Robust stability of diamond families of polynomials with complex coefficients

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Like the interval model of Kharitonov, the diamond model proves to be an alternative powerful device for taking into account the variation of parameters in prescribed ranges. The robust stability of some kinds of diamond polynomial families with complex coefficients are discussed. By exploiting the geometric characterizations of their value sets, we show that, for the family of polynomials with complex coefficients and both their real and imaginary parts lying in a diamond, the stability of eight specially selected extreme point polynomials is necessary as well as sufficient for the stability of the whole family. For the so-called simplex family of polynomials, four extreme point and four exposed edge polynomials of this family need to be checked for the stability of the entire family. The relations between the stability of various diamonds are also discussed.

1. Introduction

Motivated by Kharitonov's (1978) seminal theorem on robust stability for a box of polynomials, a number of recent papers have concentrated on the so-called dual problem for a diamond of polynomials (Tempo 1990, Barmish et al. 1990, Huang and Wang 1991, Kang et al. 1991, Wang and Huang 1992). Tempo (1990) showed that the stability of the whole diamond is equivalent to the stability

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2. The extreme point result

Let P^n denote the set of all nth order polynomials with complex coefficients

$$P^n = \{p(s)\,|\,p(s) = \Sigma^n_{i=0}\ (a_i + jb_i)s^i, \quad a_n + jb_n \neq 0\}$$

and let the complex diamond polynomial family be

$$W(s) = \{p(s)\,|\,p(s) = \Sigma^n_{i=0}\ (a_i + jb_i)s^i,$$

$$\Sigma^n_{i=0}\ (|a_i - \hat{a}_i| + |b_i - \hat{b}_i|) \leq r\} \subset P^n$$

where $\hat{a}_i$, $\hat{b}_i$ are the coefficients of the nominal polynomial

$$p_0(s) = \Sigma_{i=0}{}^n\ (\hat{a}_i + j\hat{b}_i)s^i$$

Define the eight extreme point polynomials of the complex polynomial family as

$$\begin{aligned}
p_1(s) &= p_o(s) + r \\
p_2(s) &= p_o(s) - r \\
p_3(s) &= p_o(s) + jr \\
p_4(s) &= p_o(s) - jr \\
p_5(s) &= p_o(s) + rs^n \\
p_6(s) &= p_o(s) - rs^n \\
p_7(s) &= p_o(s) + jrs^n \\
p_8(s) &= p_o(s) - jrs^n
\end{aligned}$$

Lemma 2

For any fixed $|\omega| \leq 1$, the value set $W(j\omega)$ is a rotated square with the two

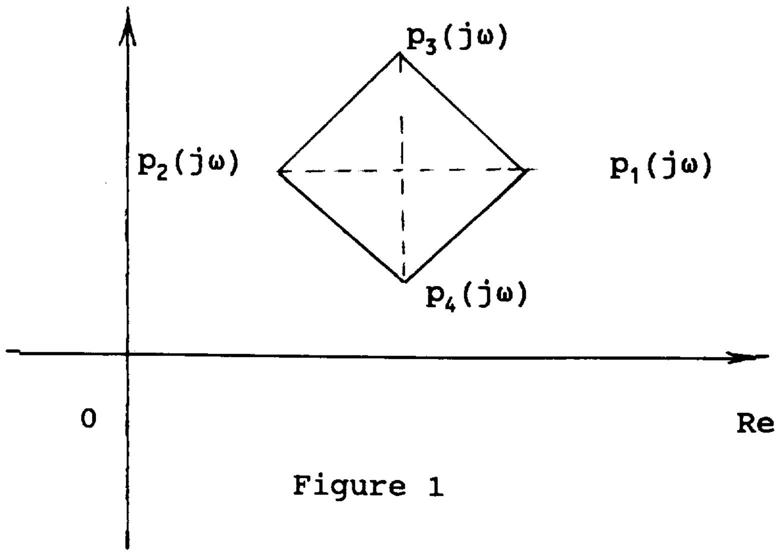

Figure 1

diagonals parallel to the two coordinate axes, and with the four vertices $p_1(j\omega)$, $p_2(j\omega)$, $p_3(j\omega)$, $p_4(j\omega)$, respectively (for illustration see fig. 1).

Lemma 3

For any fixed $|\omega| \geq 1$, the value set $W(j\omega)$ is a rotated square with the two diagonals parallel to the two coordinate axes, and with the four vertices $p_5(j\omega)$, $p_6(j\omega)$, $p_7(j\omega)$, $p_8(j\omega)$, respectively.

Lemma 4

Suppose $W(s) \cap H \neq \phi$, then $W(s) \subset H$ if

$$0 \notin W(j\omega), \quad \forall\omega \in R$$

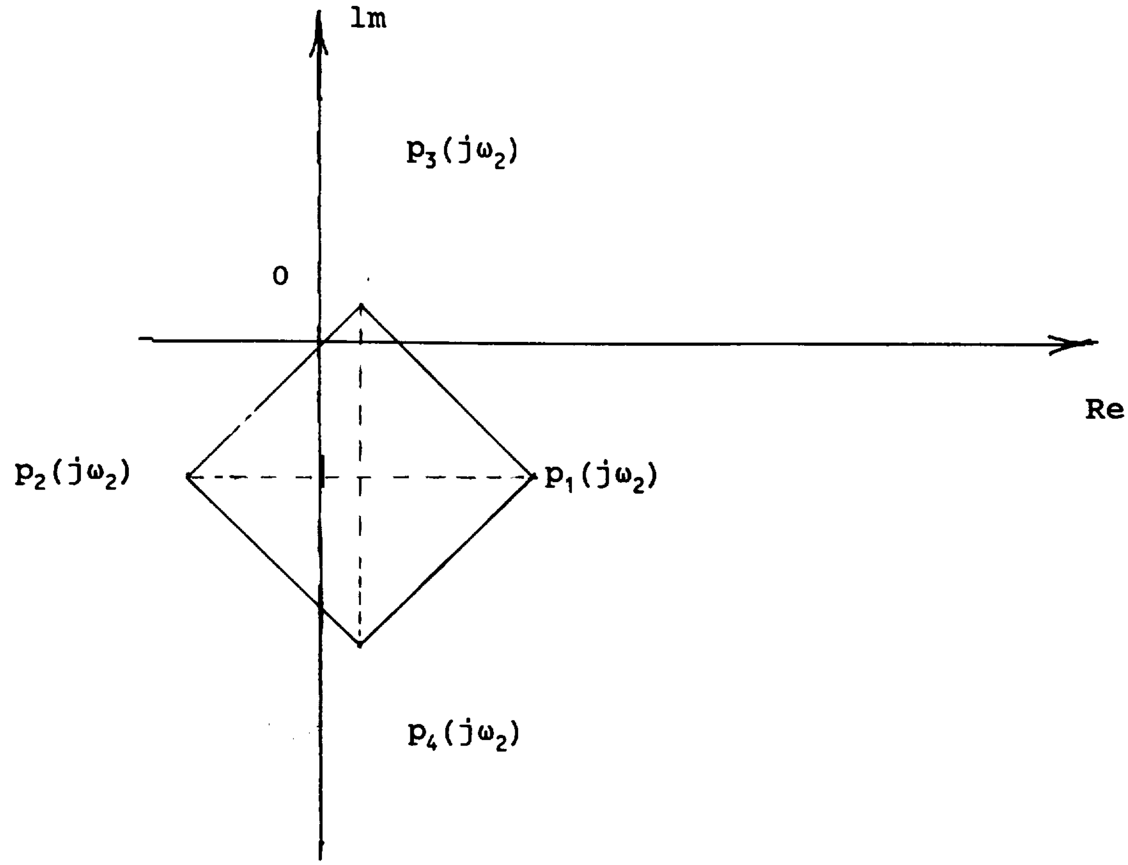

Figure 2.

Recall that $W(s) \subset P^n$. Thus if ω_∞ is large enough, then we have

$$0 \notin W(j\omega_\infty)$$

Now suppose there exists $\omega_1 \in R$ such that $0 \in W(j\omega_1)$. Then from continuity there must be $\omega_2 \in R$ such that $0 \in \partial W(j\omega_2)$. Without loss of generality, suppose $-1 \leq \omega < 1$ and $0 \in L[p_2(j\omega_2), p_3(j\omega_2)]$ (the line segment joining the two points $p_2(j\omega_2)$ and $p_3(j\omega_2)$) (see Fig.2). Since $p_2(s), p_3(s) \in H$, so $p_2(j\omega_2) \neq 0$, $p_3(j\omega_2) \neq 0$. Now consider a very small $\Delta\omega > 0$. According to Lemma 1, $p_2[j(\omega_2 + \Delta\omega)]$, $p_3[j(\omega_2 + \Delta\omega)]$ will be at the lower, upper half-planes divided by the line crossing $p_2(j\omega_2)$ and $p_3(j\omega_2)$, respectively. In this case, the two diagonals of the square $W[j(\omega_2 + \Delta\omega)]$ will no longer

It can be shown that this family is stability invariant if and only if the eight extreme point polynomials $p_0(s) \pm q_1(s)$, $p_0(s) \pm q_2(s)$, $p_0(s) \pm q_3(s)$, $p_0(s) \pm q_4(s)$ are stable, where

$$p_0(s) = \Sigma^n{}_{i=0} (\hat{a}_i + j\hat{b}_i)s^i$$

$$q_1(s) = r_0 - jr_1 s - r_2 s^2 + jr_3 s^3 + \cdots$$

$$q_2(s) = r_0 - jr_1 s - r_2 s^2 + jr_3 s^3 + \cdots$$

$$q_3(s) = jr_0 + r_1 s - jr_2 s^2 - r_3 s^3 + \cdots$$

$$q_4(s) = jr_0 + r_1 s - jr_2 s^2 - r_3 s^3 + \cdots$$

3. Result on diamonds with restrictions

In some cases, it is of interest to deal with the diamond family with the added restriction that perturbations are 'one-sided' (Wang and Huang 1992). In this section, we generalize some earlier results on the restricted diamond family of real polynomials to the complex polynomial case. Some interesting results are given.

Let us denote the so-called simplex polynomial family

$$W'(s) = \{p(s)\,|\,p(s) = \Sigma^n{}_{i=0}(a_i + jb_i)s^i, \quad a_i \geq \hat{a}_i, \quad b_i \geq \hat{b}_i,$$

$$i = 0,1, \ldots, n \text{ and } \Sigma^n{}_{i=0}[(a_i - \hat{a}_i) + (b_i - \hat{b}_i)] \leq r\} \subset P^n$$

For any fixed polynomials $p^{(1)}(s)$, $p^{(2)}(s) \in P^n$, denote

$$L[p^{(1)}(s),\ p^{(2)}(s)] = \{\lambda p^{(1)}(s) + (1 - \lambda)p^{(2)}(s) \mid \lambda \in [0,1]\}$$

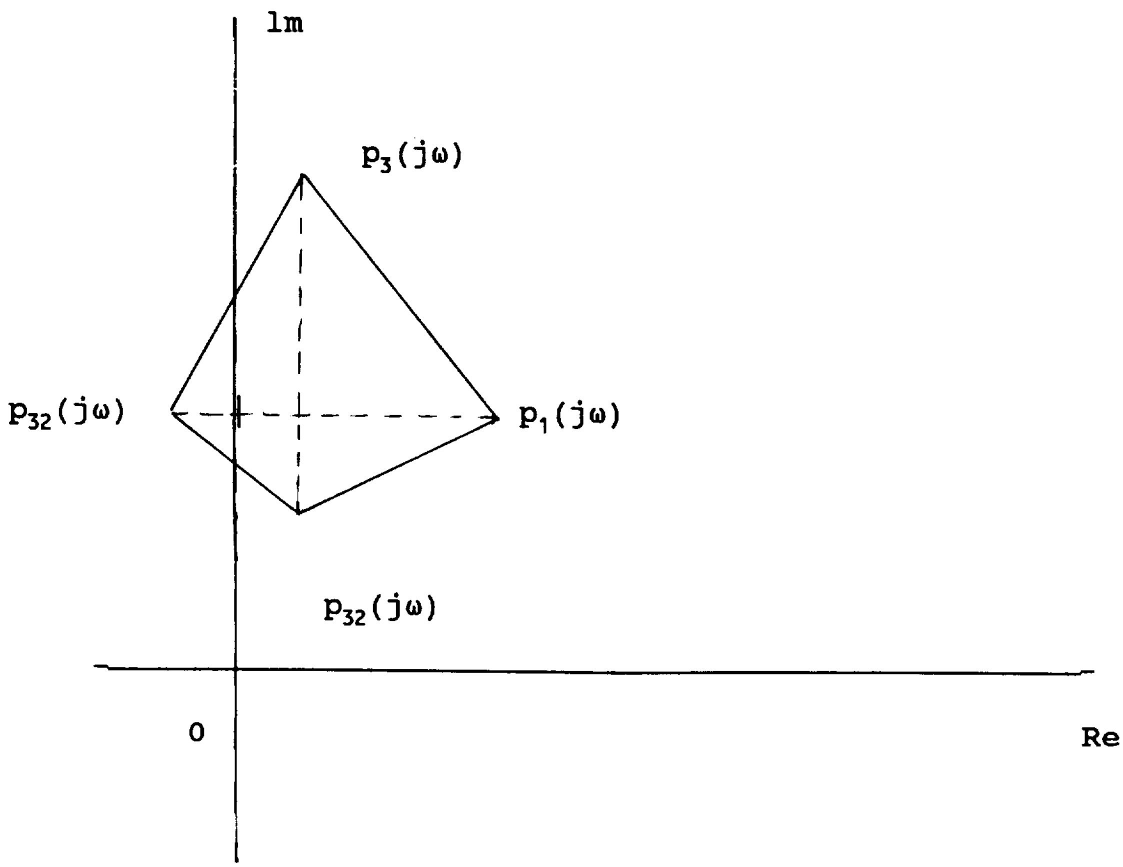

Figure 3.

Following a similar line of Tempo (1990), Huang and Wang (1991), Kang et al. (1991) and Wang and Huang (1992), we have the following.

Lemma 5

For any fixed $0 \le \omega \le 1$, the value set $W'(j\omega)$ is the convex hull

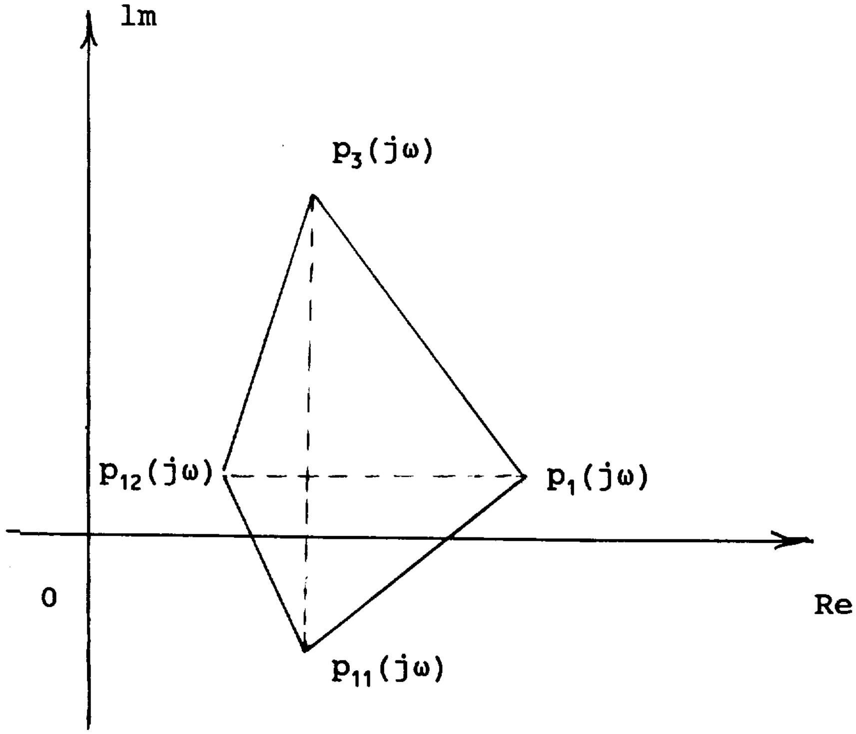

Figure 4.

Lemma 10

Let $p^{(0)}(s)$ and $p^{(1)}(s)$ be two polynomials with $\mathrm{Arg}[p^{(1)}(j\omega)]$ non-increasing with respect to $\omega \in R$. Suppose

$$p(s, \lambda) \triangleq p^{(0)}(s) + \lambda p^{(1)}(s) \in P^n, \quad \forall\lambda \in [0, 1]$$

Then $p(s,\lambda) \in H$, $\forall\lambda \in [0,1]$ if and only if $p(s, 0)$, $p(s, 1) \in H$.

Theorem 2

$W'(s) \subset H$ if and only if $p_{11}(s)$, $p_{31}(s)$, $p_{51}(s)$, $p_{71}(s) \in H$ and $L[p_1(s), p_{32}(s)]$, $L[p_3(s), p_{12}(s)]$, $L[p_5(s), p_{72}(s)]$, $L[p_7(s), p_{52}(s)] \subset H$.

When $r = r_i + r_2$ and the family

$$W'_2(s) = \{p(s)\,|\,p(s) = \Sigma^n_{i=0}\ (a_i + jb_i)s^i,\ a_i \geq \hat{a}_i,\ b_i \geq \hat{b}_i,$$

$$(a_i - \hat{a}_i) + (b_i - \hat{b}_i) \leq r_i,\ i = 0,\ 1,\ \ldots,\ n\} \subset P^n$$

when $r = \Sigma^n_{i=0} r_i$. It can be shown that results which involve checking exposed edges exist for the stability of $W'_1(s)$ and $W'_2(s)$.

Remark 4

The value set $W'(j\omega)$ exhibits some interesting behavior on the complex plane, for example, the angle spanned by $p_3(j\omega)$, $p_{31}(j\omega)$, $p_{32}(j\omega)$ is a right angle and the lengths of the line segments $L[p_0(j\omega),\ p_1(j\omega)]$, $L[p_0(j\omega),\ p_3(j\omega)]$ are equal. They are longer than $L[p_0(j\omega),\ p_{31}(j\omega)]$, $L[p_0(j\omega),\ p_{32}(j\omega)]$ and they remain constant when ω goes from 0 to 1.

Remark 5

The above result, which involves checking both extreme points and exposed edges, is a new type of criterion. From Theorem 2, we see that the restricted diamond $W'(s)$ is more 'irregular' than the diamond $W(s)$. It stands some where between the 'regular' families for whom there exist extreme point results and the ordinary polytopes for whom there exist exposed edge results. For families that are more 'irregular' than polytopes, the 'relative boundaries' need to be checked (Huang and Wang 1991). In addition, we see that

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