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COLLISION-INDUCED SQUEEZING IN A HARMONIC OSCILLATOR

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The concept of squeezing has so far been applied mainly to light, as is evidenced by numerous research works on the subject of squeezed light. Since in quantum mechanics both light and the simple harmonic oscillator are described within the same mathematical framework, there is of course no difficulty in applying the concept to the simple harmonic oscillator as well. In fact, theoretical development of squeezed states and squeezed light owes much to physical insights one obtains as the analogy between light and the harmonic oscillator is exploited [1, 2]. There, however, exist only a few publications [3, 4, 5] that deal directly with generation of squeezing in a harmonic oscillator. Since the two quadrature operators for a simple harmonic oscillator carry the physical meaning of position and momentum operators apart from constants, a squeezed oscillator, i.e., a simple harmonic oscillator in a squeezed state, exhibits squeezing in actual position or momentum. Thus, a squeezed oscillator once generated can play an important role in atomic or molecular experiments that require precise initial determination of the position or momentum of the particles involved.

In our previous work [5], it was shown that squeezing can be generated in a harmonic oscillator by subjecting it to collisional interaction. The model chosen for this study is one-dimensional collision between a helium atom, taken as a structureless particle, and a hydrogen molecule, approximated as a simple harmonic oscillator. The harmonic oscillator was assumed to be prepared in its ground state before the collision. Thus,

$$|\psi(t=0)>=|0>, \tag{1}$$

and the initial quadrature variances are given by

$$(\Delta X_1)^2 = (\Delta X_2)^2 = \frac{1}{4}. \tag{2}$$

As the collision proceeds, the oscillator develops into a superposition state,

$$|\psi(t)>=\sum_n a_n(t)|n>=\sum_n |a_n(t)|e^{i\phi_n(t)}|n>. \tag{3}$$

The quadrature variances at time t are then given by [6]

$$\begin{aligned}(\Delta X_1)^2 &= \frac{1}{4}+\sum_n n|a_n|^2+\frac{1}{2}\sum_n \sqrt{n+2}\sqrt{n+1}|a_n||a_{n+2}|cos(\phi_{n+2}-\phi_n) \\ &\quad -[\sum_n \sqrt{n+1}|a_n||a_{n+1}|cos(\phi_{n+1}-\phi_n)]^2, \end{aligned}\tag{4}$$

and a similar expression for $(\Delta X_2)^2$. Wodkiewicz et. al. [7] have shown that a superposition state consisting of a finite number of eigenstates $|n>$ can exhibit squeezing for appropriate values of the magnitudes $|a_n|$ and phases ϕ_n of probability amplitudes, and thus there is a possibility of squeezing in the collision state given by Eq.(3). Our calculations, as reported earlier [5], show that there occurs a relatively strong squeezing near the time of minimum separation and a weak squeezing altanately in position and momentum after the collision is over.

It should be noted that, in most of the collision studies in the past, attention was focused on the magnitudes $|a_n|$ of the probability amplitudes as they yield the transition probabilities. For our study of collision-induced squeezing, however, the question of how the phases develop in time as the collision proceeds is also an important issue, because the variances $(\Delta X_1)^2$ and $(\Delta X_2)^2$ depend not only on the magnitudes $|a_n|$ but also on the phases ϕ_n, as can be seen from Eq.(4). Even if the magnitudes $|a_n|$ are fixed, the variances can take on different values for different phases ϕ_n.

In order to emphasize the importance of the phases, we show below that squeezing can be achieved from a coherent state simply by changing the phases alone. Let us consider a harmonic oscillator in a coherent state $|\alpha>$ at time $t=0$. If we let the oscillator develop freely in time, its state at time t is given by

$$|\psi(t)>= e^{-|\alpha|^2/2}\sum_n \frac{\alpha^n}{\sqrt{n!}}e^{-in\omega t}|n> . \tag{5}$$

The variances $(\Delta X_1)^2$ and $(\Delta X_2)^2$ remain $\frac{1}{4}$ throughout. Let us now assume that the phases of the coherent state are changed at time $t=0$ so that the oscillator develops in time according to

$$|\psi(t)>= e^{-|\alpha|^2/2}\sum_n \frac{\alpha^n}{\sqrt{n!}}e^{i\theta_n}e^{-in\omega t}|n> . \tag{6}$$

As compared with the coherent state, Eq(5), the state represented by Eq.(6) has additional constant phase factors θ_n. Althogh this state is not identical with the coherent state, it has the same Poissonian state distribution as the coherent state and may thus be called a "quasi-coherent state". It is our purpose to show that, with appropriate values of θ_n, the quasi-coherent state can show squeezing in X_1 or X_2. To illustrate this, let

$$\theta_n = \begin{cases} 0, & \text{if n is even,} \\ -\frac{\pi}{2} & \text{if n is odd.} \end{cases} \tag{7}$$

The state represented by Eqs. (6) and (7) are a linear combination of even and odd coherent states [8] with the relative phase between the even and odd states given by $\frac{\pi}{2}$. The variance $(\Delta X_1)^2$ for this state can easily be computed using Eq.(4), and similarly $(\Delta X_2)^2$. The result of the calculation is

$$(\Delta X_1)^2 = \frac{1}{4} + |\alpha|^2 - |\alpha|^2\sin^2(\phi-\omega t) - |\alpha|^2 e^{-4|\alpha|^2}\sin^2(\phi-\omega t), \tag{8}$$

$$(\Delta X_2)^2 = \frac{1}{4} + |\alpha|^2 - |\alpha|^2\cos^2(\phi-\omega t) - |\alpha|^2 e^{-4|\alpha|^2}\cos^2(\phi-\omega t). \tag{9}$$

The variances oscillate between v_{max} and v_{min} where,

$$v_{max} = \frac{1}{4} + |\alpha|^2, \quad v_{min} = \frac{1}{4} - |\alpha|^2 e^{-4|\alpha|^2} \tag{10}$$

It is evident that the quasi-coherent state with the phases given by Eq.(7) exhibits squeezing because $v_{min} < \frac{1}{4}$.

The example presented above shows clearly that two states with different phases in general have different degrees of squeezing, even if they have the same state distribution. This means that, even if one considers collision processes that produce the same state distribution, the degree of squeezing obtained during and after the collisions can be quite different, depending on how the phases ϕ_n of the probability amplitudes develop in time as the collisions proceed. It is therefore evident that, for a detailed study of collision-induced squeezing, further study on the time development of the phases in collisions and its relation to collision parameters such as potential energy surfaces and collision energy is needed.

Acknowledgments

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