NASA Contractor Report 4543

Transition Prediction and Control in Subsonic Flow Over a Hump

Jamal A. Masad
High Technology Corporation
Hampton, Virginia

Venkit Iyer
ViGYAN, Inc.
Hampton, Virginia

Prepared for
Langley Research Center
under Contracts NAS1-19299 and NAS1-19672

NASA

National Aeronautics and
Space Administration

Office of Management

Scientific and Technical
Information Program

1993

Summary

The influence of a surface roughness element in the form of a two-dimensional hump on the transition location in a two-dimensional subsonic flow with a free-stream Mach number up to 0.8 is evaluated. Linear stability theory, coupled with the e^N transition criterion, is used in the evaluation. The mean flow over the hump is calculated by solving the interacting boundary-layer equations; the viscous-inviscid coupling is taken into consideration, and the flow is solved within the separation bubble. The effects of hump height, length, location, and shape; unit Reynolds number; free-stream Mach number; continuous suction level; location of a suction strip; continuous cooling level; and location of a heating strip on the transition location are evaluated. The N-factor criterion predictions agree well with the experimental correlation of Fage [Fage, A., *Brit. Aero. Res. Council*, 2120, 1943]; in addition, the N-factor criterion is more general and powerful than experimental correlations. The theoretically predicted effects of the hump's parameters and flow conditions on transition location are consistent and in agreement with both wind-tunnel and flight observations.

TABLE OF CONTENTS

1. INTRODUCTION

Roughness elements of varying shapes and dimensions exist on different aerodynamic surfaces. These elements contribute to an increase in drag, so that the main issue becomes the allowable shapes and sizes of these roughness elements such that the flow remains laminar. The dimensions of some unavoidable roughness elements on aerodynamic surfaces can be reduced, others cannot.[1] Therefore, control of the flow in the presence of roughness elements of various shapes and dimensions and under different flow conditions is important. Manufacturing and installation tolerances have not been developed to address this issue.

In addition to surface waviness, equally important types of surface roughness elements include steps (both backward- and forward-facing), gaps, and three-dimensional roughness elements such as flush screw-head slots and incorrectly installed flush rivets. For example, steps exist at the joints between the wing and control surfaces on airplane flaps. The influences of the compressibility, the shapes of roughness elements, and the wing sweep on manufacturing tolerances for laminar-flow surfaces are still essentially unknown.

The mechanisms by which surface imperfections and roughness elements contribute to the transition to turbulence in two-dimensional flows include enhancement of receptivity of free-stream turbulence and acoustic disturbances;[2] linear amplification of Tollmien-Schlichting (T-S) waves and shear-layer instability for separated flows;[3,4,5] Gortler instability; enhancement of secondary parametric excitations of both the subharmonic[6,7] and fundamental types; nonlinear interactions that have been captured partially by numerical simulation;[8,9] and finally, the interaction between two or more of the above mentioned mechanisms. Furthermore, as was pointed out by Spence and Randall,[10] in the presence of multiple, closely spaced surface waves, the possibility of a resonance between the critical T-S frequency and the surface waviness frequency exists. Klebanoff and Tidstrom[11] conducted an experiment to study the mechanisms by which a two-dimensional roughness element induces boundary-layer transition. They found sufficient evidence to conclude that the effect of a two-dimensional roughness element on boundary-layer transition can be regarded as a stability-governed phenomenon. An interesting experimental study on transition enhancement mechanisms, including the secondary instability caused by distributed roughness, was conducted by Corke et al.[12]

The difficulty in studying the stability characteristics of flows over roughness elements that might induce flow separation is in solving the mean-flow problem. After the velocity and temperature profiles are calculated, the stability analysis of the computed mean flow is almost standard. The mean-flow problem can be solved with a triple-deck formulation, an interacting boundary-layer (IBL) theory, or a Navier-Stokes (NS) solver. For flow over smooth roughness elements with separating and reattaching boundary layers, the IBL can be used to solve for the mean flow. If the edges of the roughness element are sharp or if the size is large enough to induce massive global breakaway separation and vortex shedding, then the triple-deck formulation and the IBL are not applicable, and a NS solver must be used. To accurately predict the flow field with an NS solver in the presence of roughness elements that might induce separation, the grid must be fine enough so that important flow structures are not smeared by the truncation errors and the artificial dissipation. However, the number of flow cases that must be investigated is very large, which makes this method very expensive. Because sharp roughness elements exist on aerodynamic surfaces, the numerical study of the stability of the flow over these surfaces and the prediction of the transition location require the use of the full NS equations. We point out that the nonsimilar boundary layer (NSBL) theory, which is capable of predicting the location of separation, fails to march through it. Moreover, the NSBL fails to accurately predict the mean flow over roughness elements that do not even induce separation because of the abrupt change in the geometry that causes viscous-inviscid coupling and an upstream influence that is not accounted for by the parabolic NSBL equations.

The mean-flow profiles generated by IBL and the stability characteristics compare well with those generated by an NS solver when a fine grid was used.[13] The IBL was less computationally demanding than the NS solver by one to two orders of magnitude. Large discrepancies between the IBL computations and the NS results were found when a coarse grid was used for the NS computations. Moreover, the IBL was used to compute incompressible and compressible flows over smooth steps, wavy surfaces and humps, convex and concave corners, suction or blowing slots, heating or cooling strips, and finite-angle trailing edges. In most of these applications, separation bubbles and upstream influences exist; the comparisons of the IBL results with the solutions of the NS equations and the experimental data showed good agreement.

Previous investigations of the stability and transition to turbulence in boundary-layer flow over roughness elements have been primarily experimental. However, the purpose of many of these studies has been the determination of only the location of transition in a naturally occurring disturbance environment under different flow conditions. Thus, neither the spectral content nor the growth and properties of instability waves were examined. In the early experiments, the transition location was identified as the appearance of turbulent bursts downstream of a roughness element. Some of these natural transition experiments were, in fact, flight experiments performed on swept and unswept wings; therefore, they include the effects of pressure gradients, compressibility, and occasionally surface suction, multiple roughness elements, three-dimensional roughness elements, and sharp roughness elements. In spite of these complications, these studies provide some empirical criteria for the prediction of transition location in the flow over roughness elements.[14,15] However, these criteria do not provide an understanding of the physical mechanisms involved in order to eventually control them. Moreover, these criteria are valid only for the specific configurations and conditions relevant to that particular experiment.

Nayfeh et al.[3] conducted theoretical research on the stability characteristics of two-dimensional incompressible flows over two-dimensional humps and dips on a nominally flat surface. They compared their results with the natural transition experimental data of Walker and

Greening reported in Fage.[14] Nayfeh et al.[3] followed a primary wave with a fixed physical frequency from the onset of instability (branch I) up to the experimentally determined location of transition, computed the value of the N-factor at that location, then changed the frequency to another value and repeated the calculation. The frequency that lead to the maximum value of the N-factor at the experimentally determined transition location was taken as the numerically predicted frequency of the disturbance wave that causes transition. Nayfeh et al.[3] compared their results with 14 sets of experimental results for humps and 6 sets of experimental results for dips. The calculated N-factor values at the experimentally determined location of transition in the case of the humps varied from $N = 7.4$ to $N = 10.0$, with an average value of $N = 8.5$. In the case of dips, they varied from $N = 6.7$ to $N = 9.2$, with an average value of $N = 8.0$. This comparison increases confidence in the e^N method as a tool for predicting the transition location. As we mentioned earlier, the calculations of Nayfeh et al.,[3] as well as the experimental data of Walker and Greening, are for the incompressible case. Use of the e^N method to predict the location of transition is more successful in incompressible flows than in compressible flows because the growth rates of the instability waves in incompressible flows are larger, which causes the location where N reaches a certain value to be less sensitive to variations in that value. In the presence of roughness elements that might cause separation, the growth rates are much larger than in the case of smooth flat plates; in this case, the e^N method would be expected to be more successful. Despite some differences between the approaches of Cebeci and Egan[4] and Nayfeh et al.,[3] the results from both approaches (including the comparisons with the experimental data of Walker and Greening) agree.

Dovgal and Kozlov[16] conducted a controlled (forced) experiment to study the stability characteristics of incompressible flow over roughness elements. They placed a vibrating ribbon upstream of a roughness element to introduce a two-dimensional small-amplitude disturbance into the developing boundary layer. The different shapes considered in the experiment included a hump, a forward-facing step, and a backward-facing step. In the presence of a hump, the experimental transverse variations in the magnitude of the streamwise velocity component of the disturbance measured by Dovgal and Kozlov have the same three-peak character found numerically by Nayfeh et al.[3] Furthermore, Masad and Nayfeh[5] calculated the transverse distribution of the phase of the streamwise velocity component disturbance in the presence of a hump; it exhibits the same two-jump character also found by Dovgal and Kozlov. In the presence of a step, Dovgal and Kozlov[16] reported the streamwise variation of the integral of the growth rates. Masad and Nayfeh[5] compared their results with all 12 cases presented by Dovgal and Kozlov. The overall agreement between the two studies was very good; these results support calculation of the mean flow with IBL and the use of quasi-parallel linear stability theory for flows that separate in the presence of a roughness element.

In this work, we study the effect of a single two-dimensional roughness element (a hump) on the predicted transition location in the subsonic boundary-layer flow over the roughness element. The transition location is correlated with the location at which the amplification factor reaches a value of 9 within the context of the empirical e^N transition criterion. The separated or attached mean flow over the roughness element is computed with interacting boundary layers. The following effects are evaluated: the hump height, length, location and shape; the flow free-stream unit Reynolds number; the flow free-stream Mach number; the level of continuous suction; the location of a suction strip; the level of continuous cooling; and the location of a heating strip on the predicted transition location. The variation of the predicted transition location with some of these parameters is compared with an existing experimental correlation.

2. FORMULATION AND METHODS OF SOLUTION

2.1 The Mean Flow

We consider a two-dimensional compressible subsonic flow (with a free-stream Mach number no larger than 0.8) around a single smooth two-dimensional hump on a flat plate (Figure 1). We consider a two-parameter family of symmetric hump shapes given by

$$y = y^*/L^* = (h^*/L^*)\, f(z) = h\, f(z) \tag{1}$$

where

$$z = 2(x^* - L^*)/\lambda^* = 2(x-1)/\lambda \tag{2}$$

and

$$f(z) = \begin{cases} 1 - 3z^2 + 2|z|^3, & \text{if } |z| \leq 1 \\ 0, & \text{if } |z| > 1 \end{cases} \tag{3}$$

Here, h^* is the symmetric hump dimensional height, and λ^* is the dimensional width of the hump with the center located at $x^* = L^*$.

The roughness element under consideration could produce separation bubbles behind it. In such flows, both a strong viscous-inviscid interaction and an upstream influence exist. The conventional boundary-layer formulation fails to predict such flows; therefore, we use the IBL theory to analyze them.

In the IBL theory, the Prandtl transposition theorem is used with the Levy-Lees variables to obtain the nonsimilar boundary-layer equations and the corresponding boundary conditions. The upstream initial condition is taken to be that of a flow over a smooth flat plate. To account for the viscous-inviscid interaction, the inviscid flow over the displaced surface is calculated with the interaction law, which relates the edge velocity to the displacement thickness. Then, the thin-airfoil theory is used to supply the relation between the inviscid surface velocities with and without the boundary layer; it is also used to calculate the inviscid surface velocity in the absence of the boundary layer. The continuity equation is then manipulated and combined with the interaction law to yield a single equation that can be solved simultaneously with the nonsimilar boundary-layer equations and boundary conditions.

2.2 Stability of the Mean Flow

In the stability analysis, small unsteady two-dimensional disturbances are superimposed on the mean flow quantities, which are computed with the IBL theory described in section 2.1. Next, the total quantities are substituted into the NS equations, the equations for the basic state are subtracted out, the quasi-parallel assumption is invoked, and the equations are linearized with respect to the disturbance quantities. The disturbance quantities are assumed to have the so-called normal-mode form so that a disturbance quantity $\hat{q}$ is

$$\hat{q} = \xi(y)e^{i(\alpha x - \omega t)} + cc \tag{4}$$

where cc denotes the complex conjugate of the preceding term. The streamwise coordinate is x, t is the time, and α and ω are generally complex. In the stability analysis and the computations throughout this work, the reference length is $\delta_r^* = \sqrt{\nu_\infty^* x^* / U_\infty^*}$, the reference velocity is U_∞^*, the reference time is δ_r^*/U_∞^*, the reference temperature is the free-stream temperature T_∞^*, the reference viscosity is the free-stream dynamic viscosity μ_∞^*, and the pressure is made nondimensional with respect to $\rho_\infty^* U_\infty^{*2}$, where ρ_∞^* is the free-stream density. The viscosity varies with temperature in accordance with Sutherland's formula; the specific heat at constant pressure $C_p{}^*$ is constant, and the Prandtl number Pr is constant and equal to 0.72. For temporal stability, α is real, and $\omega = \omega_r + i\omega_i$ is complex, in which the real part ω_r is the disturbance frequency and its imaginary part ω_i is the temporal growth rate. For the spatial stability considered in this work, ω is real, and $\alpha = \alpha_r + i\alpha_i$ is complex, in which the real part α_r is the streamwise wave number and the negative of the imaginary part $-\alpha_i$ is the spatial growth rate. The frequency ω is related to the dimensional circular frequency ω^* through $\omega = \omega^* \delta_r^* / U_\infty^*$, which leads, with the definition of $\delta_r{}^*$, to

$$\omega = F\ R \tag{5}$$

where

$$F = \frac{\omega^* \nu_\infty^*}{U_\infty^{*2}} \tag{6}$$

and

$$R = U_\infty^* \delta_r^* / \nu_\infty^* = \sqrt{x\ Re} \tag{7}$$

Because ω^* is fixed for a certain physical wave as it is convected downstream, F is also fixed for the same wave.

The normal-mode form given above separates the streamwise and temporal variations. The resulting equations and corresponding boundary conditions form an eigenvalue problem that can be solved numerically. For the results presented in this work, the computations were made with an adaptive, second-order accurate, finite-difference scheme with deferred correction.[17] The disturbances considered in this work are two-dimensional because, as pointed out by Mack,[18] disturbances in subsonic (M_∞ up to 0.8) boundary layers are most amplified when they are two dimensional.

The quasi-parallel assumption that we used in the stability analysis was justified by Nayfeh et al.[3,6] for a flow over a roughness element by arguing that the wavelength of the disturbance, in the presence of a roughness element, is of the same order as the disturbance wavelength in a flow over a smooth flat plate. If we consider a hump of a height h = *0.003*, a length λ = *0.2*, and a free-stream Reynolds number R = 10^6, then at the corresponding most dangerous frequency of F = 64×10^{-6} our calculations show that the streamwise wave number α_r within the domain of the hump varies in a range of 0.1 to 0.2. If we average α_r to be 0.15 in the domain of the hump, then nearly five disturbance wavelengths exist within the extension of the hump. Elli and Dam[9] questioned the validity of the quasi-parallel assumption and the validity of using the linear stability theory in the case of separating flow over a roughness element. However, Bestek et al.[8] performed direct numerical simulations of separating flow over a backward-facing step and compared the results with those obtained with the linear quasi-parallel stability theory. Both results were in very good agreement. Furthermore, this agreement between the results of Masad and Nayfeh[5] with the quasi-parallel linear stability theory and the experimental data of Dovgal and Kozlov[16] for separating flow over forward- and backward-facing steps reinforces the belief that the quasi-parallel assumption is reasonable. Similar comparisons in this paper with the experimental correlation of Fage[14] (section 3.3) also support this point of view.

3. RESULTS

The presence of a roughness element on a flat plate creates local regions of favorable and/or adverse pressure gradients. Because the pressure gradient has a direct effect on the stability of the flow, the streamwise variation of the pressure coefficient must be considered in the vicinity of the roughness element. In Figure 2, a typical streamwise distribution of the pressure coefficient for a flow over a hump is compared with that for a flow over a smooth flat plate. An adverse pressure gradient region exists ahead of the hump, which is followed by a region of favorable pressure gradient that extends over a very short distance; finally, a strong adverse pressure gradient follows, which causes the boundary layer to separate. Thus, the flow is expected to become more unstable ahead of the hump (Figure 3), become more stable over the short favorable pressure gradient region, and then become more unstable again in the separation region. For certain parameters of the hump or under certain flow conditions, three unstable regions are possible because of the two adverse pressure gradient regions and the smooth flat-plate region. The splitting of the unstable regions can also occur because of the existence of flow control devices that utilize surface suction or heat transfer.

In the next sections, we quantify the effects of the hump height, length, location, and shape, as well as the effects of the flow unit Reynolds number, compressibility, suction, and heat transfer on the predicted transition location. In discussing the effects of all of these parameters and conditions, we will distinguish between how each parameter or condition enhances separation and how it affects the location of transition. As we will see in some of the next sections, the relation between separation and transition is not always a simple one.

3.1 Effect of Roughness Dimensions

The relation between the transition location and the height of a roughness element is important. Earlier researchers who studied this problem believed that transition was located at the roughness element when the height of the element was large and that the roughness element had no influence on transition when the height of the element was small. However, Fage (see reference 14) has shown experimentally that the point of transition moves continuously upstream as the height of the roughness element is increased until it ultimately reaches the position of the roughness element itself. In a discussion of the influence of roughness on transition, Schlichting[19] pointed out the necessity of determining whether a maximum height of roughness elements exists below which no influence on transition occurs. If such a critical height exists, then the allowable tolerances on different unavoidable roughness elements on aerodynamic surfaces will be determined based on this critical height. Such a critical height is expected to depend on the free-stream unit Reynolds number, the Mach number, the length and shape of the roughness elements, the roughness location, suction level, and heat transfer level. To address this issue with linear stability theory, we use the e^N transition criterion; the predicted transition location is taken to be the point where the amplification factor (N-factor) of the disturbances reaches the value of 9 in the shortest distance measured from the leading edge. The value of Re_x at that location is denoted by $(Re_x)_{N=9}$. Thus, we calculated the values of $(Re_x)_{N=9}$ and the corresponding frequencies for several hump heights that range from a zero height (no hump) to the nondimensional hump height $h = 0.006$. Recall that the hump height is made nondimensional with respect to L^*, which is the distance from the leading edge of the plate to the center of the hump so that $h = h^*/L^*$. Variation of $(Re_x)_{N=9}$ with the hump height is shown in Figure 4, in which each point where the calculations were made is denoted by a circle; the circles were then joined. The filled circles in all figures indicate the occurrence of separation, and the hollow circles indicate that the flow remained attached.

Figure 4 clearly shows that the theoretically predicted transition location moves continuously upstream as the hump height is increased. However, the variation is far from linear. The curve that describes the movement of the location of $N = 9$ becomes increasingly steeper as the hump height increases; shortly after the flow separates, it becomes the steepest. When the hump height exceeds a certain value that is larger than the value responsible for separation, the location where N first reaches 9 moves very slowly upstream toward a point that is only a short distance downstream of the center of the hump (the location where the separation bubble starts). At such large heights of the hump, the instability becomes explosive, and transition is expected to occur via "bypass." Note in Figure 4(b) that as the hump height increases the most dangerous frequency increases. Close to separation, when the predicted transition location moves considerably upstream, the most dangerous frequency increases sharply. Klebanoff and Tidstrom[11] found that close to the roughness the fluctuation is composed of relatively higher frequencies.

In addition to the height of the hump, the length seems to influence the predicted transition location. However, the role of the hump length is opposite that of the hump height. If the nondimensional length $\lambda = \lambda^*/L^*$ of a hump at a fixed height is increased, then the location where the N-factor first reaches a value of 9 is shifted downstream (Figure 5(a)). This result implies that if the height of the roughness element cannot be reduced, then transition can be delayed by increasing the length of the roughness element. If the roughness element becomes so short that its length falls below a certain critical value (smaller than the value that induces separation), then the upstream movement of the transition location slows down considerably and the predicted transition location approaches a point shortly downstream of the center of the roughness element, but does not move upstream of it. As the predicted transition location moves upstream because of the decrease in the length of the hump, the most dangerous frequency increases (Figure 5(b)).

3.2 Effects of Unit Reynolds Number and Roughness Location

Another parameter of importance that affects the location of transition in a flow over a roughness element is the free-stream Reynolds number Re given by $Re = U^*_\infty L^* / \nu^*_\infty$, where U^*_∞ is the free-stream dimensional velocity, ν^*_∞ is the free-stream dimensional kinematic viscosity, and L^* is the dimensional distance from the leading edge to the center of the roughness element. An increase in the value of the free-stream Reynolds number at fixed h and λ causes the flow over the hump to separate at lower heights or larger lengths of the hump. The effect of *Re* is actually a combination of two separate effects: the free-stream unit Reynolds number $Re = U^*_\infty/\nu^*_\infty$, and the location of the center of the roughness element L^*. As pointed out by Morkovin,[20] the effect of the unit Reynolds number on the stability characteristics of any flow is always a factor whenever the mean flow is nonsimilar. To study the effect of the flow unit Reynolds number, *Re* is varied by varying U^*_∞/ν^*_∞, and L^* is fixed. Therefore, to maintain the same dimensional roughness height h^*, the nondimensional roughness height $h = h^*/L^*$ must also remain fixed. On the other hand, to study the effect of the roughness location, *Re* is varied by varying L^*, and U^*_∞/ν^*_∞ is fixed. To keep the dimensional roughness height h^* fixed, the nondimensional roughness height $h = h^*/L^*$ must vary in accordance with the variation of L^*. Similar arguments apply for the roughness length.

The effect of varying the unit Reynolds number on the predicted transition location in flows over two humps of different heights is shown in Figure 6(a). Low values of *Re* correspond to locations far upstream of branch I of the neutral stability curve. In the absence of roughness, the waves at these locations are strongly damped. The adverse pressure gradient induced by the roughness element causes the waves to become less damped, but they remain damped or weakly amplified. The net result is the lessening of the effect of the roughness element. Therefore, at low values of *Re*, the predicted transition location for the flow over the roughness element approaches that of a flow over a smooth flat plate. On the other hand, large values of *Re* correspond to locations far downstream; therefore, the *N*-factor reaches a value of 9 before the roughness element is reached, and the predicted transition location for the flow over the roughness element again approaches that of a flow over a smooth flat plate.

Moderate values of *Re* correspond to locations within the unstable regions; therefore, the existence of a roughness element and its adverse pressure gradient causes the predicted transition location to move upstream in comparison with the case of a flow over a smooth flat plate. This effect on transition was recognized in the flight experiments of Holmes et al.[1] In an explanation of the strong beneficial effect of higher altitudes on allowable step heights and gap widths, they noted that, "The increases in tolerances with increased altitude result directly from the decrease in unit Reynolds number. As the unit Reynolds number decreases, the length of the laminar separation regions associated with the steps decreases, reducing the growth of the inflectional instability and increasing the allowable step height."

The sharp drop in the predicted transition Reynolds number in Figure 6(a) at an Re of approximately 3 million is caused by the movement of the location where N reaches a value of 9 from the downstream unstable region (created by an adverse pressure gradient) to the upstream unstable region (created by the adverse pressure gradient and the smooth flat-plate instability). Because the two regions are separated by a stable region that is caused by the favorable pressure gradient (created by the roughness element), the value of the predicted transition location is expected to jump. Note in Figure 6(a) that at large values of the unit Reynolds number the flow separates. However, this separation is not harmful as far as the transition location is concerned because at such values of Re transition occurs before the separation bubble is reached.

By comparing Figures 6(a) and 6(b), we note a strong correlation between the variation of the predicted transition location with Re and the variation of the corresponding most dangerous frequency with Re. As the predicted transition location moves upstream, the most dangerous frequency increases, which also occurs in the region of the jump. At both small and large values of Re, the most dangerous frequency approaches 26×10^{-6}, which is the most dangerous frequency for incompressible flow over a smooth flat plate.

The effect of varying the hump location on the predicted transition location is shown in Figure 7(a). The value of L^* equal to L^*_0 is the reference location for the center of the hump.

With the hump centered at L^*_0, $Re_0 = U^*_\infty L^*_0/\nu^*_\infty$ is equal to 1 million, the hump height $h_0 = h^*/L^*_0$ is equal to either 0.0014 or 0.00195, and the hump length $\lambda_0 = \lambda^*/L^*_0$ is equal to 0.2. By moving the center of the hump upstream or downstream of L^*_0 so that the hump's center is at L^*, Re becomes $Re_0 L^*/L^*_0$, the hump height h becomes $h^*_0 L^*_0/L^*$, and the hump length λ becomes $\lambda_0 L^*_0/L^*$. The center of the hump is moved to keep the dimensional hump height, the hump length, and the unit Reynolds number fixed. By increasing L^*, Re increases, but the nondimensional hump height and length decrease. At hump locations far upstream or far downstream, the predicted transition location approaches that of a flow over a smooth flat plate, which can be explained by arguments similar to those made earlier in this section in regard to the results of Figure 6(a). The upstream movement of the predicted transition location is associated with an increase in the most dangerous frequency, as shown in Figure 7(b).

3.3 Comparison with Experimental Correlations

Fage[14] used his own experimental data on the effects of surface roughness on transition, as well as the experimental data of Walker and Greening, Walker and Cox, and Hislop (as reported in Fage[14]), to correlate the transition location with the height and length of the roughness element and the Reynolds number at the edge of the boundary layer. If we replace the nondimensional velocity at the edge of the boundary layer $u_e = u^*_e/U^*_\infty$ with unity, then Fage's criterion can be written as

$$(Re_x)_{tr} = \frac{9 \times 10^6}{h}\sqrt{\frac{\lambda}{Re}} \quad \text{when} \quad \frac{h\,Re^{1.5}}{9 \times 10^6} > 0.09 \tag{8}$$

and

$$(Re_x)_{tr} = \left(\frac{13.5 \times 10^6 \sqrt{\lambda}}{h}\right)^{2/3} \quad \text{when} \quad \frac{\lambda^{1/6}\,Re\,h^{2/3}}{(13.5)^{2/3} \times 10^4} < 0.09 \tag{9}$$

where $(Re_x)_{tr}$ is the value of $Re_x = U^*_\infty x^*/\nu^*_\infty$ at the transition location. Fage's criterion is valid in a range of $(Re_x)_{tr}$ that extends from 1 million to 3.5 million. Fage's criterion accounts for the effects of roughness height, length, and the free-stream Reynolds number. The criterion is applicable in two-dimensional incompressible flow over a single roughness element on a flat plate or an airfoil, but does not account for the effects of suction or heat transfer. To compare the results of the e^N transition criterion with Fage's criterion for predicting transition location for a flow over a hump on a flat plate, we considered several combinations of hump height h, hump length λ, and free-stream Reynolds number Re. The mean flow problem was then solved, and the stability calculations were performed. The hump used in our computations has the same shape that was used in Fage's experiments. For each considered combination, the most dangerous frequency (at which the N-factor reaches a value of 9 in the shortest distance from the leading edge) was determined within $\Delta F = 1 \times 10^{-6}$ with the corresponding predicted transition location. Variation of the predicted transition Reynolds number with the shape free-stream Reynolds-number parameter $\sqrt{\lambda}/h\sqrt{Re}$ is shown in Figure 8(a) and compared with Fage's experimental correlation. The agreement is very good; in fact, the scatter of the N-factor correlation points with respect to Fage's correlation is less than the scatter of the experimental points with respect to the experimental correlation.

In the experimental data (the basis for Fage's criterion), the unit Reynolds number varied from 0.5 million to 1 million, which was the range considered in the calculations that were performed to produce the N-factor correlation points in Figure 8. Note that in Fage's experimental correlation the transition Reynolds number varies with Re in accordance with $1/\sqrt{Re}$ and, therefore, decreases as Re increases. In Figure 6(a), we note that although this correlation might be the case for Re between 0.5 million and 1 million, $(Re_x)_{N=9}$ increases as Re increases over a wide range of high values of Re. This observation shows that the e^N approach for predicting the transition location in flow over a roughness element is more generally applicable and more powerful than the experimental correlations. Furthermore, to develop an experimental criterion for the transition location in a flow over a roughness element, an extensive number of cases must be considered to account for all of the parameters of the roughness element and the flow conditions. Therefore, computation of the predicted transition location with the N-factor criterion for those roughness-element parameters and flow conditions that arise is much easier. However, experimental data and additional correlations are still needed to verify and calibrate the e^N method for different configurations and flow conditions.

A second experimental correlation that is available is that of Carmichael.[15] Carmichael's criterion applies for single and multiple bulges or sinusoidal waves above the nominal surface of a swept or unswept wing. Carmichael's criterion partially accounts for the effects of compressibility, suction, pressure gradient, wing sweep, and multiple waves, which makes a quantitative comparison of theoretical results with this criterion a difficult task. However, a quantitative comparison of our results from the N-factor criterion with the predictions of Carmichael's criterion for unswept wings showed that those transition locations predicted by the N-factor method are far upstream of those predicted by Carmichael's criterion. This result is expected because Carmichael's criterion accounts for compressibility, suction, and the favorable pressure gradient on the unswept wing; these effects tend to move the transition location downstream, as will be shown in the next sections.

3.4 Effect of Compressibility

The effect of compressibility on the stability characteristics of two-dimensional flow over roughness elements is complicated by the fact that although an increasing Mach number stabilizes the flow in the attached regions, it increases the size of the separation bubble. An increase in the value of the free-stream Mach number M_∞ at subsonic and supersonic speeds causes the flow over the hump to separate at lower hump heights because compressibility makes the pressure gradient more adverse and enhances separation. In their experimental work, Larson and Keating[21] noticed a large increase in the streamwise length of the separation region when the Mach number of the flow over the roughness element was increased. We point out here that what Larson and

Keating[21] refer to as the transition Reynolds number in the case of separation is actually the product of the flow unit Reynolds number and the streamwise length of the separation bubble. Therefore, at the same unit Reynolds number, an increase in what they refer to as the transition Reynolds number is actually an increase in the streamwise length of the separation bubble.

The widening of the separation region because of the increase in M_∞ partially offsets the stabilizing effect of compressibility. Overall, in two-dimensional flow, the stabilizing effect of compressibility in the attached regions overcomes the destabilization caused by the increase in the size of the separation bubble (Figure 9(a)). The downstream movement of the transition location of a flow over a step as the Mach number increases was noticed and reported by Chapman et al.[22] Furthermore, the stability of a laminar shear layer (that develops in the case of separation) was found by Lin[23] to increase markedly as the Mach number increases. At supersonic speeds in wind-tunnel operation, larger wire diameters are required to trip the boundary layer (make it turbulent) as the Mach number increases. At large heights of the roughness element, compressibility has almost no effect on the movement of predicted transition location (Figure 9(a)).

In boundary-layer flow over smooth surfaces ($h = 0$), an increase in the Mach number shifts the most dangerous frequency toward lower values (Figure 9(b)). On the other hand, as we saw in section 3.1, an increase in the height of a roughness element at the same Mach number increases the value of the most dangerous frequency. Figure 9(b) shows that at large heights of the hump an increasing Mach number still reduces the value of the most dangerous frequency. The general trend in Figure 9 is that as the predicted transition location moves downstream the most dangerous frequency decreases.

3.5 Effect of Roughness Shape

The influence of varying the shape of a roughness element on the transition location is a controversial issue. Based on the experiments that he conducted, as well as on other available experimental data, Fage[14] reported that the shape of a roughness element has almost no effect on the transition location. In fact, the same experimental criterion of Fage applies for smooth bulges, smooth hollows, flat ridges, and arched ridges. Fage's criterion is applicable for roughness elements on two-dimensional configurations such as flat plates and airfoils.

On the other hand, in some of their flight experiments on an unswept wing, Holmes et al.[1] compared the effect of a rounded forward-facing step close to the leading edge on its allowable (critical) height with the effect of a square step on such a critical height. An increase of 50 percent in the critical step height was possible when the step was rounded with a radius approximately equal to the step height. In these experiments, the critical height was established based on the conditions where the first turbulent bursts occurred far downstream from the roughness element, as in the experiments used to develop both Fage's[14] and Carmichael's[15] criteria. However, we emphasize that what we mean by varying the shape of the roughness element here is a variation in contour and not in length or height. When a square step is rounded, its length is expected to increase somewhat; therefore, as shown in section 3.1, the transition location moves downstream. In their wind-tunnel experiments, Dovgal and Kozlov[16] showed that by tapering the forward face of a square hump, the amplitude of the disturbances is reduced. They also showed that by tapering both faces of the square hump, the amplitudes of the disturbances were reduced considerably. In both cases, the hump was tapered by increasing its length. As mentioned, this section examines the effect of varying only the contour of the hump on the predicted transition location; the height and length of the hump remain fixed.

To evaluate the effect of varying the contour of the hump on the predicted transition location, we considered two hump shapes with the same height and length, but different contours. The first shape is referred to as shape A and is given by equation (1); the second shape is referred to as shape B and is given by equation (1), where f is now given by

$$f(z) = \begin{cases} 1 - z^2, & \text{if } |z| \leq 1 \\ 0, & \text{if } |z| > 1 \end{cases} \tag{10}$$

and z is given by equation (2). The contours of shapes A and B are shown in Figure 10. Shape A is rounded in comparison with shape B, but the length of both humps is fixed. Furthermore, shape B is fuller in comparison with shape A. The effect of varying the height of both humps A and B on the predicted transition location is shown in Figure 11(a). Clearly, the flow over hump B separates at a lower hump height than the flow over hump A. As the hump's height is increased from the zero value (smooth flat plate), the effect of the shape on the predicted transition location increases and then starts to decrease. At large heights of the hump, the effect of the hump's shape on the predicted transition location is negligible. The corresponding most dangerous frequencies (Figure 11(b)) are higher for hump B except at large heights of the hump, where the frequencies become the same.

3.6 Effect of Continuous Uniform Suction

Although continuous suction thins the boundary layer, (which makes the boundary layer more sensitive to roughness), continuous suction also reduces the size of the separation bubble. In fact, suction can be used in applications to remove the decelerated fluid from the boundary layer before it causes separation. This technique makes the boundary layer capable of overcoming a stronger adverse pressure gradient. The reduction in the size of the separation bubble by suction was observed and reported in the experimental work of Hahn and Pfenninger[24] for the case of flow over a backward-facing step.

In two-dimensional flow, continuous uniform suction might affect the flow in the separation region differently than the flow in the attached regions. This possibility might be attributed to the coexistence of both viscous and shear-layer instability mechanisms in the separation region, whereas in the attached regions only the viscous instability mechanism exists. Although continuous suction might increase the growth rate of disturbances within the reduced separation bubble, the overall effect of continuous suction on the stability of flow over a roughness element of low to moderate height is stabilizing, as shown in Figure 12(a). Carmichael et al.[25] and Carmichael and Pfenninger[26] performed flight experiments on the wing of an airplane in the presence of single and multiple roughness elements and suction. Their results show that the allowable sizes of the roughness elements increase when embedded in the suction region. Despite a different configuration, our results for a flat plate are in qualitative agreement with these experimental findings. The overall stabilizing effect of suction on the flow over surface waves was demonstrated in the theoretical asymptotic work of Spence and Randall.[10]

At large roughness-element heights, continuous suction has little effect on the movement of the location of transition unless the suction level exceeds a certain value, at which the predicted transition location for the flow over the roughness element moves sharply to the predicted transition location for a flow over a smooth flat plate (Figure 12(a)). This result is significant for laminar-flow control applications. The existence of such a threshold level of continuous suction means that in the presence of a large-height roughness element on a smooth surface the applied suction level needs to exceed this threshold value to delay transition. We are not aware of any experiments on the effect of continuous suction on the transition location for a flow over a roughness element in which the variation of the transition location with the mass flow rate was measured. Therefore, the design of an experiment to verify the existence of a threshold level of suction at which the transition location moves considerably downstream is of practical interest. However, in their experiment on the effect of suction on the stability of flow over a backward-facing step, Hahn and Pfenninger[24] noticed that at weak suction rates turbulent bursts remained in flow that was mostly laminarized. Also, a further increase in the suction rate entirely suppressed the turbulent bursts.

In sections 3.1 and 3.4, we have shown that as the hump height reaches a certain large value the variation of the predicted transition location with the hump height "saturates." An increase in the hump height beyond this value has virtually no effect on the upstream movement of the predicted transition location. However, the threshold levels of continuous suction, beyond which the predicted transition location moves considerably downstream, are different for different hump heights in the saturation region. For example, in Figure 12(a) for $h = 0.003$, the threshold-continuous suction level is close to $v_w = -7.5 \times 10^{-5}$; for $h = 0.004$, the threshold suction level was not reached even at values of v_w up to -15×10^{-5}, beyond which the IBL code failed to converge.

For a boundary-layer flow over a smooth flat plate ($h = 0$), and in the presence of a roughness element, the application of suction shifts the most dangerous frequency toward a lower value, which is shown in Figure 12(b).

3.7 Effect of a Suction Strip

Previous theoretical and experimental studies have shown that for the same amount of mass flow rate the application of suction through discrete porous strips on smooth flat plates is more effective for laminar-flow control than for continuous suction. Reed and Nayfeh[27] studied the effect of suction through discrete porous strips on the stability of incompressible flow over a smooth flat plate. In the work of Reed and Nayfeh, the mean flow was calculated with the triple-deck theory. The calculations were performed both with and without suction at the same disturbance frequency; the results were in reasonable agreement with the results of the companion forced experiment of Reynolds and Saric.[28] The major conclusion of the work of Reed and Nayfeh is that at the same disturbance frequency the optimal location of a suction strip is shortly downstream of branch I of the neutral stability curve. Masad and Nayfeh[29] extended the work of Reed and Nayfeh[27] to compressible subsonic flow and found Reed and Nayfeh's conclusion to hold in these flows as well. Furthermore, Masad and Nayfeh showed that after the possibility of a shift in the most dangerous frequency (caused by the presence of a suction strip) is taken into account the optimal location of a suction strip is not necessarily close to branch I of the neutral stability curve. As pointed out by Masad and Nayfeh, accounting for the shift in frequency corresponds to the situation in a natural transition experiment, whereas the conclusion about the optimal location of a suction strip at the same disturbance frequency corresponds to the situation in a controlled (forced) experiment such as that of Reynolds and Saric.[28] In the work of Masad and Nayfeh, the compressible mean flow over the smooth flat plate with a suction strip was calculated with IBL theory.

Calculations similar to those in reference 29 were performed with and without a hump after the possibility of the shift of the most dangerous frequency in the presence of a suction strip was taken into account. The results shown in Figure 13(a) demonstrate the variation of the predicted transition location with the Reynolds number Re_x, based on the distance from the leading edge of the flat plate to the center of the suction strip. The horizontal dashed lines in Figure 13(a) indicate the predicted transition location in the absence of the suction strip. The four dashed lines that proceed downwards correspond to $h = 0, 0.001, 0.002$ and 0.003, respectively. Clearly, the application of suction through a strip has an overall stabilizing effect both with and without a hump. The optimal location for the suction strip is downstream of the center of the hump and moves upstream and toward the strong adverse pressure gradient region as the hump's height increases. Note that the center of the hump in Figure 13(a) is at an Re_x of 1 million. These results with an optimally located suction strip agree with the experimental findings of Hahn and Pfenninger:[24] the optimal location for a suction strip in a separating flow over a backward-facing step is near the reattachment region that occurs without suction. In Figures 13(a) and 13(b), note the shift in the most dangerous frequency toward lower values as the predicted transition location moves downstream.

3.8 Effect of Continuous Cooling

Although thinning the boundary layer by continuous cooling makes it more sensitive to surface roughness, continuous cooling also delays separation until the hump height becomes large and reduces the size of the separation bubble when separation occurs. Therefore, cooling can actually be used to make the boundary layer capable of overcoming stronger adverse pressure gradients before separation occurs. A consistent decrease in the streamwise length of the separation region with cooling in a flow over a roughness element was noticed in the experiment of Larson and Keating.[21] Furthermore, Larson and Keating observed that in some instances cooling caused reattachment of the separated boundary layer onto the surface.

Cooling of subsonic air boundary layers on smooth flat plates is known to cause the streamwise velocity profile to become fuller and to make the boundary layer thinner. Both of these changes in the attached mean flow have stabilizing effects through the viscous mechanism. However, the existence of a roughness element induces an adverse pressure gradient that might cause the velocity profile to develop an inflection point near the wall, which has a destabilizing effect on the flow. The existence of a roughness element in the flow field might also cause the flow to separate, which introduces a free shear-layer instability mechanism that is inviscid and different from the viscous instability mechanism.

From practical and experimental points of view, fixing the wall temperature is easier than fixing the heat flux through the wall. Therefore, we express the level of heat transfer by specifying the ratio of the actual wall temperature to the adiabatic wall temperature T_w/T_{ad}. For $T_w/T_{ad} = 1$, we have an adiabatic condition; values of $T_w/T_{ad} < 1$ indicate cooling.

Although continuous cooling increases the growth rate in the reduced separation bubble, the stabilizing effect of continuous cooling in the attached flow regions overcomes that destabilizing effect. Therefore, application of continuous cooling moves the predicted transition location downstream in a flow over a roughness element as shown in Figure 14(a). Figure 14(a) clearly shows that by applying sufficient continuous cooling, the predicted transition location in a flow over a hump moves to the predicted transition location in a flow over a smooth flat plate, as in the case with continuous suction. The local destabilization of flow over a roughness element within the separation bubble by continuous cooling was demonstrated in the theoretical works of Al-Maaitah et al.[30] and Masad and Nayfeh.[31] However, an optimal level of continuous cooling, beyond which the overall effect of continuous cooling on the flow over a roughness element becomes destabilizing,[30] was not encountered in the present study. The existence of the optimal level of continuous cooling that was found in reference 30 could possibly result because no computation was performed at the most dangerous frequencies either with or without continuous cooling. The downstream movement of the predicted transition location attributed to continuous cooling is associated with an increase in the corresponding most dangerous frequency (Figure 14(b)).

3.9 Effect of a Heating Strip

The use of a heating strip placed close to the leading edge of a flat plate to stabilize the air boundary-layer flow is not new in Russian literature. Dovgal et al.[32] and Fedorov et al.[33] referenced several theoretical and experimental studies (in Russian) on this method of laminar-flow control. Although previous studies considered the fixed-frequency condition of a forced experiment, Dovgal et al.[32] showed experimentally that the method also works under natural transition conditions. In their natural transition experiments, Dovgal et al.[32] showed that by placing a 100–mm heating strip at a temperature of 382 K at the leading edge of a flat plate with a temperature of 301 K elsewhere, the transition Reynolds number was increased from 1.7 million in the adiabatic case to 2.9 million in the presence of the heating strip. The possibility of using localized surface heating to relaminarize the turbulent flow on a smooth flat plate was demonstrated in the work of Maestrello and Nagabushana.[34]

By placing a heating strip in the air flow close to the leading edge of a smooth flat plate, the flow that leaves the heating strip encounters a relatively cooler surface, which has the same stabilizing effect as continuous cooling applied elsewhere. Masad and Nayfeh[29] conducted a theoretical study on the effect of placing a heat-transfer (heating or cooling) strip in the flow field of an air flat-plate boundary layer on the stability of that flow. This work considered subsonic flows up to a free-stream Mach number of 0.8. Criteria for the optimal stabilizing location of a heating or a cooling strip for the fixed-frequency condition of a forced experiment were also obtained by these investigators.

Masad and Nayfeh[29] found that by placing a heating strip before branch I of a certain frequency, the disturbance becomes destabilized in the region of the strip and then stabilized over a short region. Finally, in a third region, the growth rates decrease in comparison with disturbance growth rates for adiabatic flow. The optimal stabilizing location of the heating strip corresponded to the location where the accumulated growth in the first unstable region was barely compensated for by the accumulated decay in the stable region that followed. This criterion for the optimal location at a fixed frequency was accurate for different subsonic flow parameters and different heating-strip parameters.

In reference 29, no critical length of the heating strip was found to exist beyond which the stabilization (or destabilization) effect of the heating strip became active. However, an increase in the length of the strip or in the level of heating within the strip while the location of the heating strip is kept fixed causes the effect of the heating strip to become more pronounced. If the heating strip is placed at locations that are downstream of branch I of a certain frequency, then the disturbance that has that frequency becomes destabilized.

In this section, we consider the effect of moving a heating strip on a flat plate with and without a hump on the predicted transition location for air boundary layers. These conditions considered here simulate those in a natural transition experiment.

To study the effect of the location of a heating strip on the predicted transition location with and without a hump, we considered a heating strip with a fixed length of $\Delta x = 0.4$ and with a heating level of $T_w/T_{ad} = 1.3$ within the heating strip and 1.0 (adiabatic conditions) elsewhere.

The predicted transition location was calculated with the heating strip at a location close to the leading edge; the heating strip was then moved downstream and the predicted transition location was recalculated, and so on. Variations of the predicted transition Reynolds number, with the Reynolds number based on the center of the heating strip in the presence and absence of a hump, are shown in Figure 15(a). The horizontal centerlines indicate the predicted transition Reynolds numbers in the absence of a heating strip for a smooth flat plate (the upper centerline) and a flow over a hump with $h = 0.002$ (the lower centerline). The figure clearly shows that a heating strip placed in the adverse pressure gradient region created by a hump enhances separation. Placing a heating strip at locations close to the leading edge has a stabilizing effect in comparison with the case of a fully adiabatic plate (the centerlines in Figure 15(a)). If the heating strip is moved downstream of a certain location on the plate, the flow is destabilized. Note in Figure 15(a) that when the heating strip is placed at certain downstream locations, two values for the predicted transition Reynolds number correspond to the same location of the heating strip. The large value of the predicted transition Reynolds number corresponds to low frequencies; the low value corresponds to high frequencies (Figure 15(b)). The two domains of frequencies are separated by a third domain of frequencies where the N-factor does not reach a value of 9. Therefore, in a situation where initial disturbances with frequencies within the whole domain of frequencies exist, the predicted transition Reynolds number is expected to follow the lower branch in Figure 15(a). If the initial disturbances have only relatively low frequencies, the transition location is expected to follow the upper branch. Note that by moving the heating strip further downstream both branches meet and the predicted transition location becomes unique.

3.10 Relation Between Transition Location and Most Dangerous Frequency

We have studied the effects of the height, length, shape, and location of the hump; the unit Reynolds number; the free-stream Mach number; continuous suction; a suction strip; continuous cooling; and a heating strip on the predicted transition location. We have seen a consistent increase in the most dangerous frequency when the predicted transition location moves upstream. In Figure 16, we plotted the variation of the predicted transition location with the most dangerous frequency for all the data points generated in the previous sections, as well as other data points computed for subsonic flow over a smooth flat plate with different boundary conditions. Figure 16 clearly shows a strong correlation between the predicted transition Reynolds number (with the e^N method) and the most dangerous frequency. The value of the predicted transition Reynolds number increases as the value of the most dangerous frequency decreases.

4. CONCLUSIONS

The effect of a surface hump on the transition location in a subsonic flow is analyzed with the use of the linear quasi-parallel stability theory coupled with the e^N transition criterion. The mean flow, which might separate and reattach, is computed by solving the interacting boundary-layer equations. The effects of the height, length, location, and shape of the hump; the free-stream unit Reynolds number; the free-stream Mach number; the levels of continuous suction and cooling; and the effects of the locations of a suction and a heating strip on the predicted transition location are evaluated. Results with the N-factor criterion are compared with the experimental correlation of Fage.[14] Based on this study, the following conclusions are reached:

1. As the hump height increases from the zero value (no hump), the predicted transition location moves continuously upstream. Furthermore, when the hump height is close to the value that causes the flow to separate, the predicted transition location moves considerably upstream.

3. When the hump height reaches a certain large value that exceeds the value that causes the flow to separate, a further increase in the hump's height has virtually no effect on the upstream movement of the predicted transition location. An increase in the hump's height only enhances the length of the separation region at this stage.

3. The effect of the hump's length is opposite that of the effect of the hump's height. An increase in the hump's length moves the predicted transition location downstream. A decrease in the length of the hump enhances separation.

4. At low and high values of the free-stream Reynolds number Re based on the distance from the leading edge to the center of the hump L^*, the effect of the hump on the predicted transition location diminishes. The effect continues when Re is varied by changing either the free-stream unit Reynolds number or the distance L^*. An increase in Re enhances separation.

5. Results of the N-factor criterion agree well with the experimental correlation of Fage.

However, the N-factor criterion is more generally applicable and more powerful because it is not restricted to a certain range of free-stream or transition Reynolds numbers and it accounts for compressibility, roughness location and shape, wall suction, and heat transfer.

6. An increase in the Mach number of a subsonic flow over a hump enhances separation, which partially offsets the stabilizing effect of compressibility in the attached flow region. However, the overall effect of compressibility is stabilizing. At large hump heights, compressibility has virtually no effect on the movement of the predicted transition location.

7. The effect of making the hump fuller and eliminating the rounding at the leading and trailing edges of the hump (while the length remains fixed) on the upstream movement of the predicted transition location is moderate at small to medium hump heights. At large heights of the hump (which cause separation), the effect of varying the shape (contour) of the hump on the predicted transition location is negligible.

8. Continuous suction has an overall stabilizing effect on the flow over a hump of small to moderate height and moves the predicted transition location downstream. At large hump heights, suction has no influence on the predicted transition location unless the suction level reaches a threshold value beyond which the predicted transition location for the flow over the hump moves sharply to the predicted transition location of a flow over a smooth flat plate.

9. The application of suction through a strip has an overall stabilizing effect on the flow over the hump. The optimal stabilizing location of a suction strip is downstream of the center of the hump. As the hump height increases, the optimal stabilizing location for a suction strip moves upstream.

10. Continuous suction, suction through a strip, and continuous cooling delay the occurrence of separation to larger hump heights or reduce the size of the separation bubble when the flow separates.

11. Although continuous cooling might increase the growth rates of the disturbances within the reduced separation bubble, its overall effect is stabilizing and moves the predicted transition location downstream.

12. By placing a heating strip close to the leading edge of a flat plate with or without a hump, the occurrence of natural transition can be delayed to downstream locations. The optimal stabilizing location for a heating strip is close to the leading edge of the flat plate.

13. If a heating strip is placed at a downstream location, the predicted transition location moves considerably upstream. Placement of a heating strip in the adverse pressure gradient region shortly downstream of the center of the hump enhances separation.

14. As the predicted transition location in a flow over a smooth flat plate or a flat plate with a hump moves upstream because of any of the effects discussed in this work, the associated most dangerous frequency increases.

Acknowledgements

The fruitful discussions and comments of Drs. M. R. Malik, C. L. Streett, A. Kumar, and M. Choudhari are greatly appreciated. This research is supported by the Theoretical Flow Physics Branch, Fluid Mechanics Division, NASA Langley Research Center, Hampton, VA, under contract numbers NAS1-19299 (first author) and NAS1-19672 (second author).

References

[1] B. Holmes, C. O'Bara, G. Martin, and C. Domack, "Manufacturing requirements," NASA CP 2413, 1986, pp. 171-183.

[2] M. Choudhari and C. Street, "A finite Reynolds number approach for the prediction of boundary layer receptivity in localized regions," *Phys. Fluids A*, Vol. 4, No. 11, 1992, pp. 2495–2514.

[3] A. Nayfeh, S. Ragab, and A. Al-Maaitah, "Effect of bulges on the stability of boundary layers," *Phys. Fluids*, Vol. 31, No. 4, 1988, pp. 796–806.

[4] T. Cebeci and D. Egan, "Prediction of transition due to isolated roughness," *AIAA Journal*, Vol. 27, No. 7, 1989, pp. 870–875.

[5] J. Masad and A. Nayfeh, "Stability of separating boundary layers," *Proceedings of the Fourth International Conference of Fluid Mechanics*, Alexandria, Egypt, Vol. I, 1992, pp. 261–278.

[6] A. Nayfeh, S. Ragab, and J. Masad, "Effect of a bulge on the subharmonic instability of subsonic boundary layers," *Phys. Fluids A*, Vol. 2, No.6, 1990, pp. 937–948.

[7] J. Masad and A. Nayfeh, "Effect of a bulge on the subharmonic instability of subsonic boundary layers," *AIAA Journal*, Vol. 30, No. 7, 1992, pp. 1731–1737.

[8] H. Bestek, K. Gruber, and H. Fasel, "Numerical investigation of unsteady laminar boundary layer flows over backward-facing steps," *The Fourth Asian Congress of Fluid Mechanics*, Hong Kong, Aug. 19–23, 1989.

[9] S. Elli and C. Van Dam, "The influence of laminar separation bubble on boundary-layer instability," AIAA-91-3294, 1991.

[10] D. Spence and D. Randall, "The influence of surface waves on the stability of a laminar boundary layer with unifom suction," Aeronautical Research Council Note, No. Aero 2241, 1954.

[11] P. Klebanoff and P. Tidstrom, "Mechanisms by which a two-dimensional roughness element induces boundary-layer transition," *Phys. Fluids*, Vol. 15, No. 7, 1972, pp. 1173–1188.

[12] T. Corke, A. Bar-Sever, and A. Morkovin, "Experiments on transition enhancement by distributed roughness," *Phys. Fluids*, Vol. 29, No. 10, 1986, pp. 3199–3213.

[13] S. Ragab, A. Nayfeh, and R. Krishna, "Stability of compressible boundary layers over a smooth backward-facing step," AIAA 90–1449, 1990.

[14] A. Fage, "The smallest size of spanwise surface corrugation which affects boundary layer transition on an airfoil," British Aeronautical Research Council, 2120, 1943.

[15] B. Carmichael, "Surface waviness criteria for swept and unswept laminar suction wings," Northrop Aircraft Report No. NOR–59–438 (BLC-123), 1957.

[16] A. Dovgal and V. Kozlov, "Hydrodynamic instability and receptivity of small scale separation regions," *Laminar-Turbulent Transition*, (D. Arnal and R. Michel ed.), Springer, Berlin, 1990, pp. 523–531.

[17] V. Pereyra, "PASVA3: An adaptive finite difference Fortran program for first order nonlinear, ordinary boundary problems," *Lecture Notes in Computer Science*, Vol. 76, 1976, pp. 67–88.

[18] L. Mack, "Boundary-layer stability theory," Jet Propulsion Laboratory, Pasadena, CA, Document No. 900–277, Rev. A, 1969.

[19] H. Schlichting *Boundary-Layer Theory*, McGraw-Hill, Seventh Edition, 1979, pp. 536–537.

[20] M. Morkovin, "Critical evaluations of transition from laminar to turbulent shear layers with emphasis on hypersonically travelling bodies," AFFDL-TR-68–149, Wright-Patterson Air Force Base, Ohio, Air Force Flight Dynamics Laboratory, March 1969.

[21] H. Larson and S. Keating, "Transition Reynolds numbers of separated flows at supersonic speeds," NASA TN D-349, Dec. 1960.

[22] D. Chapman, D. Kuehn, and H. Larson, "Investigation of separated flows in supersonic and subsonic streams with emphasis on the effect of transition," NACA Report No. 1356, 1958.

[23] C. Lin, "On the stability of laminar mixing region between two parallel streams in a gas," NASA TN 2887, 1953.

[24] M. Hahn and W. Pfenninger, "Prevention of transition over a backward step by suction," *Journal of Aircraft*, Vol. 10, No. 10, 1973, pp. 618–622.

[25] B. Carmichael, R. Whites, and W. Pfenninger, "Low-drag boundary-layer suction experiment in flight on the wing glove of a F–94A airplane," Northrop Aircraft Rept. No. NA1–57–1163 (BLC-101), 1957.

[26] B. Carmichael and W. Pfenninger, "Surface imperfection experiments on a swept laminar suction wing," Northrop Aircraft Rept. No. NAR–59–454 (BLC-124).

[27] H. Reed and A. Nayfeh, "Numerical-perturbation technique for stability of flat-plate boundary layers with suction," *AIAA Journal*, Vol. 24, No. 2, 1986, pp. 208–214.

[28] G. Reynolds and W. Saric, "Experiments on the stability of flat-plate boundary layer with suction," *AIAA Journal*, Vol. 24, No. 2, 1986, pp.202–207.

[29] J. Masad and A. Nayfeh, "Laminar flow control of subsonic boundary layers by suction and heat-transfer strips," *Physics of Fluids A*, Vol. 4, No. 6, 1992, pp. 1259–1272.

[30] Al-Maaitah, A. Nayfeh, and S. Ragab, "Effect of wall cooling on the stability of compressible subsonic flow over smooth humps and backward-facing humps," *Physics of Fluids A*, Vol. 2, No. 3, 1990, pp. 381–389.

[31] J. Masad and A. Nayfeh, "The influence of imperfections on the stability of subsonic boundary layers," in *Instabilities and Turbulence in Engineering Flows*, edited by D.E. Ashpis, T.B. Gatski, and R. Hirsch, Kluwer Academic Publishers, 1993, pp. 65–92.

[32] A. Dovgal, V. Levchenko, and V. Timofeev, "Boundary-layer control by a local heating of the wall," *Laminar-Turbulent Transition*, edited by D. Arnal and R. Michel, Springer-Verlag, 1990, pp.113–121.

[33] A. Fedorov, V. Levchenko, and A. Tumin, "Problems of laminar-turbulent transition control in a boundary layer," *Russ. J. Theor. Appl. Mech.*, Vol. 1, 1991, pp.85–101.

[34] L. Maestrello and K. Nagabushana, "Relaminarization of turbulent flow on a flat plate by localized surface heating," AIAA Paper No. 89–0985, 1989.

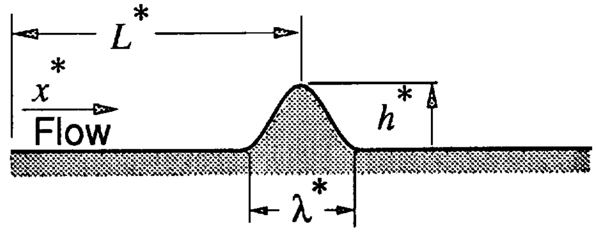


Figure 1. A hump on a flat plate.

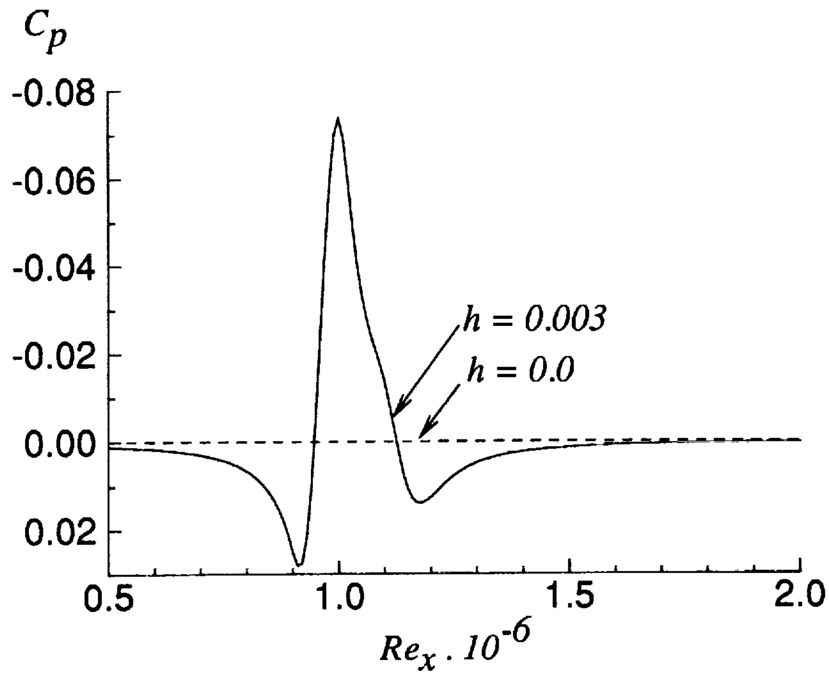


Figure 2. Variation of pressure coefficient with streamwise location for separating incompressible flow with and without a hump at $\lambda = 0.2$ and $Re = 10^6$.

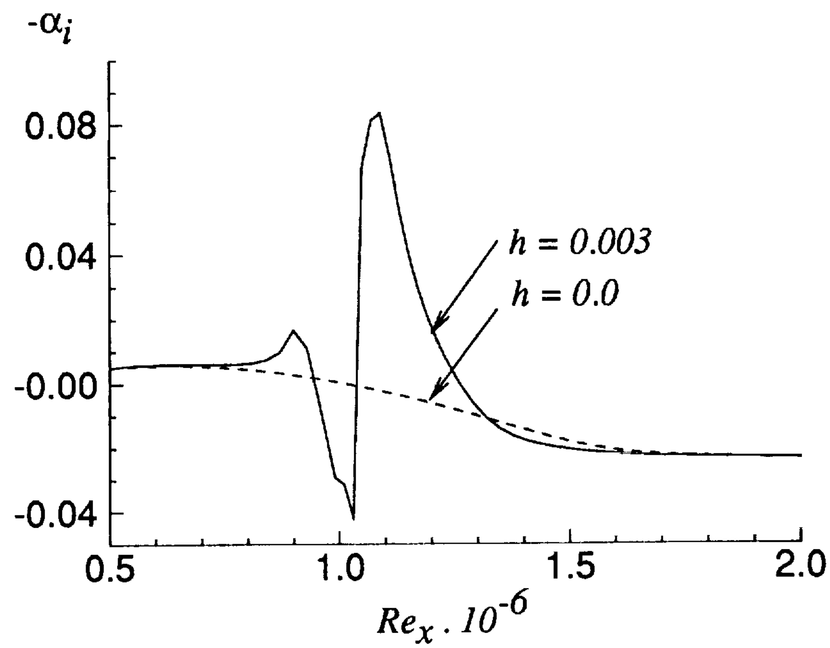
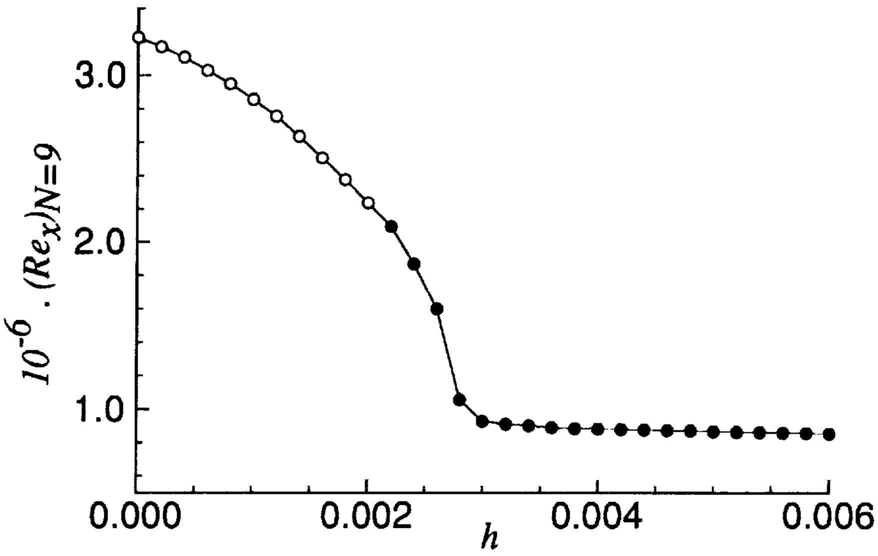
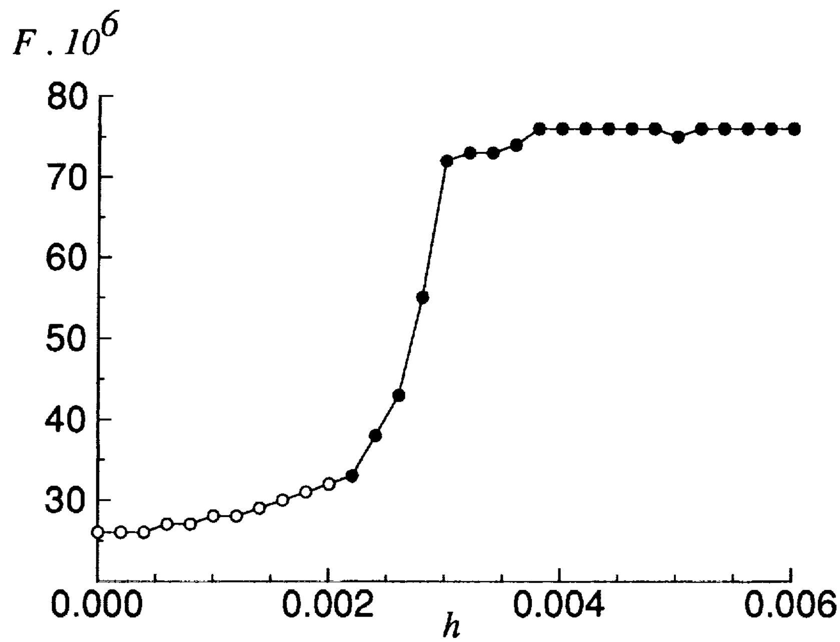


Figure 3. Variation of growth rates with streamwise location for flows shown in Figure 2 and at most dangerous frequency (in presence of hump) $F = 64 \times 10^{-6}$.

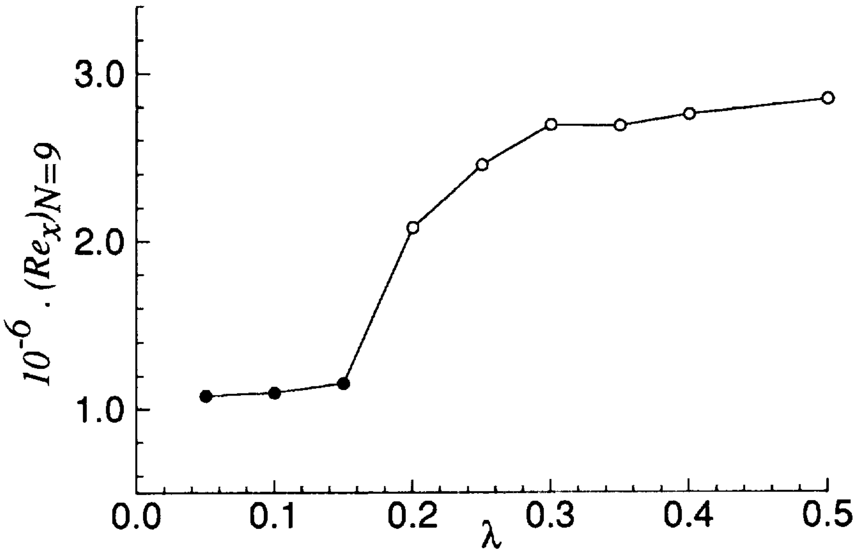


(a)

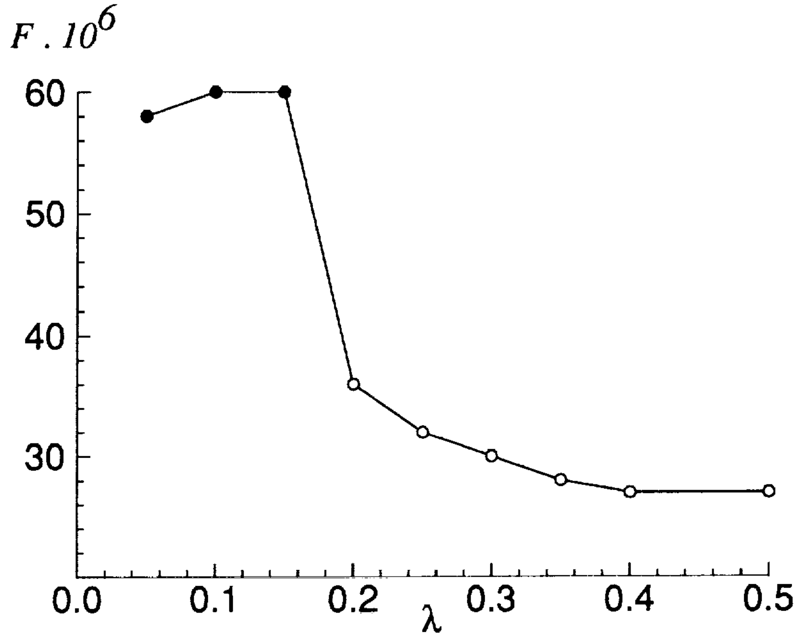


(b)

Figure 4. (a) Variation of predicted transition Reynolds number with hump height for incompressible flow at $Re = 0.8 \times 10^6$ and $\lambda = 0.2$. (b) Corresponding variation of frequencies.



(a)



(b)

Figure 5. (a) Variation of predicted transition Reynolds number with hump length for incompressible flow at $Re = 10^6$ and $h = 0.002$. (b) Corresponding variation of frequencies.

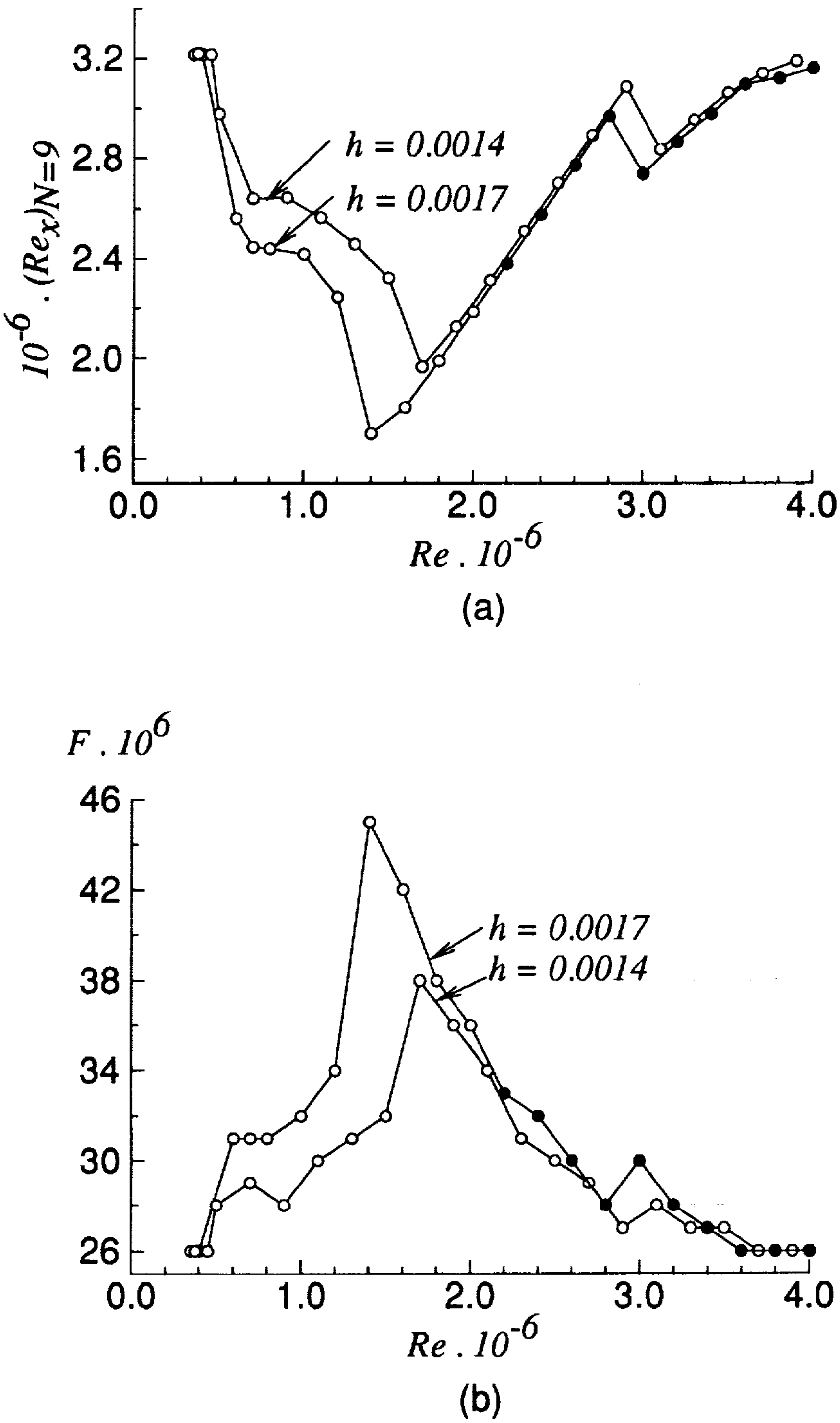


Figure 6. (a) Variation of predicted transition Reynolds number with free-stream Reynolds number for incompressible flow over hump and for two hump heights at $\lambda = 0.2$ (b) Corresponding variation of frequencies.

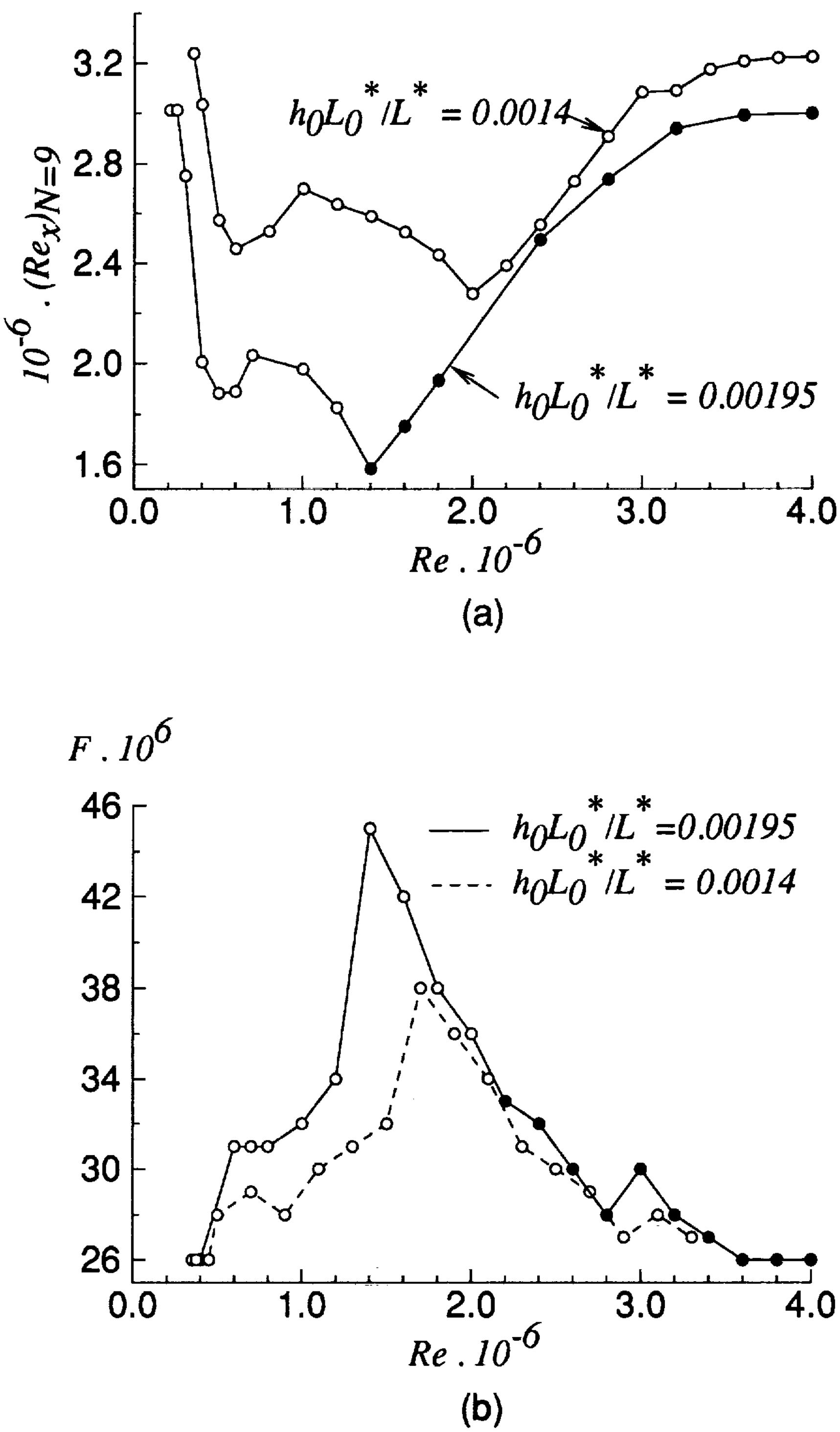


Figure 7. (a) Variation of predicted transition Reynolds number with free-stream Reynolds number for incompressible flow over hump and for two hump heights. (b) Corresponding variation of frequencies.

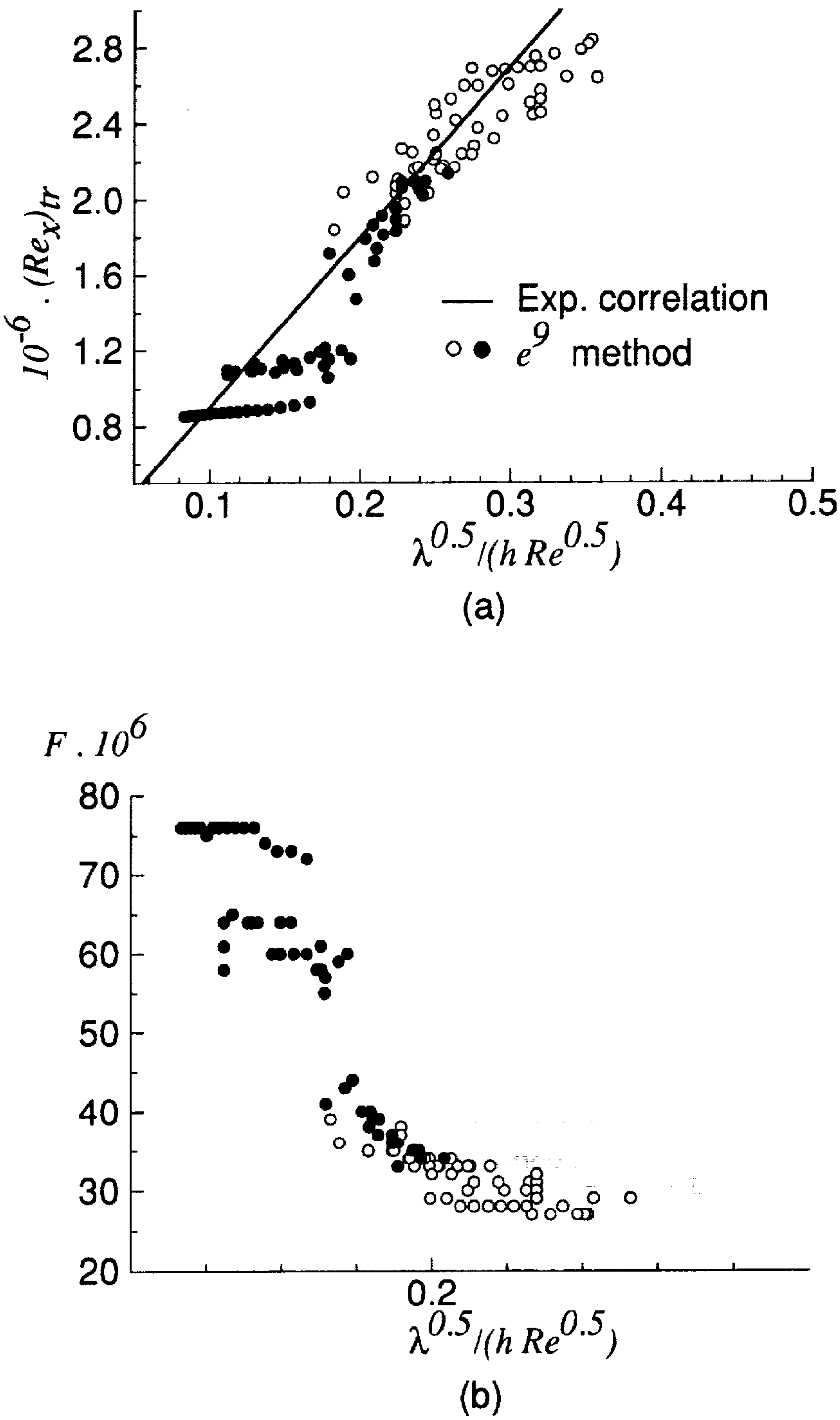


Figure 8. (a) Variation of predicted and experimentally correlated transition Reynolds number with shape free-stream Reynolds number parameter for incompressible flow over hump. (b) Corresponding variation of frequencies.

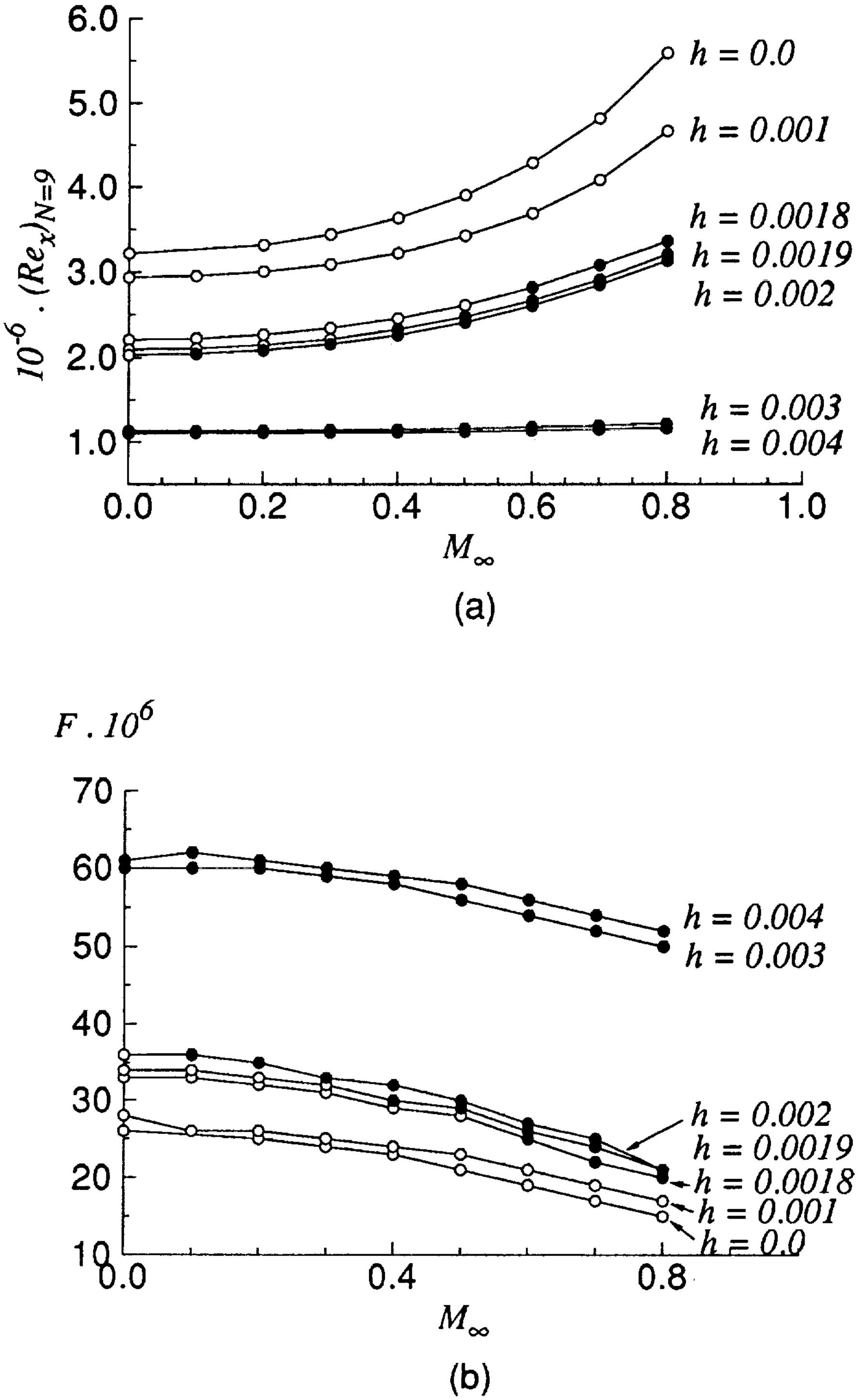


Figure 9. (a) Variation of predicted transition Reynolds number with free-stream Mach number for flow over hump at $\lambda = 0.2$, $Re = 10^6$, T_∞ =300° K, and Pr=0.72 for several heights. (b) Corresponding variation of frequencies.

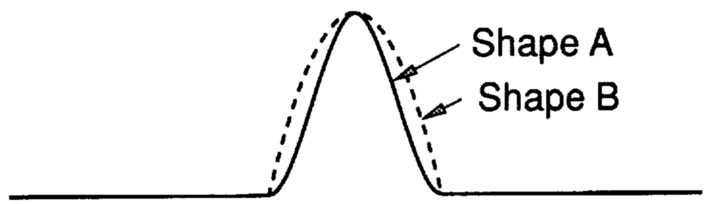


Figure 10. Shapes (contours) of two humps with same height and length.

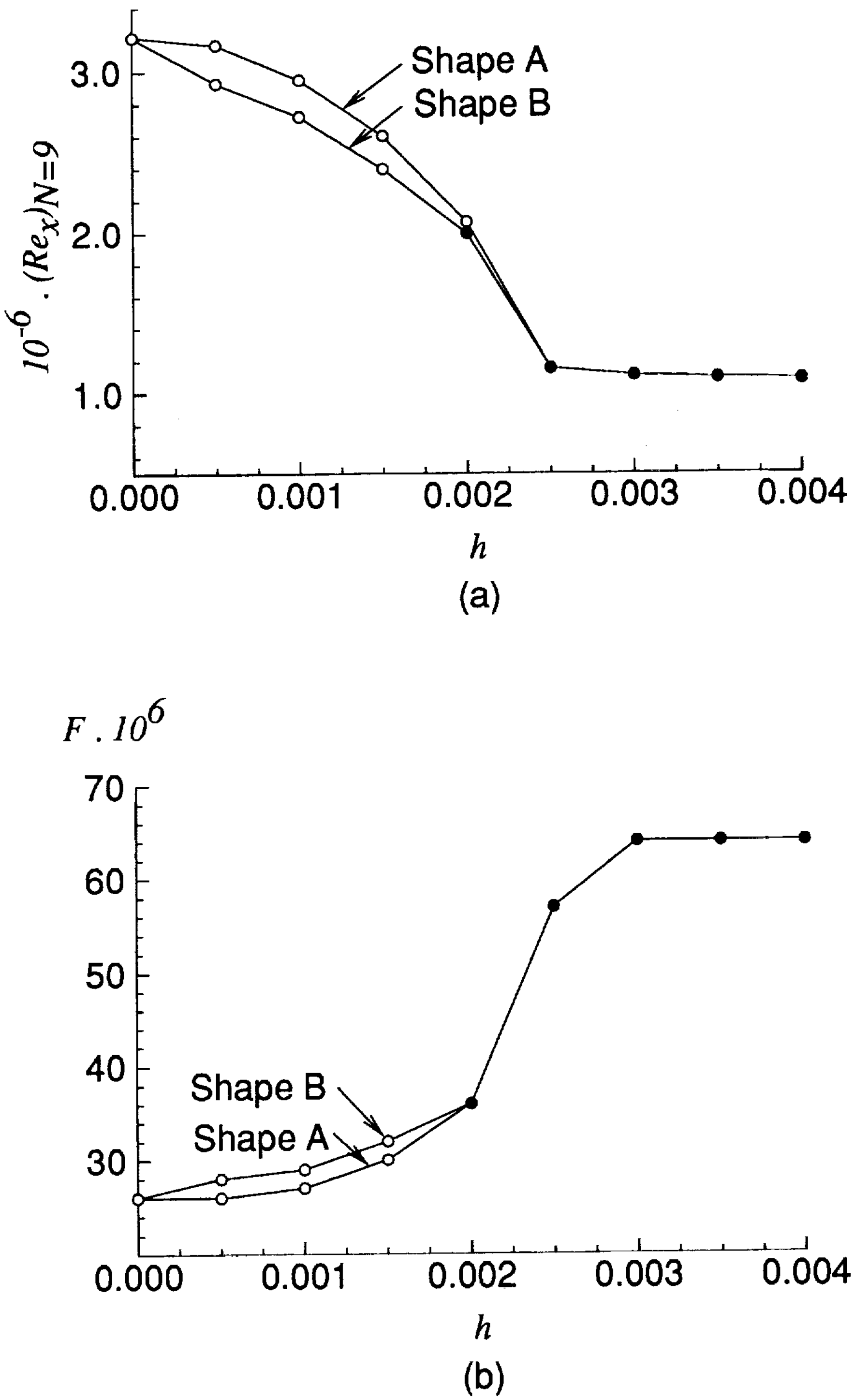


Figure 11. (a) Variation of predicted transition Reynolds number with hump height for incompressible flow at $\lambda = 0.2$, $Re = 10^6$ for two hump shapes. (b) Corresponding variation of frequencies.

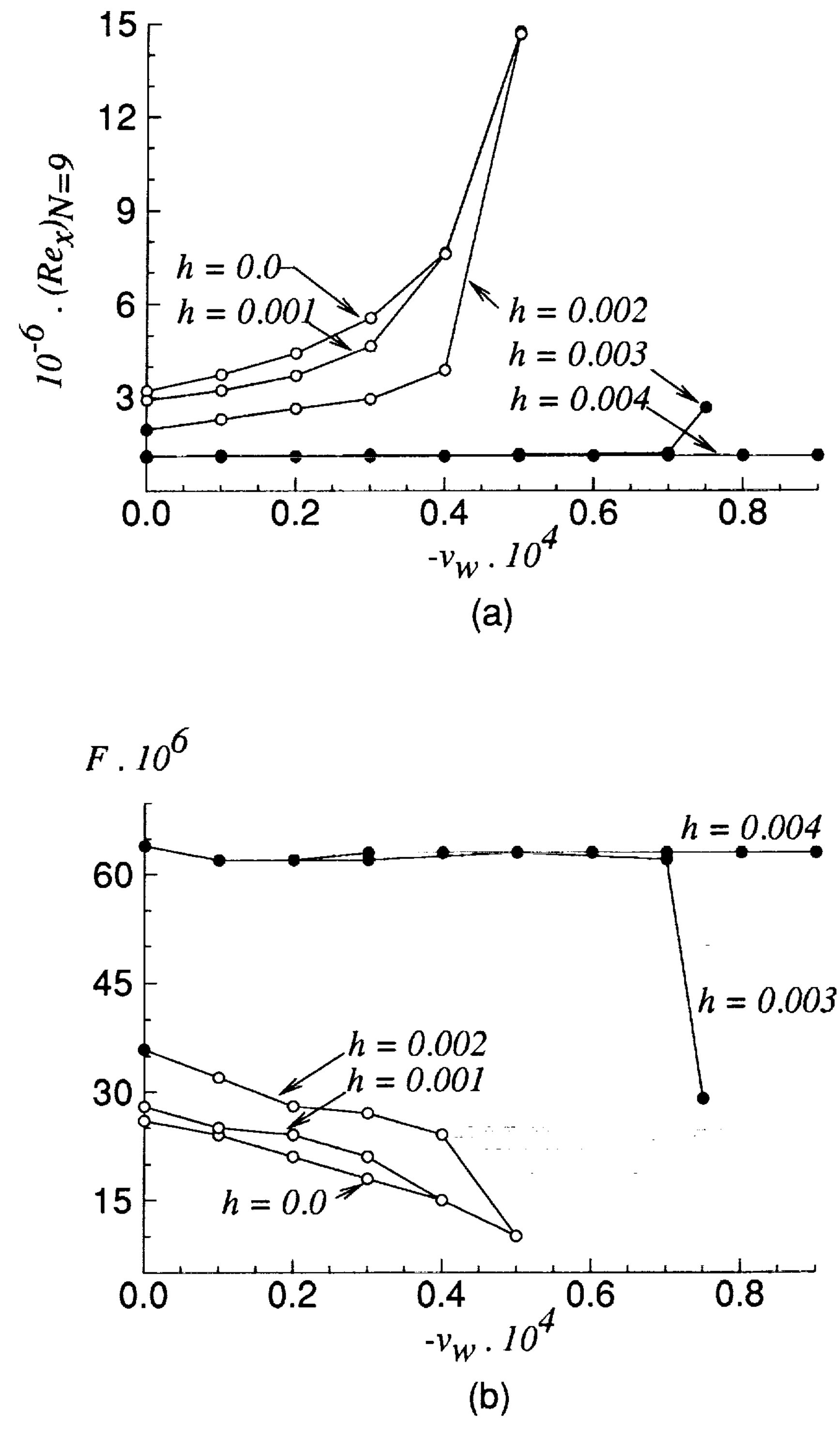


Figure 12. (a) Variation of predicted transition Reynolds number with uniform suction level for incompressible flow over hump at at $\lambda = 0.2$, $Re = 10^6$ for several heights. (b) Corresponding variation of frequencies.

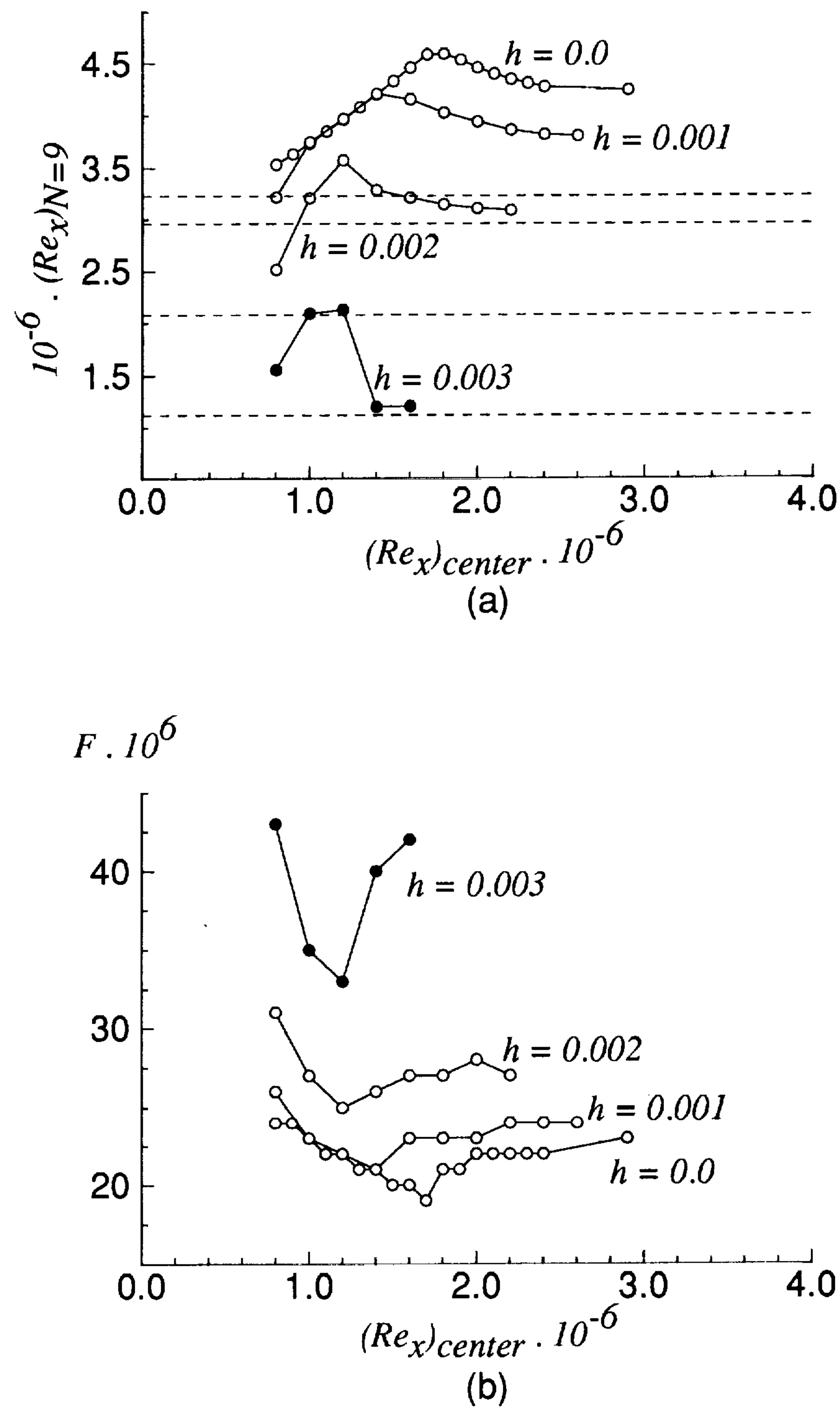


Figure 13. (a) Variation of predicted transition Reynolds number with location of center of suction strip of length $\Delta x = 0.4$ for incompressible flow over hump at $\lambda = 0.2$, $Re = 10^6$, $v_w = -2 \times 10^{-4}$ within strip for several heights. (b) Corresponding variation of frequencies.

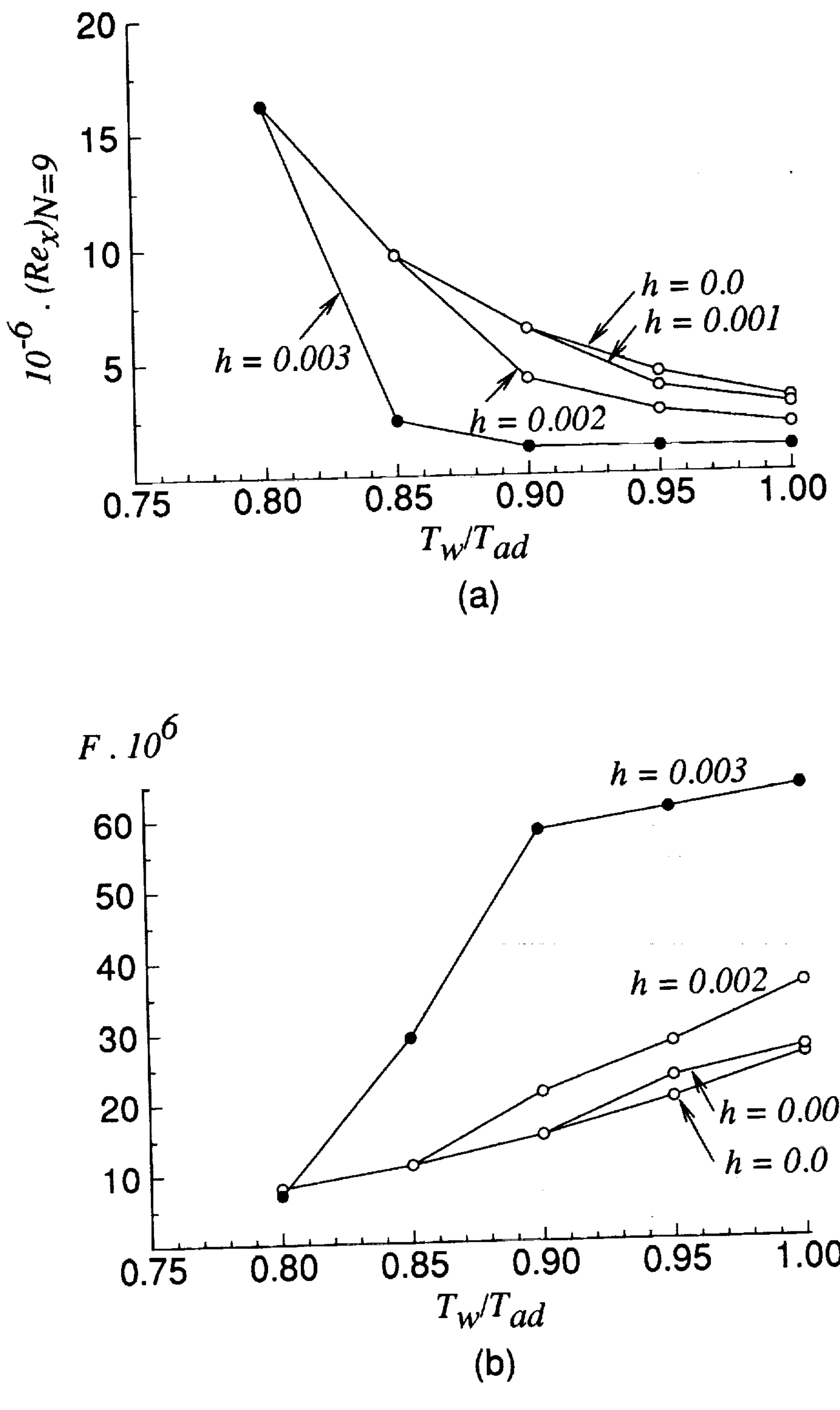


Figure 14. (a) Variation of predicted transition Reynolds number with continuous cooling level for incompressible air flow over hump at $\lambda = 0.2$, $Re = 10^6$, $T_w = 300$ °K, Pr=0.72 for several heights. (b) Corresponding variation of frequencies.

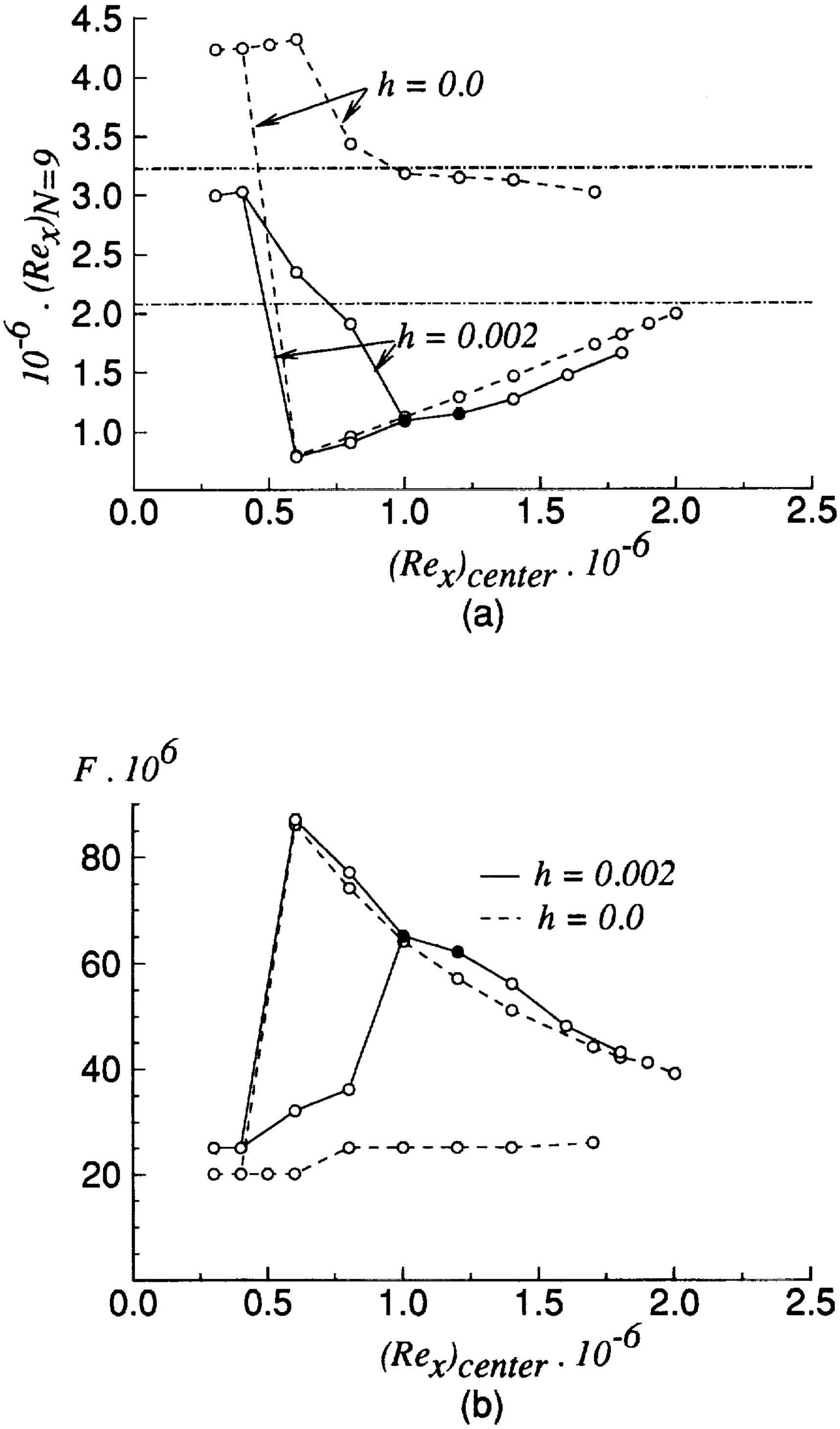


Figure 15. (a) Variation of predicted transition Reynolds number with location of center of heating strip of length $\Delta x = 0.4$ for incompressible flow over hump at $\lambda = 0.2$, $Re = 10^6$, $T_w/T_{ad} = 1.3$ within strip, T_∞=300 °K, Pr=0.72 for several heights. (b) Corresponding variation of frequencies.

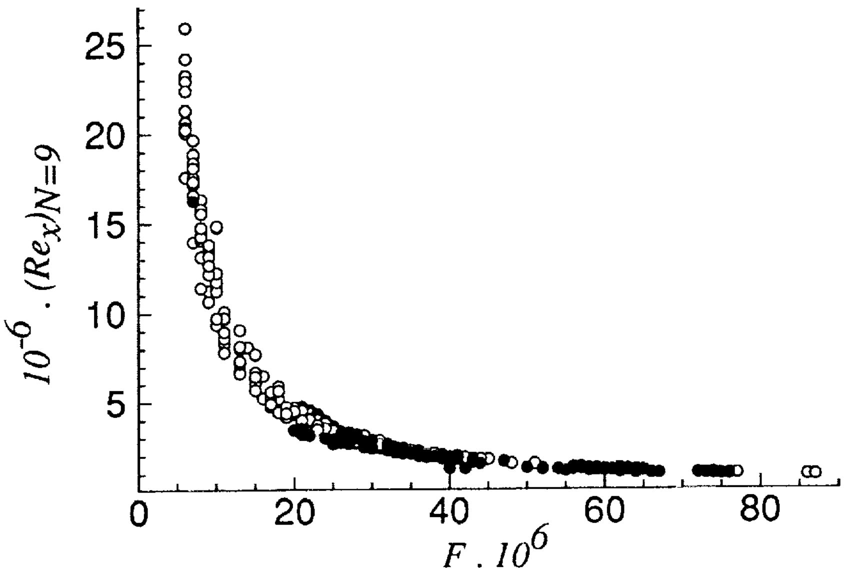


Figure 16. Variation of predicted transition Reynolds number with corresponding most dangerous frequency in subsonic flow with or without hump and for different velocity and thermal boundary conditions.

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	September 1993	Contractor Report

4. TITLE AND SUBTITLE
Transition Predicition and Control in Subsonic Flow Over a Hump

5. FUNDING NUMBERS
NAS1-19299
NAS1-19672

537-03-23-03

6. AUTHOR(S)
Jamal A. Masad
Venkit Iyer

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
High Technology Corporation
28 Research Drive
Hampton, VA 23666-1325

ViGYAN, Inc.
30 Research Drive
Hampton, VA 23666-1325

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
National Aeronautics and Space Administration
Langley Research Center
Hampton, VA 23681-0001

10. SPONSORING / MONITORING AGENCY REPORT NUMBER
NASA CR-4543

11. SUPPLEMENTARY NOTES
Langley Technical Monitor: Julius E. Harris

12a. DISTRIBUTION / AVAILABILITY STATEMENT
Unclassified/Unlimited

Subject Category 34

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

The influence of a surface roughness element in the form of a two-dimensional hump on the transition location in a two-dimensional subsonic flow with a free-stream Mach number up to 0.8 is evaluated. Linear stability theory, coupled with the *N*-factor transition criterion, is used in the evaluation. The mean flow over the hump is calculated by solving the interacting boundary-layer equations; the viscous-inviscid coupling is taken into consideration, and the flow is solved within the separation bubble. The effects of hump height, length, location, and shape; unit Reynolds number; free-stream Mach number; continuous suction level; location of a suction strip; continuous cooling level; and location of a heating strip on the transition location are evaluated. The *N*-factor criterion predictions agree well with the experimental correlation of Fage; in addition, the *N*-factor criterion is more general and powerful than experimental correlations. The theoretically predicted effects of the hump's parameters and flow conditions on transition location are consistent and in agreement with both wind-tunnel and flight observations.

14. SUBJECT TERMS
transition prediction, laminar flow control, roughness

15. NUMBER OF PAGES
60

16. PRICE CODE
A04

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	