the same circulation at the point $Z = 0$. It accordingly retains as its complex potential in the neighborhood of this point

$$\Omega = - i \frac{\Gamma}{2\pi} \log_e Z . \quad (7)$$

This appears to be confirmed by means of calculation. We substitute in (5) by means of (3)

$$z = c \frac{e^Z + 1}{e^Z - 1},$$

thus we obtain a complex potential

$$\Omega = i \frac{\Gamma}{2\pi} \log_e \left(c \frac{e^Z + 1}{e^Z - 1} \right).$$

The function $\frac{e^Z + 1}{e^Z - 1}$ has a pole of the first order at the point $Z = 0$; it is

$$\frac{e^Z + 1}{e^Z - 1} = \frac{2}{Z} + \frac{Z}{6} + a_2 Z^2 + a_3 Z^3 + \dots =$$

$$\frac{1}{Z} \left[2 + \frac{Z^2}{6} + a_2 Z^3 + a_3 Z^4 + \dots \right] ; \quad (8)$$

by the omission of a regular flow in the neighborhood of $Z = 0$, we obtain the equation (7).

The circulations Γ_1 and Γ_2 around both the circles K_1 and K_2 are transformed by means of the conformal representation into flows Γ_1 and Γ_2 along the sides of the rectangle $X = \alpha$ and $X = -\beta$, which correspond to the two circles. If Γ_1 and Γ_2 are positive, then they occur in the negative direction around the rectangle.

The flow in the z -plane must be free of singularities in the finite portion of A ; however, a velocity at infinity $w_\infty = u_\infty - iv_\infty$ may be permitted. The resulting translatory flow is obtained as

$$\Omega = w_\infty z \quad (9)$$

thus the transformed flow in the neighborhood of the point $Z = 0$ as

$$\Omega = w_{\infty} c \frac{e^Z + 1}{e^Z - 1} \quad (10)$$

or after the omission of a regular flow by means of (8) as

$$\Omega = \frac{2 w_{\infty} c}{Z} \quad (10')$$

It possesses a doublet at the point $Z = 0$.

Thus we can set up the complex potential Ω and the complex velocity $W = \frac{d\Omega}{dZ}$ of the transformed flow, in the neighborhood of the point $Z = 0$. If $P(Z)$ represents a function which is regular in the neighborhood of the zero point, then by means of (7) and (10'), it is

$$\Omega = -i \frac{\Gamma}{2\pi} \log_e Z + \frac{2 w_{\infty} c}{Z} + P(z);$$

$$W = -i \frac{\Gamma}{2\pi Z} - \frac{2 w_{\infty} c}{Z^2} + P'(z) \dots \quad (11)$$

At all other points of the transformed rectangle Ω and W are regular.

4. Formation of the complex potential.— The complex potential of the transformed flow is determined according to (11) except for a function which is regular in the transformed rectangle. The complete determination succeeds in the following manner. The complex velocity in the z -plane is

$$w = \frac{d\Omega}{dz} = \frac{d\Omega}{dZ} \frac{dZ}{dz} = W \left[\frac{1}{z+c} - \frac{1}{z-c} \right].$$

Since w is a single-valued function in A , the same applies to W ; that is, W must as a function of Z possess the period $2\pi i$, thus it has the same value at all the transformed points of z . Further, since $X = \alpha$ and $X = -\beta$ are streamlines, therefore all the same singularities of W situated within the transformed rectangle and beyond must be symmetrically located with respect to both these lines. Thus the whole flow in the transformed rectangle will be repeatedly reflected over both these lines. In every case W must thus possess the period $2(\alpha + \beta)$.

W is thus a doubly periodic function with the periods

$$2\omega_1 = 2(\alpha + \beta); \quad 2\omega_2 = 2\pi i. \quad (12)$$

The periodic rectangle has double the width of the transformed rectangle; for the following considerations a periodic rectangle is basically assumed which is composed of the transformed rectangle and its reflection on the line $X = -\beta$. In it W has besides the singularity at the zero point an additional reflected singularity at the point -2β . (Fig. 3.) Thus W is an elliptic function which may be put in the form

$$W = -i \frac{\Gamma}{2\pi} \frac{1}{Z} + i \frac{\Gamma}{2\pi} \frac{1}{Z + 2\beta} - \frac{2c(u_\infty - i v_\infty)}{Z^2} - \frac{2c(-u_\infty - i v_\infty)}{(Z + 2\beta)^2} + R(Z) \quad (13)$$

where $R(Z)$ represents a function which is regular in the periodic rectangle. This elliptic function is determined by means of its poles except for an additive constant; we may put for brevity

$$u_\infty + i v_\infty = \bar{w}_\infty \quad \text{with} \quad u_\infty - i v_\infty = w_\infty,$$

so that*

$$W = -i \frac{\Gamma}{2\pi} [\zeta(Z) - \zeta(Z + 2\beta)] - 2c[w_\infty p(Z) - \bar{w}_\infty p(Z + 2\beta)] + i\kappa. \quad (14)$$

The additive constant is introduced in the imaginary form $i\kappa$ for a later evident useful basis; it could be determined by means of the substitution of the value of the velocity at a point of the flow field; κ is thereby proven to be real; however, a better way for the determination of κ will be shortly pointed out.

The complex potential will be

$$\Omega = -i \frac{\Gamma}{2\pi} \log_e \frac{\sigma(Z)}{\sigma(Z + 2\beta)} + 2c [w_\infty \zeta(Z) - \bar{w}_\infty \zeta(Z + 2\beta)] + i\kappa Z + \kappa'. \quad (15)$$

Here κ' is a new additive constant which can be chosen as we please. For simplicity we shall put $\kappa' = 0$. However, it should be noted that Ω is also an infinitely multiple-valued function; if $\log_e \frac{\sigma(Z)}{\sigma(Z + 2\beta)}$ changes by $\pm 2\pi i$ by a revolution around a pole, then Ω itself changes by $\pm \Gamma$.

*Burkhardt-Faber, "Elliptische Funktionen," section 30.

In order to make Ω single-valued in the periodic rectangle, we must prevent the revolution around both the poles 0 and -2β ; this is done by the introduction of a branch-cut along the combined segments of the x-axis.

5. Relationship between circulation and flux.— The function $\Omega(Z)$ is an elliptical transcendental one and changes by an additive constant if the argument of Z increases by a period $2\omega_1$ or $2\omega_2$, written combined as 2ω .

In order to calculate these changes, $\Omega(Z)$ must be put in the following form, in which it is to be noted that the factor of u_∞ is an elliptic function:

$$\Omega(Z) = -i \frac{\Gamma}{2\pi} \log_e \frac{\sigma(Z)}{\sigma(Z+2\beta)} + 2c u_\infty [\zeta(Z) - \zeta(Z+2\beta)] - 2ic v_\infty [\zeta(Z) + \zeta(Z+2\beta)] + i\kappa Z. \quad (16)$$

Then

$$\Omega(Z+2\omega) - \Omega(Z) = -i \frac{\Gamma}{2\pi} \log_e \frac{\sigma(Z+2\omega)}{\sigma(Z+2\beta+2\omega)} \frac{\sigma(Z+2\beta)}{\sigma(Z)} - icv_\infty [\zeta(Z+2\omega) + \zeta(Z+2\beta+2\omega) - \zeta(Z) - \zeta(Z+2\beta)] + 2i\kappa\omega.$$

From the properties of the ζ and σ functions*

$$\zeta(Z+2\omega) - \zeta(Z) = 2\eta, \quad \frac{\sigma(Z+2\omega)}{\sigma(Z)} = e^{-2\eta(Z+\omega)}$$

where η is written for $\eta_1 = \zeta(\omega_1)$, $\eta_2 = \zeta(\omega_2)$ combined, it follows immediately for both the periods written separately

$$\Omega(Z+2\omega_1) - \Omega(Z) = i [2/\pi \Gamma \beta \eta_1 - 8c v_\infty \eta_1 + 2\kappa\omega_1], \quad (17a)$$

$$\Omega(Z+2\omega_2) - \Omega(Z) = i [2/\pi \Gamma \beta \eta_2 - 8c v_\infty \eta_2 + 2\kappa\omega_2]. \quad (17b)$$

It should be noted here that ω_1 and η_1 are real, ω_2 and η_2 purely imaginary, that therefore the right side of (17a) is purely imaginary, that of (17b) is real. This remark brings in the hydrodynamical meaning of the result. We decompose the complex potential $\Omega = \phi + i\psi$ into real and imaginary parts; thus ϕ represents the velocity potential, ψ the stream function. We connect the points P_1 and P_2 in a region in which Ω is single-valued, by any path we please, and decompose the difference $\Omega_2 - \Omega_1$ in the same way:

*Burkhardt-Faber, sections 25-26.

$$\Omega_2 - \Omega_1 = (\varphi_2 - \varphi_1) + i (\psi_2 - \psi_1),$$

so that $\varphi_2 - \varphi_1$ is the flow along the path P_1P_2 and $\psi_2 - \psi_1$ is the flux through this path, calculated as positive from the right.

Thus $\Omega(Z + 2\omega_1) - \Omega(Z)$ is, except for the factor i , the flux through the horizontal sides of the periodic rectangle. We put $\Omega(Z + 2\omega_1) - \Omega(Z) = -2iF$, so that F is the flux through the horizontal sides of the transformed rectangle, calculated as positive from the left, or also, because of the invariance of Ω under a conformal transformation, the flux through a chosen line joining two points of the circles K_1 and K_2 , calculated as positive from the left.

Similarly $\Omega(Z + 2\omega_2) - \Omega(Z)$ is the flow along one of the vertical sides of the periodic rectangle, hence $-\Gamma_1$; or, except for the sign, the circulation around the circle K_1 . (Fig. 3.)

Thus it follows from (17a,b)

$$-2iF = i [2/\pi \Gamma \beta \eta_1 - 8c v_\infty \eta_1 + 2\kappa \omega_1], \quad (18a)$$

$$-\Gamma_1 = i [2/\pi \Gamma \beta \eta_2 - 8c v_\infty \eta_2 + 2\kappa \omega_2]. \quad (18b)$$

Each of these equations would permit the calculation of the constant κ with the aid of the quantities $\Gamma_1, \Gamma, F, v_\infty$, on each of which the flow around the two circles depends. We eliminate κ on the other side of both of the equations by the application of the Legendre relation*

$$\eta_1 \omega_2 - \eta_2 \omega_1 = \frac{\pi i}{2}, \quad (19)$$

so that we obtain an interesting relation between the hydrodynamical quantities:

$$-2iF \omega_2 + \Gamma_1 \omega_1 = -\Gamma \beta + 4\pi c v_\infty.$$

This is simplified by means of (6) and (12) into

$$2\pi F = 4\pi c v_\infty - \Gamma_1 \alpha + \Gamma_2 \beta. \quad (20)$$

This equation gives the flux passing between the two circles if the velocity at infinity and the circulations around both the circles are given.

*Burkhardt-Faber, section 25, (14).

6. Discussion of the possible flows.- If the opposite conditions for two circles are given, by means of the three constants α , β , and c , then the flow in the region surrounding both the circles is therewith nonuniquely determined; likewise, also if we have given the velocity at infinity $w_\infty = u_\infty - i v_\infty$. According to (16) a doubly infinite number of different flows occur by means of the choice of the constants Γ and κ , or also, if we take from (18b),

$$i \kappa = \frac{i}{2\pi} \Gamma_1 - \frac{1}{\pi^2} \Gamma \beta \eta_2 + \frac{4}{\pi} c v_\infty \eta_2$$

by the choice of the constants Γ and Γ_1 .

We introduce further, besides $\Gamma = -(\Gamma_1 + \Gamma_2)$, the quantity

$$\Gamma' = \Gamma_1 - \Gamma_2$$

thus putting

$$i \kappa = \frac{i}{4\pi} \Gamma' - \frac{i}{4\pi} \left(1 - \frac{4 i \beta \eta_2}{\pi} \right) \Gamma + \frac{4}{\pi} c v_\infty \eta_2,$$

so that the complex potential (16) appears in the following form (out of which the unsymmetric advantages of the quantity β before α could be set aside by means of (12) and (19)):

$$\Omega(Z) = \frac{i}{4\pi} \Gamma' Z - \frac{i}{4\pi} \Gamma \left[2 \log_e \frac{\sigma(Z)}{\sigma(Z + 2\beta)} + \left(1 - \frac{4 i \beta \eta_2}{\pi} \right) Z \right] + 2 c u_\infty [\zeta(Z) - \zeta(Z + 2\beta)] - 2 i c v_\infty \left[\zeta(Z) + \zeta(Z + 2\beta) + \frac{2 i \eta_2}{\pi} Z \right]. \quad (21)$$

The four-term aggregate on the right side appears to show that the whole flow arises from the linear supposition of four separate flows, for each of which only one of the four quantities Γ' , Γ , u_∞ , and v_∞ is different from zero.

By the first partial flow $\frac{i}{4\pi} \Gamma' Z$, both the circulations are equal and opposite, if we return to the z -plane: $\Gamma'_1 = -\Gamma'_2 = \Gamma'/2$. The streamlines are the circles of the pencil; the velocity at infinity is zero.

By the second partial flow with the factor Γ , the two circulations are equal: $\Gamma_1 = \Gamma_2 = -\Gamma/2$; the velocity at infinity is zero.

The third partial flow with the factor u_∞ is a noncirculatory translatory flow in the direction of the line of centers of both the circles.

The fourth partial flow with the factor v_∞ is a non-circulatory translatory flow perpendicular to the direction of the line of centers of both the circles.

If we introduce the constants Γ and Γ' in (20), then we may recognize in this equation the portion of the partial flows in the flux passing through the gap between the two circles.

$$2 \pi F = 4 \pi c v_\infty + \frac{\alpha - \beta}{2} \Gamma - \frac{\alpha + \beta}{2} \Gamma'. \quad (22)$$

For a noncirculatory flow, (22) reduces to

$$F = 2 c v_\infty; \quad (22')$$

this is also the flux of an uninterrupted flow through a canal of width $2c$; the flux which passes between the two circles is independent of the actual distance of both the circles, and does not increase if we replace both of the circles by others of the same pencil; the effective width of the gap between two circles is the distance of the zero circles of the pencil determined by the two circles.

7. Introduction of the radii and center-to-center distance of both circles. The use of bipolar coordinates has the disadvantage that both the circles K_1 and K_2 are not given by their radii R_1 and R_2 and the abscissas a_1 and $-a_2$ of their centers, or the center-to-center distance $d = a_1 + a_2$, but separately by three constants α , β , and c , which determine the circles only indirectly. It remains only to add the equations which serve to introduce R_1 , R_2 and d . (Fig. 2a.)

The equation of the first circle K_1 in bipolar coordinates is $\frac{r_2}{r_1} = e^\alpha$; in order to bring in rectangular coordinates we can replace $\frac{r_2}{r_1}$ by $\frac{|z+c|}{|z-c|}$. A simple transformation gives the equation of the circle

$$x^2 + y^2 - 2 x c \coth \alpha + c^2 = 0,$$

from which

$$a_1 = c \coth \alpha; \quad R_1 = \frac{c}{\sinh \alpha}$$

appears to be taken.

For the second circle, K_2 , there follows from $\frac{r_2}{r_1} = e^{-\beta}$, the corresponding values

$$-a_2 = -c \coth \beta; \quad R_2 = \frac{c}{\sinh \beta}.$$

Thus if both the radii R_1 , R_2 and the center-to-center distance d are given, we have the three equations

$$R_1 = \frac{c}{\sinh \alpha}; \quad R_2 = \frac{c}{\sinh \beta}; \quad d = c(\coth \alpha + \coth \beta) \quad (23)$$

for the calculation of α , β , and c . From these three simple and admirably detached equations, the introduction of any (more-over conformally invariant) auxiliary quantity is permitted, namely, the intersecting angle of both the circles.

In general the intersecting angle of two circles is given by

$$\cos \omega = \frac{d^2 - R_1^2 - R_2^2}{2 R_1 R_2};$$

for $d > R_1 + R_2$, the angle will be imaginary. We put $\omega = i J$ so that $\cos \omega = \cosh J$, thus we have for the calculation of the auxiliary quantity J , the equation

$$\cosh J = \frac{d^2 - R_1^2 - R_2^2}{2 R_1 R_2}. \quad (24)$$

J appears also to be expressed by the system of quantities α , β , and c ; from (23) and (24) it follows that

$$\cosh J = \cosh (\alpha + \beta); \quad \text{thus } J = \alpha + \beta.$$

Now this gives the solution of the equation (23) in simplest form

$$c = \frac{R_1 R_2}{d} \sinh J; \quad \sinh \alpha = \frac{R_2}{d} \sinh J, \quad \sinh \beta = \frac{R_1}{d} \sinh J \quad (25)$$

where J is to be taken from (24).

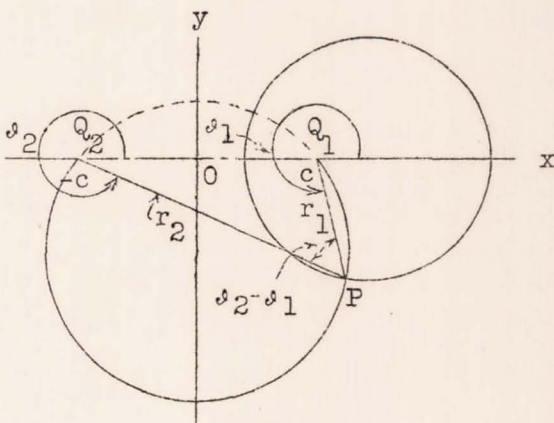


Fig. 1a

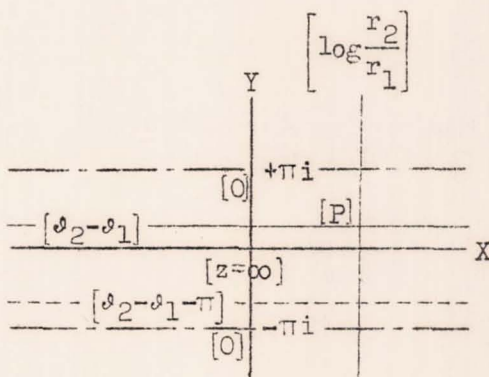


Fig. 1b

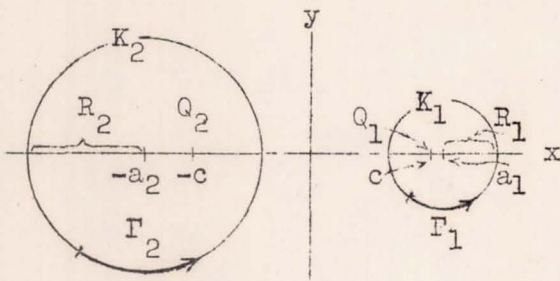


Fig. 2a

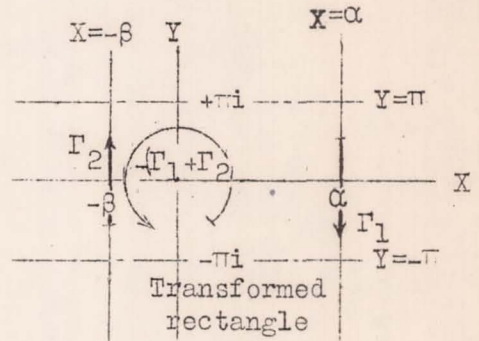


Fig. 2b

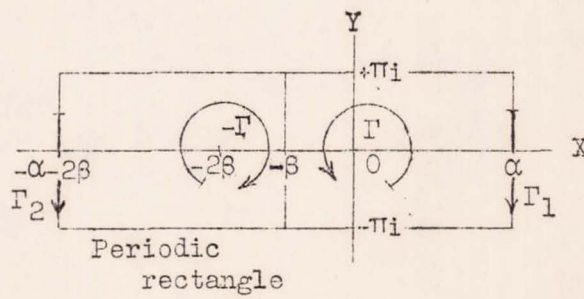


Fig. 3