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EXPERIMENTAL RESEARCH ON THE FRICTION OF PIVOTS

By A. Jaquerod, L. Defossez, and H. Mügeli

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Introduction

Even though the friction between solids plays a fundamental role in natural phenomena as well as in applied mechanics, its peculiarities are not yet known sufficiently well, and one is reduced in most cases to the use of laws which represent only a very rough approximation.

In horology the problem is of the greatest importance; one is limited, however, to the application of the laws of Coulomb which, as we well know, do not at all correspond to reality.

We have undertaken a research on the friction of pivots, a particular case of sliding friction, the results of which we shall publish later. We wish, at the present time, to introduce the subject by briefly reviewing our knowledge in a very complex field, where experimentation is particularly delicate by reason of the difficulty which one finds in attempting to reproduce the same conditions identically. This difficulty is without doubt one of the principal reasons for the neglect in which the works

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on rational mechanics still leave the question of friction. In this rapid review, we do not pretend to be complete; this would be superfluous since most of the recent researches, scattered in the technical periodicals, treat of particular cases which are hardly realized in horology.

It is necessary from the beginning to distinguish carefully between immediate or dry friction, which is produced when the solids in question are directly in contact, and intermediate friction characterized by the presence of a lubricant. The observed phenomena are at moderate speeds very different in the two cases. It is a well-known fact that lubricants diminish friction but, what is not so well known, is that they modify completely the laws which apply to the phenomena, notably the variation of friction with speed.

In immediate or dry friction, the theory of which is yet to be completed, the roughness of the surfaces in frictional contact play a considerable role. Every one knows that the degree of polish influences the friction enormously; it is supposed that the small irregularities of the surfaces are mutually entangled, tending thus to oppose any sliding. Even on a surface which appears to us to be perfectly polished, irregularities with microscopic dimensions, of such a sort that their role remains effective, still persist.

Further, the molecular forces of cohesion are also effective at the points where the two bodies are in very close contact.

If one interposes a lubricant, the two surfaces moving with respect to one another tend, each one, to entrain the fluid which reacts by reason of its vicosity. One may also conceive that at the points where the lubricating layer is exceedingly thin an immediate or dry friction is again produced for the moment which thus complicates matters. Lastly, the least traces of abrasives, or of impurities which accidentally soil the surfaces, modify the conditions. In consequence one cannot in general define in an explicit manner the actual nature of the forces of friction, and one is limited to the measurement of their total effects.

One understands thus why it is so difficult to always obtain the same experimental conditions identically.

In what follows we shall consider only the case of dynamic friction, that is to say, the friction which takes place between two surfaces in relative motion. When this relative velocity is zero, one has to do with static friction, which ceases then to be determinate, and which simply comprises a value between two limits, the one zero and the other which is only acquired at the instant when sliding begins.

The force necessary to place an immobile body in motion is called the starting friction; in general, it is admitted to be a little greater than the dynamic friction.

I. Immediate or Dry Friction

This follows, at least approximately, the laws established in 1781 by Coulomb (Reference 1) and confirmed in 1833 by Morin, (Reference 2), but these experiments were made in the very limited range of velocities from 40 to 350 centimeters per second.

These laws may be stated as follows:

1. The friction F is proportional to the normal force N which presses the bodies in frictional contact against each other.
2. It is independent of the areas of the surfaces in contact.
3. It is independent of the relative velocity of the bodies in frictional contact.

These three laws are condensed in the expression

$$F = f\,N \tag{1}$$

where f is a constant coefficient which depends on the nature of the surfaces in contact. It is called the coefficient of friction.

If one considers a body sliding on an inclined plane making an angle α with the horizontal, in such a manner that the motion is uniform, it can be seen from Figure 1 that

$$\tan\alpha = \frac{F}{N} = f = \tan\Phi \tag{2}$$

where F is the force of friction, N the normal component and Φ the angle which the resultant R makes with the normal N.

In a general way, one can say that the laws mentioned are not applicable in certain cases of translations at low speeds and under low pressures.

According to Morin, the first law is subject only to a few exceptions relative to the case where the surfaces in contact undergo too great a deformation, due to a considerable pressure.

On the contrary, the second law is not applicable except in an approximative manner, and lastly, the third law cannot be confirmed even in very constrained limits.

Let us review rapidly the recent experiments which have shown these general laws to be in default.

In 1858, Bochet (Reference 3) by measurements made at high speeds on railway brakes, found that the force of friction varied with the speed, and in particular that it diminishes when the speed increases. In order to take account of the variation of the coefficient of friction f with the speed v, Bochet proposed the relation

$$f = \frac{f_o - f_\infty}{1 + a\,v} + f_\infty \qquad (3)$$

where f_o is the coefficient of friction for zero speed, f_∞ the coefficient for a very high speed, and a, a constant depending on the units chosen.

Bochet brought out the fact that the results obtained oscillate about an average value, and that under the same experimental conditions, it was impossible to obtain always the same coefficient of friction. Instead of being located on a curve, the points fell between two limiting curves, which the author interpreted by stating: "To the variation of a variable (here the velocity) corresponds not a curve but a zone in the plane."

These results have been confirmed by other experimenters, among others by D. Galton (Reference 4) who found that friction diminishes when the speed increases and that he could represent it by an expression of the form

$$f = \frac{a}{b + v} \tag{4}$$

Further, he observed that the friction diminished with time, which he attributed to heating and to the disappearance of certain asperities.

Continuing his study, Bochet found that the second law of Coulomb is only approximately correct, and that the coefficient of friction varies with the extent of the surface of contact (so that it is therefore a function of the load per unit area) following a curve with a distinct minimum.

The American A. S. Kimball (Reference 5), working within very wide limits, completed the results mentioned, and found that at very low speeds, the coefficient of friction is in general, small. It increases quite rapidly when the speed increases, passes through a maximum, and then decreases and reaches a certain limiting value at high speeds (Fig. 2).

It is seen that in working in the neighborhood of the maximum, one should find a coefficient of friction sensibly independent of the speed; and in this neighborhood, approximately, are the conditions realized in the experiments of Coulomb and Morin.

To conclude, we will mention the interesting work of Mll. Ch. Jakob (Reference 6) on sliding friction at very low speeds.

Previously, F. Jenkin and J. A. Ewing (Reference 7) under the same conditions, had found that there existed a continuity between static and dynamic friction.

Pushing these researches further, and operating in well-defined physical conditions, in particular taking scrupulous precautions against all traces of dust, Mlle. Jakob arrived at the following results, using the inclined plane method.

For a very small inclination of the plane, the sliding weight moved with a uniform speed which began to accelerate when the inclination of the plane was increased. The smallest angle measured was 1.5 degrees, the weight being a glass plate, and the smallest speed recorded was 0.0006 millimeter per second (movement observed by microscope). The friction increased with the speed, rapidly at first, and then more and more slowly, becoming at last practically constant. It is this constant value which had been observed by previous investigators.

The least impurities placed between the surfaces in contact, the humidity of the air for hygroscopic bodies, modified completely the character of the phenomenon. In order to make sure that dust was absent, and also to be able to experiment in a vacuum or in different gases, the apparatus was placed under a bell-jar. The presence of humidity, of alcohol, or ether vapors, played the role of impurities, and introduced a discontinuity between the static and the dynamic friction.

The influence of temperature has also been studied; heating first produces a diminution and then a rapid augmentation of the dynamic friction. If one passes a certain temperature, the phenomenon becomes irreversible, which can be attributed to a change in the surfaces of contact.

II. Intermediate Friction, or Friction with Lubrication

It is this which particularly interests the technician, since in all machines, and watches in particular, the frictional surfaces are always carefully oiled. The role of the lubricant is of a very complex nature and has not been at all well understood until recent years. In the past it was thought that the lubricant modified the surfaces in contact in a very simple manner. In 1883 Tower made the remark that it created a fluid layer between the frictional surfaces, which annulled the influence of the physical properties of the bodies in question.

The chief works are those of Reynolds (Reference 8) in England and of Petroff (Reference 9) in Russia. Recently A. Sommerfeld (Reference 10) has taken up the question, and also Lord

Rayleigh (Reference 11).

We will not speak of the works which consider the physical aspect of the problem. A very great number of researches have been made by engineers or technicians, but we cannot consider here their detailed studies; besides, the conditions which obtain in large machines are not in general those encountered in horology; our own work shows nevertheless, that in their general aspects the phenomena are comparable.

The works of Reynolds and Petroff place the question in its proper domain, by showing that it can be referred directly to the field of hydrodynamics. According to these theories, if the surfaces in contact are carefully polished, if the lubrication is complete, and if the load is not too great, in such a fashion that the frictional surfaces are not in direct contact, only the viscosity of the lubricant enters; the characteristics of the frictional bodies are without influence. Under these conditions the friction is proportional to the coefficient of viscosity of the lubricant, to the area of the surfaces in contact and to the relative speed of the two bodies, and inversely proportional to the thickness of the fluid layer, the result being expressed by the formula

$$F = \frac{\mu v S}{e} \qquad (5)$$

in which μ is the coefficient of viscosity of the lubricant (it varies considerably with temperature and, of course, with the nature of the liquid), v is the linear speed, S the area of the surface in question, and e the thickness of the lubricating layer or the "play."

If the total load is called P and the unit pressure p, one has, naturally

$$P = p\,S \tag{6}$$

One can thus replace S by P/p in equation (5) which becomes

$$F = \frac{\mu\,v}{e\,p}\,P \tag{7}$$

It is customary to regard the coefficient of friction as a result of the laws of Coulomb and Morin given above, even when this coefficient is not a constant quantity, but depends on various factors, notably the speed.

It will be, therefore, always defined by the relation

$$f = \frac{F}{P} \tag{8}$$

F being the force of friction and P the normal pressure exerted by the two surfaces against each other. Comparing with (7), one can in consequence say that in the case of a lubricant, there exists a coefficient of friction which has the value

$$f = \frac{\mu\,v}{e\,p}. \tag{9}$$

We assert emphatically the fact that this coefficient f necessarily varies with conditions and varies over a considerable range.

The theory of Reynolds and Petroff is directly applicable to the case of friction of pivots; to this end, we replace S in equation (5) by $2 \pi r$, the area over unit length of the pivot. There results

$$F = 2 \pi r \mu \frac{v}{e} \tag{10}$$

whence for the moment of the force:

$$\text{Moment of force} \equiv M = 2 \pi r^2 \mu \frac{v}{e} \tag{11}$$

r being the radius of the pivot and e the thickness of the lubricating layer, assumed uniform; this thickness is nothing else than the play which exists between the pivot and the bearing, or the difference between the two radii.

Sommerfeld has continued this study, taking into account the fact that in reality the pivot or journal is never centered in its bearing, and in consequence, that the thickness of the lubricating layer is not uniform.

When the speed is zero or very small, it is natural to suppose that the two solid surfaces are in direct contact; when the speed increases, the motion makes the lubricant penetrate between the two surfaces and the friction then, and only then, becomes intermediate. The pivot is at the very first eccentric, and as soon as the speed increases it tends to center itself more and more, that is to say, that the fluid layer tends to assume a uniform thickness.

One observes at the same time the following phenomenon: At rest the point of contact between pivot and bearing is found at the lowest possible position, at A, for example, (we suppose the axis of rotation to be horizontal).

During the movement, the point of contact, or better, the point of least thickness of the lubricating layer, is displaced. The old theory (Coulomb) asserted that this displacement occurred in the sense inverse to the rotation (Fig. 3 at B, for example). In effect, in rotating, the pivot tends to climb the slope of the bearing up to the point where the inclination to the horizontal is such that $\tan \Phi = f$, so that one can identify this case with that of an inclined plane.

Practice shows, on the contrary, that the displacement occurs in the opposite sense, that is to say that the pivot tends to climb the slope of the bearing in the same sense as the rotation.

The cause of this unexpected phenomenon seems to be that the lubricant, entrained by the movement of the shaft, tends to form a sort of fluid wedge between the two frictional surfaces. The compression of the fluid in this wedge exerts a normal component on the pivot which is separated from its support.

The theory of this phenomenon is very abstruse, and we shall only refer to the memoirs of A. Sommerfeld (Reference 12). These establish in particular that the center of the pivot is displaced upward and the more so the more rapid the rotation; at the same time the eccentricity diminishes and tends towards zero at very great speeds, that is to say, that the pivot tends to turn concentrically in its bearing.

One can easily see that under these conditions the friction is entirely different from that which is calculated on the basis of the old theories.

The laws of Coulomb gave for the moment of the frictional force:

$$\text{Moment of force} \equiv M = P\ r\ \sin\Phi \tag{12}$$

P being the load, r the radius of the pivot and Φ the angle of friction. This relation which is still currently used in practice, is completely false.

Sommerfeld, taking into account the complex phenomena which we are describing, arrived at the following equation, representing the moment of the force of friction per unit length of the pivot

$$M = 2\pi r^2 \mu \frac{v}{h}, \tag{13}$$

a formula identical with equation (11), but where the play e is replaced by h, which this time represents the variable thickness of the lubricating layer, and should not be considered as constant, and is equal to e only at very high speeds since then the pivot is centered.

Taking account of the variation of the thickness h with eccentricity, on the basis of dimensional theory, Sommerfeld by theoretical considerations arrived at some more complicated relations which we cannot consider in this brief study.

We refer the reader to the original work of the author (Reference 13) or to that of H. Bock (Reference 14), who has given a resume of this theory with relation to its application to horology.

We shall only state that the relations established by Sommerfeld permit us to take account, in their general aspects, of the experimental results of R. Stribeck (Reference 15) and of other experimenters who have made technical researches in this field.

Stribeck, in particular, found that the coefficient of friction with lubrication varied considerably with speed (Fig. 5).

This coefficient diminished as the speed increased, passed through a very well-defined minimum, and then increased just as indicated in the figure referred to. The minimum of the coefficient of friction appeared to be independent of the pressure at a given temperature and for a given lubricant. The theory of Sommerfeld predicts the existence of this minimum and shows the nature of the curve beyond the minimum. It predicts further that in the neighborhood of this minimum, the quotient v/P of the speed by the load should be a constant, which experiment verifies approximately. On the contrary, at low speeds, as Sommerfeld himself said, the theory is not in accord with experiment. One can perhaps interpret the matter by assuming that the lubricating layer becomes too thin and that immediate contact intervenes directly.

As we shall see in the course of this work, the experiments made at the laboratory for horological research show that the friction of steel pivots on jewels follows similar laws in every respect, and that in particular, curves of the same characteristics as those of Stribeck are found even for journals (pivots) of very small dimensions. This is perhaps surprising, considering how different is the form of the bearing (a pierced jewel) from the theoretical cylinder with horizontal axis.

In conclusion we remark that the smaller the frictional bodies the greater will be the effect of very small surface irregularities, grains of dust or other impurities, as well as faults in the geometrical forms of the pieces (not perfectly round, etc.).

It should not be expected, therefore, that with pivots of the dimensions of those used in horology, the same uniformity of results will be found as when one works with journals of 5, 10, or more centimeters in diameter, and experience completely confirms this supposition.

We think that this very rapid review will suffice to show the interest and nature of the researches which we have undertaken, and the results of which will be published in later articles.

Method of Measurement

The study of friction in horology is rendered difficult by the small dimensions of the parts, the pivots in particular, and the smallness of the forces coming into play. It is required to find a simple method capable of being adapted to these special conditions.

In a preliminary series of tests, the frictional couple was measured directly by the work absorbed by the friction. For this purpose, a staff supported by two pivots in jewels and furnished with a flywheel was turned horizontally between two plates. A thread of silk, about 1 meter in length, was fixed by one end to the staff and rolled up on it, and a weight P of several decigrams was attached to the free end of the thread.

Under the influence of this weight, the staff was set into accelerated motion and, on account of the kinetic energy acquired by the flywheel, the thread after unrolling completely, rolled up in the opposite direction and the weight was lifted. Because of the friction, the weight did not return to its original but to a lower level. Let h be the difference in level; the work of friction is measured by P h.

Representing by θ the total angle, expressed in radians, through which the staff turned, and by M the average frictional couple, the work absorbed will be $M\theta$, whence

$$M\theta = p\ h, \quad \text{and} \quad M = \frac{ph}{\theta}$$

This method, difficult to handle and not very sensitive, did not give us satisfaction. For one thing, it did not permit a determination of the frictional couple for a given speed; and further, the speeds attained were such that the resistance of the air on the flywheel was not negligible. Finally, the determination of the friction in the case of a vertical staff was practically impossible.

On the contrary, the method which we adopted after these preliminary unfruitful tests, has given good results. In principle it consists of placing a movable bearing at the extremity of the pivot to be studied. The rotation of the pivot tends to drag the bearing with it on account of the friction; it is restrained by the opposing force of a spiral spring, and takes a new position of equilibrium corresponding to equality of the elastic moment of the spring and the frictional moment. For a given speed, the frictional couple is determined by the angle of deviation of the bearing, if the elastic moment of the spring is known. The resistance of the air does not enter.

Description of the Apparatus

The apparatus evolved after several attempts was constructed with great care at the horological school of Locle, to which we express our appreciation. It is represented schematically in Figure 6. It is composed of two circular brass plates M and N about 10 centimeters in diameter and 45 millimeters thick; they are connected by three bars B about 2 centimeters long. Through the center of each plate passes an axle supporting a pulley P and terminating exteriorly in a pivot p carefully turned and polished. (m is a metal mirror set at 45 degrees, the use of which will be described in the third section of this work.)

The axle is set into rotation by means of a cord passing over the pulley of an electric motor. The speed of the motor can be varied over a very large range. Uniform running is assured by an electromagnetic brake consisting of an aluminum disk fixed on the shaft of the motor and turning between the poles of a permanent magnet.

The pivot p is capped by a brass cylinder C which we will call the "cap." This serves as the movable bearing mentioned above; it is shown in section in Figure 7.

The cap is jeweled at E and F and carries a jewel mounting D furnished with a cap-jewel H which can be removed when desired by means of the screws V and V'. The flange R serves as support for the movable weights which can be slid on the cap to increase its weight and also serve to determine the elastic moment of the spiral spring.

These weights and the cap were balanced by the method used in horology to adjust balance wheels.

Finally, the apparatus is fixed on a support which permits the axle to be placed vertically or horizontally to determine the friction in these two conditions.

Figure 8 is a view of two instruments placed in these two positions; the different parts we have described are easily recognizable.

The spiral spring which produces the resisting force is represented by r in Figure 6; one of its extremities is fixed to the cap and the other to the plate N by means of the pin n.

At each instant the frictional couple is measured by the elastic reaction of the spring which is proportional to the angle the cap has turned. This angle is indicated by the pointer a on the circular scale S graduated in degrees.

To measure the speed of rotation of the axle the driving cord was marked distinctly at one point; the time required for this mark to pass a given position ten times was measured by a stop watch. Let t be this time and l the length of the cord; the speed of the cord which is the same as that of the periphery of the pulley of radius s should be

$$V = \frac{10\,l}{t_1} \tag{14}$$

whence for the angular velocity ω of the pivot

$$\omega = \frac{V}{s} = \frac{10\,l}{s\,t_1} \tag{15}$$

From this relation is deduced the value of the linear speed v of the surface of a pivot of radius r:

$$v = \omega\,r = \frac{r}{s}\,\frac{10\,l}{t_1}. \tag{16}$$

Calculation of the Friction and Calibration of the Apparatus

We have said in the introduction that according to Coulomb, the friction is given by the relation

$$F = f N \quad \text{(see equation 8)}$$

whence for the moment of the frictional forces in the case of a pivot placed horizontally

$$M = P r f \tag{17}$$

In our case P represents the total weight of the cap (including the loading weights), r the radius of the pivot, and f the coefficient of friction which Coulomb takes as a constant; this coefficient is <u>variable</u> and in particular a function of the speed.

On the other hand, if one designates by C the elastic moment of the spiral spring for an angle of one radian, the elastic moment for a deviation θ of the pointer, expressed in radians will be $C \theta$.

We shall have for equilibrium:

$$f P r = C \theta \tag{18}$$

Thus at each instant, the coefficient of friction will be given by the relation

$$f = \frac{C \theta}{P r} \tag{19}$$

If α is the value in degrees observed for the angle θ, we have for the value of θ in radians

$$\theta = \frac{\pi \alpha}{180} \tag{20}$$

and equation (19) becomes

$$f = \frac{C \pi \alpha}{180 P r} . \tag{21}$$

For a given series of tests C, P, and r are constants, so that we can write

$$f = K \alpha \tag{22}$$

K being a constant whose numerical value should be determined. Now P, the weight of the cap, and r, the radius of the pivot, are easily measured. The simplest and best way of determining the constant C, that is the elastic moment of the spiral spring, is by the dynamical method. The pivot is placed vertically and the cap is made to oscillate under the influence of the spring; a certain period of oscillation T is obtained which is given by the relation

$$T = 2\pi \sqrt{\frac{I}{C}} \tag{23}$$

where I represents the unknown moment of inertia of the cap and C the couple which is to be determined. Next the loading weight of known moment of inertia I_1 is placed on the cap and a new period of oscillation is obtained:

$$T_1 = 2\pi \sqrt{\frac{I + I_1}{C}} . \tag{24}$$

From these two relations there results the value

$$C = \frac{4 \pi^2 I_1}{T_1{}^2 - T^2} \tag{25}$$

The loading weight used was a carefully turned cylindrical brass ring, adjusted on the cap, the dimensions of which were:

External radius $R = 1.098$ cm

Internal radius $r = 0.593$ cm

Mass $m = 8.062$ g

The moment of inertia I_1 of this ring is given by the formula

$$I_1 = \tfrac{1}{2} m (R^2 + r^2)$$

or in C.G.S. units

$$I_1 = 6.277 \text{ g cm}^2.$$

The measurements of the periods of oscillation gave the following data (averages of 30 measurements):

Period without loading weight $T = 0.6268$ sec,

Period with loading weight $T_1 = 1.5426$ sec,

whence for the value of C calculated according to equation (25):

$$C = 124.74 \frac{\text{g cm}^2}{\text{sec}^2} .$$

Procedure of Testing

To make a series of measurements, the apparatus is first adjusted by means of three leveling screws so that the shaft is horizontal; then the zero is determined, that is, the position of the pointer when the shaft is not running. Since there is always a static friction, this position should be found by trial by tapping lightly on the apparatus until the pointer reaches a fixed position. This operation is repeated one or more times and the average of the values found gives the zero sought. This determination should be made with care for any uncertainty in the zero systematically affects all of the results, and renders all intercomparisons illusory.

The motor is then started and readings of the angle of deviation and of the speed are made in the order indicated in the table below. In general, one starts at a low speed which is increased progressively to a maximum value; the speed is then decreased and check readings made at intermediate points.

Example of a Determination of Friction

As a practical example, we give the determinations of friction as a function of the speed, made on March 6, 1922.

The experimental conditions were the following:

Axis horizontal,

Lubricant: Ezra Kelley oil

Thickness of jewel at bearing point: 1.3 mm

Play: 1/100 mm

Load, weights and cap: 12.328 g

Radius of pivot: 0.045 cm

Radius of pulley: 2.51 cm

Zero of pointer: 100.8 degrees.

These values give:

$K = 0.00400$ (equation 22)

$\omega = V/251$ (" 15)

$v = 0.0179V$ (" 16)

Proceeding as described above, we have found the values reported in the following table:

No. of reading	Position of pointer = β	α = β-100.8	t_1 s	V cm/s	v cm/s	ω rad/s	f Coef. of friction
1	131.5	30.7	414	11.5	0.206	4.6	0.123
2	125.5	24.7	236	20	0.358	8.0	0.099
3	113.2	12.4	146	33	0.591	13.1	0.050
4	109.2	8.4	95	51	0.913	20.3	0.034
5	106.4	5.6	64	75	1.342	30.0	0.022
6	105.8	5.0	47.6	101	1.808	40.2	0.020
7	106.2	5.4	34	141	2.524	56.2	0.022
8	107.0	6.2	25.2	191	3.419	76.1	0.025
9	107.8	7.0	18.2	265	4.744	105.6	0.028
10	111.2	10.4	13	370	6.623	147.4	0.042

No. of reading	Position of pointer = β	α = β-100.8	t s	V cm/s	v cm/s	ω rad/s	f Coef. of friction
11	113.5	12.7	9.4	513	9.165	203.9	0.051
12	106.8	6.0	25.8	187	3.347	74.5	0.024
13	106.0	5.2	39	124	2.220	49.4	0.021
14	106.2	5.4	64	75	1.342	30.0	0.022
15	108.4	7.6	92	52	0.931	20.7	0.030
16	112.2	11.4	138	35	0.627	13.9	0.046
17	124.5	23.7	224	21	0.376	8.4	0.095

Diagram 1 gives the values of the coefficient of friction f in function of the speed.

It is evident that this coefficient is far from being a constant. It varies greatly with the speed and passes through a very pronounced minimum. Other measurements which we shall publish later have given similar results.

We have made our measures up to considerable speeds, which at first sight seem to be of no interest for horology. We remark, however, that certain parts of a watch attain greater speeds than one would believe.

Thus for a balance wheel which makes 18,000 oscillations an hour, each of $1\frac{1}{2}$ rotations, the angular velocity varies from zero when the balance is at the extremity of its course to 74 radians per second at its passage through the position of equilibrium. The maximum speed at the periphery of a pivot of 0.01

cm diameter is equal to 0.370 cm/s; for a pivot of 0.012 cm, it is 0.444 cm/s.

Later we shall give in detail the results obtained by the method which has been described.

Results

We have previously mentioned the difficulties encountered in the study of the friction of parts of the dimensions used in horology. Therefore, it should not be surprising that we have not yet been able to determine clearly the influence of certain factors such as the quality of the lubricant, the pressure on the pivot, etc. This partial insuccess is due to causes which, although negligible when one works with large machines as, for example, Stribeck, who utilized a shaft 7 centimeters in diameter with continuous lubrication, must be considered when one is concerned, as in most of our experiments, with a pivot of hardened steel 0.09 centimeters in diameter turning in a jeweled bearing. We refer to the grains of dust which are very difficult to eliminate completely, to impurities in the lubricant and particularly to the air bubbles which the lubricant contained or which formed during rotation, to the variable composition of the oil, etc. It is only by using very pure lubricants and taking all possible precautions in experimentation that we have been able to obtain the results given below.

We hope to be able to restudy certain questions by the same methods but using larger pivots - 5 mm, for example. This procedure appears legitimate since the phenomena we have observed with very small dimensions are, in these general aspects, identical with those observed by Stribeck on shafts 80 times as large.

Dry, or Immediate, Friction

In the absence of lubrication, the measurements are particularly difficult; they become practically impossible at high speeds because of the very rapid wear of the parts and because of seizure. On several occasions, we have tried to work at considerable speeds, but each time the bearing seized and the spiral spring, which served to determine the frictional couple, was damaged or even broken.

It was interesting to try to determine the position of the point of contact of the pivot with the bearing during rotation. According to the classical theory, and in the absence of lubrication, this point of contact should be displaced in the inverse sense to that of the direction of rotation (see Fig. 3,B). Further, one should have at each instant

$$f = \tan \Phi$$

f being the coefficient of friction and Φ the angle between the normal at the point of contact and the vertical.

By observing the angle Φ, the coefficient of friction may thus be determined in a second way, and the values obtained by this method can be compared with those furnished by the determination of α.

In order to observe the point of contact we have proceeded in the following manner: A mirror M (Fig. 6) of polished steel, pierced to permit passage of the pivot, and making an angle of about 45 degrees with the pivot, is placed back of the cap. This mirror reflects a ray of light parallel to the axis, and this illuminates the space, or play, between the pivot and the bearing.

By means of a microscope mounted in front of the apparatus the relative position of the two parts can be observed. The play presents the appearance of a very brilliant crescent terminating in two points C and C' (Fig. 9). The point of contact is at the center of this interval CC'.

Figure 9 is reproduced from photographs made at an enlargement of about 44 diameters. Number 1 represents the pivot in its position of repose; the crescent CC' is then symmetrical with respect to the vertical; number 2 refers to a rotation in the sense indicated by the arrow. The lines drawn show the position B of the new point of contact making an angle Φ with the vertical. In order to determine the position of the point of contact B, the ocular of the microscope is furnished with a reticule and is rotatable about its axis; it carries a pointer traveling over a graduated circle. The reference line of the reticule is successively placed in coincidence with the points

C and C' of the crescent, as indicated by the dotted lines in Figure 9, and in each case the corresponding position of the pointer is noted. The average of the two readings gives the position of the point B and hence the angle Φ.

These measures are somewhat uncertain, especially at high speeds, on account of the vibrations and oscillations of the cap which indicate that the pivot is never absolutely cylindrical.

All the dimensions of dry friction have been made with the pivot horizontal, the only position for which the coefficient of friction can be calculated.

In our first tests we have used bearings of rubies. Later, in order to measure more easily the deviation of the point of contact, we replaced the first jewel by a well-polished brass bearing. By this means it is possible to control the circular opening of the bearing more accurately so that the reflections which are inevitably produced by the surface of the jewels will cause no trouble.

Diagram 2 shows the variation of the angle α as a function of the speed. This angle is proportional to the coefficient of friction f, (equation 22) and also that of the angle Φ determined simultaneously. In this set of measurements, the polished steel pivot was 0.091 cm in diameter, and the average play was about 0.03 mm. Unfortunately, in trying to obtain measurements at high speeds, the spiral spring was broken on account of seizure in the bearing so that it was impossible to determine the constant C of the spring; the measurements of the angle therefore, furnish only relative values of the coefficient of friction.

These results nevertheless present a certain interest. The deviation Φ of the point of contact is in the predicted direction; it is given as a function of the speed by the curve B. Up to the point L, the bearing is very unsteady; from L to M it is steady, and between M and N the zone of contact diminishes little by little until at the end it shows only as a well-defined point. On further increasing the speed, the pivot separates from the bearing and one can see light all around the pivot. Should this phenomenon be attributed to the air, which at high speeds acts as a lubricant? Or is it caused by the vibrations due to imperfect roundness of the pivot? This is a question which we cannot at present answer. The tests im progress on pivots of greater dimensions which are easier to make will perhaps furnish a solution to this problem.

But the general form of the curves is interesting. Note that the angle α and in consequence the coefficient of friction f to which it is proportional, increases rapidly at first, passes through a maximum, and finally diminishes (curve A).

These results are comparable with those obtained under entirely different conditions, by Mlle. Ch. Jakob, in her study on sliding friction made by means of the inclined plane. This author has made a very careful study of this phenomenon at low speeds and the curves which she has published agree very well with those which we have obtained, at least in their general aspects.

The table below contains the results of a series of measurements in which care was taken not to raise the speed too high so that complete results were obtained. It contains all the necessary data for comparison of the coefficient f furnished below by measurements of the angle α and of the angle Φ.

t_1 s	ω rad/s	v cm/s	α deg.	Φ deg.	$f = K\alpha$ K=0.0274	$f = \tan\Phi$
176	11.1	0.51	6.7	7.7	0.18	0.14
103	19	0.86	7.6	10.4	0.21	0.18
61.4	32	1.45	8.5	12.5	0.23	0.22
40	49	2.23	10.0	14.0	0.27	0.25
27.3	72	3.26	12.5	15.8	0.34	0.28
22	89	4.05	12.5	16.9	0.34	0.30
16.8	117	5.30	14.5	18.2	0.40	0.33

Considering the experimental difficulties the values of f given in the last column are quite close to those obtained by measuring the angle α but always slightly less. This difference is perhaps connected with the phenomenon we have just mentioned; that is, the progressive separation of the pivot from the bearing.

Other series of measurements have given similar results; we do not think they are of much interest since large variations of the coefficients of f are obtained for identical speeds. We hope to obtain more conclusive results by using a pivot of large diameter; furthermore, friction without lubrication is of little importance in practical horology, except for gear teeth, since one always tries to eliminate it on account of the wear which it produces.

Results - Friction with Lubrication

In this general study we have tried to determine the influence on the coefficient of friction of certain factors such as speed of rotation, the nature and age of the oil, load on the pivot, the form of the jewel bearing, the diameter of the pivot and its position, whether vertical or horizontal, but have not been able to complete the entire program.

At the same time we have studied the displacement of the point of contact during rotation which we have examined in detail by means of a microscope. This examination has revealed to us the presence of dust and of air bubbles entrained by the lubricant, the presence of which it is very difficult to avoid completely and which complicates the measurement to a surprising extent. This explains why we have never been able to obtain identical results when the apparatus has been dismounted for adjustment and oiling, although the essential conditions remain the same.

The curves in diagram 3 are characteristic in this respect. They represent the coefficient of friction as a function of the speed for identical conditions; axis horizontal, cap without loading weights, and jewel bearing 0.9 mm in diameter, play 0.01 mm, Ezra Kelley oil. The series represented by the curve 2 was obtained after adjusting and re-oiling the apparatus; in consequence, we find values for the coefficient of friction differing from those previously measured as in curve 1. Curve 3 was obtained six months later, after a new adjustment and under the same conditions as the other tests. These differences can be explained by the presence of greater or less quantities of dust and air bubbles in the oil as well as by variation in the composition of the oil from one test to another.

It frequently happens that the friction changes abruptly during a test. Such changes manifest themselves by a jump of the pointer which may be as much as 5 degrees. Diagram 4 shows an abrupt variation of this kind. The points from 1 to 7 have been obtained while progressively increasing the speed; the points 8 to 10 correspond to decreasing speeds. All these points lie on the same curve. While still decreasing the speed, the pointer abruptly changed to position (point 11), and from this time on the new points determined lie on a similar curve but displaced with respect to the first (points 12 to 16).

By means of a microscope we have been able to find the reason for these abrupt changes. The lubricant as we have mentioned always contains small bubbles of air. It sometimes happens that these bubbles unite to form one large one which as it moves down into the point of the crescent, it makes a free space between the pivot and the bearing. Whenever the air bubble rests near the point of contact the friction is diminished (points 11 to 16, diagram 4).

If on account of the rotation, the bubble passes to the other side of the zone of contact, or is forced out at the edge of the bearing, the pointer returns to its original position. The formation or the disappearance of this bubble can be artificially produced by slightly displacing the cap in the direction of the axis of the pivot.

We have made a systematic study of these perturbations proceeding in the following manner: The jewel was pierced by a small hole permitting the insertion of a very fine glass tube. By blowing lightly into the tube small air bubbles in the lubricant can be created at will, and simultaneously the coefficient of frictiom changes its value.

Also we have tried to eliminate the air bubbles which form during the course of a test by placing the oil under reduced pressure in a bell-jar for 30 to 60 minutes before using. In this way we were able to obtain greater regularity in a series of measurements made without dismounting the apparatus.

On another occasion a grain of dust in the oil became lodged between the pivot and the bearing, occasioning an abrupt increase in friction. These solid impurities (grains of silica, particles of brass, etc.) are visible with the microscope and their irregular motions in the bearings and displacements under the pivot can be followed.

The amount of solid impurities and air bubbles is never the same after dismounting and recoiling the apparatus so that it is very difficult always to obtain the same values of the coefficient of friction for a given speed. On the contrary, one finds practically the same curves when the apparatus has not been dismounted. For example, in diagram 5 are four curves taken under the same conditions (without any change whatever in the apparatus) over an interval of two days. The agreement is satisfactory, and we are justified in comparing the results of tests made without dismounting the apparatus.

Friction as a Function of the Speed

The general nature of the friction curve, determined by more than 120 series of measurements made under very varying conditions, has always been the same. It is characterized by a rapid decrease in friction at low speeds, passing through a minimum more or less accentuated according to the viscosity of the lubricant, and increasing progressively at high speeds. This is shown clearly in the diagrams given. We shall return in a future article to the interpretation of these curves. The position of the minimum and the slope of the curve on each side of the minimum depend upon conditions which can be specified to some extent.

Note that the coefficient of friction may decrease from the value of 0.14 to 0.20 at low speeds to 0.03 or 0.02 at the minimum, and then increase indefinitely.

Influence of the Viscosity

Diagram 6 pertains to three series of measurements made with oils of different viscosities. Curve 1 was obtained with Kelley oil of a viscosity of 0.45 C.G.S. Curve 2, "Autol" oil, viscosity about 11.9, and curve 3, castor oil, viscosity 10.9, show the influence of very viscous lubricants. The general nature of the curves is the same but the position of the minimum is different; further, the increase of friction from the minimum point is very rapid and at a velocity of about 70 radians per second corresponding to the minimum of curve 1, the coefficient of friction is 0.23 for the Autol oil and 0.40 for the castor oil.

Note particularly that at low speeds the coefficient of friction is considerably less for a very viscous oil. We shall return to the practical importance of this observation.

Ageing of the Oil

The pivot was lubricated with extra fine Cuypers chronometer oil and several series of observations were made at intervals during several weeks, without touching the apparatus. The curves obtained from June 13 to August 22, are given in diagram 7. These tests were made with the axis vertical. For this reason we have plotted the angle α as a function of the speed since the coefficient of friction cannot be calculated for this position of the pivot.

A comparison of these curves with those of diagram 6 shows clearly that the ageing produces the same effect as an increase of viscosity, as is naturally to be expected.

Influence of the Amount of Oil

A number of tests were made to try to determine the influence of this factor. The results are yet uncertain but from about 30 series of measurements we believe that it can be shown that a large quantity of oil tends to increase the value of the coefficient of friction.

Diagram 8, giving the angle α as a function of the speeds, shows the results for three series of measurements made with the axis vertical. Curve 1 refers to a small quantity, curve 2 to a normal quantity, and curve 3, to a large quantity of oil. The differences between the three curves are not very great; they are, however, appreciable and in apparent agreement with experience which demonstrates that it is useless to put too much oil on the pivots. Too much oil introduces other difficulties of a different nature well known to watchmakers.

Influence of the Load

We have tried to determine this factor by placing different weights on the cap without dismounting the apparatus. In spite of all possible precautions during the work, we have not been able to demonstrate the exact manner in which the coefficient varies with the load. These experiments will be repeated later to clarify certain very close results which we believe we have obtained.

One thing seems to be certain, however; it is that the load changes the value of the coefficient of friction and in particular it tends to increase it up to a certain limit and according to the lubricant used (see diagram 9).

Displacement of the Point of Contact

The displacement of the point of contact as a function of the speed of rotation of the pivot has been determined by the method already indicated.

In the case of dry friction the point of contact is displaced in the direction opposite to that of the rotatiom of the pivot. It is not the same in the presence of a lubricant. The successive phases of the displacement are shown schematically in Figure 10. Position 1 represents the pivot at rest. At very low speeds the point of contact is displaced, at least for oils of low viscosity, in the direction which we have indicated for dry frictiom, position 2. At a speed of about 3 radians per second, the point of contact returns to its position at rest, 3; then it passes to the other side; 4; and is displaced in the same direction as the rotation of the pivot. At high speeds the pivot tends little by little to separate from the bearing and finally turns almost concentrically in the bearing, position 5. At this position the radius passing through the point of least thickness makes an angle Φ of about 60 or 70 degrees with the vertical. It appears that if very great speeds could be obtained the pivot would be exactly centered.

As in the case of dry friction, we have determined the angle Φ and simultaneously the angle α which is proportional to frictional couple; the values have been plotted in diagram 10. The minimum of the curve for α coincides approximately with the sharp break in the curve for Φ.

An interesting phenomenon, which may be of importance in estimating the lubricating qualities of an oil, has been established by numerous measurements. At very low speeds and for certain oils, the pointer of the apparatus is never steady, but oscillates irregularly, the amplitude of the oscillation sometimes being greater than 10 degrees. This motion, which naturally increases the difficulties of reading, is connected with the adherence of the lubricant and probably with the surface tension of the oil.

Pure paraffin oil, for example, adheres neither to the jewel nor to the pivot; once placed in the bearing it separates into patches along the sides of the jewel. During the rotation of the pivot the pointer is then very agitated; it is not until a certain speed is reached that it becomes steady. At the same time, one can see all the small patches disappear little by little between pivot and bearing as if some suction were taking place at high speeds; the lubrication then becomes normal. When the pivot is stopped the oil again separates into small patches and a number of air bubbles can be seen on the surface of the liquid.

Other oils have more pronounced lubricating qualities; The oil adheres well to the metal, the lubrication is good even at small velocities, and the pointer of the apparatus remains remarkably steady. In this case, the point of contact deviates almost immediately in the direction of the rotation of the pivot (Sine Dolo oil, for example). The influence on the friction of the adhesion of the oil to the surfaces in contact is not a new phenomenon; but this manner of demonstrating its importance will perhaps lead to a practical method of specifying certain qualities of lubricants at present not well defined.

Pivot Vertical

Thirty-six series of friction determinations were made with the pivot vertical. The curves found were of the same general nature as in the case of the horizontal axis (diagrams 7 and 8).

In the present case the calculation of the coefficient of friction is impossible since the form and extent of the frictional surfaces are not exactly defined. It is even very difficult to estimate the order of magnitude of the coefficient of friction. Therefore, in the curves which we shall discuss, we have simply plotted the deviations α, which are directly proportional to the frictional couple, as ordinates.

A comparison of diagrams 7 and 8 with the others published in this article shows an appreciable difference between the two positions of the pivot. When it is horizontal the curve, after passing through the minimum, is in general concave downwards. With the pivot vertical this part of the curve is more nearly straight or slightly convex downward.

It has been known for a long time that the total friction is considerably less for a vertical than for a horizontal pivot. It is this difference in friction which produces the variation in the amplitude of the balance wheel of a watch in the flat and pendant positions, a difference well known to all watchmakers. It can be seen on diagram 11 that this difference is considerable. Curve 1, axis vertical, is always less than curve 2, axis horizontal. Except for position the two series of measurements were made under exactly the same conditions. Further, it can be seen that the difference in friction is extremely pronounced at low speeds, becomes less and less as the speed increases, and the two curves tend to approach each other at high speeds.

Finally, the same diagram 11 shows that the speed corresponding to the minimum friction is considerably less with the axis vertical than with the axis horizontal.

In these tests the point of the pivot was rounded off to an approximately spherical shape and rested against a flat cap jewel of synthetic ruby. It would be of interest to repeat these tests with pivots of other shapes and dimensions. In a final article we will show what interpretation can be made of the phenomena described and try to draw a number of practical conclusions.

Interpretation of the Results, Empirical Formula

When one examines the curves which we have published representing the variation of friction with speed, one fact is immediately apparent: The ascending part of all these curves is very nearly a straight line passing through the origin. This is illustrated in diagram 12, where the straight line has been produced by the dotted line. It seems therefore that one part of the friction varies proportionally to the speed.

One may consider the total friction as the sum of two effects, one increasing practically proportionally to the speed, the other beginning with a fixed value and continually decreasing until it becomes practically zero at a certain angular speed. This second part is represented by the dotted curve 2 of diagram 12. It has been calculated by subtracting the ordinates of the straight line which form those of the experimental curve.

How can this be interpreted? It is known that fluids offer a resistance to motion of the sort which we are considering here, a resistance called viscous which is proportional to the speed of the movement. It seems natural then to suppose that the curve 1 is determined by the lubricant and measures the effect of its viscosity.

We have seen further that during the rotation the pivot tends to separate more and more from the bearing and to center itself. At very low speeds, the contact between the pivot and the bearing is probably immediate, or the layer of lubricant is so thin that it cannot prevent solid friction being partially produced. As the speed increases, the lubricating layer also increases in thickness and the friction progressively diminishes.

It may be assumed that the solid friction does not cease immediately. The surfaces of the parts in contact, no matter how well finished, have certain irregularities which have the effect of causing immediate friction to intervene from time to time or periodically during a rotation. Solid particles of dust act in a similar manner but in measure as the speed increases, this solid friction becomes less and less perceptible and its effect becomes zero when the lubricating layer reaches a sufficient thickness.

Evidences for the validity of this interpretation can be found in certain phenomenon which have been observed in the course of this study and described above. Thus, for example, the pointer of the apparatus is very uncertain at slow speeds which can be attributed to solid friction produced in an intermittent manner. Further, with a slightly viscous lubricant the point of contact is displaced immediately even at slow speeds in the same direction as in the case of immediate or solid friction, returns to its initial position at medium speeds, and then is displaced in the opposite direction at the same time that the pivot begins to separate from the bearing. We do not wish to be too positive in asserting that the descending portion of the curve is due entirely to the progressive diminution of solid friction. It is quite evident that the phenomenon is more complex and that, in particular, as shown by Sommerfeld, the increase in thickness of the lubricating layer from the point where it is thinnest, acts in the same sense and causes a decrease of the viscous resistance. The separation of the total friction into two parts which we propose is entirely schematical. In order not to risk misinterpretation, we shall say that everything happens as if the solid friction diminishes progressively in proportion as the speed increases.

The dotted curve 2, which diminishes to zero, represents therefore the effect of this solid friction.

A complete theory will certainly be extremely complex. The works of Sommerfeld cited above show this. We shall not even attempt to formulate it, but it seems that an empirical formula representing the results of our tests should be of service.

The straight line passing through the origin and representing the viscous friction will be written

$$f_v = A\,\omega \qquad \text{(a)}$$

For the descending curve, measuring the solid friction, one can try several mathematical expressions. For very small velocity, this friction should be practically the same as that which one observes without a lubricant. It should therefore have a finite value comprised probably between 0.20 and 0.30. We shall call this friction for zero or very small velocity f_o. The solid friction f_s decreases continually and tends toward zero. It seems that the exponential form

$$f_s = f_o e^{-\lambda \omega} \qquad (b)$$

is the most simple which can be used. This as we shall see agrees well with the results. The coefficient f for an angular speed ω will then be given by

$$f = f_o e^{-\lambda\omega} + A\omega \qquad (c)$$

The constant coefficients λ and A should depend, probably in a complicated manner, on the form of the bearing, on the play, on the perfection with which the cylindrical surfaces have been made, on the presence or absence of a cap jewel, and lastly on the properties of the lubricant, especially its viscosity.

How may the expression (c) be verified?

For the straight line passing through the origin diagram 12 is sufficient evidence. Let us consider now the dotted curve 2 on the same graph:

$$f_s = f_o e^{-\lambda\omega}$$

One may write:

$$\log f_s = \log f_o e^{-\lambda \omega}$$

This may be represented by a straight line if one plots the logarithm of f_s as a function of the angular velocity ω, as we have done on diagram 13. Note how closely the experimental points lie on a straight line. This verification has been repeated in many cases with good approximation.

The following tables show the agreement between the experimental values of the coefficient of friction and the values calculated according to our formula, for a number of values of ω.

ω	f observed	f_v	f_s	log f_s	f calculated
7.5	0.138	0.0012	0.1368	-0.864	0.180
16	0.120	0.003	0.117	-0.932	0.118
25	0.083	0.0045	0.0785	-1.105	0.068
29	0.069	0.0052	0.0638	-1.195	0.063
37	0.042	0.0068	0.0352	-1.454	0.043
50.5	0.027	0.0092	0.0178	-1.750	0.027
60.0	0.022	0.011	0.0110	-1.959	0.022
90.5	0.017	0.017	0.0	$-\infty$	0.017
138.5	0.027	0.027	0.0	"	0.026
195.0	0.035	0.035	0.0	"	0.036

The formula (c) has been applied with the following values of the constants:

$$f_o = 0.27$$

$$\lambda = 0.0537$$

$$A = 0.0001855$$

It is seen that the agreement between the calculated and experimental values is very good. The greatest discrepancies occur at the low speeds, but this is precisely where the precision is least and the measurements are particularly delicate.

In a considerable number of tests, the ascending part of the friction curve is concave; it has the appearance of a parabola passing through the origin. (See diagram 15, curve 1.) This is especially noticeable when a very viscous lubricant is used. It must be admitted then that for very viscous oil the friction f_v is not exactly proportional to the speed, or better yet, that the progressive thickening of the lubricating layer at the point of contact corresponds to a relative decrease of the friction, such as can be predicted by the hydrodynamic theory.

We will therefore write:

$$f_v = A\omega - B\omega^2 \qquad (a')$$

B being a positive constant. The solid friction retains the same form, as shown by the logarithmic graph in diagram 14 which has been drawn exactly in the manner explained above.

The total friction is thus given by

$$f = f_o\, e^{-\lambda\omega} + A\omega - B\omega^2 \qquad (c')$$

The following table shows how this expression accords with the experiments.

ω	f observed	f_v	f_s	$\log f_s$	f calculated
1.4	0.191	0.0013	0.1897	-0.722	0.195
3	0.174	0.0028	0.1712	-0.767	0.171
7	0.127	0.007	0.120	-0.921	0.123
11	0.093	0.011	0.082	-1.086	0.091
15	0.081	0.015	0.066	-1.181	0.070
18	0.060	0.018	0.042	-1.377	0.060
22	0.046	0.021	0.025	-1.602	0.050
25	0.042	0.024	0.018	-1.745	0.046
34	0.039	0.033	0.006	-2.222	0.042
50	0.046	0.046	0.0	$-\infty$	0.0463
78	0.067	0.067	0.0	"	0.0675
109	0.087	0.087	0.0	"	0.091
153	0.115	0.115	0.0	"	0.115
219	0.151	0.151	0.0	"	0.147

$f_o = 0.22$ $\qquad$ $A = 0.00100$

$\lambda = 0.0922$ $\qquad$ $B = 0.0000015$

If the interpretation which we have given to these phenomena is correct, it is possible to predict, in a general way, how f_s and f_v will vary with the viscosity of the lubricant. For a lubricant only slightly viscous f_s will diminish slowly since the pivot will not separate from the bearing until high speeds are reached. The constant λ will therefore have quite a small value. With a very viscous lubricant, λ will be greater, which is well confirmed by examples given above.

The initial value f_o of the friction, corresponding to an infinitely small speed or rotation, should be constant and independent of the nature of the lubricant. This has been roughly verified (in the two cases above, f_o has the values 0.27 and 0.22) and considering the experimental difficulties at low speeds and the irregular variation of the friction, a rigorous verification ought not to be hoped for.

As a first approximation, the coefficient A ought to be proportional to the viscosity of the liquid. It is very much greater for a very viscous lubricant than for a fluid oil. We do not give numerical values as the accidental variations of the friction make a quantitative comparison impossible at the present time. The curves of diagram 6 relative to Autol and castor oils seem at first sight to contradict our statement since the Autol oil is slightly more viscous than the castor oil while the curve corresponding to the castor oil increases more rapidly than the other. However, curve 3 for the castor oil was determined using a cap jewel which was not the case with curve 2. Now the cap jewel evidently augments the surface of friction and in consequence the influence of the viscosity.

Theory of Sommerfeld

Sommerfeld, in his study on friction with lubrication, formulated laws for the variation of friction with speed. Quantitative comparison with experimental facts is difficult as the author himself has stated. One result, however, is susceptible of variation: this is the constancy of the quotient v/p of the velocity by the load at the point where the friction is a minimum.

We have calculated this ratio for several of our curves and the following table gives the following values:

Speed, r.p.m.	60	90	100	110	140
Load (g)	6.64	9.90	12.29	14.33	16.65
v/p	9.05	9.10	8.15	7.7	8.4

It is evident that the constancy of the ratio is somewhat roughly realized. Is this due to chance? However, the variations of the constant are in the same sense as those given by the measurements of Stribeck.

Practical Conclusions

When one attempts to draw from our research any conclusions of practical value from the standpoint of horology, one finds immediately that these conclusions can only be extremely general. The measurements were obtained at uniform speeds, but in a watch one never has to do with uniform movements. The movements are either oscillating as in a balance wheel or impulsive as in the gears, and it is not at all evident that the friction in these two cases follows the laws corresponding to uniform movement; it is quite possible that they do not.

In any case one conclusion is evident: The laws of Coulomb still generally used in calculations are not at all exact; for the friction of pivots with lubrication, one cannot even apply them approximately.

In the choice of a lubricant, however, according to our results one should take account of the average speed of frictional bodies. For fairly large speeds (balance wheels, escape wheels), one should choose a slightly viscous lubricant. For slow speeds, on the contrary, one should choose a very viscous lubricant since then the friction is considerably less than in the first case (see diagram 6).

Also it is necessary to take into account the pressure between the frictional surfaces which tends to expel the lubricant. A lubricant of considerable consistency, almost solid, may perhaps give better results than an oil. Experience has already shown watchmakers that oils of different qualities should be chosen according to the conditions. It will perhaps be advantageous for slow-moving gears transmitting considerable power to use greases analogous to those which are used on machinery, prepared naturally with special care. The wear will then be considerably lessened.

Finally, for parts moving at high speeds, it is necessary to take into account not only the viscosity but also the adhesion of the lubricant to the jewel and the pivot. Here again a special study of this factor will be necessary.

Is it possible to apply the empirical formula which we propose to the oscillating movement of a balance wheel? This movement is represented by

$$\theta = \theta_0 \sin 2 \pi n t$$

n being the frequency, or the number of complete oscillations per second. In a watch, n should be 2.5.

The angular speed ω is given by

$$\omega = \theta_0 2 \pi n \cos 2 \pi n t$$

If r is the radius of the pivot, P the load on the staff, the energy dissipated by the friction during the half period for the case of a horizontal axis will be

$$W = \int_0^{T/2} P r f \omega d t$$

an expression readily computed by replacing f by

$$f_o\, e^{-\lambda\omega} + A\omega$$

and ω by the value above. Will this conclusion be in accord with experimental results? Or, in other words, have we the right to suppose that in the case of oscillating motion the friction is the same for each velocity as in the case of uniform motion? It is impossible to answer this question at the present time. The experiments which we intend to undertake will perhaps permit us to decide this important point, but it can be readily foreseen that the friction will be less in the case of an oscillating motion than when the motion is uniform.

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11.	Rayleigh, Lord	:	Phil. Mag. 6, 35, 1918, p.1 and p.157
12.	Sommerfeld, A.	:	Memoires. See also the work of H. Bock, on the friction of pivots in the Deutsche Uhrm. Ztg. No.5, 1920, p.39 and 46.
13.	Sommerfeld, A.	:	Zur Theorie der Schimer mittel reibung, Z. f. tech. Phys. 1, 1921, p.58.
14.	Bock, H.	:	Deut. Uprm. Ztg. 44, 1920, p.39 and p.46.
15.	Stribeck, R.	:	Z. d. V. D. Ing. 46, 1902, p.1341 and p.1463.

Translation by K. H. Beij,
Bureau of Standards.

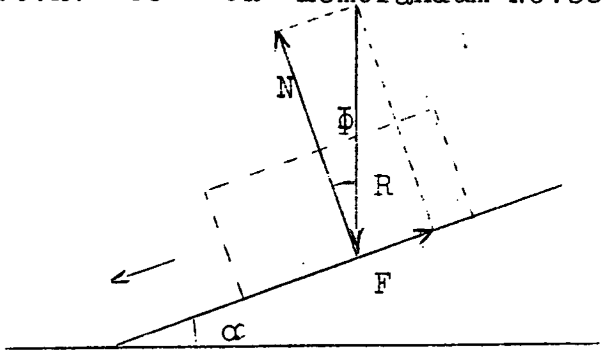
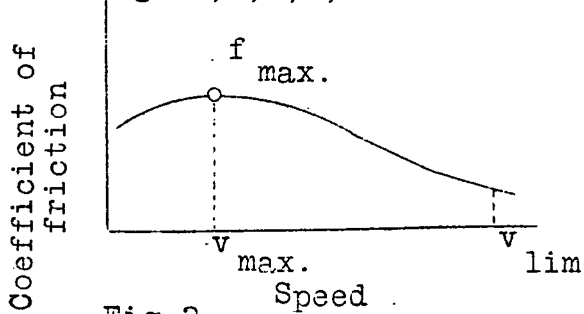


Fig.1



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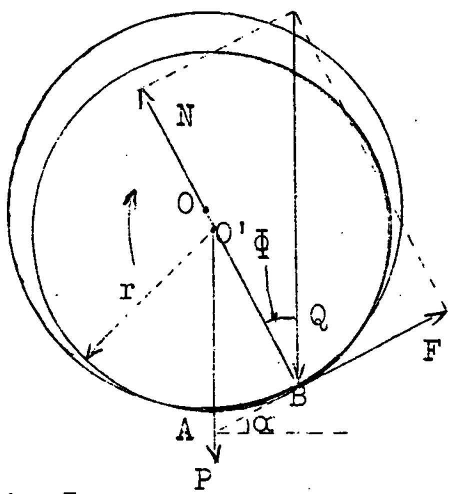

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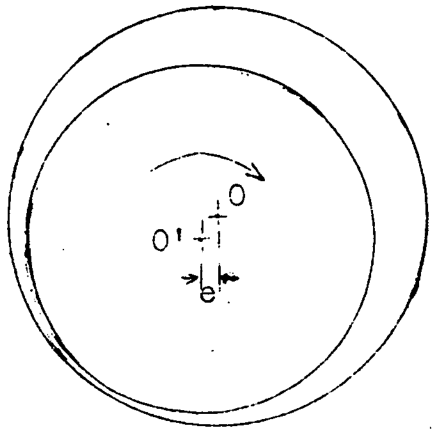

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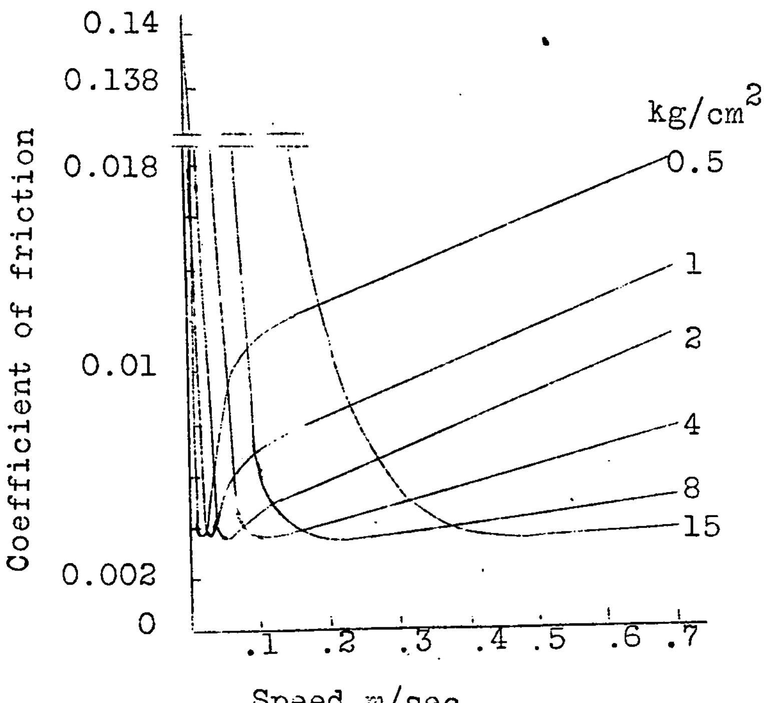
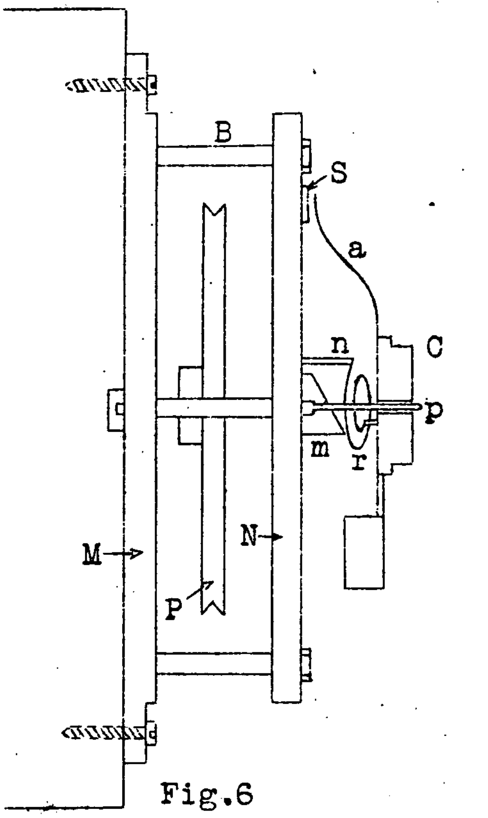


Fig.5 (after Stribeck)



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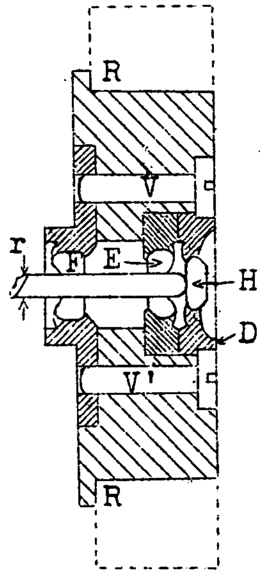

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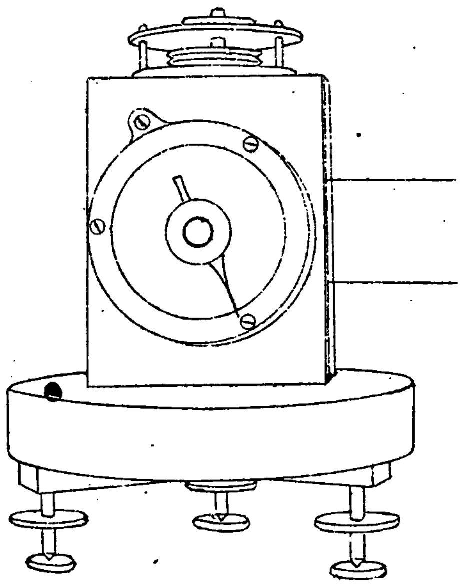

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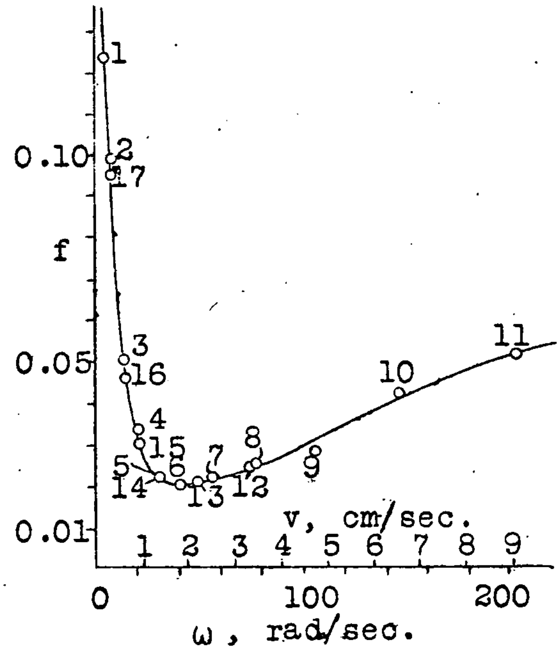


Diagram 1

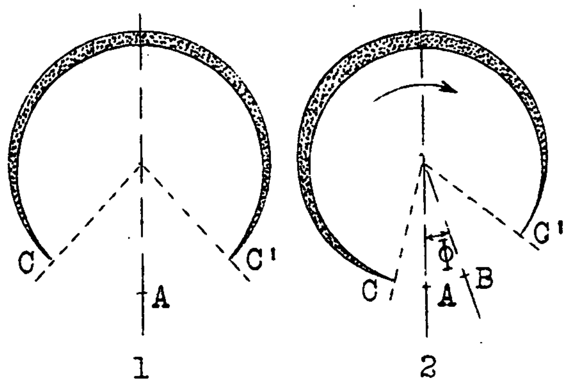
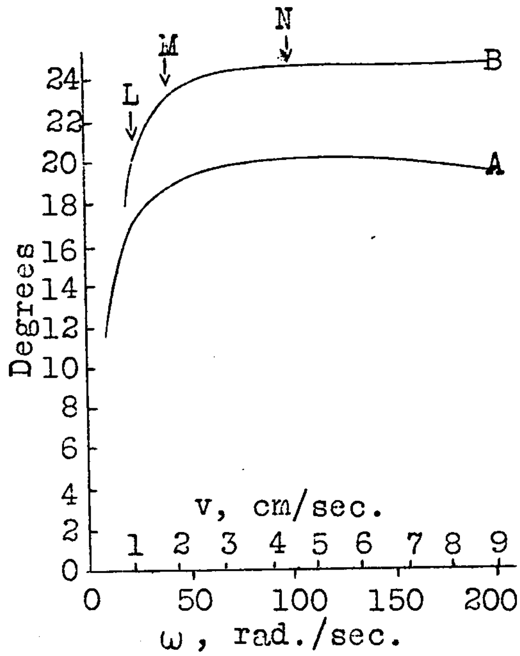


Fig.9



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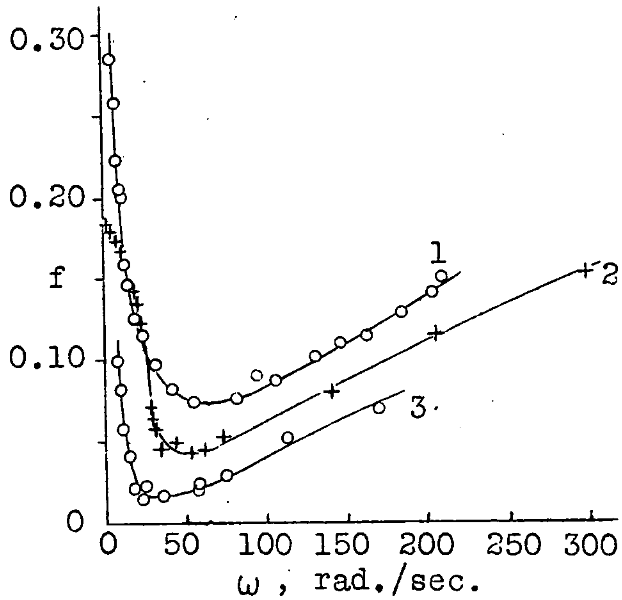

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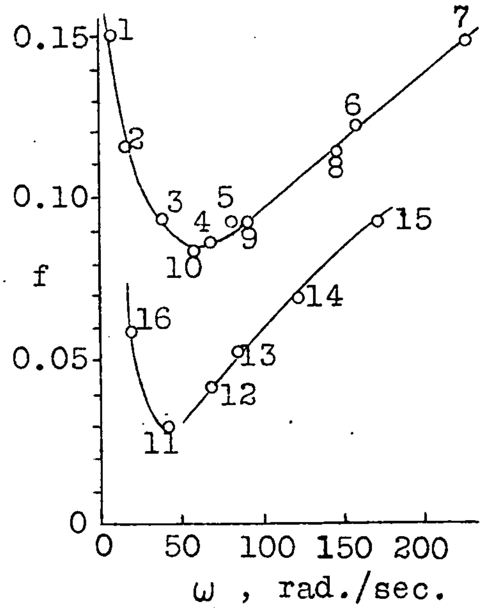

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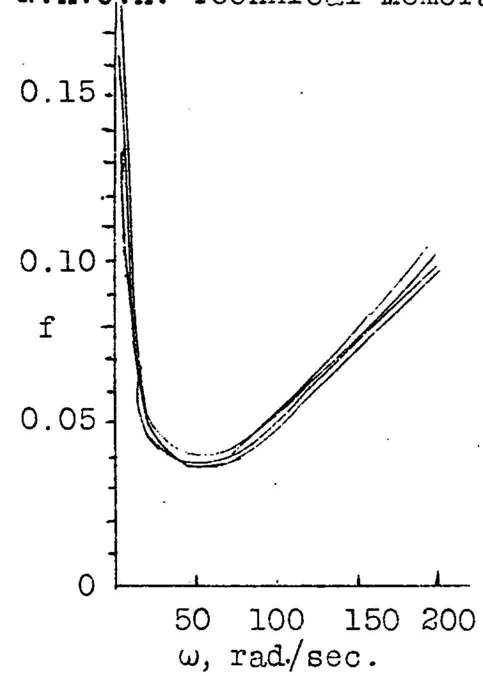

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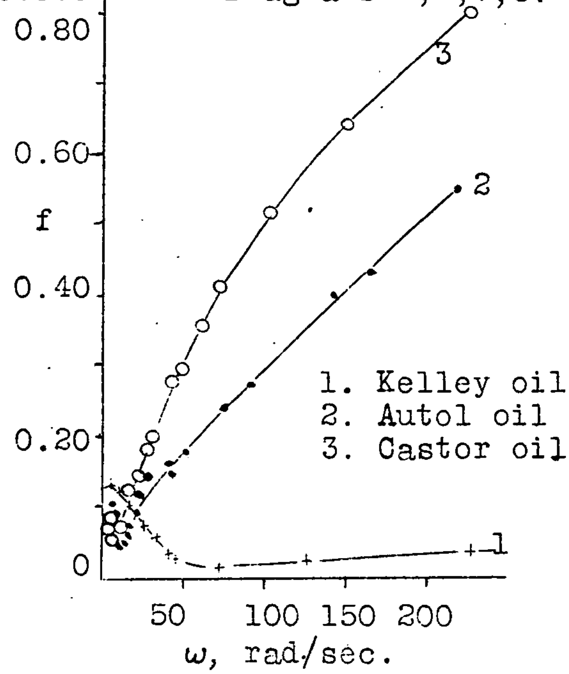

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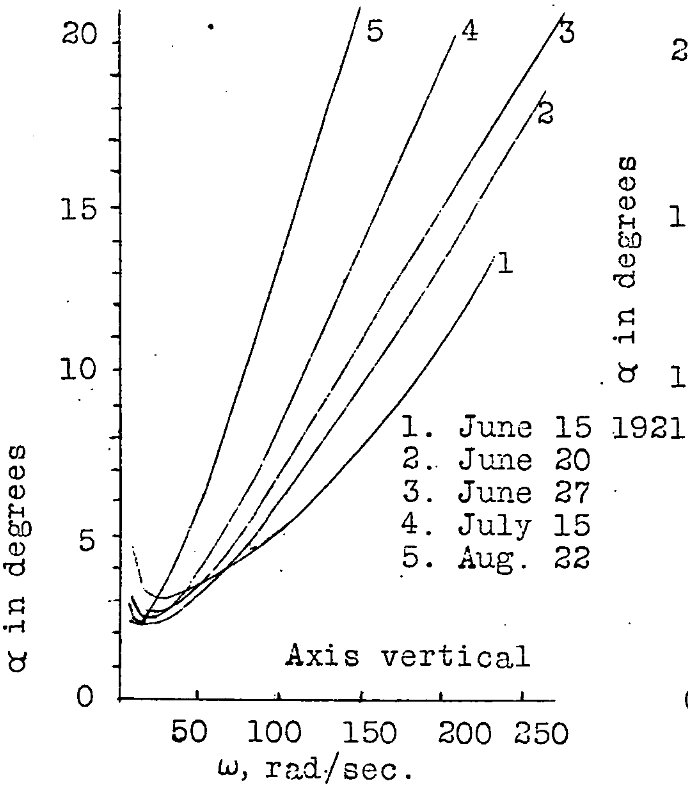

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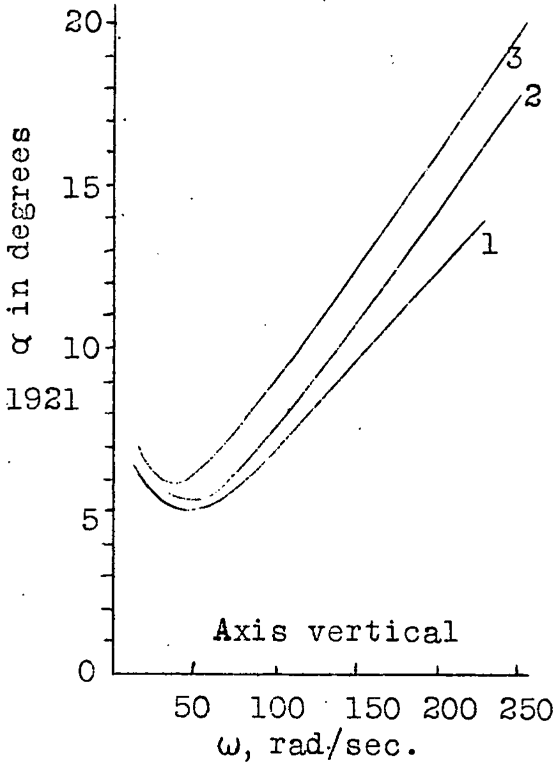

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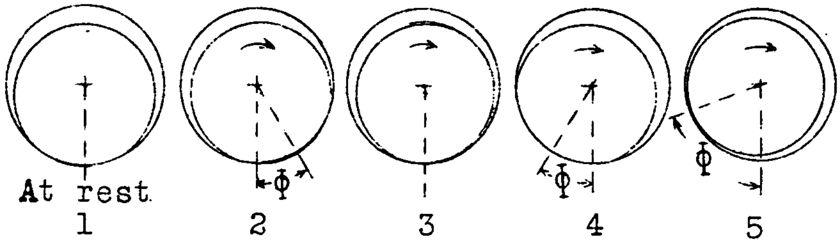
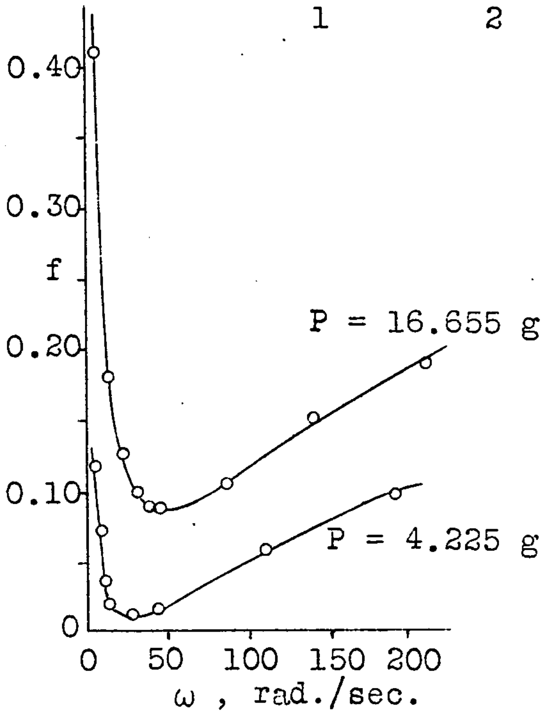


Fig.10



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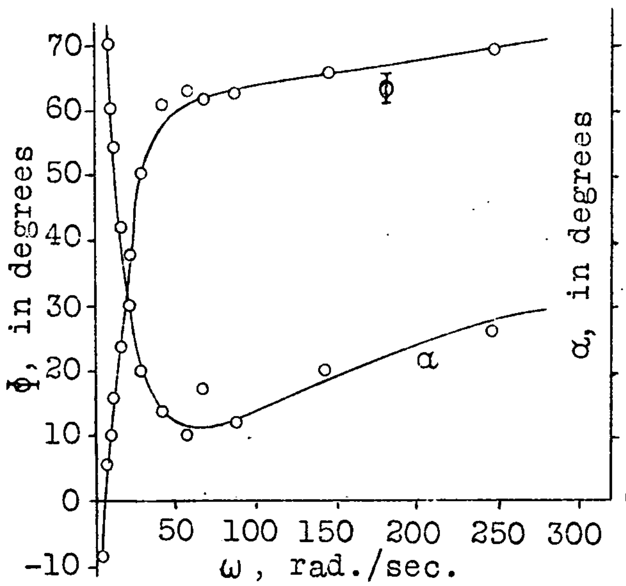

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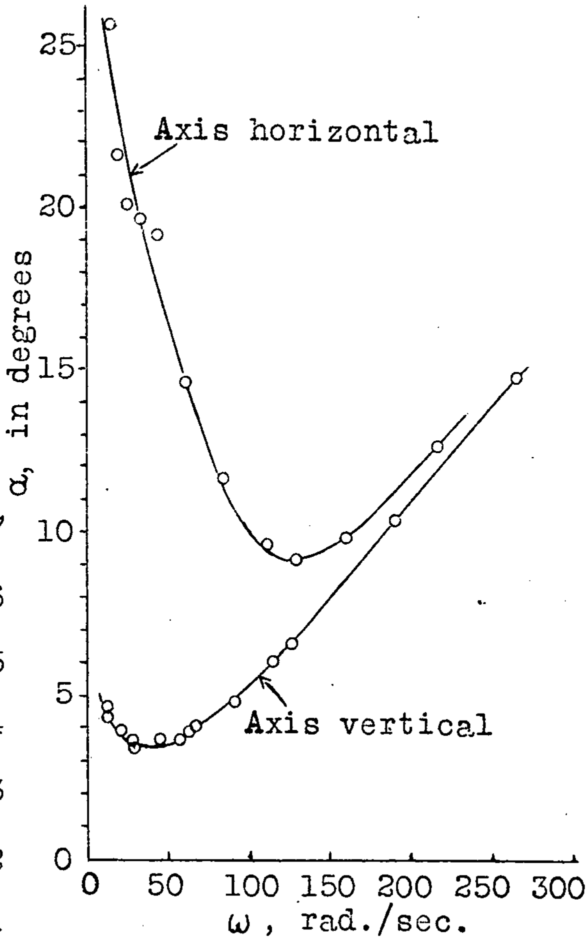

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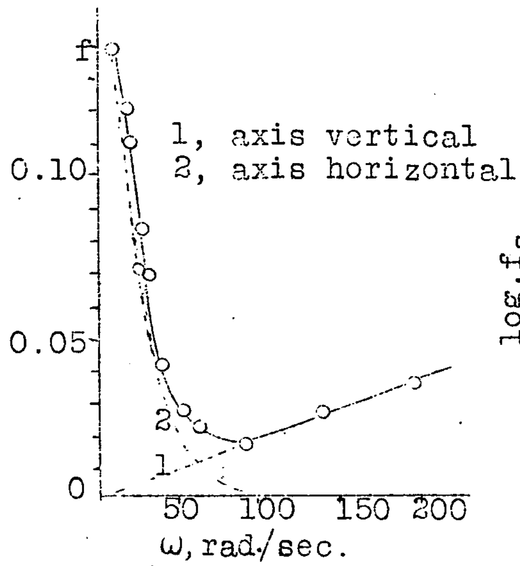

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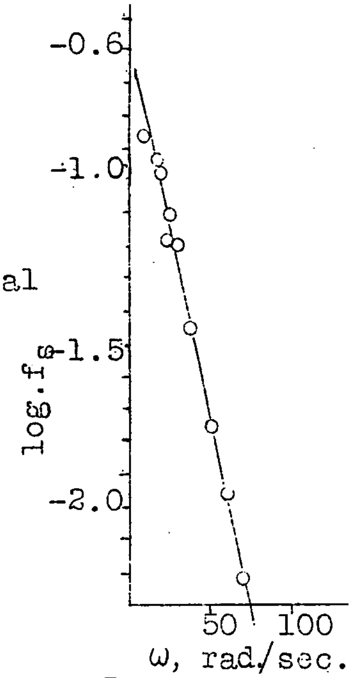

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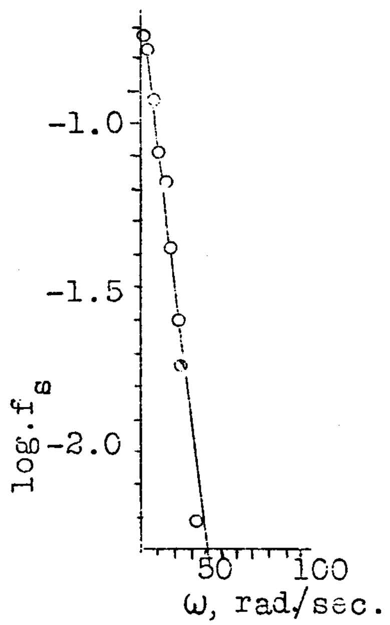

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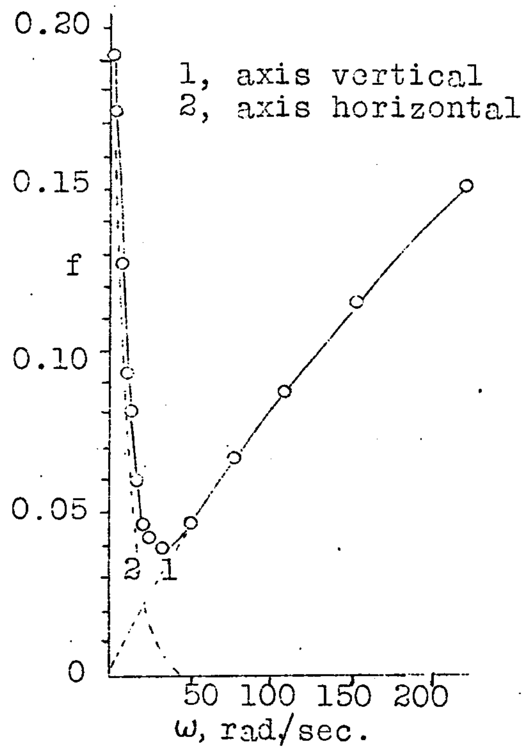

Diagram 15