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No. 619  
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DETERMINATION OF RESISTANCE AND TRIMMING MOMENT  
OF PLANING WATER CRAFT

By P. Schröder

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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM NO. 619

DETERMINATION OF RESISTANCE AND TRIMMING MOMENT  
OF PLANING WATER CRAFT\*

By P. Schröder

I. Theory

Notation

$W$  = resistance  
 $M$  = trimming moment  
 $A$  = lift

} of water craft at  
planing speed  $v$ .

$\epsilon = \frac{W}{A}$  = lift-drag ratio.

$\mu_0 = \frac{M}{A}$  = trimming moment with respect to lift.

In conformity with previous reports\*,  $\epsilon = \text{constant}$  and  
 $\mu_0 = \text{constant}$  when  $\kappa = \frac{v^2}{A} = \text{constant}$ .

Thus it becomes possible to interpret the resistance and the trimming moment for any loading of a planing aircraft when these values are given for one load. This application of the new theory forms the basis of the present paper.

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\*"Über die Bestimmung von Widerstand und Trimmoment bei gleitenden Wasserfahrzeugen," Zeitschrift für Flugtechnik und Motorluftschiffahrt, Vol. 21, No. 22, November 28, 1930.

\*\*P. Schröder, "Take-Off of Seaplanes Based on a New Theory of Reduction in Hydrodynamics," Zeitschrift für Flugtechnik und Motorluftschiffahrt, January 14, 1931 (to be issued as N.A.C.A. Technical Memorandum No. 621).

P. Schröder, "A Reduction Theory for Planing Water Aircraft and Its Experimental Verification," Bericht der Hamburgischen Schiffbauversuchsanstalt, July 14, 1930.

## Derivation of Various Conversion Formulas

Case 1 - for different loads of hydrovanes

Given:  $W_1$  and  $M_1$  for a load  $A_1 = \text{constant}$ ;

Find:  $W_2$  and  $M_2$  for a load  $A_2 = \text{constant}$ .

The equation defining the corresponding speeds now becomes:

$$\frac{v_1^2}{A_1} = \frac{v_2^2}{A_2} \quad (1)$$

or

$$v_2 = v_1 \sqrt{\frac{A_2}{A_1}} \quad (2)$$

By virtue of  $W = c A$  and  $M = \mu_0 A$ , these speeds are

$$\frac{W_1}{A_1} = \frac{W_2}{A_2} = c \quad \text{and} \quad \frac{M_1}{A_1} = \frac{M_2}{A_2} = \mu_0$$

and  $W_2$  and  $M_2$  are now expressed as

$$W_2 = W_1 \frac{A_2}{A_1} \quad (3)$$

and

$$M_2 = M_1 \frac{A_2}{A_1} \quad (4)$$

These three equations (2), (3), and (4), now assume a general significance, inasmuch as they retain their validity even when  $A_1$  and  $A_2$  are arbitrary speed functions.

Case 2 - Conversion of resistance  
and trimming moment curves of a hydrovane  
for application as seaplane float

Given:  $W_1$  and  $M_1$  for a load  $A_1 = \text{constant}$ ,

Find:  $W_2$  and  $M_2$  for a load  $A_2 = A(v)$ .

This case of  $A$  being a prescribed function of the speed is always applicable when the pertinent vane is to be used as seaplane float. If  $A_1$  is the gross weight of the aircraft, the load formula by fixed trim is:

$$A_2 = A_1 \left( 1 - \frac{v_2^2}{v_s^2} \right) \quad (5)$$

where  $v_s$  = getaway speed. Now we write the quotient  $A_2/A_1$  in (2), resolve it according to  $v_2$ , and obtain the relation for computing the corresponding speeds

$$v_2 = \frac{v_1}{\sqrt{1 + \left(\frac{v_1}{v_s}\right)^2}} \quad (6)$$

Then we insert the quotient  $A_2/A_1$  of (5) in (3) and (4), and use (6) for eliminating  $v_2$  which yield the formulas for  $W_2$  and  $M_2$  as

$$W_2 = \frac{W_1}{1 + \left(\frac{v_1}{v_s}\right)^2} \quad (7)$$

$$M_2 = \frac{M_1}{1 + \left(\frac{v_1}{v_s}\right)^2} \quad (8)$$

### C a s e 3 -

Given:  $W_1$  and  $M_1$  for a load  $A_1 = A_1 (v)$ ,

Find:  $W_2$  and  $M_2$  for a load  $A_2 = A_2 (v)$ .

This case has practical significance when resistance and moment of a seaplane flotation gear have been measured for a certain wing and it is subsequently desired to apply these values to some other gross weight and other wing. In addition, it contains the possibility of embodying the effect of the wind in take-off investigations of seaplanes.

a) Conversion of the take-off resistance curves and trimming moment curves of a seaplane flotation gear to different wings and gross weights:

We assume  $W_1$  and  $M_1$  measured for

$$A_1 = G_1 \left( 1 - \frac{v_1^2}{v_{s1}^2} \right) \quad (9)$$

where  $G_1$  = gross weight of aircraft upon which the measurement was based, and  $v_{s1}$  = relevant getaway speed. The new gross

weight is to be  $G_2$  and the getaway speed with the new wing is to be  $v_{s2}$ . Then the loading

$$A_2 = G_2 \left( 1 - \frac{v_2^2}{v_{s2}^2} \right) \text{ is requested} \quad (10)$$

A calculation analogical to the preceding case yields with abbreviation

$$N^2 = \frac{G_1}{G_2} \left[ 1 - \left( \frac{v_1}{v_{s1}} \right)^2 + \left( \frac{v_1}{v_{s2}} \right)^2 \right] \quad (11)$$

the following results:

$$v_2 = v_1 N^{-1} \quad (12)$$

$$W_2 = W_1 N^{-2} \quad (13)$$

$$M_2 = M_1 N^{-2} \quad (14)$$

β) Effect of wind in take-off studies of seaplanes:

Assume as measured,  $W_1$  and  $M_1$  for take-off with zero wind by lift

$$A_1 = G \left( 1 - \frac{v_1^2}{v_s^2} \right) \quad (15)$$

where  $G$  = gross weight and  $v_s$  = getaway speed in zero wind.  $W_2$  and  $M_2$  are to be computed for  $w$  m/s head wind. Because of the greater unloading by the wings the loading of the flotation gear remains as

$$A_2 = G \left[ 1 - \left( \frac{v_2 + w}{v_s} \right)^2 \right] \quad (16)$$

The calculation of  $v_2$  reveals

$$v_2 = - \left( \frac{v_1}{v_s} \right)^2 w + v_1 \sqrt{1 - \left( \frac{w}{v_s} \right)^2 \left[ 1 - \left( \frac{v_1}{v_s} \right)^2 \right]} \quad (17)$$

and the elimination of  $v_2$  sets forth the quotient  $A_2/A_1$  as:

$$\frac{A_2}{A_1} = 1 + \frac{v_1^2 w^2}{v_s^4} - \frac{2v_1 w}{v_s^2} \sqrt{1 - \left( \frac{w}{v_s} \right)^2 \left[ 1 - \left( \frac{v_1}{v_s} \right)^2 \right]} \quad (18)$$

which, substituted in (3) and (4), supplies the desired values for  $W_2$  and  $M_2$ .

### Graphic Method

The preceding analytical problems can equally be solved by graphical means. Given curve  $W_1$  or  $M_1$  under the secondary assumption that  $A_1 = A_1(v)$ , find  $W_2$  or  $M_2$ , respectively, for the assumption  $A_2 = A_2(v)$ . Equations (2), (3), and (4) may be written as

$$\frac{W_2}{W_1} = \frac{M_2}{M_1} = \frac{A_2}{A_1} = \frac{v_2^2}{v_1^2}$$

on which the following method can be based:

Plot  $A_1$  and  $A_2$ , as well as  $W_1$  and  $M_1$  over  $v^2$ . For the arbitrarily chosen speed  $v_1$  on the  $W_1$ ,  $M_1$ , and  $A_1$  curves, we have point  $P_1$ , which we connect with the origin of the coordinates  $O$ . The connecting line intersects the prescribed lift  $A_2$  in point  $P_2$ . The ratio of the distances  $OP_2$  to  $OP_1$  is the same in all three diagrams, namely,  $W/v^2$ ,  $M/v^2$ , and  $A/v^2$ , thus revealing  $P_2$  as the point of the desired resistance and moment curve. This representation yields  $W_2$  and  $M_2$  in simple fashion for any stage of the prescribed lift.

## II. Examples of Application

The examples have been selected so as to allow the comparison of the mathematical figures with the test data. This applies to all but the last example where the lack of suitable data makes this impossible.

The solutions are given in graphical form. The points defined by calculation and graphic construction, respectively, are indicated by small circles in the first four examples. The pertinent curve in question was included for comparison in conformity with measurements made prior to the development of the conversion method.

### 1. Example - Conversion of the resistance of a hydrovane:

Given resistance curve  $W_1$  (Fig. 1) of a 1:8 scale model of a flat-bottom hydrovane (designed by Engineer Ellinghausen, Bremen), pertaining to loading  $A_1 = 6.5$  tons;

Find the resistance curve  $W_2$  for loading  $A_2 = 5.5$  tons. The lines emanating from origin  $O$  define the corresponding speeds. Each intersection point  $P_1$  of such a line with curve  $W_1$  has a relevant point  $P_2$  on curve  $W_2$ , so that

$$OP_2 : OP_1 = 5.5 : 6.5 = 0.847.$$

This defines  $W_2$ . The portion of the  $W_2$  curve from 2.5 to 4.5 m/s was measured direct (Test No. 2681 of the Hamburg seaplane channel laboratory, January, 1928).

2. Example - Compute the resistance curve of twin floats:

Given resistance curve  $W_1$  (Fig. 3) of a 1 : 6 scale model of twin floats for a seaplane with constant loading  $A_1 = 2920$  kg;

Find the resistance curve for loading

$$A_2 = A_1 \left( 1 - \frac{v^2}{v_s^2} \right)$$

with 100 km/h getaway speed. For the model we obtain

$$A_1 = 2920 : 6^3 = 13.52 \text{ kg}$$

$$v_s = \frac{100}{3.6 \times \sqrt{6}} = 11.34 \text{ m/s}$$

Figure 2 exhibits the construction of the lift ratio  $A_2/A_1$ . The formula  $OP_2 : OP_1 = A_2 : A_1$  reveals a point  $P_2$  of the desired resistance curve on each line emanating from  $O$ . The plotted curve  $W_2$  was measured direct. (Report Jf 39/2 of the D.V.L. (Deutsche Versuchsanstalt für Luftfahrt), January 21, 1929, page 39).

3. Example - Compute the trimming moments of twin floats:

Given trimming moment curve  $M_1$  (Fig. 5) of a 1 : 6 scale model of the twin floats for a seaplane with constant loading  $A_1 = 2400$  kg;

Find the moment curve for the same trim for loading  $A_2$  of the floats:

$$A_2 = A_1 \left( 1 - \frac{v^2}{v_s^2} \right)$$

for a seaplane with 84 km/h getaway speed. We find:

$$A_1 = 2400 : 6^3 = 11.12 \text{ kg,}$$

$$v_s = \frac{84}{3.6 \sqrt{6}} = 9.53 \text{ m/s}$$

Figure 4 represents the lift ratio  $A_2/A_1$ . The points  $P_2$ , which in the moment diagram (Fig. 5) reveal the desired moment  $M_2$ , are defined by

$$OP_2 : OP_1 = A_2 : A_1.$$

The  $M_2$  curve was measured direct. (Report Jf 63/1 of the D.V.L., June 12, 1929, pages 38 and 39 for 5° trim run).

#### 4. Example - Conversion of resistance curve of an aircraft to a different gross weight:

Given resistance curve  $W_1$  of a 1 : 6 scale model of twin floats for a prescribed loading

$$A_1 = G_1 \left( 1 - \frac{v^2}{v_s^2} \right).$$

The gross weight is  $G_1' = 2400 \text{ kg}$  and the getaway speed is 84 km/h, so that, as in the preceding example,  $G_1 = 11.12 \text{ kg}$ , and  $v_s = 9.53 \text{ m/s}$ .

Find the resistance curve for the same twin floats with a smaller wing and a lower gross weight, that is,

$$G_2' = 0.75 G_1' = 1800 \text{ kg}$$

by identical wing loading.

The latter implies that  $v_s$  does not change. The new lift is

$$A_2 = G_2 \left( 1 - \frac{v^2}{v_s^2} \right),$$

where  $G_2 = 1800 : 6^3 = 8.34 \text{ kg}$ , as illustrated in Figure 6 for  $A_2/A_1$ . The small circles on curve  $W_2$  of Figure 7 again conform to the condition

$$OP_2 : OR_1 = A_2 : A_1.$$

The part curves shown are from direct measurements (Report Jf



63/1 of the D.V.L., June 12, 1929, page 36, and Report Jf 63/1 of the D.V.L., June 14, 1929, page 30, 5° trim run).

5. Example - Including the effect of the wind in take-off studies of seaplanes:

Given resistance curve  $W_1$  (Fig. 9) of twin floats for take-off with no wind;

Find therefrom the resistance curve for take-off at  $w_2 = 5$  m/s, and  $w_3 = 10$  m/s head wind.

Figure 8 contains the prescribed loading  $A_1$  for  $w_1 = 0$  and the construction of the lift ratios  $A_2/A_1$  and  $A_3/A_1$ . Figure 9 reveals the desired resistance  $W_2$  and  $W_3$ . The twin floats selected produced unfavorable resistances at high speeds, and the lowered resistance, owing to the wind, is particularly noticeable.

Translation by J. Vanier,  
National Advisory Committee  
for Aeronautics.

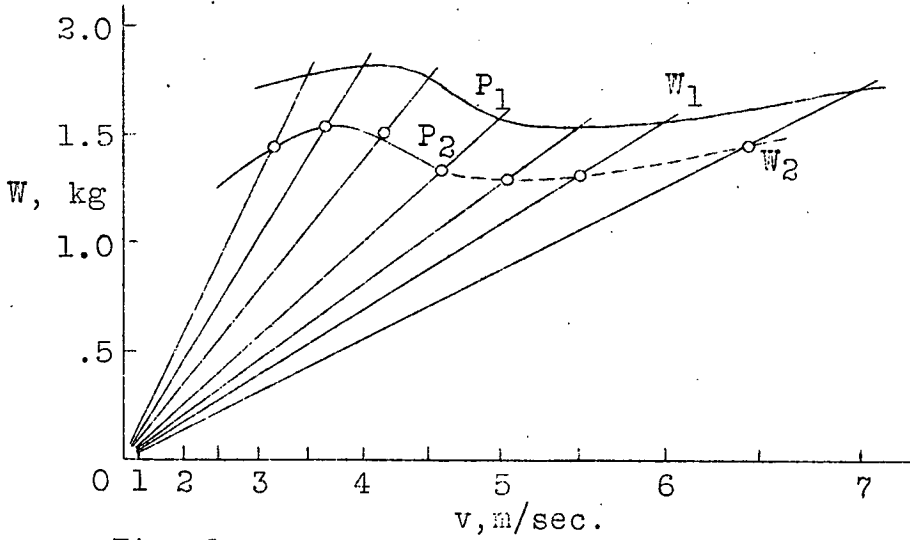


Fig. 1

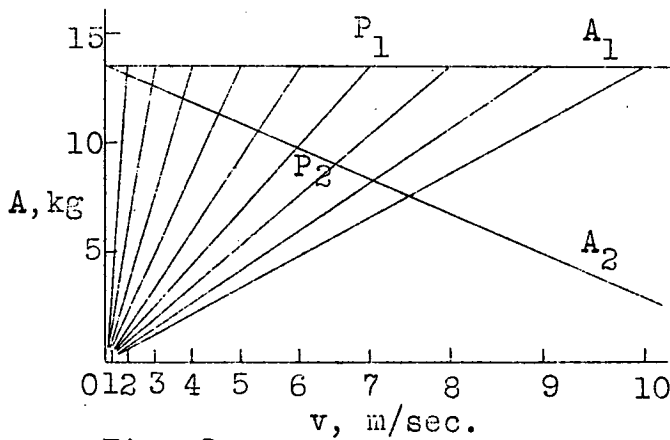


Fig. 2

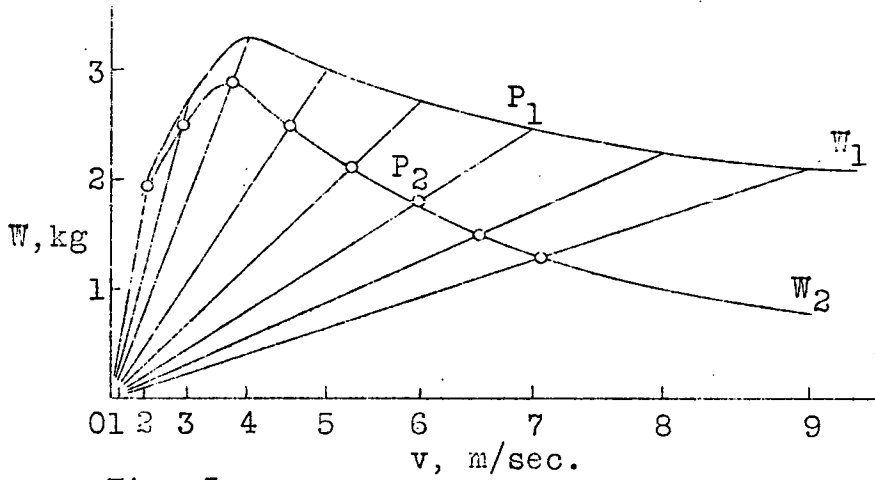


Fig. 3

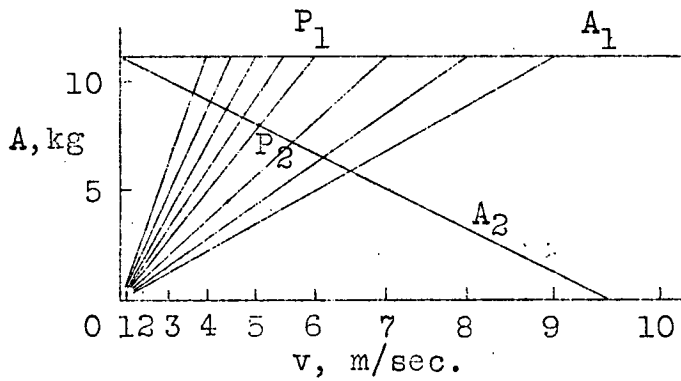


Fig. 4

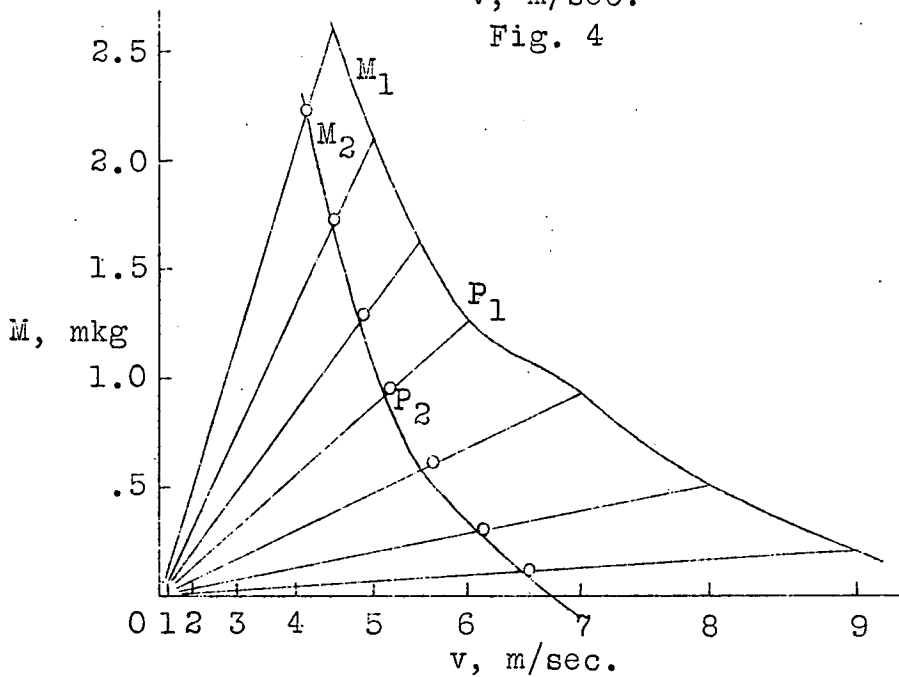


Fig. 5

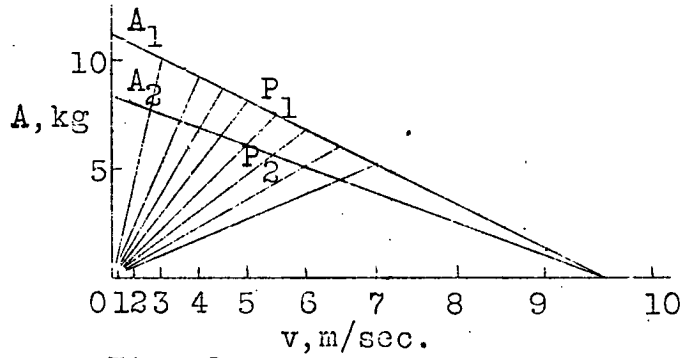


Fig. 6

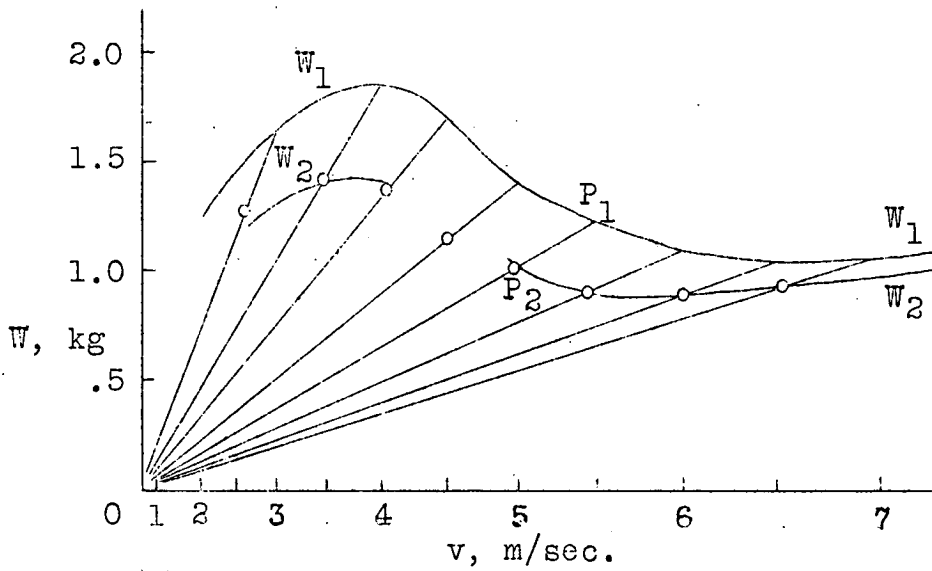


Fig. 7

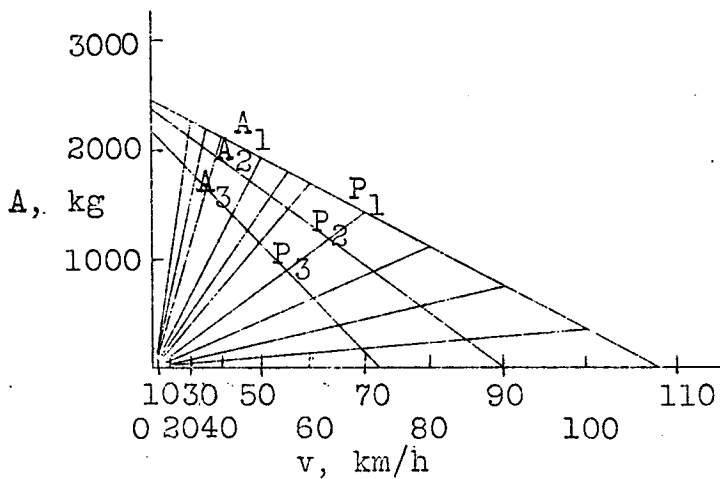


Fig. 8

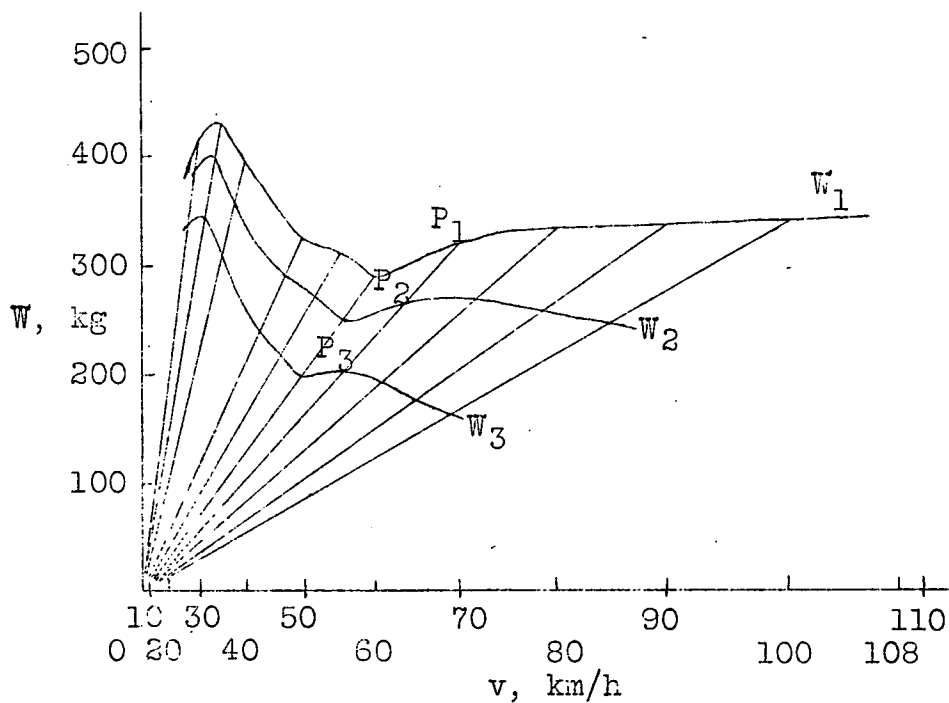


Fig. 9