Mathematical Modeling for Diffractive Optics

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Abstract

We consider a "diffractive optic" to be a biperiodic surface separating two half-spaces, each having constant constitutive parameters; within a unit cell of the periodic surface and across the transition zone between the two half-spaces, the constitutive parameters can be a continuous, complex-valued function. Mathematical models for diffractive optics have been developed, and implemented as numerical codes, both for the "direct" problem and for the "inverse" problem. In problems of the "direct" class, the diffractive optic is specified, and the full set of Maxwell's equations is cast in a variational form and solved numerically by a finite element approach. This approach is well-posed in the sense that existence and uniqueness of the solution can be proved and specific convergence conditions can be derived. An example of a metallic grating at a Wood anomaly is presented as a case where other approaches are known to have convergence problems. In problems of the "inverse" class, some information about the diffracted field (e.g., the far-field intensity) is given, and the problem is to find the periodic structure in some optimal sense. Two approaches are described: phase reconstruction in the far-field approximation; and relaxed optimal design based on the Helmholtz equation. Practical examples are discussed for each approach to the inverse problem, including array generators in the far-field case and antireflective structures for the relaxed optimal design.

Mathematical Modeling for Diffractive Optics

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Outline

- Need
- Statement of Problem
- Overview of Approaches
- Examples

Mathematical Modeling for Diffractive Optics

Classes of Problems

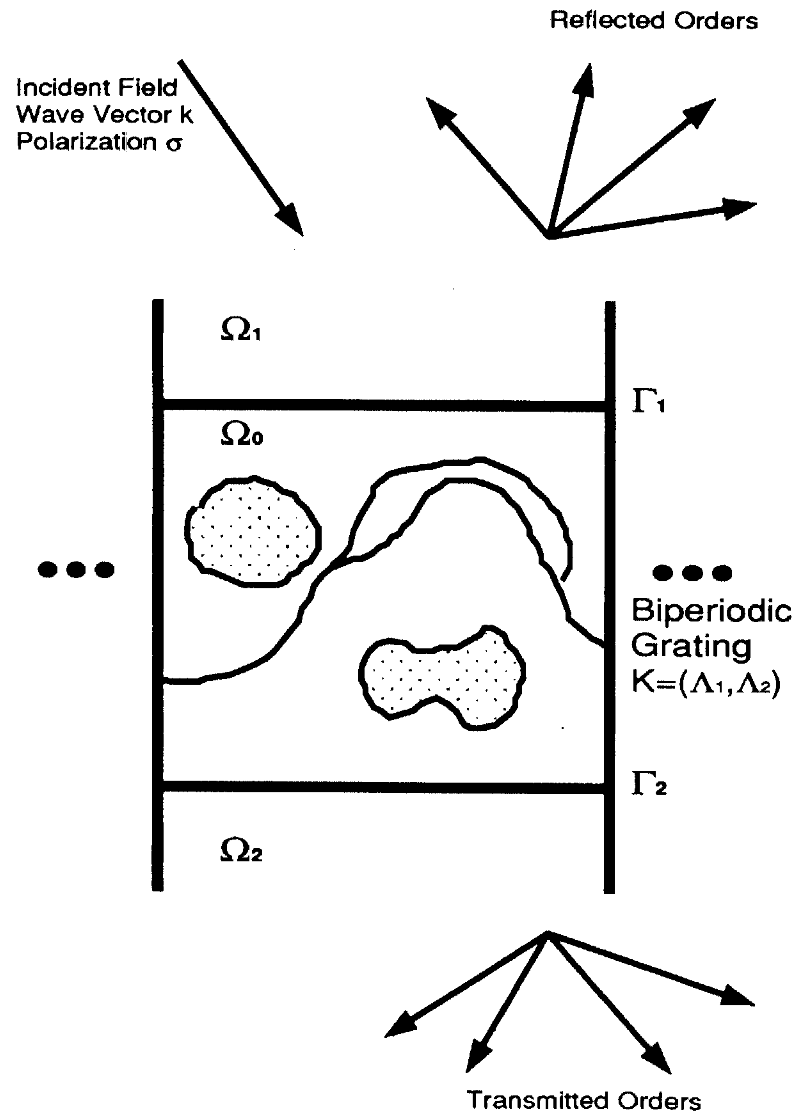
- **The Direct Problem**

  Given the incident field and grating structure
  Predict the behavior of the outgoing fields
  Solve Maxwell's equations rigorously

- **The Inverse Problem**

  Given the incident field and the desired output field
  Calculate the optimum structure
  Model a scalar wave equation with simplifications

Definition of the Direct Problem



Time Harmonic, Source-free Maxwell's Eqs

$$\nabla \times E - i\omega H = 0$$

$$\nabla \times H + i\omega\varepsilon E = 0$$

$\varepsilon \in L_\infty(\Omega_0)$ $\quad \varepsilon = \varepsilon_1$ in Ω_1 $\quad \varepsilon = \varepsilon_2$ in Ω_2

Find

Quasiperiodic Solutions with Bounded Outgoing Waves

Survey of Approaches to the Direct Problem

Approach	Comments
1. Integral Method 2. Differential Method (coupled waves) 3. Coupled Modes 4. Variational Method 5. Riemann-Hilbert Problem	Discretized grating profile PDE embedded in infinite set of coupled linear eqs Numerical implementation - truncate set of linear eqs - solve $Ax = b$
6. Analytic Continuation	Smooth grating profile Infinite Taylor series for Rayleigh coef. (recursion) Padé approximant sum of series

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Honeywell / IMA Program

The Direct Problem

1. Integral Method (Maxcoll)	Dobson & Friedman	Singly periodic grating Simple profile(graph)
2. Variational Method (Maxfelm)	Dobson	Biperiodic grating General profile
3. Analytic Continuation (TBD)	Bruno & Reitich	Biperiodic grating Simple profile(function)

The Inverse Problem

1. Phase Reconstruction (Phaseopt)	Dobson	Scalar field / Fraunhofer approx Nonperiodic structures Nonlinear least squares method
2. Relaxed Optimization (Profopt)	Dobson	Scalar field / Helmholtz eq Singly periodic grating Complex profile

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Examples

The Direct Problem

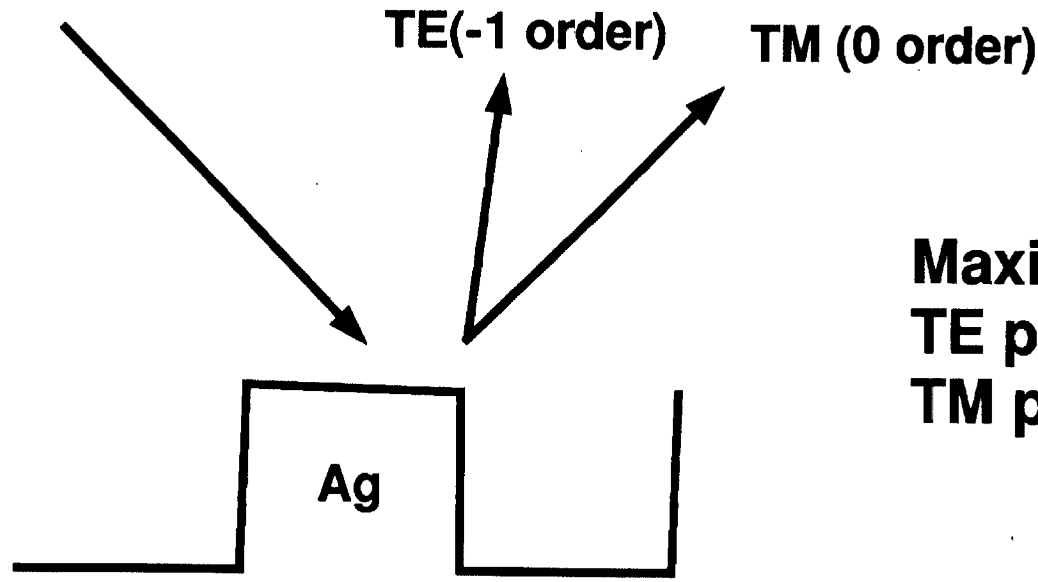
1. Reflective Polarization Beamsplitter
2. LIGA Grating
3. Mixed Index Biperiodic Grating

The Inverse Problem

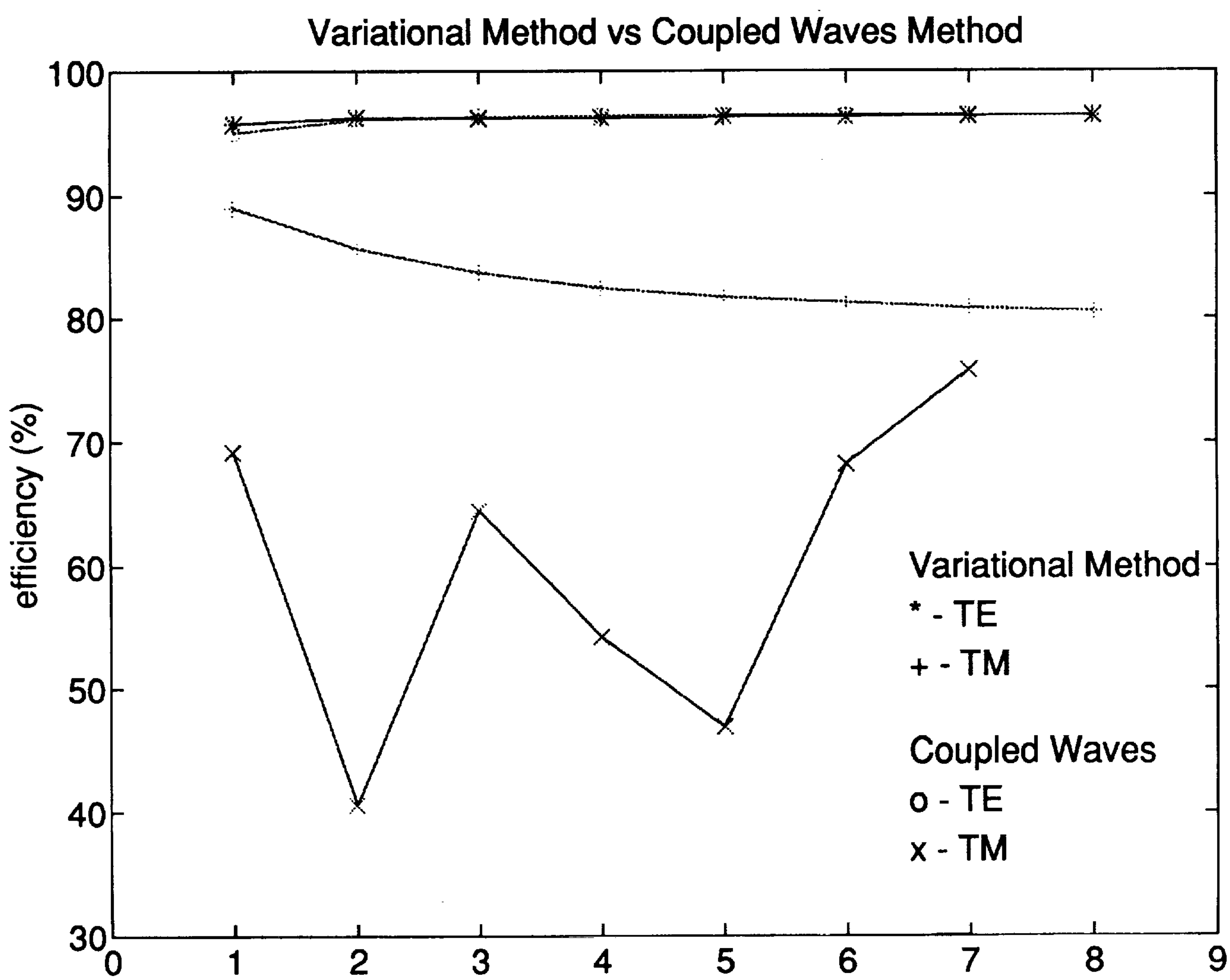
1. Phase Reconstruction - Hypercube Beamsplitter
2. Relaxed Optimization - Angle Optimized Motheye Structure

Reflective Polarization Beamsplitter

$\lambda = 0.78\ \mu m$
$\theta = 45°$

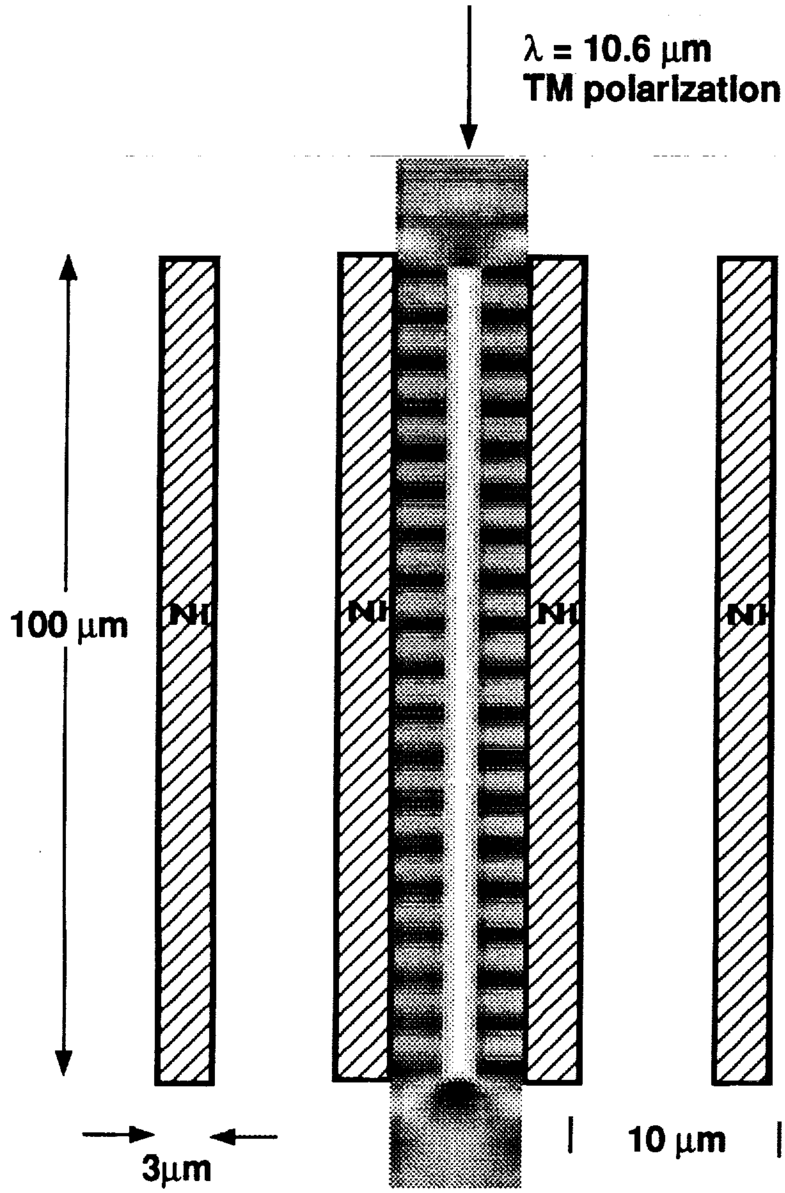


Maximize reflectivity
TE polarization (-1 order)
TM polarization (0 order)


Variational Method vs Coupled Waves Method
efficiency (%)
100
90
80
70
60
50
40
30
0
1
2
3
4
5
6
7
8
9
Variational Method
* - TE
+ - TM
Coupled Waves
o - TE
x - TM

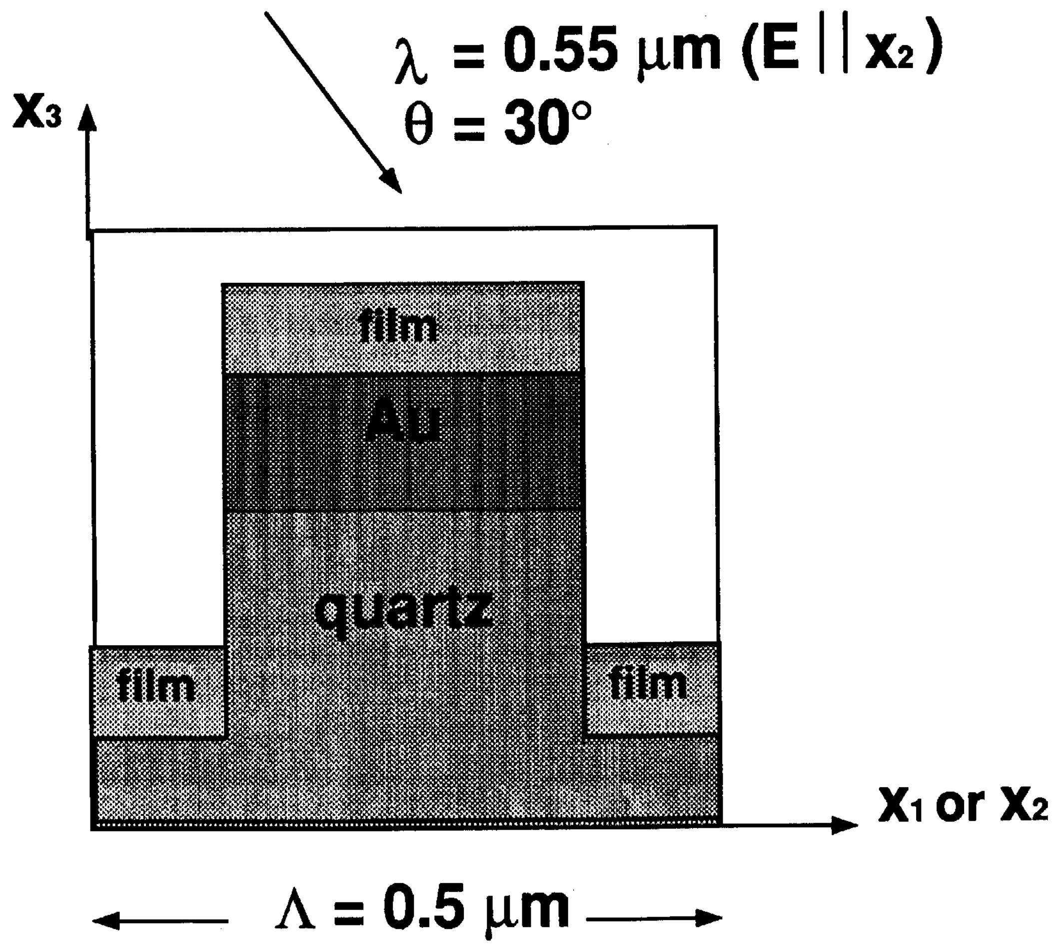
Variational Method (Maxfelm) Example

LIGA Grating



Variational Method (Maxfelm) Example

Mixed Index Biperiodic Grating



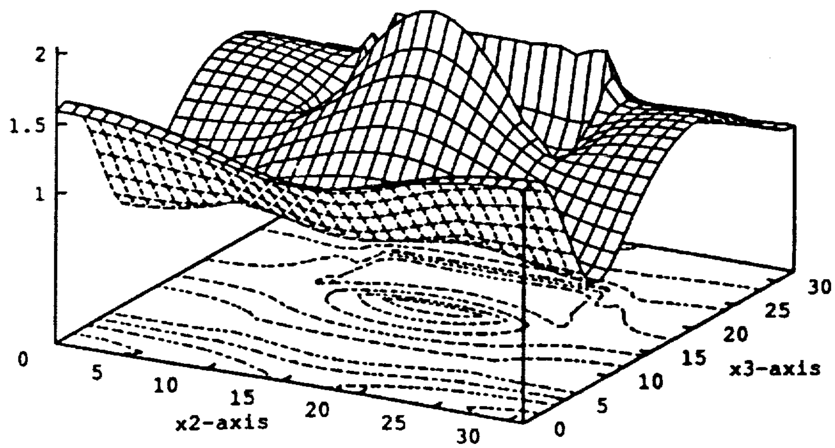


FIG. 2. *Cross-section of the amplitude $|H|$, taken through the metal region in the (x_2, x_3) plane.*

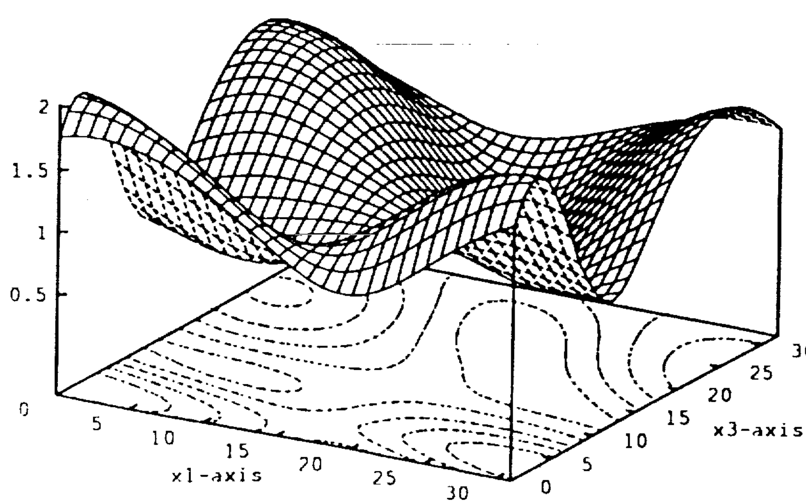


FIG. 3. *Cross-section of the amplitude $|H|$, taken through the non-absorbtive region in the (x_1, x_3) plane.*

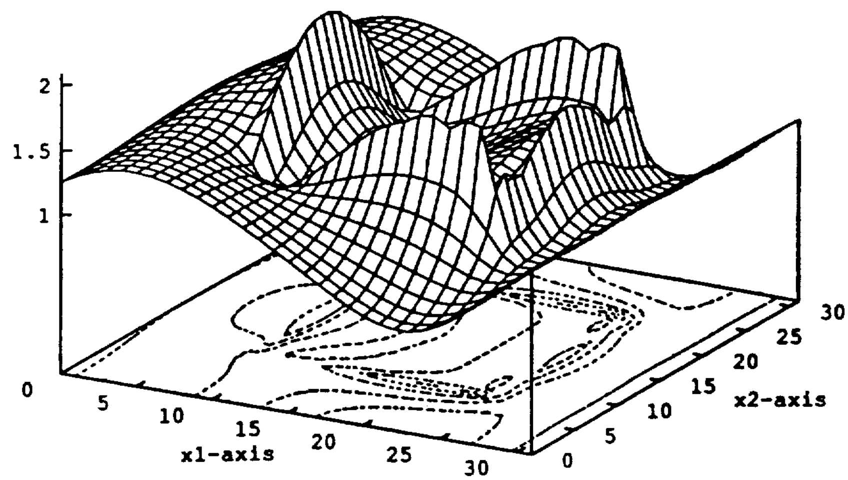


FIG. 4. *Cross-section of the amplitude $|H|$, taken through the metal region in the (x_1, x_2) plane.*

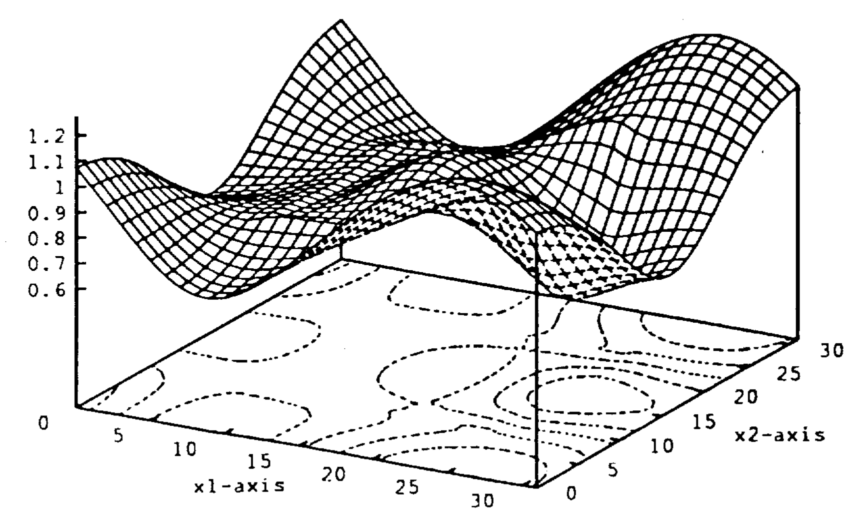
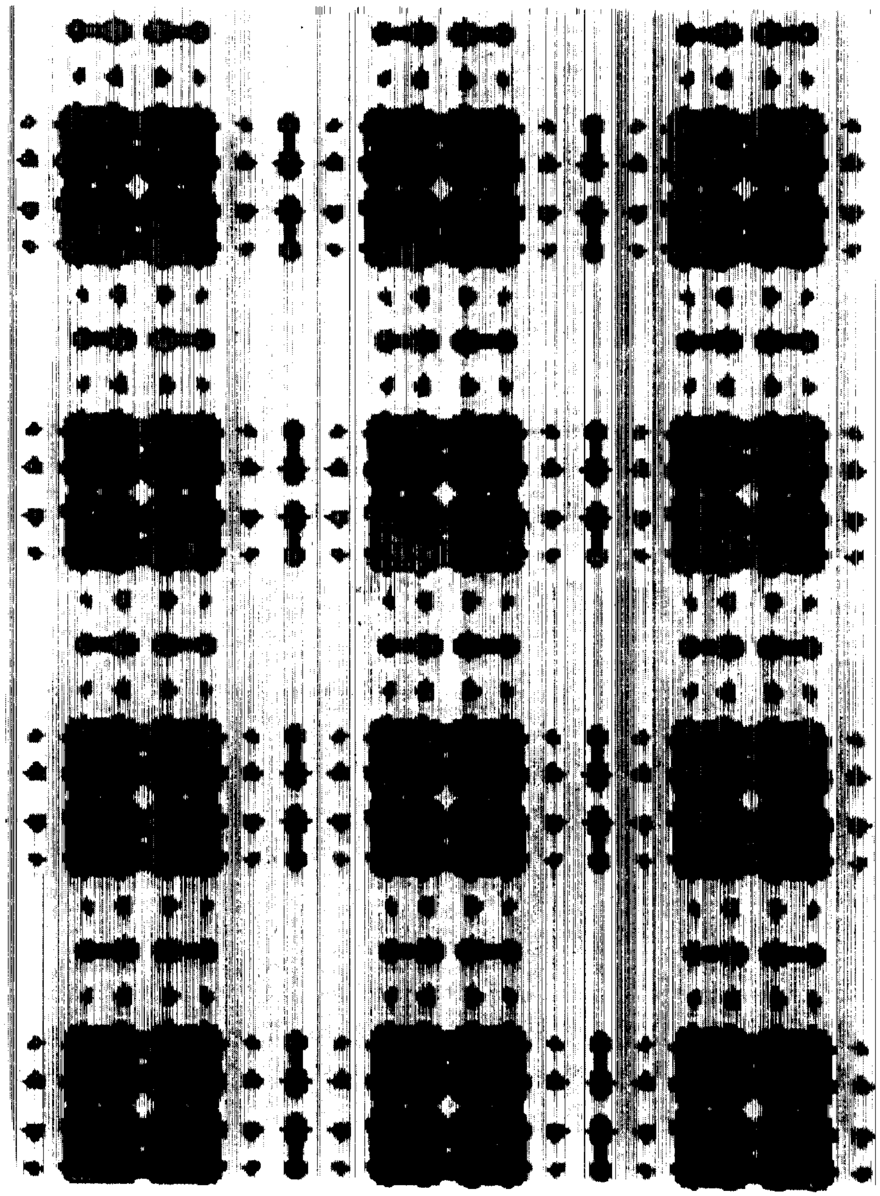
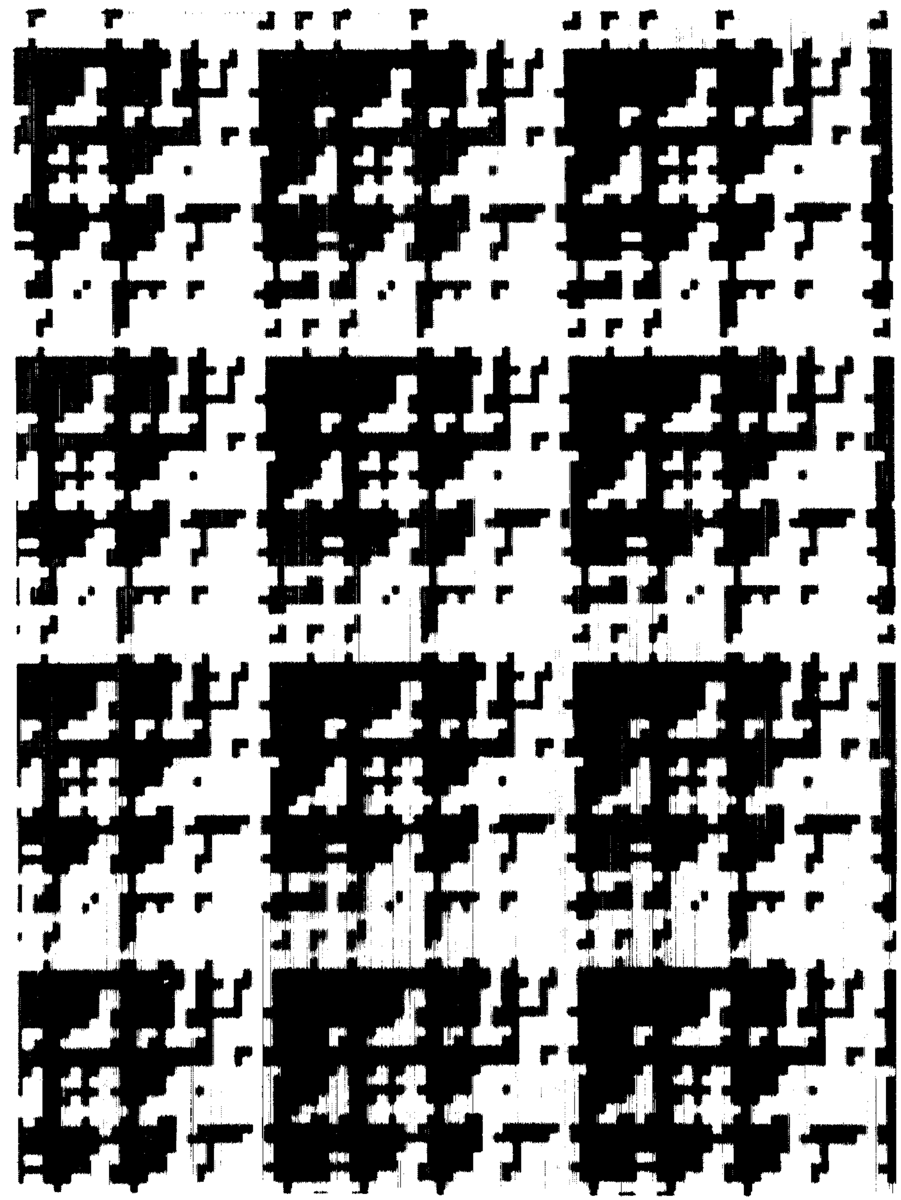


FIG. 5. *Cross-section of the amplitude $|H|$, taken below the metal region in the (x_1, x_2) plane.*



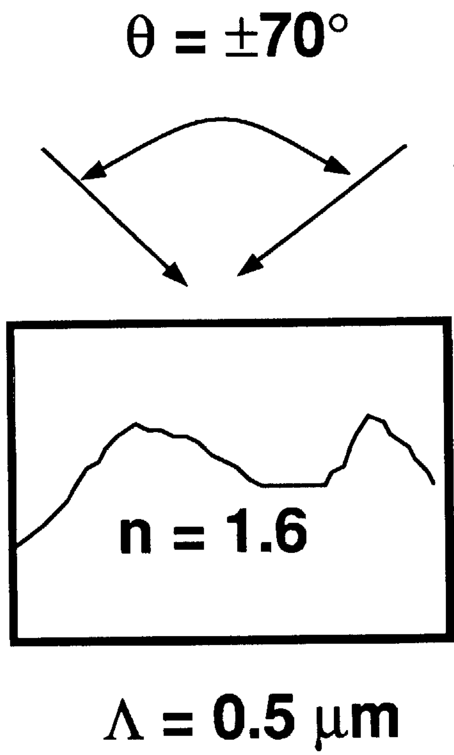
Dobson Method



Gerchberg Saxton Method

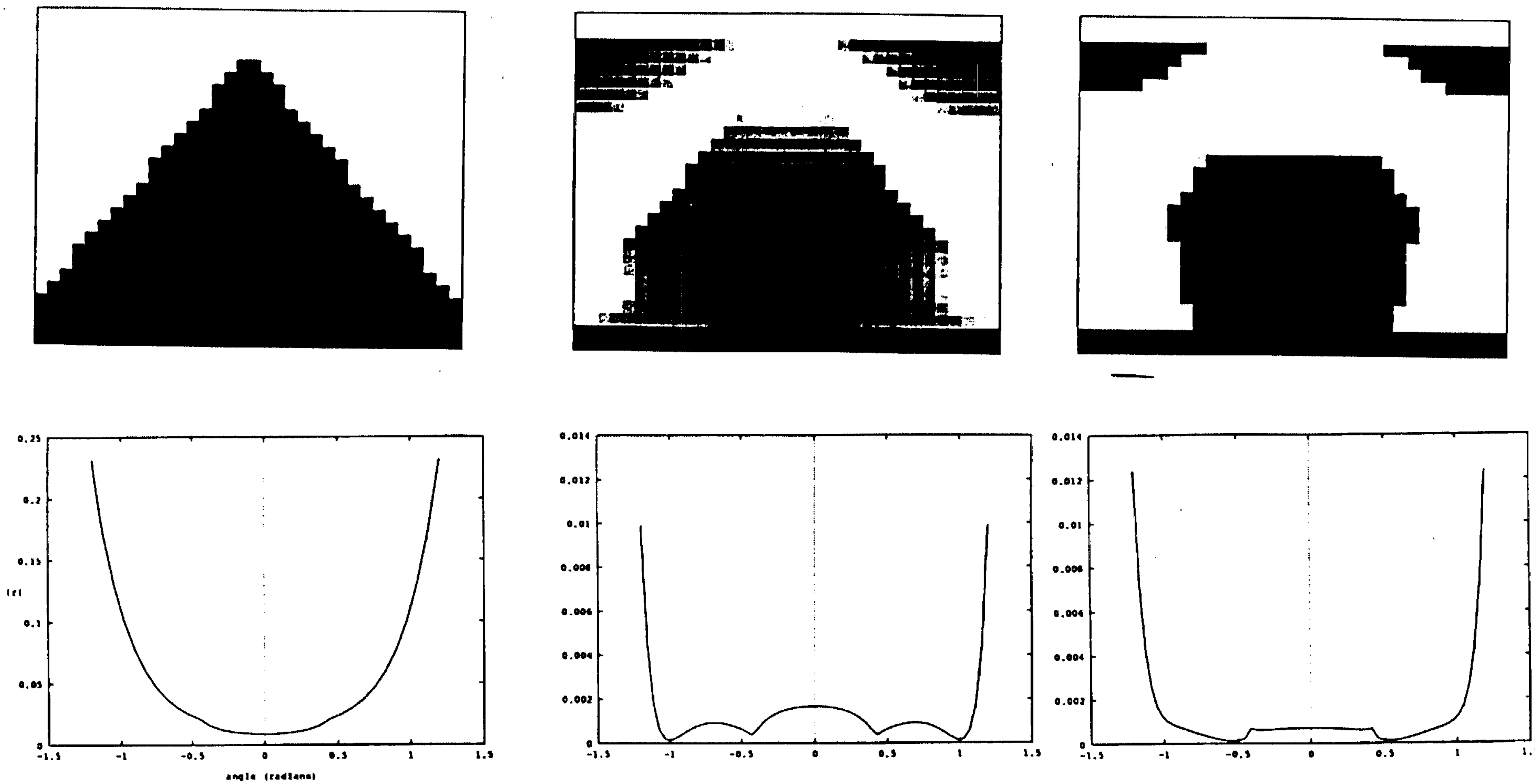
Relaxed Optimization (Profopt) Example

Optimized Moth Eye Grating



Find structure of zero order grating to minimize reflectivity over range of incident angles

Relaxed Optimization (Profopt) Example



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Summary

The Direct Problem

Variational Approach with Finite Elements Method
- exhibits good convergence, numerical stability
- treats complicated biperiodic structures
- can be computationally intensive

Analytic Continuation Approach
- elegant solution
- limited domain of convergence and biperiodic structures
- computationally very fast

The Inverse Problem

Phase Reconstruction - comparable to other approaches

Relaxed Optimization - potential to identify new structures