185287 **335**

N94-12311

Mixing in a stratified shear flow: energetics and sampling

By G. N. Ivey[1], J. R. Koseff[2], D. A. Briggs[2] AND J. H. Ferziger[2]

Direct numerical simulations of the time evolution of homogeneous stably stratified shear flows have been performed by Holt (1990), Holt *et al.* (1992), and Itsweire *et al.* (1992) for Richardson numbers from 0 to 1 and for Prandtl numbers between 0.1 and 2. The results indicate that mixing efficiency R_f varies with turbulent Froude number in a manner consistent with laboratory experiments performed with Prandtl numbers of 0.7 and 700. However, unlike the laboratory results, for a particular Froude number, the simulations do not show a clear dependence on the magnitude of R_f on Pr. The observed maximum value of R_f is 0.25. When averaged over vertical length scales of an order of magnitude greater than either the overturning or Ozmidov scales of the flow, the simulations indicate that the dissipation rate ϵ is only weakly lognormally distributed with an intermittency of about 0.01 whereas estimated values in the ocean (Baker and Gibson (1987)) are 3 to 7.

1. Introduction

The specification of the buoyancy flux in a fluid with a stable density gradient and subjected to forcing by either mean shear or a mechanical means of generating turbulent kinetic energy is crucial to the understanding of mixing processes in a wide variety of geophysical applications. Quantifying the rate of vertical mixing in the ocean, for example, is central not only to parameterizing vertical heat and salt fluxes, but also those of passive tracers such as pollutants and chemicals. Thus, predicting the mixing efficiency or flux Richardson number R_f is of central importance in turbulent mixing in density stratified flows. Ivey and Imberger (1991) (hereafter II) introduced a generalized definition of R_f based on the full turbulent kinetic energy equation and demonstrated that available laboratory measurements had peak values of $R_f = 0.20$ for fluids with Prandtl number of 700 and 0.15 for fluids with Prandtl number of 0.7. In the laboratory work, turbulence was generated with grids and, in the case of Rohr *et al.* (1988), by both grids and a mean shear. This work has been extended by the direct numerical simulation work of Holt (1990), Holt *et al.* (1992), and Itsweire *et al.* (1992), who numerically studied the temporal evolution of homogeneous turbulence subjected to a constant mean velocity and density gradients using the pseudo-spectral method of Rogallo (1981). The Boussinesq form of the Navier Stokes equations were solved for the three dimensional velocity and density fields on a $128 \times 128 \times 128$ grid. The velocity fields

1 Centre for Water Research, University of Western Australia, Nedlands, Western Australia
2 Stanford University

were initially isotropic with Reynolds numbers based on the integral scale and the turbulent kinetic energy ranging from 52 to 104, Richardson numbers ranging from 0.058 to 1.0, and Prandtl numbers of either 1 or 2. In the oceanic case, the density stratification is predominantly due to vertical temperature gradients. Therefore, in order to investigate the differences in the behavior of R_f with Pr observed by II, we have extended the earlier work of Holt *et al.* (1992) by performing additional simulations at values of Ri from 0.075 to 1.0 with Prandtl numbers as low as 0.1; these results are described in Section 3 below.

Oceanic turbulence is intermittent in character, and there exists considerable controversy about how to correctly sample the flow to determine the volume averaged dissipation (eg. Baker and Gibson (1987)). We examine this question by averaging over sub-sections of our $128 \times 128 \times 128$ domain and calculating the intermittency, defined as the variance of the logarithm of a quantity: the results are summarized in Section 4. Finally, Section 5 summarizes our conclusions.

2. Dynamics of mixing in stratified flows

The full turbulent kinetic energy equation can be written as:

$$-\frac{\partial}{\partial t}(\frac{\overline{q^2}}{2}) - \frac{\partial}{\partial x_j}(\frac{\overline{u_j q^2}}{2}) - \frac{\partial}{\partial x_j}(\frac{1}{\rho_o}\overline{u_j p}) - \overline{u_i u_j}\frac{\partial \overline{U_i}}{\partial x_j} = \frac{g}{\rho_o}\overline{\rho u_3} + \epsilon = B + \epsilon \quad (1)$$

where $\overline{q^2} = \overline{u_1^2} + \overline{u_2^2} + \overline{u_3^2}$ is twice the turbulent kinetic energy, ρ_o is the background density, $\epsilon = \nu\overline{\frac{\partial u_i}{\partial x_j}\frac{\partial u_i}{\partial x_j}}$ is the rate of dissipation of turbulent kinetic energy, and velocity component u_3 is in the direction of the gravity vector. For homogeneous turbulence at steady state, (1) can be written as $P = B + \epsilon$ where P is the production of turbulent kinetic energy (the last term on the left hand side of (1)) Then, II defined

$$R_f = \frac{B}{P} = \frac{1}{1 + (\epsilon/B)} \quad (2)$$

and also introduced the concept of a turbulent Froude number associated with the energy bearing eddies defined by

$$Fr_T = \frac{(\overline{q^2})^{1/2}}{NL_c} = (\frac{L_o}{L_c})^{2/3} \quad (3)$$

where L_c is the scale of the most energetic overturns, $L_o = (\epsilon/N^3)^{1/2}$ represents the Ozmidov scale, and N is the buoyancy frequency characterizing the stable background stratification. An alternate form of (2) is

$$R_f = \frac{1}{1 + Fr_T^2/R_{\rho w}} \quad (4)$$

where $R_{\rho w}$ is the correlation coefficient between the density and vertical velocity fluctuations. II found the available laboratory measurements were in accord with
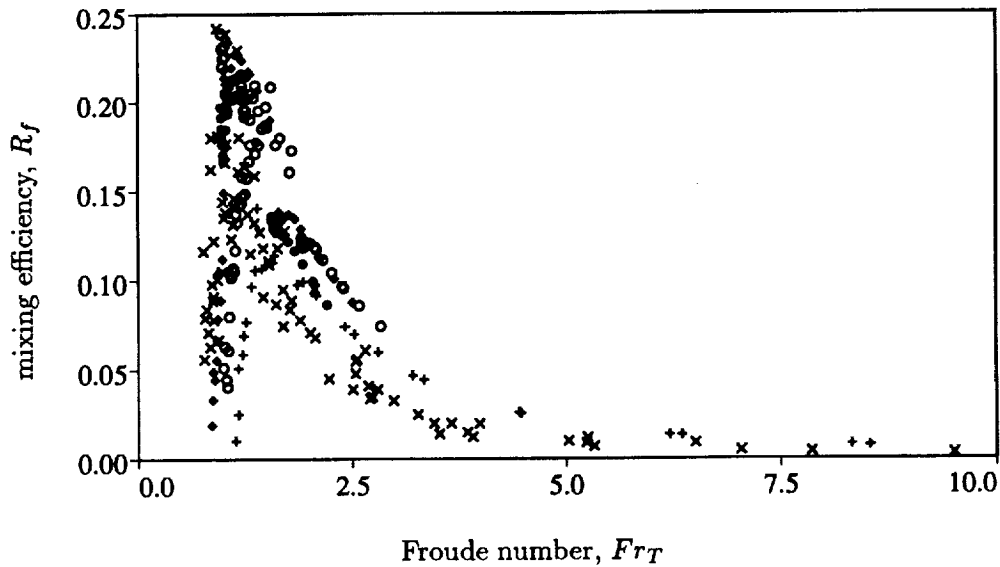
0.25

0.20

mixing efficiency, R_f

0.15

0.10

0.05

0.00

0.0   2.5   5.0   7.5   10.0

Froude number, Fr_T

FIGURE 1. R_f as a function of $Fr_T = (L_o/L_e)^{2/3}$ for simulations with $Pr = 0.1$ (o), $Pr = 0.5$ (◇) and $Pr = 2.0$ (●). Experimental data is provided by Stillinger *et al.*, Itsweire *et al.*, and Rohr *et al.* for $Pr = 700$ (×) and Lienhard and Van Atta for $Pr = 0.7$ (+).

the simple predictions in (4). Additionally, II introduced two further dimensionless numbers, the turbulent Reynolds number Re_T and small scale Froude number Fr_γ defined by

$$Re_T = \frac{uL_c}{\nu} = (\frac{L_c}{L_k})^{4/3} \tag{5}$$

and

$$Fr_\gamma = (\frac{\epsilon}{\nu N^2})^{1/2} = (\frac{L_o}{L_k})^{2/3}, \tag{6}$$

where $\epsilon/\nu N^2$ is a measure of the range of overturning scales when buoyancy strongly affects the flow and $L_k = (\nu^3/\epsilon)^{1/4}$ represents the Kolmogorov scale. The dimensionless numbers (3), (5), and (6) are related by

$$Fr_T = (\frac{1}{Re_T})^{1/2} Fr_\gamma. \tag{7}$$

One final length scale is used in discussing the numerical results below. The Ellison scale L_e is defined as

$$L_e = -\frac{(\overline{\rho^2})^{1/2}}{\frac{\partial \overline{\rho}}{\partial z}} \tag{8}$$

where $\overline{\rho}(z)$ is mean density. As Itsweire *et al.* (1992) show, except at very high Richardson numbers (based on mean shear and density gradient), L_c and L_e are the same scale to within a constant of $O(1)$.
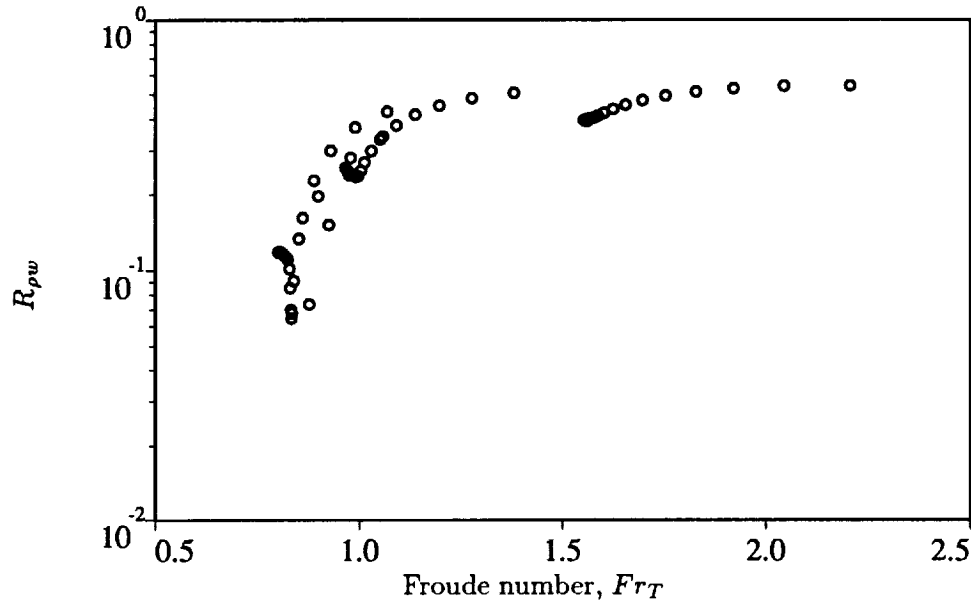
FIGURE 2. $R_{\rho w}$ as a function of Fr_T for $Pr = 2$.

3. Mixing efficiency

All results in this section were computed by ensemble-averaging data over the entire computational domain at each time step. In order to eliminate transient behavior associated with the initial conditions, only the data for $St \geq 2$ are considered, where the shear time, St, is a dimensionless time with $S = \partial \overline{U}/\partial z$. R_f was computed according to the definition in (2), as were the dimensionless numbers Fr_T, Re_T, and Fr_γ from their respective definitions in equations (3), (5), and (6). Figure 1 summarizes the results for the simulations with $Pr = 0.1$, 0.5, and 2. Mixing efficiency, R_f, is plotted against Fr_T to facilitate direct comparison with the laboratory observations (presented by II) which are also shown in Figure 1. The numerical results show the same general distribution as the laboratory measurements. As Fr_T becomes very large, the overturning scale is small compared to the Ozmidov scale, the mixing becomes inefficient because there is much more turbulent kinetic energy than necessary to mix the scalar gradient, and R_f rapidly decreases. Similarly, for very small values of Fr_T when buoyancy effects dominate and reduce the mixing, R_f again sharply decreases. At an intermediate value of Fr_T in the range of 1 to 1.5, the mixing is optimal with peak values of $R_f = 0.25$.

The behavior of $R_{\rho w}$ as a function of Fr_T given in Figure 2 for $Pr = 2$ (as an example) shows a similar trend. For $Fr_T > 1$, the correlation coefficient $R_{\rho w}$ tends to a constant value of about 0.6; hence, from Equation (4), R_f will rapidly decrease with increasing Fr_T. Conversely, for $Fr_T < 1$, $R_{\rho w}$ decreases rapidly as the density and velocity fluctuations become uncorrelated, and R_f tends to zero as a consequence of the denominator in (4) being much greater than unity.

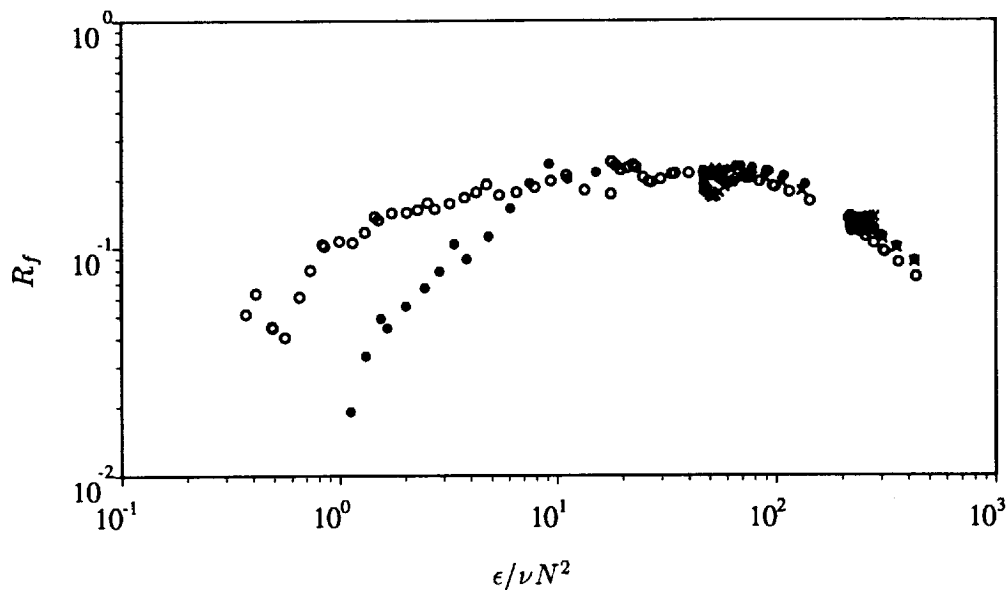While the overall behavior in the numerical and laboratory results is the same

FIGURE 3. R_f as a function of $\epsilon/\nu N^2$ for simulations with $Pr = 0.1$ (○), 0.5 (●), 2.0 (×).

in Figure 1, there are differences in detail. Unlike the laboratory data, for a given Fr_T, the numerically derived values of R_f are independent of Prandtl number and do not show the experimental tendency of R_f to increase with Pr. Nevertheless, recalling that the laboratory data are derived from time-averaged statistics in a steady mean flow whereas the numerical value are ensemble-averaged values in an evolving flow, the differences are remarkably small. The implication is that the peak mixing efficiency is 0.25 for Fr_T 1 to 1.5, irrespective of Prandtl number.

In field measurements of oceanic turbulence, the overturning scales L_e or L_c are not usually measured while ϵ invariably is. Substituting (7) into (4) yields the more practical form

$$R_f = \frac{1}{1 + \beta(\epsilon/\nu N^2)} \tag{9}$$

where $\beta = R_{\rho w}^{-1} Re_T$.

In Figure 3, we re-plot the results from the simulations against $\epsilon/\nu N^2 (= Fr_\gamma^2)$. The minimum dissipation needed to sustain a vertical buoyancy flux, and hence positive R_f, is clearly a strong function of Pr. For $Pr = 0.1$, $\epsilon/\nu N^2$ may be as little as 0.4 and still sustain a positive buoyancy flux ,whereas for $Pr = 2$, the minimum $\epsilon/\nu N^2$ for a positive buoyancy flux is about 20. The other interesting feature of Figure 3 is that R_f becomes independent of Pr for large $\epsilon/\nu N^2$. For $\epsilon/\nu N^2 \gg 10$, a best fit is $\beta = (\epsilon/\nu N^2)^{-0.6}$, hence (9) simplifies to

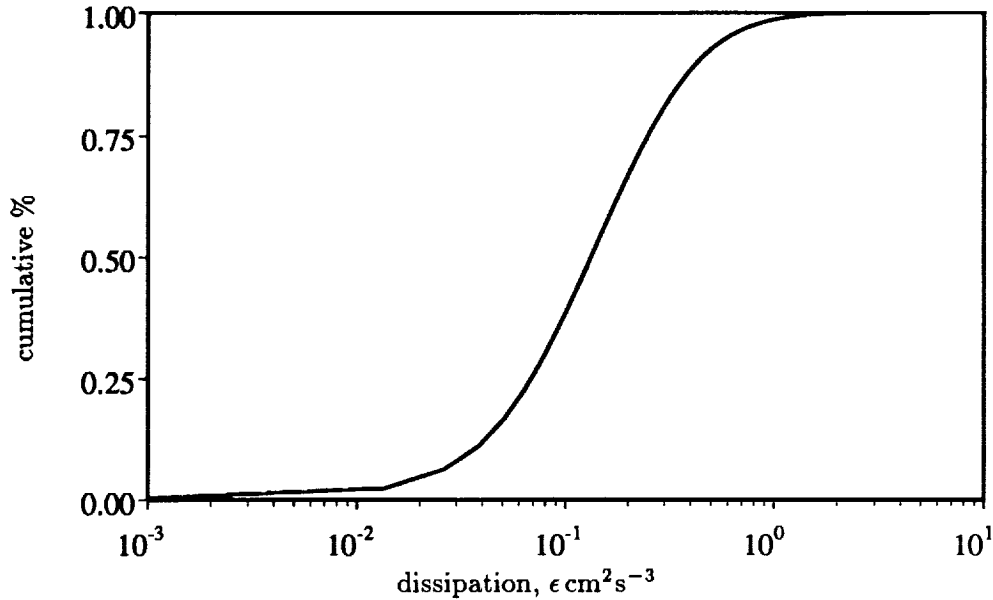$$R_f = \frac{1}{1 + (\epsilon/\nu N^2)^{0.4}} \tag{10}$$

FIGURE 4. Lognormal plot of dissipation estimates with no averaging ($Ri = .075$, $Pr = 2$, $St = 6$).

and using (2)

$$B = \epsilon^{0.6}(\nu N^2)^{0.4} \tag{11}$$

which provides an approximate but simple means of computing buoyancy flux from the dissipation for relatively energetic flows.

4. Sampling turbulence in a stratified fluid

Turbulence measurements are made in the ocean with either vertically falling microstructure instruments or, less commonly, horizontally towed instruments. The buoyancy flux B is not directly measured but, as indicated above, dissipation estimates are made and then B is computed by estimating R_f as outlined in Section 3 (see also Itsweire *et al.* 1992). For falling probes, dissipation estimates are typically made by measuring two turbulent velocity components and computing total dissipation using models (see Itsweire *et al.* 1992). This procedure produces estimates of dissipation averaged over about 2 meters in the vertical, and these estimates are further averaged to characterize the dissipation on much larger scales such as the oceanic thermocline (for example see review of Gregg 1987). Gibson and Baker (1987) and Gibson (1991) have argued that dissipation in oceanic turbulence is lognormally distributed with an intermittency, σ^2, in the range of 3 to 7. Furthermore, they claim that, due to the large scales and the limited sampling, the dissipation is greatly undersampled. Given the large intermittency, Baker and Gibson maintain that to estimate of ϵ to within $\pm 10\%$ one would need to average the dissipation calculated from thousands of independent sampling profiles! Gurvich and Yaglom

(1967) (see also Yamazaki and Lueck (1990)) developed three criteria which must be satisfied in order for dissipation to be a lognormally distributed quantity in the ocean.

(i) The turbulence must be homogeneous,

(ii) the averaging scale, r, must be small compared to the domain scale, L, and

(iii) the averaging scale must be large compared to the Kolmogorov scale L_k.

Yamazaki and Lueck (1990) suggest that (iii) can be satisfied with r as low as $3L_k$, but in all oceanic datasets, there is uncertainty about meeting criterion (i) given the patchy nature of the turbulence away from regions of highly energetic forcing such as the near-surface region. We investigated the sampling question by analyzing the results from one typical simulation. In particular, we examined the numerical results corresponding to $Ri = 0.075$, $Pr = 2$ at $St = 6$. For this data set, $L = 25$ cm, the grid scale, $r = L/128$, is 0.195 cm, the ensemble-averaged dissipation is 0.213 cm^{2}s^{-3}, $L_k = 0.0465$ cm, $L_c = 2.68$ cm, and $L_e = 1.37$ cm. For the full data set of 128^3 points without averaging, all three criteria of Yamazaki and Lueck (1990) are met since $r = 4.2L_k$. The corresponding distribution of dissipation should, therefore, be lognormal, and in Figure 4 the strong lognormality of the data with $\sigma^2 \simeq .75$ is evident. However, for comparison with typical oceanic sampling, of greater interest are the consequences of averaging estimates in the vertical. We chose to examine the effect of averaging over the full depth of the computational domain (25 cm), which is equal to 18.2 L_e or 9.3 L_o. Even though we are averaging over the depth, we still have a statistically significant sample of 128^2 points. Note that we do not satisfy criterion (ii) because $r = L$.

Figure 5 shows the distribution of vertically averaged ϵ for four different Richardson numbers. The effect of averaging is quite dramatic and the value of σ^2 is reduced to 0.01 and becomes essentially independent of Ri. Thus, when averaged over vertical scales significantly greater than L_e or L_o, dissipation is only weakly lognormal with σ^2 less than the value of 3 to 7 suggested for the ocean. Using techniques developed in Baker and Gibson, this translates to about 4 to 8 required profiles to obtain an estimate of the mean value of ϵ within $\pm 10\%$.

In order to more fully explore the second criterion of Gurvich and Yaglom, we averaged the data in the vertical but over length scales r which were smaller than L, e.g. 16 points or about $2.5L_e$. The distribution of ϵ averaged over this scale is plotted in Figure 6 which shows that the data is indeed lognormal but with $\sigma^2 \simeq .1$, much smaller than the 3 - 7 obtained by Baker and Gibson (1987). Again, the implication is that, if the data are averaged over length scales comparable to the overturning scale, the intermittency will not be as high as that described by Baker and Gibson (1987). This implies that much fewer sampling profiles are necessary to obtain a reasonable estimate of ϵ. Finally, we should note that since oceanic samples are likely to be drawn from a domain with significant regions of minimal dissipation the homogeneity criteria may be the most difficult restriction to satisfy.
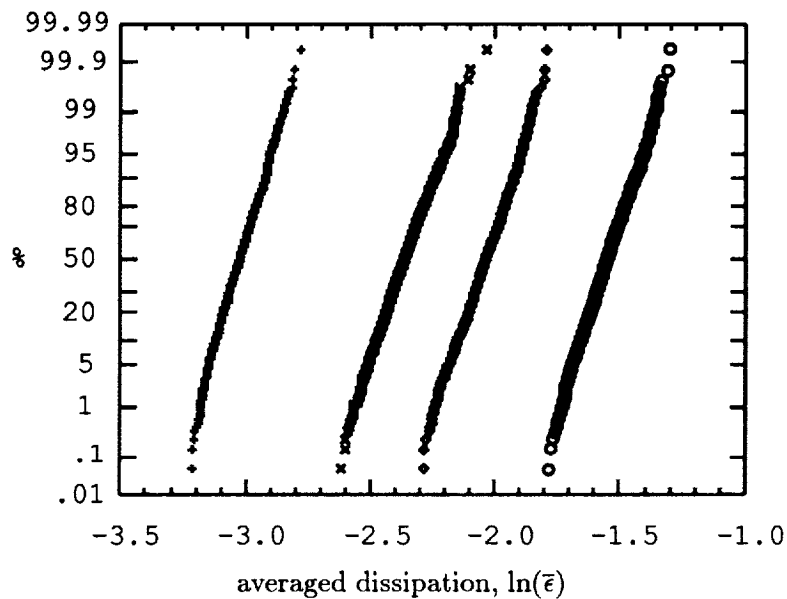
FIGURE 5.   Lognormal probability plot of $\bar{\epsilon}$ for $Ri = 0.075$ (o), 0.21 (◇), 0.37 (×) and 1.0 (+) with $Pr = 2.0, St = 6$.
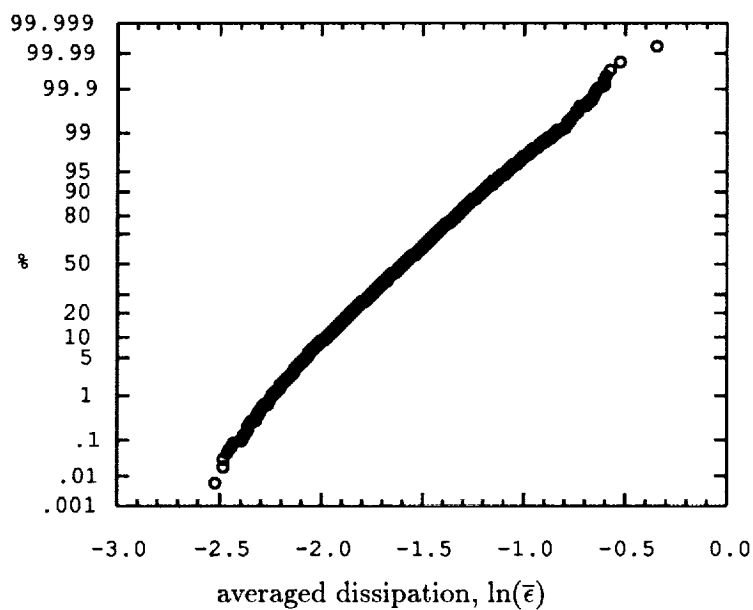

FIGURE 6.   Lognormal probability plot of $\ln(\bar{\epsilon})$ with the averaging scale $r \simeq 2.5L_e$, $Ri = .075$.

5. Conclusions

Direct numerical simulations of homogeneous turbulence in stably stratified shear flows confirm the variation of flux Richardson number R_f with turbulent Froude number Fr_T and $\epsilon/\nu N^2$ observed in laboratory experiments. With R_f defined as the buoyancy flux divided by the production of turbulent kinetic energy, there appears to be no systematic dependence of R_f on Pr in the range of Pr from 0.1 to 2. This result is not consistent with the laboratory observations; however, the differences in R_f between the simulations and the experiments are small, and the data from all sources indicate that R_f has a peak of 0.25, independent of Pr. Sub-sampling of the computational domain of 128^3 points was investigated to examine the distribution of the dissipation. The results indicate that when dissipation is estimated by averaging over vertical scales of an order of magnitude greater than either the Ellison or Ozmidov scales, the distribution is very weakly lognormal with an intermittency, $\sigma^2 \simeq 0.01$. This value is considerably smaller than some estimates in the oceanic literature and suggests sampling restrictions may not be as severe as previously suggested provided the sampling and averaging are performed over domains where the turbulence is homogeneous.

Acknowledgements

REFERENCES

BAKER, M. & GIBSON, C. G. 1987 Sampling turbulence in the stratified ocean: statistical consequences of strong intermittency. *J. Phys. Oceanogr.* **17**, 417-440.

GIBSON, C. H. 1991 *Turbulence and Stochastic Processes: Kolmogorov's ideas 50 years on*, Hunt, J. C., Phillips, O. M. & Williams, D (eds). The Royal Society, London.

GREGG, M. C. 1987 Diapycnal mixing in the thermocline: A review. *J. Geophys. Res.* **92**, 5249-5286.

GURVICH, A. S. & YAGLOM, A. M. 1967 Breakdown of eddies and probability distributions for small scale turbulence. *Phys. Fluids.* **10**, 59-65.

HOLT, S. E., KOSEFF, J. R. & FERZIGER, J. H. 1992 The evolution of turbulence in the presence of mean shear and stable stratification. *J. Fluid Mech.* **237**, 499-539.

LIENHARD, J. H. & VAN ATTA, C. W. 1990 The decay of turbulence in thermally stratified flow. *J. Fluid Mech.* **210**, 57-112.

ITSWEIRE, E. C., HELLAND, K. N. & VAN ATTA, C. W. 1986 The evolution of grid-generated turbulence in a stably stratified flow. *J. Fluid Mech.* **162**, 299-338.

ITSWEIRE, E. C., KOSEFF, J. R., BRIGGS, D. A. & FERZIGER, J. H. 1992 Turbulence in stratified shear flows: Implications for interpreting shear-induced mixing in the ocean. *J. Phys. Oceanogr, (accepted for publication)*.

IVEY, G. N. & IMBERGER, J. 1991 On the nature of turbulence in a stratified fluid. Part I: Energetics of mixing. *J. Phys. Oceanogr.* **21**, 650-658.

ROGALLO, R. S. 1981 Numerical experiments in homogeneous turbulence. *NASA Tech. Memo 81315*.

ROHR, J. J., ITSWEIRE, E. C., HELLAND, K. N. & VAN ATTA, C. W. 1988 An investigation of the growth of turbulence in a uniform-mean-shear flow. *J. Fluid Mech.* **187**, 1-33.

STILLINGER, D. C., HELLAND, K. N. & VAN ATTA, C. W. 1983 Experiments on the transition of homogeneous turbulence to internal waves in a stratified fluid. *J. Fluid Mech.* **131**, 91-122.

YAMAZAKI, H. & LUECK, P. 1990 Why oceanic dissipation rates are not lognormal. *J. Phys. Oceanogr.* **20(12)**, 1907-1918.