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NDING OF SEAPLANES

By Herbert Wagner

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LANDING OF SEAPLANES*

By Herbert Wagner

I wish to preface my report with a brief resume of a theoretical consideration and its results.** Then I intend to describe the processes at the moment a keeled bottom strikes the water and give the formulas and solutions. Lastly, I shall discuss the application of these results to practical problems.

Let us assume that a long prismatic body, having a keeled bottom, moves vertically downward at an initial speed V_0, its mass being m per unit of length. Now (Fig. 1, top) it touches the flat surface of the water; an instant later it is already immersed. By nosing under the body imparts a downward motion to the water, whereby it experiences an upward blow P, itself.

The water is pushed aside (See Fig. 2), and as a result its level raises on the sides of the body. Now, since the air pressure is the same at any point of the surface, the pressure gradient must be perpendicular to the surface, so that the acceleration of the particles on the water level is perpendicular to the surface, and the velocity on the surface is approximately upward. This velocity is low at the outer edge but increases toward the impact area. At the edge of the impact surface (Fig. 3) the water is pushed outward and upward at enormous speed, is finally inverted from the bottom and disappears as spray to the sides. The pressure on the bottom is enormous at this point of inversion and the flung-off spray corresponds to the motion energy dissipated during the impact.

The pressure distribution over the body (Fig. 1, center) becomes apparent if we realize that the water pressure is essentially the reaction of the water against the incipient downward motion. In the middle of the bottom the water already has a downward speed V. Whereas width c of the impact surface increases with the time the water at the edge of the impact sur-

*"Über die Landung von Seeflugzeugen," Zeitschrift für Flugtechnik und Motorluftschiffahrt, Vol. 22, No. 1, Jan. 14, 1931, pp. 1-8, Verlag von R. Oldenbourg, München und Berlin.
**This treatise was originally given in the physical seminary of the Danzig Institute of Technology (Jan., 1930), and at the International Congress for Mechanics, at Stockholm (Aug., 1930). A detailed description of the mathematical formulas and results is to appear in the Zeitschrift für angewandte Mathematik und Mechanik.

face, which previously exhibited an upward velocity, is set in downward motion; the pressure is highest at the edge of the impact area, and spreads toward the middle of the bottom.

Since the body on impact experiences an upward force its initial speed V_0 drops to V and with it the water previously set in motion,

Although the body embraces an infinitely large mass, that is, strikes with constant speed, this slowing up of the water calls forth a suction (or better, pressure component), which is greater in the middle than at the outside and reduces the pressure in the middle still more. Lastly, it may happen by relatively small body mass that toward the end of the blow (Fig. 1, bottom) this suction induced by the retardation exceeds the overpressure in the middle caused by expansion of the impact surface. In that case negative pressure prevails in the middle of the bottom, while at the edge of the impact surface the high and positive pressures predominate in their combined effect, which retard the blow still more.

As the impact surface finally reaches the side edge of the boat bottom $(c = \frac{1}{2} b)$, it terminates the impact. In the further immersion of the body its static lift which, during the actual impact process remained practically ineffective, makes itself manifest.

If c is the half-width of the impact surface, figured from center of bottom to the point where the spray breaks away, then dc/dt is the rate at which the breadth of the impact surface expands. The most prominent mathematical quantity is vested in the speed ratio $u = V/\frac{dc}{dt}$. This comparative value is defined as follows (Fig. 4).

Let the shape of the bottom be given as $y = y(x)$. Then we assume the bottom at rest and the water with initially level surface as moving upward at variable speed V. At the surface the water particles are additionally accelerated, that is, as already stated, perpendicular to the surface. For the moment we infer the slope of the bottom to be (infinitely) flat, so that the y ordinates are very (infinitely) small. Then it becomes apparent that the inclination of the surface of the water (apart from the surface of the spray) is very (infinitely) small, and that, in consequence, the adduced accelerations and the ultimate speeds on the surface are approximately vertically upward.

On the bottom itself the speed of the surface falls in direction of the bottom; on the free surface it is vertically upward. This defines the fluid motion without ambiguity; it agrees (very closely) with the fluid motion about a flat plate (Compare Fig. 4, bottom). The speed v_y on the free surface at a stated point x, is

$$v_y = \frac{V}{\sqrt{1 - c^2/x^2}} \tag{1}$$

The rise η of the water particle at this pertinent point is

$$\eta = \int_0^t v_y \, dt = \int_0^t \frac{V dt}{\sqrt{1 - c^2/x^2}} \tag{2}$$

Then we introduce c as new integration variable, that is, $t = t\ (c)$ and $dt = \frac{1}{\frac{dc}{dt}} dc$, that is,

$$\eta = \int_0^c \frac{V}{\frac{dc}{dt}} \frac{dc}{\sqrt{1 - c^2/x^2}} \tag{3}$$

This is rise η, when the width of the impact has attained

$c < x$; whereas c increases with the time, η always becomes greater at point x. When the water particles finally reach the bottom surface $\eta = y$ and simultaneously $c = x$. Consequently

$$y = \int_{c=0}^{c=x} u \frac{dc}{\sqrt{1 - c^2/x^2}} \tag{4}$$

where u is the abbreviated speed ratio $\frac{V}{\frac{dc}{dt}}$, from which u is quickly determined.

The mathematical representation of the bottom shape is by series

$$y = \beta_0 \ x + \beta_1 \ x^2 + \beta_2 \ x^3 + \beta_3 \ x^4 + \beta_4 \ x^5 + \ldots, \tag{5}$$

where β constant is defined conformably to the given body shape. Then equation (4) yields for u

$$u = \frac{V}{\frac{dc}{dt}} = \frac{2}{\pi} \beta_0 + \beta_1 \ c + \frac{4}{\pi} \beta_2 \ c^2 + \frac{3}{2} \beta_3 \ c^3 + \frac{16}{3\pi} \beta_4 \ c^4 + \ldots \tag{6}$$

u is of the order of magnitude of the keel angle, which means it is small when the keel is flat.

Now we come to the spray where at its point of origination, the slope of the water level is not small and where our previous considerations become inapplicable as a result in the vicinity of this point. A searching study of the flow reveals that within range of large angles of slope the speed on the surface may be translated as a constant tangential speed (Fig. 3) $\frac{dc}{dt} = \frac{V}{u}$ and a superposed horizontal speed of like magnitude; $\frac{dc}{dt}$ is the previously mentioned rate of expansion of the impact area. The velocity in the spray near its origin is precisely twice as great, that is, $2\,\frac{dc}{dt}$, and the spray water is flung off at this rate.

The thickness δ of the spray at its point of inception is

$$\delta = \frac{\pi}{8}\, c\, u^2 \tag{7}$$

hence, of the order of u^2, that is, very small. On impact of a flying boat this spray is several millimeters thick.

The pressure distribution in the spray is illustrated in Figure 3. The maximum pressure is equivalent to a dynamic pressure of $\frac{dc}{dt}$ velocity; hence (Compare (12))

$$p_{max} = \frac{\rho}{2}\, \frac{V_0^2}{(1 + \mu)^2}\, \frac{1}{u^2} \tag{8}$$

Now we introduce the abbreviations:

$$\mu = \frac{\pi\, \rho\, c^2}{2m} \tag{9}$$

The fluid pressure is given by

$$p = \frac{\rho\, V_0^2}{(1 + \mu)^2}\, \frac{1}{u} \left[\frac{1}{\sqrt{1 - x^2/c^2}} - \frac{2\,\mu}{1 + \mu} \sqrt{1 - \frac{x^2}{c^2}} - \frac{u}{2\left(\frac{c^2}{x^2} - 1\right)} \right] \tag{10}$$

The pressure distribution in this formula is applicable to within the vicinity of the edge of the impact area (to within about $p = \frac{3}{4}\, p_{max}$). There the distribution conformal to (10) changes to that shown in Figure 3; the latter is decisive for the extreme edge.

The first term in (10) corresponds to the distribution ushered in by the growth in impact area. The second term corresponds to the retardation of the water; there are negative pressures with elliptical distribution. The third term, owing to the square of the velocity, is small in the middle range with respect to the other two terms (since it appears multiplied by u), and becomes nominal at the edge of the impact area where c approaches x.

By integration over the immersion width we obtain (disregarding infinitely small terms) the force of impact per unit length (l = length of impact area).

$$P/l = \frac{\pi \rho V_0^2}{(1 + \mu)^3} \frac{c}{u} \qquad (11)$$

Besides, the velocity is

$$V = \frac{V_0}{1 + \mu} \qquad (12)$$

The depth of immersion of the keel with respect to the original water level is computed at

$$T = \int_0^c V \frac{dt}{dc} dc = \int_0^c u dc \qquad (13)$$

The results are withal applicable to calculations of the force effect on impact of a flat-bottom hull against the crest of a wave (Fig. 5). In this case y is the relative rise between bottom and undisturbed water level at the inception of blow on impact.

E x a m p l e s

First we compare a flat keel bottom with one having twice as much keel (Fig. 6), but of similar shape otherwise. The mass of the boat body and the initial speed of impact are to be the same in both cases.

In the middle range of the boat the pressures for the sharp keel are (very nearly) half as high as in the flat keel bottom (the pressure here is, aside from the third term in formula (10), proportional to $1/u$). But the pressure maximum at the edge of the impact area drops to one-fourth of that for the flat keel (the pressure here is proportional to $1/u^2$; compare equation (8)). A flat keel is very disagreeably demonstrative on this point, even though the range of high pressure is limited. The total force of impact drops approximately to half of that of the flat keel.

Now I would like to illustrate the process of impact on a certain bottom for two different mass loadings. Once the seaplane is to take the blow over a great length of the body, so as to make the mass per unit length of impact area small, the same mass loading can occur when the boat settles comparatively far forward on a limited area; then the "reduced mass" is small. In the second case the mass loading is to be high; the boat is to take the blow on a comparatively small area near the step. The speed of immersion is high at the beginning of the impact (Fig. 7). Although our bottom has a pronounced keel in the middle, the pressure is nevertheless relatively low. As the impact progresses the retardation is so pronounced by the small mass loading that the pressures do not become extraordinarily high in spite of the fact that the keel is flat at the side of the bottom. By great mass loading, however, the barely retarded boat strikes with the flat keel side portion, so the pressures become very high.

If, as in our case, the bottom slopes downward, a point is reached at last where the slope of the bottom equals the slope of the water. Then the forces of impact are extraordinarily violent (mathematically, infinitely great). This happens in our case when the lateral edge of the bottom strikes first $(c = \frac{1}{2} b)$.

Figure 8 exhibits the force of impact for several bottom shapes with different mass loadings. The indicated pressure scale applies to $V_0 = 3$ m/s; time t since inception of impact, force of impact P and speed V (relative to initial speed V_0) for a weight per unit length of $G = 4000$ kg/m, $2G = 8000$, and $G = \infty$ are plotted against width c (m) of the impact area. The shape of the central portion of the bottom is such that the total force P remains constant during the whole impact for the given mass loading. A bottom of this kind is flat in the middle and has the shape of a parabola. The keel flattens out toward the edge of the impact area. In spite of this flat slope the force is just as high as at the beginning of the impact, because the speed of impact is already lower.

Now we extend our considerations to cover five effects hitherto disregarded:

1) Effect of acceleration of gravity;
2) Variation in bottom shape and depth of immersion over the length of impact;
3) Deviations from the plane flow problem (the water escapes to the front and back);
4) Finite size of keel angle;
5) Elasticity of hull bottom.

1) The effect of the acceleration of gravity manifests itself in the static lift of the water displaced by the bottom.

2) A flying boat (Fig. 9) with mass M and variable bottom shape strikes against a wave of given form and speed.

The force effect is explained by assuming plane flow in each length element $d\,l$, and to which the previously enumerated relations are applicable. Then we define the width of the impact area from (13) for each depth of immersion T (with respect to the undisturbed form of the surface), and use series (6) to demonstrate u at each point. Then the force becomes

$$P = \frac{\pi\ \rho\ V_0^2}{(1+\mu)^3} \int^{l} \frac{c}{u}\, d\ l \tag{14}$$

Should the impact area have reached the side edge of the hull bottom in some region, we must write $c = b/2$ and $u = \infty$, and express μ by

$$\mu = \frac{\pi\ \rho}{2\ M} \int^{l} c^2\ d\ l \tag{15}$$

so that pressure distribution, speed, etc., are now resolvable from the preceding formulas.

3) Hitherto the flow was presumably plane, and the escape of the water forward and backward was disregarded, which, however, is permissible as long as the impact length is materially greater (at least $1\frac{1}{2}$ times) than the width of the impact. Even the case of appreciably smaller impact length than width (perhaps smaller than 2/3 the width) can be suitably explained by these relations. But if width and length are nearly alike the exact force of impact and, particularly, the pressure distribution, become difficult to define, although Pabst supplies some pertinent data in his report on the vibration of rectangular plates in fluids.

4) From other considerations, not discussed in this report, it follows that the force of impact is smaller at greater keel angles than our equations reveal. The ratio of the actual force of impact P_W to our theoretical P is approximately

$$\frac{P_W}{P} = 1 - \frac{\beta}{\pi} - 0.15\,\frac{u}{\pi} - \frac{u}{\pi}\ln\frac{1}{u} \tag{16}$$

with β = angle of slope of hull bottom at edge of impact area.

Figure 10 exhibits this P_W/P ratio for a bottom with keel angle β_o (rectilinear keel), and $\frac{1}{\beta}\frac{P_W}{P}$ as scale of the reduced impact caused by this keel. The value $\frac{P_W}{P} = \frac{A_\beta}{A_o}$ indicates the decrease in planing lift at start due to keel. Inserting the computed value for P in (11) reveals that the force of impact diminishes with $\frac{1}{\beta_o}\frac{P_W}{P}$ as the keel increases.

Unfortunately, I was unable to finish my exact calculation on the force effect of bottoms with straight keel; but I mention the fact that, for the case of constant rate of impact, the speed v on the free surface is in very simple relation to the form of the surface. If t is the time elapsed since the start of the impact, the second end point of vector vt for a point of the surface can be obtained by drawing length s of the surface from the lowest point of the bottom, figured from the end point of the spray, parallel to the tangent (Fig. 11). In this manner the form of the free surface and the velocity field are determinable, as illustrated in Figure 12, where the velocities on the surface are shown graphically.

The effect of finite keel in contrast to our assumedly infinitely small keel may be summarized to the extent that our equations (6) to (12) are applicable up to 0.5 (about 30°) keel angle. In more accurate calculations the effect of finite keel is determined by means of (16) if the keel angles are 10° or over.

5) In hulls with very flat keel the elasticity is of great importance, while in those with sharp keel the effect is slight.

For the local deflection of the hull bottom between the side members, the following holds true. If the mean curvature of the bottom is small compared to the height h of the keel (Fig. 13), the temporary mean value of force of impact P is not changed (or at least, negligibly) by the elasticity. The force of impact is somewhat less at incipience of the immersion but slightly greater at the termination of the emersion, which is obviously true, because a flexible flat bottom is comparable to a rigid cambered bottom (Fig. 13).

Moreover the pressure maximum at the edge of the impact area in an elastic bottom is approximately like the dynamic pressure of a fluid with speed dc/dt (Compare (8)), with dc/dt as the rate of expansion of the impact area for the elastic bottom. This pressure maximum likewise is lower at the beginning of the blow but higher at the end than for a rigid bottom.

*In that case the value μ in (11) must be raised in ratio to P/P_W, and the thus computed P value decreased in ratio to P_W/P.

I would like to reiterate several statements on seaway made last year in one of my reports.* The rate of penetration V_0 of the hull bottom into the wave (Fig. 14) is

$$V_0 = v \varphi + (v - w) \kappa + w_e \upsilon \qquad (17)$$

where v = flight speed, φ = angle of inclination of flight path relative to the air, w = wind velocity, κ = angle of attack of hull bottom with respect to the horizontal, $w_e = \sqrt{\frac{gL}{2\pi}}$ = wave velocity (L = wave length) and υ = slope of the wave (Fig. 14). In a fresh breeze the wave velocity is lower than that of the wind (perhaps half as great), but approaches it when the wind is steady. The angle of slope υ of the wave also depends on the wind velocity. In a fresh breeze, when the waves foam, it amounts to about 30°.

The effect of the square of speed of blow on the force of impact is shown in Table I for various wave lengths with the height of drop $H = V_0^2/2g$ proportionate to this square. In conformity with it the specific impact pressure seems to be affected in a greater degree by the ratio of wind to wave velocity and form of wave than by its size.

TABLE I

Height of drop $H = \frac{V_0^2}{2\,g}$ in m

$v = 30$ m/s, $\varphi = 1:20$, $\kappa = 1:10$, $\upsilon = \frac{4 \times \text{wave height}}{\text{wave length}}$

No seaway	H = 1.0		
Seaway Wave velocity Wave length	 W_e = 4 l = 10	 6 22.5	 8 40
Fresh breeze ($W = 2\ W_e$; $\upsilon = 1:1.75$) ($W = 2\ W_e$; $\upsilon = 1:3$)	 H = 1.8 H = 1.25	 2.2 1.4	 2.8 1.55
Steady wind ($W = W_e$; $\upsilon = 1:5$)	 H = 1.2	 1.3	 1.4
Sudden calm ($W = 0$; $\upsilon = 1:5$)	 H = 1.4	 1.6	 1.9

*Herbert Wagner, "The Mechanics of Take-Off and Landing of Seaplanes," Schiffbau, Vol. 30, No. 14.

The total force of impact depends on the ratio of size of seaplane and wave. In relatively small waves (Fig. 15) only a small portion of the bottom hammers on the crest of the wave, but this becomes larger as the waves become greater; then the force of impact P is higher.

If the waves are small compared to the seaplane, the crest of the wave strikes the fore-and-aft step more or less simultaneously; but in the reverse case the fore step touches the water first (Fig. 15) and the seaplane bounces forward and upward. The shock has lowered the speed of the aircraft, making it unable to fly in this steep position; the control surfaces are not very effective. The subsequent blow may endanger the seaplane because the pilot is unable to control it.

If the stern touches the large wave first the retardation is small because of the distance of the force from the C.G. The seaplane undergoes a forward turn and the nose of the hull bottom strikes the water at greater speed and almost simultaneously with the entire forward bottom area.

The processes at take-off prior to going on the step cannot be repeated by the given methods of calculation because of the primary role of the static lift and of the marked dependence of these forces on the time of breakaway at the step.

The planing lift A_{g_0} which decides the speed at which the boat goes on the step is for the flat keel

$$A_{g_0} = \frac{\pi\rho}{8} b^2 v^2 \kappa \qquad (18)$$

where b = width of step, v = rate of rolling, κ = angle of attack of hull bottom at step; A_{g_0} is unaffected by the size or shape of keel, but A_g decreases for sharp keels. In that case

$$A_g = A_{g_0} \frac{A_g}{A_{g_0}} \quad \text{(approximately)},$$

with A_g/A_{g_0} as given in (16), and shown in Figure 10 for rectilinear keel. Whereas a keel lowers the impact pressures quite considerably, the planing ability of the boat diminishes but slowly. Consequently, the best technical compromise lies in pronounced keeling, as every practical seaplane designer knows and, in particular, ahead of the fore step.

The planing resistance (Fig. 16) embraces the dynamic planing resistance and the frictional resistance:

$$W = W_g + R = \kappa A_g + c_f \frac{1}{2} \rho v^2 b l$$

l = length of wetted planing area. The friction coefficient $c_f = 0.074 R^{-1/5}$ is around 0.025 for a smooth bottom. When we insert this and A_g according to (18), we obtain

$$W = \frac{1}{2} \rho v^2 b^2 \left(\frac{\pi^2}{4} \kappa^2 + 0.025 \frac{l}{b}\right) \tag{19}$$

The frictional resistance is about as high as the dynamic planing resistance. Unfortunately, the formulas do not permit the safe calculation of the effect of a certain bottom shape on the planing lift by really pronounced keel (keel angle > 30°).

Suggestions for the Practical Calculation of Seaplanes

If M_r is the reduced mass of the seaplane for a definite position of the force of impact, equation (11) interprets the total force of impact as

$$P = \frac{\pi \rho V_0^2}{(1 + k l)^3} \frac{c}{u}$$

where

$$k = \frac{\pi \rho c^2}{2 M_r} .$$

The differentiation conformal to l yields the length and the maximum force of impact itself for the maximum value of the blow, that is,

$$P_{max} = 0.3 V_0^2 M_r \frac{1}{u c} \tag{20}$$

and P_{max} can be corrected with the aid of (16) also.

The respective length of impact l is

$$= \frac{M_r}{\pi \rho c^2} \tag{21}$$

Thus each width of impact area c has a maximum value of force of impact, and to find the maximum of the maximum forces we write the smallest possible values of $u c$ in (20).

Still it remains to be seen whether the force of impact of the bottom ahead or aft of the chosen position retains the half-length l according to (21), or whether this length of impact area is possible, according to the wave length for which the seaplane is still to be seaworthy. Otherwise we must define c from (21) for the actually existing l conformably to this value u c and then compute P_{max} according to (20).

If u c assumes its minimum at $c = 0$, and another at $c = b/2$, we obtain two maximum values of P_{max} - one for great length but small width of impact area, and one for small length but great width of impact area. This simple computation yields very reliable data for the static calculation of seaplanes.

Now we illustrate the various effects on Figure 16. The boat weighs 15 tons and has an inertia radius of 3.75 m the calculation was made for two different step widths $b = 2$ m and $b = 3$ m. The forces in Figures 17 and 18 apply at V_o = m/s rate of impact. Figure 17 shows the forces of impact for a boat with rectilinear keel ($\beta_o = 0.25$, $V_o = 5$ m/s, $G = 15$ tons, example I) with $b = 2$ m and $b = 3$ m over the length of the boat to the first step. The heavy curves P_{max} ($l = 3$ m) and P_{max} ($l = 1.5$ m) apply to both step widths. The P_{max} curves indicate the highest possible forces which are apt to occur at this point for the respective boat. Fig. 18 exhibits the impact forces for the plotted rib forms (example II) for $b = 2$ m (bottom) and $b = 3$ m (top). The P_{max} curves reveal the highest possible impact forces at each point. The highest P_{max} (for $b = 3$ m) occurs near the step for small impact length and great impact width, contrary to the other examples; the impact forces for great impact lengths predominate at the forebody of the boat.

Example I.- The keel is rectilinear, its angle is $\beta_o = 0.25$ and constant over the whole length of the forebody of the boat. The $l = 3$ m width of Figure 17 is just about the possible impact length in waves corresponding to wind velocity 4. For $l = 1.5$ m and $l = 3$ m the greatest impact force is reached before half-width c of the impact area has arrived at the edge of the narrow boat ($c < 1$ m), so that even the wider boat is not subjected to greater blows. The impact force on the wider boat does not exceed that on narrower boats till the impact area has become considerably smaller. However, it should be remembered that, given equal starting characteristics, the wide boat can presumably keel somewhat more than the small one, in which case its impact forces are less than for the narrow boat.

Whereas u c is minimum for $c = 0$ in the rectilinear keel, the highest possible force occurs by greatest possible impact length in our chosen form of bottom.* Even this greatest possible force exceeds that of the narrow boat at very few points and then only a trifle.

The highest of all these impact forces for a boat with constant keel angle is

$$P_{max} = 0.835 \frac{V_o^2}{\beta_o} \sqrt{\rho \, l_{max} \, M_r}$$

with l_{max} as greatest possible length of impact area conformal to the chosen seaway (at the highest equal to the length of the bottom ahead of the fore-step). This force, whose magnitude is unaffected by the width of the boat, is applied at $\frac{1}{2} l_{max}$ ahead of the fore-step. The reduced mass M_r must be computed for this point also. This formula can equally be applied with a mean β_o value to variable keel. In our example this maximum force of impact amounted to about 85 tons, or about 5.7 times the weight of the seaplane.

The impact forces decrease slowly forward - further forward decrease being prevented by the fact that the position of the aircraft relative to the waves does not facilitate as extended impact lengths forward as on the step. The stresses on the boat can be lowered by enlarging the keel toward the bow. This effect as well as that of other impact speeds V_o can be taken into account by changing the forces illustrated in Figure 17 at each point to conform to the relevant V_o^2/β_o (Compare (11) or (20)).

<u>Example II</u>.- The equation $y = x \, [0.35 - 0.4 \, (x/b_s)^2]$ applies to the whole length of the bottom ahead of the fore-step. The width of the step b_s is again 2 m and 3 m, respectively. The resulting rib forms are geometrically similar for both widths (Fig. 18). The height of the lateral bottom edge above the keel is respectively, 0.25 and 0.375 m, as in the preceding example.

In the small boat the greatest impact forces occur when the width of the impact area has reached the side edge of the bottom. They appear at impact lengths of from 3 to 4 m, although they are not much lower when these lengths are still smaller (Compare the forces for 1.5 m and 3 m impact lengths in Figure 18).

A bottom of this kind is subjected to enormous forces even in small waves, while no large forces set in until great impact

*The eccentric effect of the impact force turns the boat, making immersion speed V and acceleration dV/dt variable over the length of the impact area. I checked this effect and found that it does not perceptibly influence the magnitude of the maximum impact force aside from a slight rearward shift of the position, which, however, is hardly noticeable on the P_{max} diagram.

lengths are approached, if the keel is rectilinear. The greatest possible force of impact is essentially higher on the narrow boat than on the straight keel bottom; it corresponds to 7.1 times the weight of the aircraft. But since this force is approximately concentric with the center of gravity, the rotary accelerations are not excessive and the stresses are comparatively low. The rapid forward decrease in force of impact is attributable to the increase in the middle keel of the forebody.

The wide boat is subjected to decidedly lower maximum impact pressures than the narrow one; it is already sufficiently decelerated when the lateral flat portion of the bottom surface hits.

The maximum impact pressures occur now by very short impact lengths, a strong seaway manifests itself merely in accelerated impact speed V_0 and in a longer duration of the forces. In very long waves the P_{max} values may become slightly higher than with very short impact lengths (Fig. 18).

In conclusion, I recapitulate the comparison of the wide and the narrow boat. Given equal take-off capability, a wide boat presumably is subjected to lower impact pressures than a narrow one, so the first blow does not produce any exorbitant stresses on the aircraft, but the actual bottom construction becomes heavier because of the greater width. Another vital factor is the duration of the forces on the wide boat, lasting till they reach the lateral bottom edge, so that the total retardation is greater for the wide than for the narrow boat. In consequence a wide boat is more liable to be tossed upward by the wave and by its own forward speed than a narrow boat, which may even continue to nose under after the blow.

The speed at which the blow tosses the boat upward is $V_0 - V$; since its previous speed $v\,\varphi$ was downward (Fig. 14), the imparted upward speed predominates when $V_0 - V > v\,\varphi$. Inserting (See equations (12) and (17)) the previously computed values, we obtain as condition that the boat be <u>not</u> flung upward:

$$\left(1 - \frac{w}{v}\right) \kappa + \frac{w_e}{v}\, \upsilon < \frac{\varphi}{\mu} ;$$

so, for example:

$$\frac{w}{v} = 0.3; \quad \kappa = 1:5; \quad \frac{w_e}{v} = 0.2; \quad \upsilon = 0.4; \quad \varphi = 0.1$$

$$c^2 < \frac{1}{3.5} \times \frac{G}{1000} \times \frac{1}{l}.$$

Thus, with $G = 15,000$ kg and $l = 3$ m, the boat of 2-meter width would, after the blow (the impact area has reached the lateral edge of the bottom), not yet be flung upward, but the boat of 3-meter width would have attained considerable downward speed, which, as a matter of fact, is the most dangerous part of the landing.

Finally, I wish to state that my calculations purport the possibility of designing seaworthy aircraft by correct choice of bottom form.

Translation by J. Vanier,
National Advisory Committee
for Aeronautics.

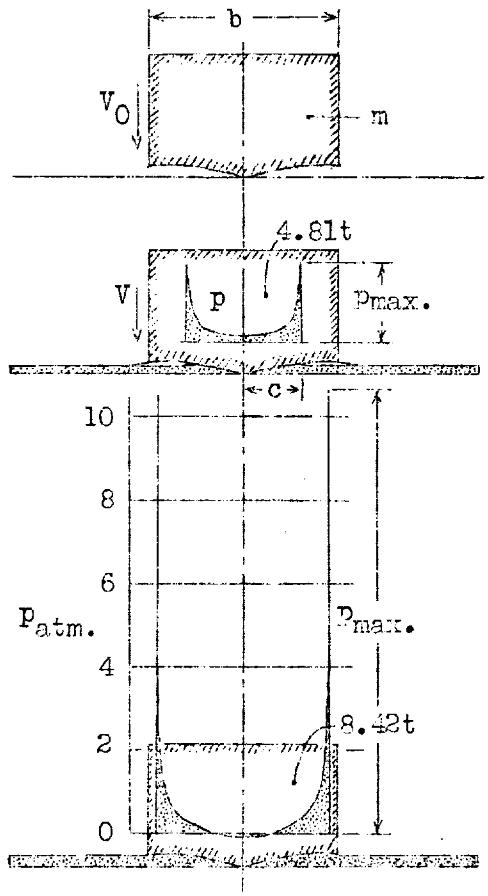


Fig. 1 The pressure scale is valid for b=2m, V_0 = 3m/s, weight per unit length 1100 kg, hence m = 110

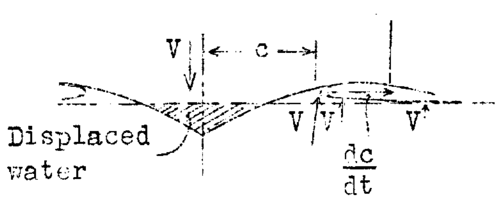
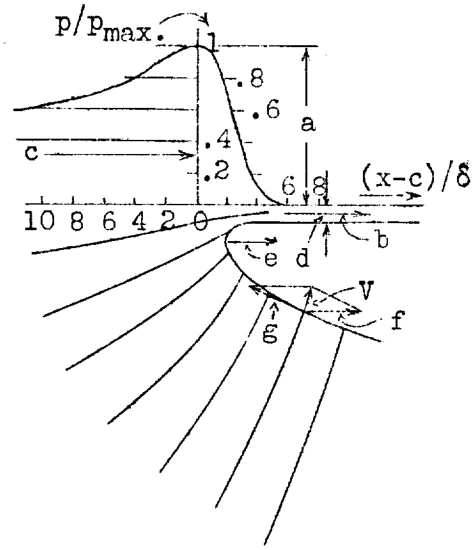


Fig. 2



a; $p_{max.} = \frac{1}{2} \rho \left[\frac{dc}{dt}\right]^2$

b; $2\frac{dc}{dt}$

d; $\delta = \frac{\pi}{8} cu^2$

e; $\frac{dc}{dt} = \frac{V}{u}$

f; $\frac{dc}{dt}$

g; $\frac{dc}{dt}$

Fig. 3 Flow and pressure distribution in region of incipient spray.

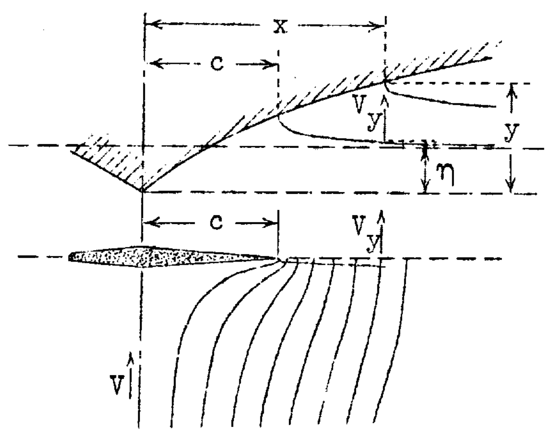


Fig. 4 Field of velocity on impact.

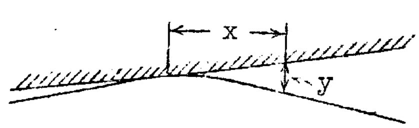


Fig. 5 Blow on impact of a flat bottom.

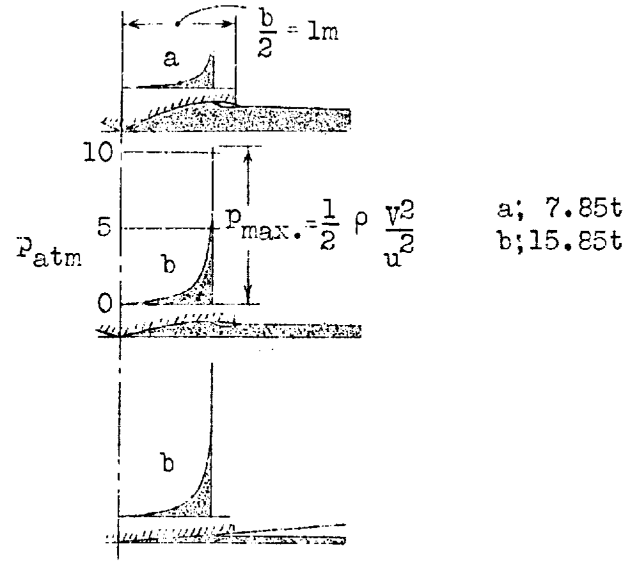


Fig. 6 Pressure scale for b=2m, V_0=5m/sec., weight per unit length 1100 kg

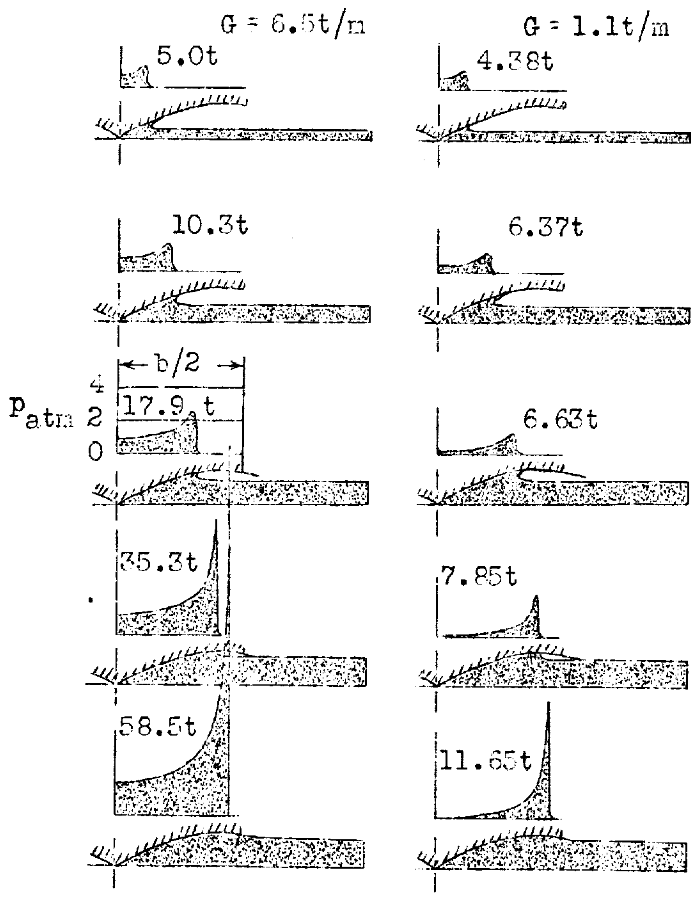


Fig. 7 The pressure scale, (center,left) is for b = 2m, V_0=5m/sec., weight per unit length 6500 kg; at right 1100 kg. Negative pressure in lower right figure.

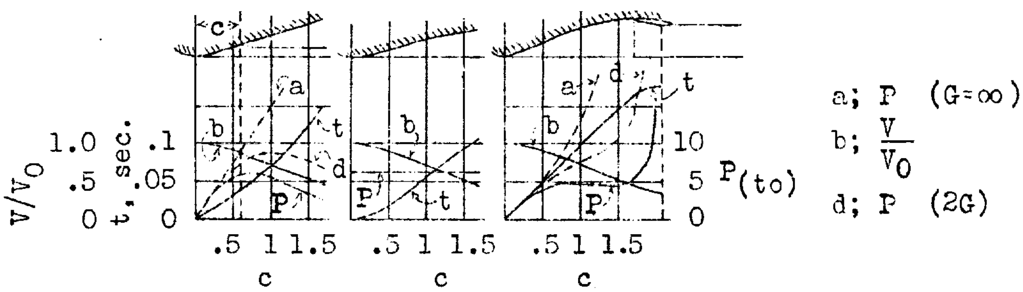


Fig. 8

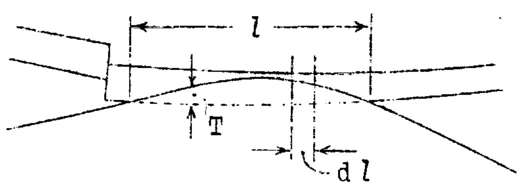


Fig. 9 Impact of keeled bottom.

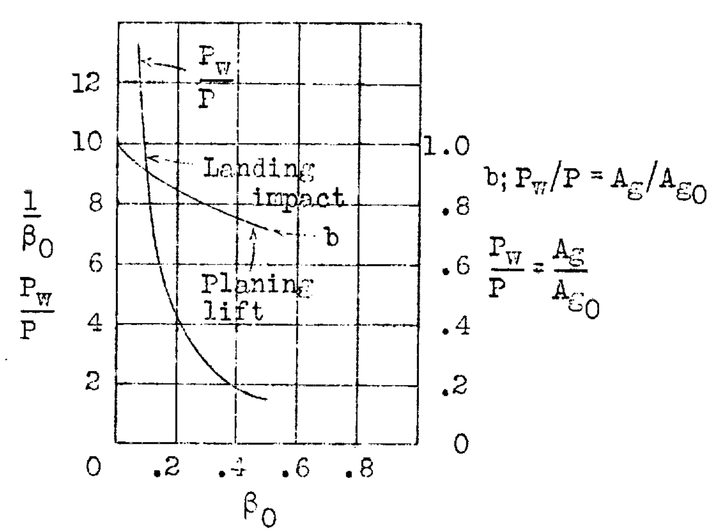


Fig. 10

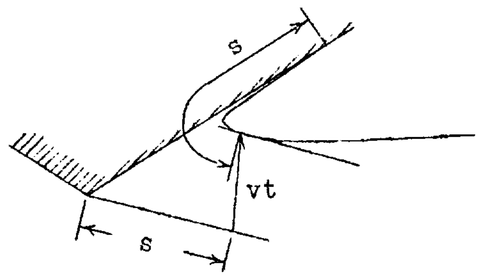
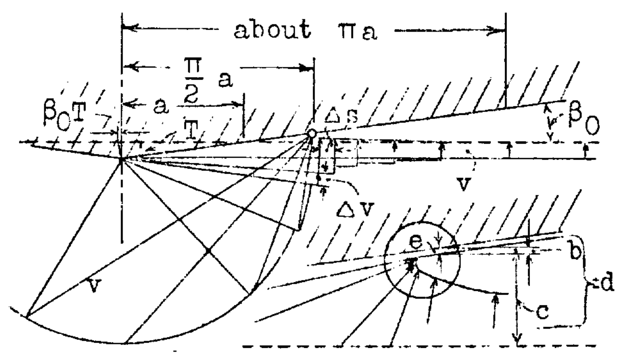


Fig. 11



b; $\frac{1}{4}\,a\beta_0$

c; $\left[\frac{\pi}{2} - 1\right]a\beta_0$

d; $\frac{1/4}{\pi/2-1}\,\beta_0$

e; $\frac{1}{4}\beta_0\,T$

Fig. 12

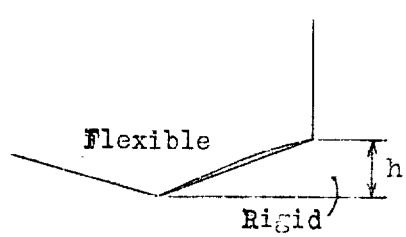


Fig. 13

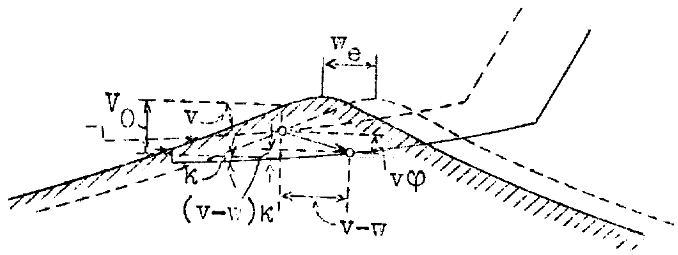


Fig. 14

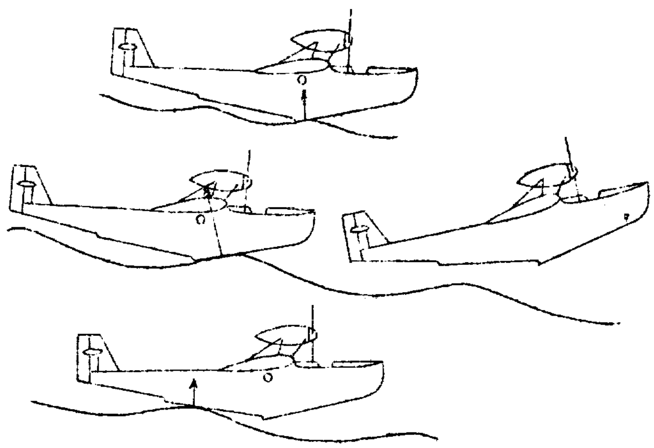


Fig. 15 Effect of size of wave on extent of impact area and on hull position after blow.

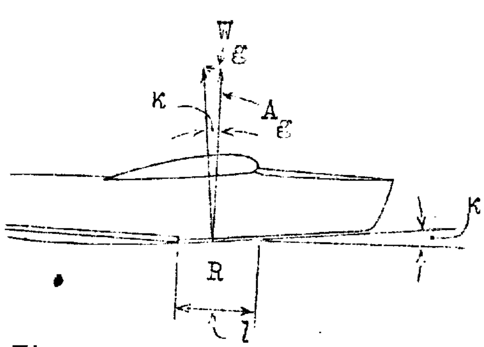


Fig. 16

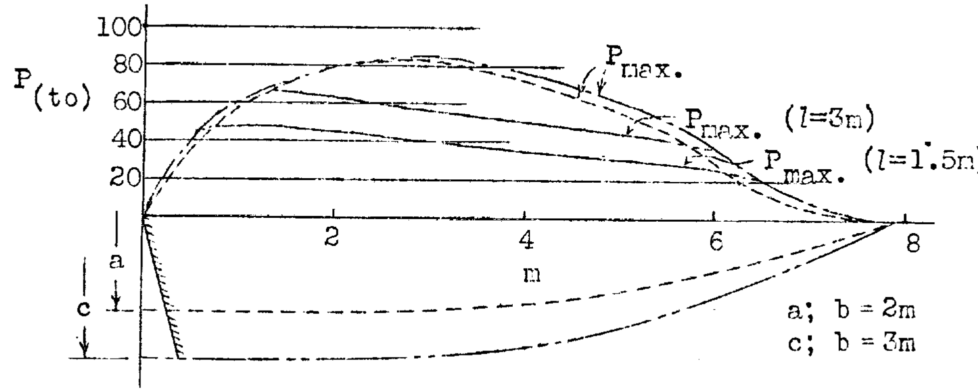

100
80
60
40
20
P(to)
Pmax.
Pmax. (l=3m)
Pmax. (l=1.5m)
2
4
6
8
m
a
c
a; b = 2m
c; b = 3m

Fig. 17

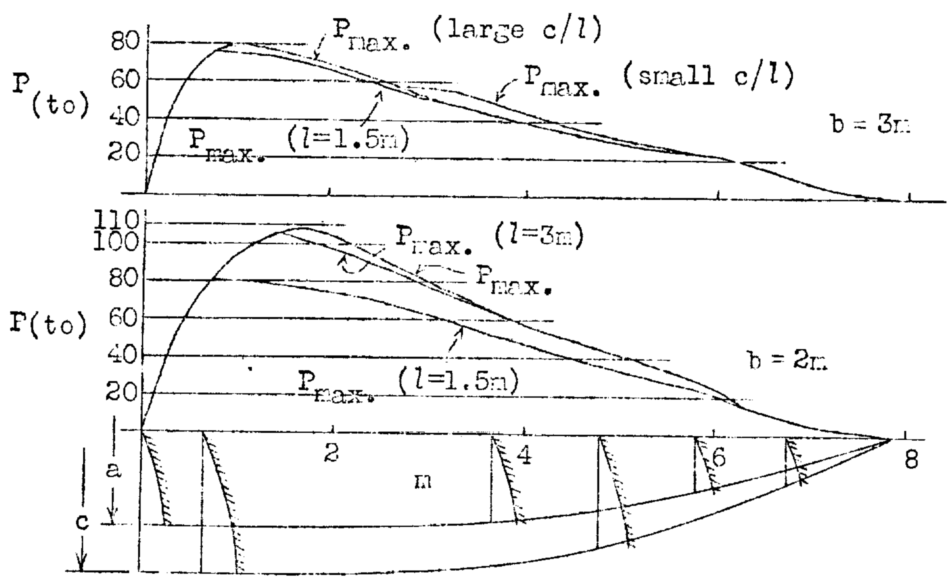

80
60
40
20
P(to)
Pmax. (large c/l)
Pmax. (small c/l)
Pmax. (l=1.5m)
b = 3m
110
100
80
60
40
20
P(to)
Pmax. (l=3m)
Pmax.
Pmax. (l=1.5m)
b = 2m
2
4
6
8
m
a
c

Fig. 18