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SWIRL as a Means of Liquid Management in Low Gravity

W. Gene Steward

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SWIRL as a Means of Liquid Management in Low Gravity

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1. INTRODUCTION

1.1 WHENCE SWIRL

Swirling of a liquid in a container may, in some applications, prove to be a better way of managing liquids in space than rotating the entire container. A transient study by the National Bureau of Standards in Gaithersburg Md.[1] showed that swirl could be started quickly by the injection of a relatively high velocity jet of liquid tangentially into the body of liquid. One unknown still remained: the quantity and velocity of jet flow, or mechanical power of a solid impeller required to maintain a given radial acceleration ("G" force).

The crux of this problem is to determine the rate of rotational energy dissipation by wall friction.

1.2 RECAP

The part of this study which ended in December '86 gave a first approximation of the jet requirements to maintain a given "G" force in a tank. A sample calculation was made for a hypothetical 5,000 liter liquid helium tank with a radial acceleration of 0.01g. The resulting estimation of jet flow was $3(10)^{-5}$ m^3/s. If the NBS (Boulder) centrifugal pump were used for transferring liquid helium from this tank, the jet flow just stated would amount to about 22% of the pump output. Since the required jet flow is a significant portion of the total pump output, the accuracy of calculation could affect a decision as to the feasibility of the jet method, or the suitability of a particular pump.

The present study investigates the possible additional effects of axial variation of tangential velocity and secondary (radial and axial) flow components.

1.3 SUMMARY OF THE PRESENT STUDY

1.3.1 Part 1 (under the assumption of negligible secondary flow)

Since the rotational energy dissipates along the cylinder walls and end closures, it seemed likely that the tangential velocity and angular momentum would diminish appreciably with axial distance from the input. Therefore, the first investigation concerned the effect of axial variation of angular momentum assuming negligible radial and axial velocity components. This assumption together with a simplified turbulent viscosity reduces the tangential equation of motion to a two dimensional (radial and axial) linear equation solvable in closed form. The solution produces both the tangential velocity radial profile and an axial decay factor.

The linear theory predicted that the velocity would decay to 1/10 of its input plane velocity in an axial distance of one radius. Simple visual experiments with a spinning horizontal disk in a vertical cylinder revealed a much different behavior characterized by:

1. A thin boundary layer on the spinning disk, cylinder walls, and bottom plate.
2. Very little axial decay of tangential velocity in the body of liquid.
3. The fluid body spinning at 1/2 to 1/6 the rotational speed of the disk (depending on the disk-bottom spacing).
4. Noticeable outward radial flow near the spinning disk, downward flow along the cylinder wall, inward flow at the bottom, and upward flow at the center.

To test the possibility that the linearization was at fault, the last attempt at "pure rotational flow" employed Prandtl's mixing length hypothesis for turbulent shear and required a numerical solution of the equation of motion. The resulting axial decay was only slightly less severe.

1.3.2 Integral momentum approach

Even though the radial and axial velocity appear insignificant, these secondary flows have a large effect as evidenced by the failure of the "pure rotation" theory to describe the actual flow. Therefore, the second approach applies a published method developed to predict the frictional torque due to the housings of rotating turbine or pump impellers (a three dimensional flow problem). For the swirl problem it was necessary to extend the theory to account for the cylinder walls and the existence of a free interior surface in the absence of gravity. The relationship between the cylinder length, drag coefficient, and the difference in speed between a spinning disk and the liquid was demonstrated in a general way by visual experiments.

The sample calculation for the 5,000 liter liquid helium dewar, repeated using the new drag coefficients, resulted in a much larger jet flow requirement. In terms of the NBS centrifugal pump, about 55% of its flow would be required to maintain 0.01g radial acceleration instead of the 22% calculated earlier.

2.0 ANALYSIS OF SWIRLING FLOW

2.1 AXIAL VARIATION OF TANGENTIAL VELOCITY ASSUMING NEGLIGIBLE SECONDARY FLOW

In the first part of this study it is assumed that radial and axial velocity components are negligible, ie. the flow is assumed to be purely rotational, but with tangential velocity varying in both radial and axial directions.

It should be noted that the final results cast doubt on the "pure rotation" hypothesis, and the work done under that hypothesis is only briefly outlined. Details are given in Appendix A.

2.1.1 A Linear Solution

The assumptions are:

1. Steady state .... $\partial/\partial t=0$
2. Pure rotation .... $v_r=0$, $v_z=0$
3. Axial symmetry .... $\partial/\partial\theta=0$
4. No external forces .... $g_\theta=0$

By the simplification of using an average value of the turbulence factor, $|\partial m/\partial z|/|\partial m/\partial r| = \varepsilon^2$, rather than considering it a variable, the θ equation of motion reduces to a linear equation in r and z, solvable in closed form. The z equation is exponential:

$$m(r,z) = m(r,0)\exp(z/z_d), \text{ where} \qquad (1)$$

z_d is the z decay constant whose numerical value comes from the solution of the equation in r.

The equation in r is a Bessel equation with the following general solution:

$$\frac{m(r,0)}{m_M} = \left[C_1 J_1\left(\frac{r\xi}{z_d}\right) + C_2 Y_1\left(\frac{r\xi}{z_d}\right)\right] \qquad (2)$$

J_1 is the Bessel function of the first kind, first order.
Y_1 is the Bessel function of the second kind, first order.

The values of the constants C_1, C_2, and z_d are dependent on the fullness of the cylinder ($r_i/r_o=0$ being full and $r_i/r_o=1$ being empty), and the Reynolds number. These constants are evaluated in tables A1 and A2. The resulting values of z_d/r_o being near 0.4 means that, in an axial distance of one radius, the tangential velocity would decay to less than 1/10 its value at z=0. A very simple visual experiment shows clearly that this is not true; in fact, it appears that z_d/r_o should be at least 2.

The calculated variation of ε from equation (2) reveals a spike near m_M, even though a constant value was used in the derivation. To test the possibility that the linearization (ε=const.) causes the unrealistically small z_d/r_o, the non-linear turbulent equation was solved numerically. The resulting z_d/r_o of 0.46 is hardly an improvement, and so it appears that the linearization was not the problem.

2.2 ADAPTATION OF A MOMENTUM INTEGRAL METHOD FOR TURBULENT DRAG DUE TO SWIRLING FLOW IN A HOUSING

The unrealistic decay is apparently caused by neglecting the secondary circulation which transfers momentum by large scale movement of fluid containing the momentum. This circulation, which takes place mostly within the boundary layers (or Ekman layers [4]), can be very powerful compared to (small scale) turbulent momentum transfer. Greenspan [4] states: "----secondary flow can significantly alter the primary motion through a slight redistribution of angular momentum and vorticity".

The drag on a pump impeller or turbine wheel in a housing is a problem similar to the present one in which secondary flows play an important role. Schultz-Grunow employed the momentum integral method as described in Greenspan [4] and Schlichting [3] and arrived at an expression for drag coefficient on a spinning disk in a housing. The connection with the present problem is that the drag on the rotating liquid, which we require, is the exact balance of the drag on the spinning disk. The spinning disk may either be a real or an imaginary device.

The adaptation of the Schultz-Grunow theory to account for an elongated cylinder and the possibility of free surfaces is <u>detailed in Appendix B</u>. The result is the following expression for frictional torque on a spinning disk (or torque of the liquid on the housing). Equations (B5) with (B9):

$$T_D = 0.041(2/f)^{1/5}\ \rho\omega_D{}^2 r_o{}^5 (1-\omega_f/\omega_D)^{7/5}\left(1-(r_i/r_o)^{23/5}\right)Re_D{}^{-1/5} \tag{3}$$

$$Re_D = \omega_D r_o{}^2/\nu$$

The remainder of the problem consists of determining the ratio ω_f/ω_D, and the factor f, both of which are functions of the geometry .

Appendix B contains the details of the modifications of the Schultz-Grunow equation to include:

1. An elongated cylindrical section
2. A possible free interior surface
3. A possible solid interior concentric cylinder wall

(as in proposed ground experiments)

4. A possible free top surface as in normal gravity swirl in a partially filled vertical cylinder.

The rotational frequency ratio ω_f/ω_D is worked out for two cases which cover the four possibilities listed above:

<u>Case 1</u>. For a partially or completely filled cylinder in zero gravity having a free surface (ie. no torque) at r_i, from (B15) and (B16):

$$\omega_D/\omega_f = \left(F(L)+1\right)^{4/7} + 1 \tag{4}$$

where,

$$F(L) = \frac{23}{5}\,\frac{L}{r_o}\left[\frac{r_{oc}}{r_o}\right]^{18/5}\frac{1}{1-(r_i/r_o)^{23/5}} \tag{5}$$

<u>Case 2</u>. Swirl in an open top vessel in a normal gravity environment. There will be no torque on the top end ($T_E=0$). The following also allows for a possible inner fixed cylinder as in a proposed swirl test apparatus:

$$\omega_D/\omega_f = \left[F(L)\left(1+(r_i/r_o)^{18/5}\right)\right]^{4/7} + 1 \tag{6}$$

The factor f arises from the increase in the boundary layer thickness due to the added frictional surfaces. The quantity $(2/f)^{1/5}$ can be considered unity except for very long cylinders.

$$f = 2 + f_c, \tag{7}$$

where f_c is the cylinder contribution. Equation (B19):

$$f_c = \frac{18}{5}\left[\frac{\omega_f/\omega_D}{1-\omega_f/\omega_D}\right]^{7/4}\frac{L}{r_o}\left[\frac{r_{oc}}{r_o}\right]^{18/5}\frac{1}{1-(r_i/r_o)^{18/5}}$$

Appendix B also presents experimental data which gives some credence to the modification.

<u>Comparison With Previous Drag Calculations</u>

Returning to the example of final report 12/86:

5,000 ℓ cylindrical tank
Arbitrarily specified radial acceleration 0.01g
Total length = 2L ----------------- 2.94 m
r_i -------------------------------- 0.635 m
r_o -------------------------------- 0.735 m
$\omega_f = v_\theta/r$ --------------------------- 0.393 s^{-1}

$r_{oc} = r_o$

$r_i/r_o=0.864$
$L/r_o=2$
Fluid: normal liquid helium
$\nu=\mu/\rho=8.28(10)^{-8}\ m^2/s$
$\rho=145\ kg/m^3$

$Re_D=1.66(10)^8$
$F(L)=18.8$ (equation (5))
$\omega_f/\omega_D=0.154$ (equation (4))
$f=2.79$ (equation (7))

$T=2T_D=0.128\ kg\ m^2/s^2$

If an actual spinning disk were used to generate the swirl:

Power to the spinning disk = $T\omega_D=0.332$ w.

The frictional torque calculated in the 12/86 report was 0.0488; the new value is 2.7 times the old for the same conditions. The jet flow requirement, being approximately proportional to the frcitional torque will also be 2.7 times the previous value, or 57% of the NBS pump output to maintain 0.01g radial acceleration.

The torque also varies with the degree of fulness of the vessel as follows:

	r_i/r_o	ω_D/ω_f	T
Full	0	4.77	0.139
	0.2	4.77	0.139
	0.4	4.80	0.139
	0.6	4.97	0.137
Example	0.864	6.51	0.130
Empty	1.0	∞	0

3.0 CONCLUSIONS

The pure rotation assumption leads to a predicted sharp decrease in tangential velocity with axial distance from the input plane. A simple experiment with swirling flow in a cylinder demonstrates that axial decay of velocity is actually very much smaller.

The unrealistic decay produced by the pure rotation assumption indicates that radial and axial circulation within the boundary layers have a strong effect on the momentum distribution and on the frictional drag. The published theory of Shultz-Grunow treats a similar subject of turbulent rotating flow with radial and axial circulation in a housing. This theory, modified to match the present application, predicts nearly three times the drag calculated previously for a sample case. A far thinner boundary layer is the reason for the increased drag. Simple visual experiments lend credence to the modified theory.

A swirl inducing jet flow was also three times as large as originally thought (57% the NBS pump's output for the example 5,000 ℓ tank) and this would mean:

1. A larger pump than the NBS pump would be required for tanks the size of the example, or
2. A smaller radial acceleration than 0.01g would be achieved.

Since frictional torque is proportional to the vessel surface area, the NBS pump suffices for a smaller tank.

Other means of producing swirl should be considered. The sample calculation shows that the mechanical power to achieve 0.01g in a 5,000 ℓ tank is only 0.34 watts.

4.0 WHITHER SWIRL

The Schultz-Grunow theory underpredicts his own experimental data by 17%, and the crude experimental observations reported here indicate that the modification for long cylinders may not be exactly correct. Therefore, it seems that theorizing has gone about as far as it should go whithout accurate experimental measurements of rotation rates and drag due to turbulent swirl in a scaled configuration similar to the application. Normal gravity experiments with a spinning disk, both at room temperature and with liquid helium, would be beneficial.

A good check of jet flow rate to produce a given swirl velocity could be performed at room temperature with an apparatus only slightly more sophisticated than the one used to visualize the rotational velocity. Experiments of this type would be a useful supplement to the planned helium swirl tests.

The findings of this swirl study regarding secondary flow, as well as the extensive studies of Greenspan, have a bearing on the spin-up of liquid in an impulsively started cylinder. It seems that spin-up time could be greatly reduced by intentionally stimulating radial and axial flow during spin-up. Design and testing of suitable vanes to accomplish this would seem desirable.

5 NOTATION

C ------------- coefficient in Prandtl's mixing length hypothesis
C_1, C_2, C_3, C_4 ------------------------- constants of integration

C_f ------------ coefficient of the expression for drag coefficient
C_s ------------------------------ constant defined in equation (B5)
f ------------------------------ drag ratio defined in equation (B2)
$F(L)$ -------------- function of geometry defined in equation (B16)
F_τ ---------------- wall shear function defined following table A1
g ------------------------------ acceleration of gravity 9.8 m/s^2
g_e ---------------------------------- external force per unit mass
J_1, J_0, Y_1, Y_0 - Bessel functions used in equations (A7) and (A13)
ℓ ------------------------------------ Prandtl's mixing length, m
L ------------------------- half length of a cylindrical vessel, m
m ------------------------- angular momentum per unit mass, kg m/s
m_M ---- maximum angular momentum per unit mass in momentum profile
p --- pressure, Pa
r, r_i, r_M, r_o, r_{oc} --------- radius, internal, at maximum momentum, outer, outer-cylinder, m
Re, Re_f, Re_D ------------ Reynolds number, based on fluid velocity, based on disk velocity
t -- time, s
T, T_{cyl}, T_D, T_E, T_I ---------------- torque, on cylinder, on disk, on end disk, on internal cylinder, kg m^2/s^2
v, v_r, v_z, v_θ, $v_{\theta M}$ ----------- velocity, radial, axial, tangential, tangential at maximum momentum, m/s
z, z_D ------------------- axial coordinate, axial decay length, m

δ ------------------------------------ boundary layer thickness, m
θ ------------------------------------ tangential angular coordinate
μ, μ_T -------------------- viscosity, turbulent viscosity, kg/m·s
ν ------------------------------ kinematic viscosity = μ/ρ, m^2/s
ξ ------------------- ratio of momentum gradients, equation (A5)
ρ -- density, kg/m^3
τ, $\tau_{r\theta}$, $\tau_{\theta z}$ -- shear stress, r-θ component, θ-z component, $kg/m \cdot s^2$
ω, ω_D, ω_f ----------- rotational frequency, of disk, of fluid, s^{-1}

6 CITED REFERENCES

[1] T. T. Yeh, "A Vortex-Induced, Gas-Liquid separation in a Cylindrical Tank at Zero Gravity", U.S. Dept. of Commerce NBSIR 86-3322 (1986)

[2] B. E. Launder and D. A. Spalding, "Mathematical Models of Turbulence", Academic Press (1972)

[3] H. Schlichting, "Boundary Layer Theory",

[4] H. P. Greenspan, "The Theory of Roatating Fluids", Cambridge University Press, Cambridge (1968)

APPENDIX A

AXIAL VARIATION OF TANGENTIAL VELOCITY ASSUMING NEGLIGIBLE SECONDARY FLOW

A.1 A LINEAR SOLUTION FOR PURE ROTATION IN TWO DIMENSIONS

The θ component equation of motion in cylindrical coordinates is, in terms of shear stress [1]:

$$\rho\left[\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r} + v_z\frac{\partial v_\theta}{\partial z}\right] = -\frac{\partial p}{\partial \theta} + \rho g_\theta$$
$$-\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\tau_{r\theta}\right) + \frac{1}{r}\frac{\partial \tau_{r\theta}}{\partial \theta} + \frac{\partial \tau_{\theta z}}{\partial z}\right] \tag{A1}$$

The following assumptions will be made:

1. Steady state .... $\partial/\partial t=0$
2. Pure rotation .... $v_r=0,\ \ v_z=0$
3. Axial symmetry .... $\partial/\partial\theta=0$
4. No external forces .... $g_\theta=0$

Then (1) becomes:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\tau_{r\theta}\right) + \frac{\partial \tau_{\theta z}}{\partial z} = 0 \tag{A2}$$

These equations apply to turbulent flow as well as laminar flow when appropriate expressions are used for the shear stresses. The expressions for turbulent shear stresses can become extremely complex, even differential equations in themselves. Launder and Spaulding [2] point out that Prandtl's mixing length hypothesis (MLH) is still useful at least for approximations, and since it is relatively simple it is used here. If, in Prandtl's intuitive development of MLH for flow over a flat plate, one substitutes angular momentum transport for linear momentum transport and substitutes torque for force, the following expressions for rotating flow result: (same assumptions as above)

$$\tau_{\theta z} = \left[\mu - \mu_{Tz}\right]\frac{1}{r}\frac{\partial m}{\partial z}\text{, where} \tag{A3}$$

$$\mu_{Tz} = \frac{C\rho\ell^2}{r}\left|\frac{\partial m}{\partial z}\right| .$$

$$\tau_{r\theta} = \left[\mu - \mu_{T\theta}\right]\frac{1}{r}\frac{\partial m}{\partial r} + 2\mu m/r^2\text{ , where} \tag{A4}$$

$$\mu_{T\theta} = \frac{C\rho\ell^2}{r}\left|\frac{\partial m}{\partial r}\right| .$$

In (A3) and (A4) ℓ is the mixing length, generally taken to be the distance from the nearest surface, and c is an empirical constant. In what follows ℓ and c are assumed to be the same in

(A3) and (A4), and the eventual results are independent of their numerical values. Except for an extremely thin laminar sub-layer, μ is negligible compared to μ_T. Then, in the turbulent region, (A3) and (A4) substituted into (A2) gives:

$$\frac{\partial^2 m}{\partial r^2} - \frac{1}{r}\frac{\partial m}{\partial r} + \xi^2 \frac{\partial^2 m}{\partial z^2} = 0 \text{ , where} \qquad (A5)$$

$$\xi^2 = \mu_{Tz}/\mu_{T\theta} = |\partial m/\partial z| / |\partial m/\partial \theta| \text{ .}$$

As a first approximation we can assign a constant value to ξ. With ξ constant equation (5) is solvable by separation of variables. The equation in z is an exponential:

$$m(r, z) = m(r, 0)\exp(z/z_0) \text{, where} \qquad (A6)$$

z_0 is the z decay constant whose numerical value comes from the solution of the equation in r.

The equation in r is a Bessel equation with the following general solution:

$$m(r, 0) = r\left[C_3 J_1\left(\frac{r\xi}{z_0}\right) + C_4 Y_1\left(\frac{r\xi}{z_0}\right)\right] \quad \text{, or}$$

$$\frac{m(r, 0)}{m_M} = \left[C_1 J_1\left(\frac{r\xi}{z_0}\right) + C_2 Y_1\left(\frac{r\xi}{z_0}\right)\right] \qquad (A7)$$

$C_1 = C_3 \quad v_{\theta M}$
$C_2 = C_4 \quad v_{\theta M}$
J_1 is the Bessel function of the first kind, first order.
Y_1 is the Bessel function of the second kind, first order.

A.2 BOUNDARY CONDITIONS

The three unknowns C_1, C_2, and z_0 require three boundary conditions.

Boundary Condition #1. Since we have eliminated the laminar sub-layer from the solution, we will make the condition at the inside edge of that layer ($r=r_o$) the first boundary condition.

The "universal velocity profile" [3] places the edge of the laminar sub-layer at $y^+ = 5$, where

$$y^+ = \frac{r_W - r_o}{\mu}\sqrt{\tau_o \rho} = 5 \text{ .} \qquad (A8)$$

Also in the laminar sub-layer, $y^+ = v^+$ where

$$v^+ = v_\theta(r_o, z)\sqrt{\frac{\rho}{\tau_o}} = 5 \qquad \text{(A9)}$$

Equation (A9) introduces the wall shear, and the Blasius expression for wall shear [3] makes the connection back to the maximum angular momentum (which is the input quantity):

$$\tau_o = 0.0225\ \rho\ v_M^2\left(\frac{1}{1-r_M}\right)^{1/4}(Re_M)^{-1/4}\ . \qquad \text{(A10)}$$

$$Re_M = \frac{\rho v_M r_o}{\mu}\ ,$$

v_M is the velocity at maximum angular momentum,
r_M is the radius at maximum angular momentum.

Substitution of (10) into (9) gives Boundary Condition #1:

$$m(r_o, 0) = 0.75\ r_o\ v_M\ \left(\frac{1}{1-r_M}\right)^{1/8}(Re_M)^{-1/8} \qquad \text{(A11)}$$

It should be noted that a guess of r_M must be used for the first trial solution, then succeeding trial values are obtained by setting the derivitive of the momentum equation (eq.(A13)) equal to 0 and solving for $r=r_M$.

Boundary Condition #2. Zero shear stress at the free surface.

At the free surface turbulent fluctuations disappear; therefore, $\mu_{T\theta} = 0$. The condition, $\tau_{r\theta} = 0$ at r_i yields:

$$(\partial m/\partial r)_i = 2m(r_i, 0)/r_i \qquad \text{(A12)}$$

Equation (A7) differentiated is:

$$\frac{\partial m}{\partial r} = \frac{r\varepsilon}{z_d}\left[C3\ J_0\left[\frac{r\varepsilon}{z_d}\right] + C4\ Y_0\left[\frac{r\varepsilon}{z_d}\right]\right]\ . \qquad \text{(A13)}$$

J_0 is the Bessel function of the first kind, 0 order,
Y_0 is the Bessel function of the second kind, 0 order.

Equations (A12) and (A13) form the second boundary condition.

Boundary Condition #3. Maximum angular momentum.

The maximum angular momentum, m_M, is considered to be a given quantity. The angular momentum profile is normalized by dividing all $m(r,z)$ by m_M.

Solution

The solution for C1, C2, and z_d (program ZGROT) depends upon the fullness of the cylinder, r_i/r_o ($r_i/r_o=0$ being full, and $r_i/r_o=1$ being empty), and the Reynolds Number range, as outlined in the following table:

TABLE A1, For $10^6 < Re < 10^8$ $Re_{ref} = 5*10^7$

r_i/r_o	C1	C2	$z_d/(r_o \varepsilon)$	r_M/r_o	F_τ *
0.01	1.926	3.317E-7	0.2748	0.6609	9.323E+8
0.2	1.935	4.535E-2	0.2730	0.6628	9.336E+8
0.4	1.958	0.5564	0.2584	0.6823	9.477E+8
0.6	0.9068	2.175	0.2065	0.7363	9.930E+8
0.8	-0.8738	-3.1489	0.1229	0.8374	1.120E+9
1.0			0.0	1.0	

* F_τ is a dimensionless quantity from which the wall shear can be calculated:

$$F_\tau = \tau_o \rho (r_o/\mu)^2 (Re_{ref}/Re)_2$$

Figure A1 shows the computed angular momentum profiles.

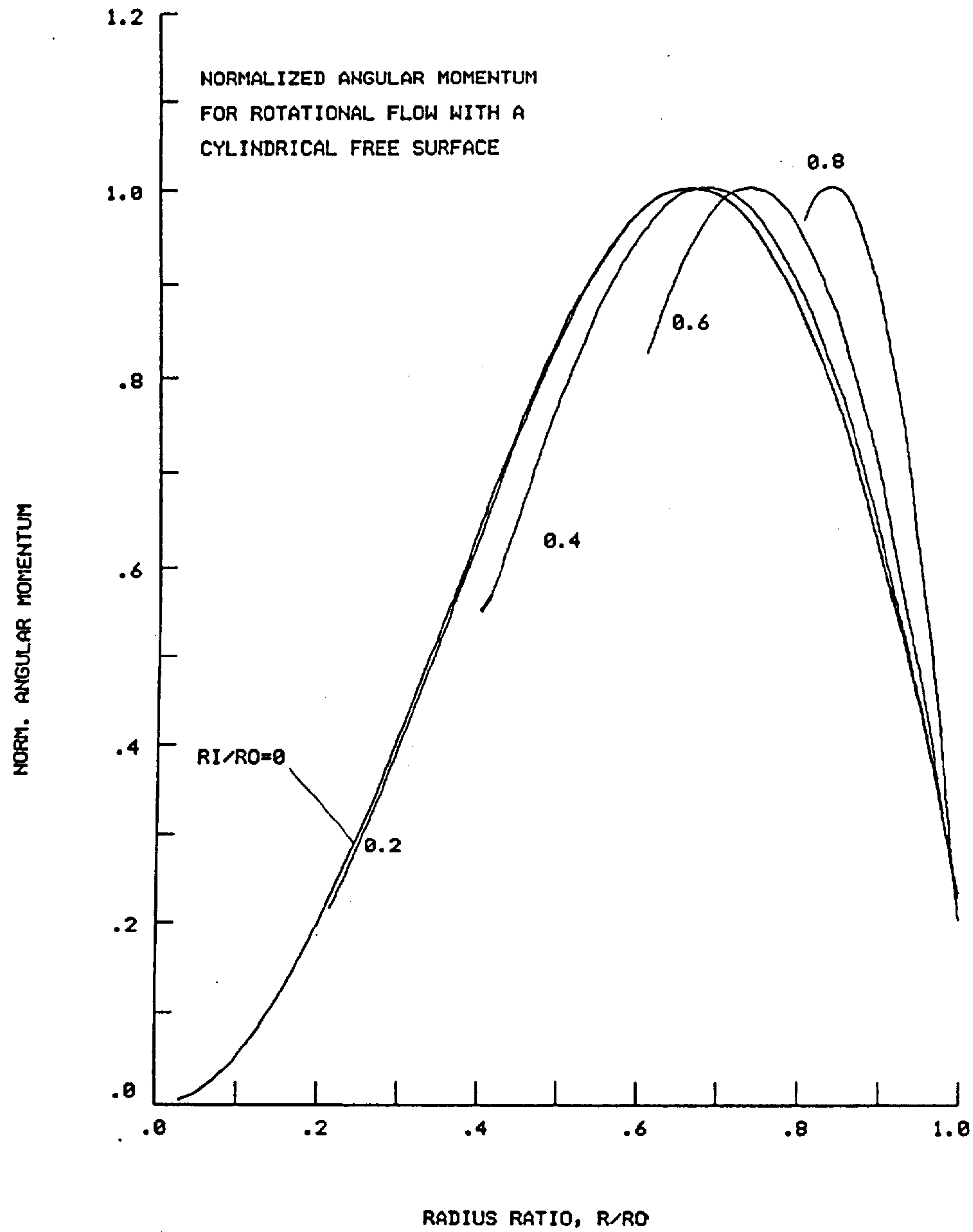

FIGURE A1.

Program ZGROT also computes the factor ε, the ratio of axial to radial angular momentum derivitives, and the average of ε over r once C1, C2, and $z_d/(r_o\varepsilon)$ are computed.

<u>TABLE A2</u>

r_i/r_o	ε(avg)	z_d/r_o
0.01	1.42	0.391
0.2	1.49	0.407
0.4	1.53	0.409
0.6	1.65	0.341
0.8	1.70	0.209

These values of z_d/r_o indicate a large decay of angular momentum in the z direction. For example, in an axial distance of one radius the angular momentum or angular velocity would decay to 1/10 its value at z=0. A simple visual experiment shows this to be incorrect.

A.2 <u>NUMERICAL SOLUTION OF THE MOMENTUM EQUATION WITH VARIABLE ε</u>

(Other boundary conditions and assumptions as in A.1)

In the linear solution of section A.1 an average (constant) value of ε was used. The radial variation of ε calculated from the resulting equation shows a spike in the region of m_M, so that the assumption is inconsistent with the results. (This is paradoxical because a solution which allows a variable ε results in a more nearly constant ε). To test the possibility that the constant ε assumption resulted in erroneously small values of z_d/r_o, the following non-linear equation (which makes no assumptions concerning the momentum derivitives) was solved numerically. The equation contains the complete expressions for $\tau_{r\theta}$ and $\tau_{\theta z}$, equations (3A) and (4A) so that the computation should be valid into the laminar regions. (To obtain the same z variation as before it is only necessary to assume similar profiles.)

z variation (equation A6).

r variation:

$$r\frac{\partial^2 m}{\partial r^2} - \frac{\partial m}{\partial r} + C\ell^2\frac{1}{\nu}\frac{\partial}{\partial r}\left|\frac{\partial m}{\partial r}\right|\frac{\partial m}{\partial r} + r\frac{m}{z^2} + 2C\ell^2\frac{1}{\nu}\frac{m^2}{z_d{}^3} = 0 \qquad \text{(A14)}$$

C and ℓ are the "mixing length" parameters which were discussed and evaluated in the midterm report.

A sample calculation for r_i/r_o=0.2 resulted in z_d/r_o=0.464 which is still too small (observations would indicate z_d/r_o > 2).

APPENDIX B

ADAPTATION OF THE SCHULTZ-GRUNOW INTEGRAL MOMENTUM METHOD

The Schultz-Grunow case of interest is one in which the axial spacing between the disk and housing is considerably greater than the combined boundary layers. Contrary to the present problem, however, the length of cylinder between the rotor and housing is not considered enough to contribute to the drag, so it is necessary to extend the theory. It is also necessary to account for partially filled cylinders ($r_i/r_o > 0$) and for the existence of a free surface as would occur in a low gravity environment. The presence of an actual spinning disk is immaterial since the flow in the liquid core does not depend on the means (such as a jet or spinning disk) used to induce the rotation.

The approach taken is that the central core of liquid rotates as a solid body and the viscous effects and secondary flows take place in the thin boundary layers next to the solid and free surfaces. This description agrees basically with the visual perception.

B.1 EXTENSION OF SCHULTZ-GRUNOW TO ACCOUNT FOR A LONG CYLINDRICAL SECTION AND AN INTERIOR FREE SURFACE

The boundary layer thickness δ is taken to be that thickness of liquid which, being dragged along with the spinning disk, has exactly the centrifugal force to balance the drag due to the radial velocity component. The tangential drag in terms of δ is that due to Blasius [3] and the 1/7 power volocity distribution:

$$\tau_o \cos(\theta) = C_f \rho [r(\omega_D - \omega_f)]^{7/4} (\nu/\delta)^{1/4} \tag{B1}$$

θ is the angle of the shear stress with respect to the tangential velocity.

The next task is to determine δ, and that is accomplished by balancing the centrifugal force in the layer δ against the drag force from (B1):

$$f \tau_o \sin(\theta) 2\pi r dr = \rho r \omega_D^2 \delta 2\pi r dr \tag{B2}$$

f = (contribution to radial drag force due to all of the surfaces)/(radial drag force on the spinning disk alone)

From (B1) and (B2) the boundary layer thickness is:

$$\delta = (C_f \tan(\theta))^{4/5} r^{3/5} (\nu/\omega_D)^{1/5} (1-\omega_f/\omega_D)^{7/5}. \qquad (B3)$$

The torque on one side of the disk is (in our case integrating from r_i instead of 0):

$$T_D = \int_{r_i}^{r_o} r(2\pi r)\tau_o \cos(\theta)\,dr \qquad (B4)$$

Substitution from (B1) and (B3) gives:

$$T_D = C_s\ \rho\omega_D{}^2 r_o{}^5 (1-\omega_f/\omega_D)^{7/5} \left(1-(r_i/r_o)^{23/5}\right) Re_D{}^{-1/5}\ . \qquad (B5)$$

Where $C_s = 1.366\ C_f{}^{4/5} \left(f\ \tan(\theta)\right)^{-1/5}$,

$Re_D = \omega_D r_o{}^2/\nu$.

Equation (B5) as developed is a more general case of the following Schultz-Grunow equation (for one side of the disk):

$$T_D = 0.0311(1/2\ \rho\omega_D{}^2 r_o{}^5) Re^{-1/5} \qquad (B6)$$

C_s can be evaluated from (B5) and (B6) by matching the conditions in (B5) to those of (B6); namely, $r_i=0$, and $\omega_f/\omega_D=0.5$ (Schlichting [3]), and f=2 (end disk drag = spinning disk drag). Then,

$$C_s = 0.041(2/f)^{1/5} \qquad (B7)$$

<u>Cylindrical Section</u>

The tangential torque on one half of the cylinder is:

$$T_{cyl} = 2\pi r_{oc}{}^2 L \tau_o \cos(\theta) \qquad (B8)$$

$$\tau_o \cos(\theta) = C_f \rho [r_{oc}\ \omega_f]^{7/4} (\nu/\delta)^{1/4} \qquad (B9)$$

Assuming δ the same as on the rotating disk we get:

$$\frac{T_{cyl}}{T_D} = \frac{23}{5}\ \frac{L}{r_o} \left[\frac{r_{oc}}{r_o}\right]^{18/5} \left[\frac{\omega_f/\omega_D}{1-\omega_f/\omega_D}\right]^{7/4} \frac{1}{1-(r_i/r_o)^{23/5}} \qquad (B10)$$

<u>Fixed End Closure</u>

The torque due to one fixed end is:

$$T_E = \int_{r_i}^{r_o} r(2\pi r)\tau_o \cos(\theta)\,dr\ , \qquad (B11)$$

and the shear stress is

$$\tau_o \cos(\theta) = C_f \rho (r\ \omega_f)^{7/4} (\nu/\delta)^{1/4}\ . \qquad (B12)$$

Again assuming the same δ we get:

$$\frac{T_E}{T_D} = \left[\frac{\omega_f/\omega_D}{1-\omega_f/\omega_D}\right]^{7/4} \tag{B13}$$

Inner Fixed Cylinder (optional)

If an inner fixed cylinder exists, its torque contribution is the same as (B10) with r_{oc} replaced by r_i:

$$\frac{T_I}{T_D} = \frac{23}{5}\frac{L}{r_o}\left[\frac{r_i}{r_o}\right]^{18/5}\left[\frac{\omega_f/\omega_D}{1-\omega_f/\omega_D}\right]^{7/4}\frac{1}{1-(r_i/r_o)^{23/5}} \tag{B14}$$

Since the torque on the disk (whether the disk is actual or imaginary) balances the fluid torque on all of the contacting surfaces, equation (B5) gives the fluid torque. Changes in the rotation ratio ω_f/ω_D reflect the changes in geometry from that assumed in the original derivation. The following cases are two of the configurations of interest:

Case 1. For a partially or completely filled cylinder in zero gravity having a free surface (ie. no torque) at r_i, the following holds:

$$1 = T_{cyl}/T_D + T_E/T_D .$$

This, combined with (B10) and (B13) yields:

$$\omega_D/\omega_f = \lfloor F(L)+1 \rfloor^{4/7} + 1 \tag{B15}$$

where,

$$F(L) = \frac{23}{5}\frac{L}{r_o}\left[\frac{r_{oc}}{r_o}\right]^{18/5}\frac{1}{1-(r_i/r_o)^{23/5}} . \tag{B16}$$

Case 2. Swirl in an open top vessel in a normal gravity environment: there will be no torque on the top end (T_E=0). The following also allows for a possible inner fixed cylinder as in a proposed swirl test apparatus:

$$1 = T_{cyl}/T_D + T_I/T_D$$

$$\omega_D/\omega_f = \left[F(L)\left(1+(r_i/r_o)^{18/5}\right)\right]^{4/7} + 1 \tag{B17}$$

Now it is possible to calculate the factor f in equation (B7) (it should be pointed out that $(2/f)^{1/5}$ can be considered unity except for very long cylinders):

$$f = 1 + f_c + f_E \tag{B18}$$

If the liquid contacts a fixed end closure f_E=1; if not, as in case 2, f_E=0.

$$f_c = \frac{18}{5}\left[\frac{\omega_f/\omega_D}{1-\omega_f/\omega_D}\right]^{7/4}\frac{L}{r_o}\left[\frac{r_{oc}}{r_o}\right]^{18/5}\frac{1}{1-(r_i/r_o)^{18/5}} \qquad \text{(B19)}$$

B.2 EXPERIMENTAL CONFIRMATION OF THE ROTATION RATIO

Since equations (B16) and (B17) involve observable rotation ratios, a visual experiment is useful for at least a crude check. For this purpose I constructed a plexiglass cylinder in which swirl was induced by a vertical axis spinning disk. Near-neutral-buoyancy particles made the liquid rotation visible. Both the liquid and disk rotation rates were timed by stopwatch. The specifics are:

Fluid ----------------	water
r_i -------------------	0
r_o -------------------	2.27 in. -- 0.0576 m
r_{oc} ------------------	2.13 in. -- 0.0544 m
L --------------------	varied from 0.5 to 5.625 in.
ω_D -------------------	3.5 rev/s (approx. constant)

Figure B1 shows the results of 30 measurements at 6 values of L/r_o.

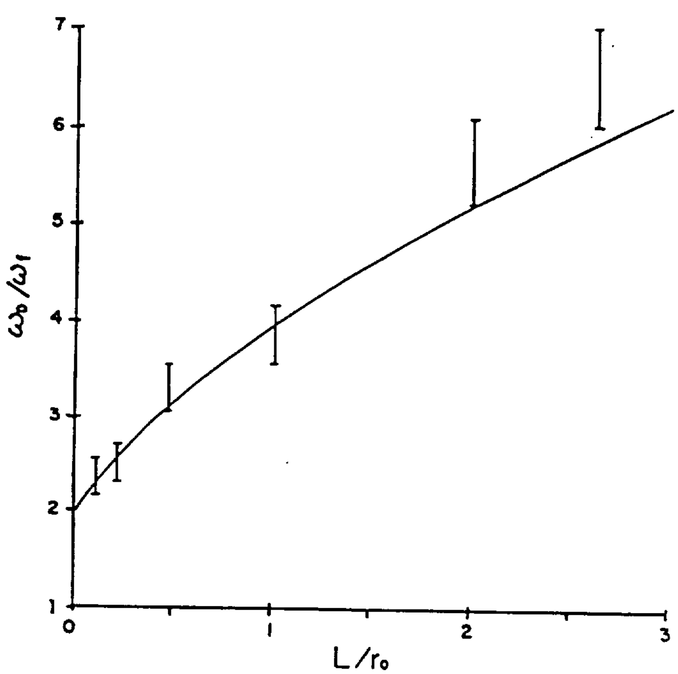

Figure B1. Disk/liquid rotation frequency ratio as a function of cylinder length.

The vertical bars show the approximate spread of the data. The main experimental difficulty was in following particles which tended to drift in and out of the main rotating core. As a result, the timing of rotation was imprecise.

The solid line represents the theory. From equation (B16)

$$F(L) = 5.76(L/r_o),$$

and the rotation ratio ω_D/ω_f was obtained from equation (B15).

These simple experiments cannot actually check the theory since the frictional torque was not measured; however, the effect of the added cylinder length can be checked since length is reflected in the change in ω_f/ω_D from the Schultz-Grunow case where $\omega_f/\omega_D=0.5$. Even though the observed ω_D/ω_f appear to be about 9% higher than the predictions it seems reasonable to conclude that the observations generally substantiate the modifications. The frictional drag has already been compared with experiments as documented in Schlichting [3], and the Schultz-Grunow predictions were found to be 17% low over a Reynolds Number range $2(10)^5$ to $(10)^7$.

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Swirling of a liquid in a container may prove to be a more desirable method of managing liquids in low gravity (space) environments than by rotating the entire container. By injecting a relatively high velocity liquid tangentially into the body of the fluid, swirl can be started rapidly, however an estimate of the quantity and velocity of jet flow, or mechanical power of a pump impeller required to maintain a given radial acceleration (G force) is needed to assess the feasibility of such a method. While the key aspect of the problem is determining the rate of rotational energy dissipation by wall friction in the container, there are other considerations, and the present study investigates the possible additional effects of axial variation of tangential velocity and secondary (radial and axial) flow components within the rotating fluid.

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