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Computational Strategies in the Dynamic Simulation of Constrained Flexible MBS

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ABSTRACT

This research focuses on the computational dynamics of flexible constrained multibody systems. At first a recursive mapping formulation of the kinematical expressions in a minimum dimension as well as the matrix representation of the equations of motion are presented. The method employs Kane's equation, FEM and concepts of continuum mechanics. The generalized active forces are extended to include the effects of high temperature conditions, such as creep, thermal stress and elastic-plastic deformation. The time variant constraint relations for rolling/contact conditions between two flexible bodies are also studied. The constraints for validation of MBS simulation of gear meshing contact using a modified Timoshenko beam theory are also presented. The last part deals with minimization of vibration/deformation of the elastic beam in multibody systems making use of time variant boundary conditions. The above methodologies and computational procedures developed are being implemented in a program called DYAMUS.

KINEMATICS OF FLEXIBLE TREE-LIKE SYSTEMS

An explicit matrix representation of the partial velocities and partial angular velocities for tree-like structures is given below. Consider a flexible body in a MBS discretized into P elements. Let the position vector to an arbitrary element i of body j w.r.t. a fixed reference frame R be given by

$$\overline{p}_{ji} = \{\sum_{h=0}^{H(j)}\{q_l\}^T[S^{l-1,0}] + \sum_{h=0}^{H(j)}\{\zeta_l\}^T[S^{l-1,0}]$$

$$+ (\{r_{ji}\}^T + \{\rho_{ji}\}^T[N]^T)[S^{j0}]\}\{\overline{n}\} \qquad (1)$$

where S denotes the shift matrix, q, ζ and r represent the body vector, the translation vector between adjacent bodies, and the position vector from the local reference frame of body j to element i, respectively. N is the shape function matrix, ρ denote the nodal coordinates, and $\overline{n}$ a set of unit vector fixed in R (see reference [1]-[2] for more detail).

The velocity of element i of body j found by differentiation of the above equation can be expressed as

$$\overline{v}_{ji} = \{\dot{x}\}^T\{V^{ji}\}+\{\dot{q}\}^T\{V_s^j\}+\{\dot{\zeta}\}^T\{V_s^j\}+\{\dot{\rho}\}^T\{V_e^{ji}\} \qquad (2)$$

Four arrays are identified in the velocity expression and found to take a special form. Note that x represent the rigid body rotation between adjacent bodies. The partial derivative of the element velocity yield the following

$$[V^{ji}] = [W]\begin{bmatrix} ([S_{q_2}] + [S_{\zeta_2}])[S^{10}] \\ ([S_{q_3}] + [S_{\zeta_3}])[S^{20}] \\ \vdots \\ ([S_{q_j}] + [S_{\zeta_j}])[S^{j-1,0}] \\ ([S_{r_{ji}}] + [S_{ji}])[S^{j0}] \\ \vdots \\ 0 \end{bmatrix} \qquad (3)$$

where W is a transformation matrix used to isolate the generalized coordinate derivatives from the generalized speeds. S_q, S_ζ, S_r and S_{ji} are skew matrices corresponding to q, ζ, r and ρ, respectively. The partial velocity array associated with element deformation is given by

$$[V_e^{ji}] = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ [N]^T[S^{j0}] \\ \vdots \\ 0 \end{bmatrix} \qquad (4)$$

The partial velocity array associated with $\dot{q}$ is expressed as

$$[V_i^j] = \begin{bmatrix} [I] \\ [S^{10}] \\ \vdots \\ [S^{j-1,0}] \\ \vdots \\ 0 \end{bmatrix} \qquad (5)$$

The bodies of the above block matrices could be achieved through a budgeting procedure where a master block is first developed then the rest of the arrays are formed through a partition and mapping technique see Table 1.

EQUATIONS OF MOTION

The governing equations of motion for flexible multibody systems can be expressed as

$$[M]\{\ddot{y}\} + [C]\{\dot{y}\} + [K]\{y\} = \{F\} \qquad (6)$$

where $[M]$ denotes the generalized mass matrix composed of 9 submatrices of the form

$$[M_{11}] = \sum_j \sum_i \int_{v_{ji}} (m_{ji}[V^{ji}][V^{ji}]^T + [\omega^j][I_{ji}][\omega^j]^T)dv \qquad (7)$$

The generalized mass is symmetric and the components of M_{ij} come directly from the kinematic bank of partial velocities and angular velocities of elements. The other mass components have similar expressions. Similarly, we can write the dynamic damping matrix, generalized stiffness matrix and force vector in a partition form with its components expressed as

$$[C_{11}] = \sum_j \sum_i \int_{v_{ji}} \{m_{ji}[V^{ji}][\dot{V}^{ji}]^T + [\omega^j]([I_{ji}][\dot{\omega}^j]^T$$

$$+ [\Omega_x^{0j}][I_{ji}][\omega^j]^T)\}dv \qquad (8)$$

and

$$\{F_1\} = \sum_j \sum_i \int_{s_{ji}} ([V^{ji}]\{f_{ji}\} + [\omega^j]\{\mathcal{M}_{ji}\})ds$$

$$+ \sum_j \sum_i \int_{v_{ji}} [V^{ji}]\{b_{ji}\}dv \qquad (9)$$

It is important to note at this stage how the kinematical expression form the bulk of all computations. In the above equations m_{ji} denotes the mass of element i in body j, I_{ji} the tensor dyadic, f_{ji} the force vector array acting on element i of body j, $\mathcal{M}_{ji}$ the corresponding moment array and b_{ji} the surface traction contribution vector.

CONSIDERATION OF HIGH TEMPERATURE, CREEP AND ELASTIC-PLASTIC DEFORMATIONS

The modeling of time-dependent forces resulting from deformable bodies when subjected high temperature conditions can be of interest in many engineering applications, which include creep, thermal stress, thermal shock, etc.. Many researchers studied material nonlinearities, in which some problems are solved, other still remain to be issues of concern.

The effects of temperature, creep and thermal stress and thermal shock can be included in the third term of the generalized force (see reference [2])

$$\{F_3\} = \sum_j \sum_i \int_{s_{ji}} ([V_e^{ji}]\{f_{ji}\} + [\omega^{ji}]\{\mathcal{M}_{ji}\})ds$$

$$+ \sum_j \sum_i \int_{v_{ji}} [V_e^{ji}]\{b_{ji}\}dv + \{F_T\} \qquad (10)$$

where the last part $\{F_T\}$ brings in the contribution from the effects of temperatures and material nonlinearities

$$\{F_T\} = \sum_j \sum_i \int_{v_{ji}} \{[B]^T[D][\{\epsilon_c\} + \alpha(\{T\}$$

$$+\sigma^*\{T_0\})] + \alpha[N]^T[D]\{T'\}\}dv \qquad (11)$$

The nonlinearities including geometric nonlinearity and material nonlinearity can be considered in the stiffness matrix. So does the elastic-plastic deformation. The material property matrix is given by

$$[D] = [D_e] + [D_p]$$

where $[D_e]$ denotes the elastic part. The second part $[D_p]$ is the contribution from the plastic deformation

$$[D_p] = [D_e] -$$

$$[D_e]\{\frac{\partial F}{\partial \sigma}\}\{\frac{\partial F}{\partial \sigma}\}^T[D_e](A + \{\frac{\partial F}{\partial \sigma}\}^T[D_e]\{\frac{\partial F}{\partial \sigma}\})^{-1} \quad (12)$$

TIME VARIANT BOUNDARY CONSTRAINT CONDITIONS

For the time variant boundary conditions, finite difference method can be used to account for the rate of change of mode shape. Consider the modal transformation

$$\{\eta\} = [\Phi]\{\rho\} \quad (13)$$

Differentiation of the above equation yields

$$\{\dot{\eta}\} = [\dot{\Phi}]\{\rho\} + [\Phi]\{\dot{\rho}\} \quad (14)$$

When substituting the nodal displacement with the nodal coordinates and taking into consideration the effects of $[\dot{\Phi}]$, then at $t = t_l$ the new terms coming from the previous and newly computed mode shapes at $t = t_l$ are seen in $[C]$ and $[K]$ as[2],[3]

$$[C_{l33}] = [C_{33}] + \frac{2}{\Delta t}[M_{33}]([\Phi_l] - [\Phi_{l-1}]) \quad (15)$$

and

$$[K_{l33}] = [K_{33}] + \frac{1}{\Delta t^2}[M_{33}]([\Phi_l]$$

$$- 2[\Phi_{l-1}] + [\Phi_{l-2}]) + \frac{1}{\Delta t}[C_{33}]([\Phi_l] - [\Phi_{l-1}]) \quad (16)$$

The method developed above has a wide range of applications for which one can easily see and analyze its dynamics.

While the time variant contact conditions can be considered as a set of constraints which can be holonomic or nonholonomic. Some constraint equations which do not contain prescribed motion terms can be factorized to minimize the dimension of the equations of the system. For the case of two flexible bodies with one rolling without slipping on the other as shown in Figure 1, we can write in R the following position vector[2]

$$\bar{p}_{ni} = \bar{p}_{n-1} + \bar{q}_n + \bar{r}_{ni} \quad (17)$$

where

$$\bar{q}_n = \bar{r}_{n-1,c} + \bar{u}_{n-1,c} - (\bar{r}_{nc} + \bar{u}_{nc}) \quad (18)$$

Differentiation of equation (17) yields the constraint equations at the velocity level

$$[J]\{\dot{y}\} = \{g\} \quad (19)$$

where

$$\{y\} = [\ x^T \ \ \zeta^T \ \ q_n^T \ \ \rho^T\ ]^T \quad (20)$$

and

$$\{g\} = [S^{n-1,n}][\Omega^{n-1,n}](\{r_n\} + [N_n]\{\rho_{nc}\}) \quad (21)$$

$[J]$ is a Jacobi matrix and a function of generalized coordinates and velocities.

In the dynamics of MBS for the case when one flexible body is rolling on another, equations (19) and (6) extended with λJ^T are solved together. The time history of the system allows us to systematically update the contact position and the reevaluation of the Jacobi matrix J.

DYNAMICS OF GEAR MESHING TEETH

For validation of the results obtained by multibody dynamics code which utilize FEM, a modified Timoshenko beam theory is presented to analyze the dynamics of gear meshing teeth in rotorcraft systems. The acting position, direction and magnitude of the external forces are assumed to time variant. The meshing tooth is considered as a cantilever beam, as shown in Figure 2, where the inertia force due to the large rotation of the tooth base, as well as the external equivalent axial force and moment are all included in the equation of motion.[2]

$$\frac{\partial^2}{\partial x^2}\{EI(x)\{\frac{\partial^2 w}{\partial x^2} - \frac{1}{kA(x)G}[\varrho A(x)\frac{\partial^2 w}{\partial t^2} + f(x,t)$$
$$+P(x,t)\frac{\partial^2 w}{\partial x^2}]\} + m(x,t)\} - \varrho I(x)\frac{\partial^4 w}{\partial x^2 \partial t^2}$$
$$+\frac{\varrho I(x)}{kA(x)G}\frac{\partial^2}{\partial t^2}[\varrho A(x)\frac{\partial^2 w}{\partial t^2} + f(x,t) + P(x,t)\frac{\partial^2 w}{\partial x^2}]$$
$$+ \varrho A(x)\frac{\partial^2 w}{\partial t^2} + f(x,t) + P(x,t)\frac{\partial^2 w}{\partial x^2} = 0 \qquad (22)$$

For the assumed model, the boundary conditions are given by:
At the fixed end $x = 0$,

$$\psi(0,t) = w(0,t) = 0 \qquad (23)$$

At the free end $x = l$,

$$V(l,t) = kA(l)G[\frac{\partial w(l,t)}{\partial x} - \psi(l,t)] = 0 \qquad (24)$$

$$M(l,t) = [EI(l)\frac{\partial \psi(l,t)}{\partial x} + m(l,t)] = 0 \qquad (25)$$

A solution to the above proposed model will result in prediction of contact forces or dynamic loading on gear teeth.

MINIMIZATION OF VIBRATION IN ELASTIC BEAMS

The minimization of vibration (deformation) of flexible bodies in mechanical systems is a major concern in dynamics and control. What follows are procedures used to minimize vibration in elastic beams. The elastic beam is modeled in two ways: one has a movable support not to exceed the lower tip, whereas the other treats the body as a hollow beam with a moving mass.

Equation of motion for the model used to minimize vibration of the flexible beam, as shown in Figure 3, is given by[2]

$$EI\frac{\partial^4 y}{\partial x^4} + m(\frac{\partial^2 y}{\partial t^2} - y\dot{\theta}^2 + g\cos\theta + a_O^t + x\ddot{\theta}) = 0 \quad (26)$$

Laplace transform gives

$$Y = \sum_{i=1}^{4} c_i e^{r_i x} + \frac{1}{ms^2}\{L(x,s) + m[sf_1(x) + f_2(x)]\} \qquad (27)$$

The functional used to minimize vibration of the beam is

$$J(x_0) = \int_0^t F(x, x_0, t)dt \qquad (28)$$

Euler-Lagrange equation

$$\frac{\partial F}{\partial x_0} - \frac{d}{dt}(\frac{\partial F}{\partial \dot{x}_0}) = 0 \qquad (29)$$

is used to solve for the problem at hand.

The solution for optimum positioning conditions is time variant and yields minimum deflection at the proposed location of the beam.

*

References

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[2] Xie, M.: *Flexible Rotorcraft System Dynamics with Time Variant Contact Conditions*, Ph.D Dissertation, University of Illinois at Chicago, Chicago, IL, March 1992.

[3] Amirouche, F.M.L. and Xie, M.: Dynamic analysis of flexible multibody systems with time variant mode shapes, *The 13th Biennial ASME Conference, Mechanical Vibration and Noise*, Miami, Florida, Sept. 1991. ASME DE *Structural Vibration and Acoustics*, 34:261-267.

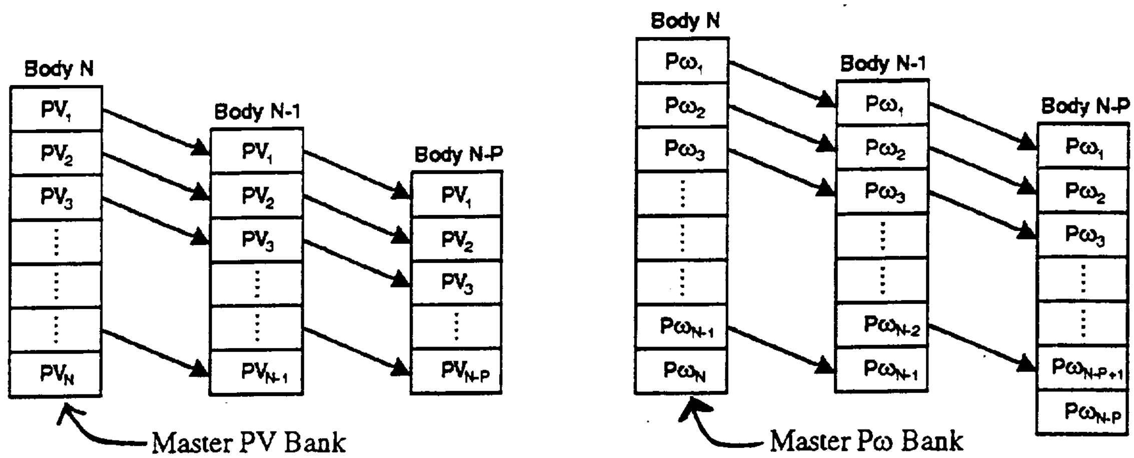

Table 1: Mapping technique used in MBS

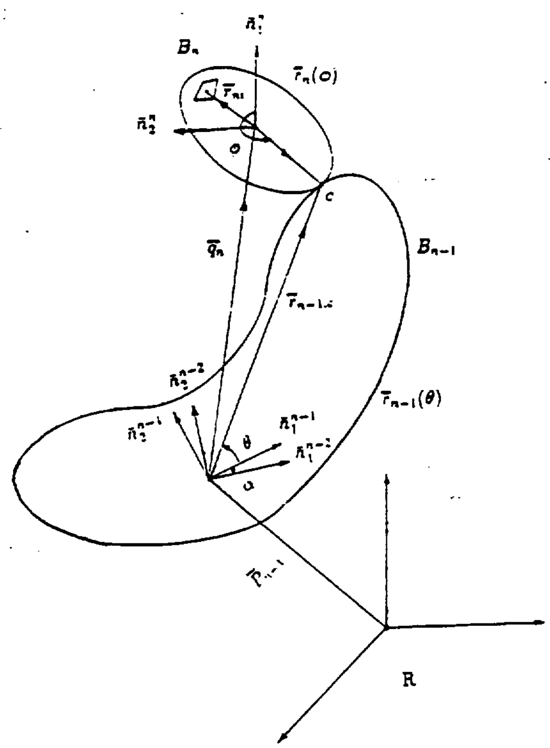

Figure 1: Model for time variant contact conditions

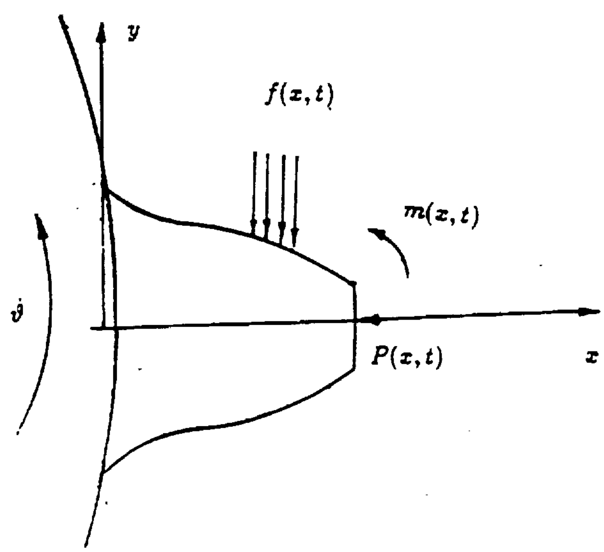

Figure 2: Model for gear meshing teeth

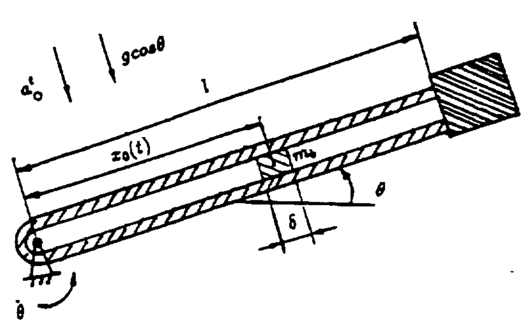

Figure 3: Models for vibration minimization