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A Noniterative Improvement Of Guyan Reduction

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ABSTRACT: In determining the natural modes and frequencies of a linear elastic structure, Guyan reduction is often used to reduce the size of the mass and stiffness matrices and the solution of the reduced system is obtained first. The reduced system modes are then expanded to the size of the original system by using a static transformation linking the retained degrees of freedom to the omitted degrees of freedom. In the present paper, the transformation matrix of Guyan reduction is modified to include additional terms from a series accounting for the inertial effects. However, the inertial terms are dependent on the unknown frequencies. A practical approximation is employed to compute the inertial terms without any iteration. This new transformation is implemented in NASTRAN using a DMAP sequence alter. Numerical examples using a cantilever beam illustrate the necessary condition for allowing a large number of additional terms in the proposed series correction of Guyan reduction. A practical example of a large model of the Plasma Motor Generator module to be flown on a Delta launch vehicle is also presented.

1. Introduction: The dynamic analysis of complicated structures often produces large finite element models. In some instances, the automated computer procedures to generate finite element meshes also lead to large models. These highly refined models are really a byproduct of the use of model generating software and they may not be needed for accuracy. A common approach to reduce the size of the eigenvalue problem for structural dynamics applications is Guyan reduction. This approximate method finds its place among other applications also. For the purposes of cost-effectiveness, Guyan reduction is employed in Coupled Loads Analysis using substructuring techniques. In the experimental modes analysis, analytical selection of retained degrees of freedom for Guyan reduction is used as a guide to select accelerometer locations on the test article. Mass weighted orthogonality computations between the test and analytical modeshapes are performed using Guyan reduction.

While Guyan reduction [1] is exact in static applications, it introduces approximations in structural dynamics. The correct relationship between the retained and omitted degrees of freedom can be expressed in the form of a series. The Guyan reduced mass and stiffness matrices, available in explicit form, are used to compute the series terms approximately. The Guyan reduced matrices provide the best possible solution without requiring any further iterations. The condition for convergence of the series and the relationship of this series transformation to the improved reduced system (IRS) introduced by O'Callahan [2] are examined in this paper.

2. Theory: The eigenvalue problem from the structural dynamic analysis is given as

$$K\mathbf{u} = \lambda M\mathbf{u} \tag{1}$$

Eq. (1) can be written in the partitioned form as,

$$\begin{bmatrix} K_{aa} & K_{ao} \\ K_{oa} & K_{oo} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_a \\ \mathbf{u}_o \end{Bmatrix} = \lambda \begin{bmatrix} M_{aa} & M_{ao} \\ M_{oa} & M_{oo} \end{bmatrix} \begin{Bmatrix} \mathbf{u}_a \\ \mathbf{u}_o \end{Bmatrix} \tag{2}$$

where $\mathbf{u}_a$ represents the eigenvector of the retained degrees of freedom and $\mathbf{u}_o$ the eigenvector of the degrees of freedom omitted in the Guyan reduction. M_{ij} and K_{ij} are the corresponding submatrices of the mass and stiffness matrices respectively and λ is the eigenvalue. The second partition of Eq. (2) can be written separately as

$$(K_{oa} - \lambda M_{oa})\mathbf{u}_a + (K_{oo} - \lambda M_{oo})\mathbf{u}_o = 0 \tag{3}$$

Expanding the vector $\mathbf{u}_o$ in terms of $\mathbf{u}_a$ from Eq. (3),

$$\begin{aligned} \mathbf{u}_o &= -(K_{oo} - \lambda M_{oo})^{-1}(K_{oa} - \lambda M_{oa})\mathbf{u}_a \\ &= -\left(I - \lambda K_{oo}^{-1} M_{oo}\right)^{-1}\left(K_{oo}^{-1} K_{oa} - \lambda K_{oo}^{-1} M_{oa}\right)\mathbf{u}_a \end{aligned} \tag{4}$$

Guyan reduction transformation leaves out the frequency dependent terms in Eq. (3). Hence, the regular Guyan reduction transformation becomes,

$$\mathbf{u}_o = -K_{oo}^{-1} K_{oa}\mathbf{u}_a \tag{5}$$

If the condition for convergence (Section 4.) is satisfied, the inverse of $\left(I - \lambda K_{oo}^{-1} M_{oo}\right)$ can be expanded in Neumann series as,

$$\left(I - \lambda K_{oo}^{-1} M_{oo}\right)^{-1} = I + \lambda K_{oo}^{-1} M_{oo} + \lambda^2 \left[K_{oo}^{-1} M_{oo}\right]^2 + \ldots \tag{6}$$

Using Eq. (6) in Eq. (4) and simplifying the terms yields,

$$\mathbf{u}_o = -\left[K_{oo}^{-1} K_{oa} + B\lambda + AB\lambda^2 + A^2 B\lambda^3 + \ldots\right]\mathbf{u}_a \tag{7}$$

where

$$A = K_{oo}^{-1} M_{oo} \quad \text{and} \quad B = K_{oo}^{-1} M_{oa} - AK_{oo}^{-1} K_{oa} \tag{8}$$

The exact relationship between $\mathbf{u}_o$ and $\mathbf{u}_a$ in Eq. (7) involves nonlinear terms of the unknown eigenvalues (λ). A practical approximation to compute these terms in Eq. (7) can be made from regular Guyan reduction by taking,

$$K_r\mathbf{u}_a \cong \lambda M_r\mathbf{u}_a \tag{9}$$

where K_r and M_r are Guyan reduced stiffness and mass matrices respectively and are given explicitly as,

$$\begin{aligned} K_r &= K_{aa} - K_{ao}K_{oo}^{-1}K_{oa} \\ M_r &= M_{aa} - M_{ao}K_{oo}^{-1}K_{oa} - K_{oa}K_{oo}^{-1}M_{oa} + K_{ao}K_{oo}^{-1}M_{oo}K_{oo}^{-1}K_{oa} \end{aligned} \tag{10}$$

From Eq. (9), it is seen that,

$$\lambda \mathbf{u}_a = M_r^{-1}K_r\mathbf{u}_a \tag{11}$$

Using Eq. (11) repeatedly, it can be shown that,

$$\begin{aligned} \lambda^2\mathbf{u}_a &= \left(M_r^{-1}K_r\right)^2\mathbf{u}_a \\ &\vdots \\ \lambda^i\mathbf{u}_a &= C^i\mathbf{u}_a, \quad C = M_r^{-1}K_r \end{aligned} \tag{12}$$

Substituting Eq. (12) into Eq. (7), the relationship between $\mathbf{u}$ in Eq. (1) and $\mathbf{u}_a$ becomes,

$$\mathbf{u} = T\mathbf{u}_a \tag{13}$$

where

$$T = \begin{bmatrix} I \\ -K_{oo}^{-1}K_{oa} + \sum\limits_{i=1,2,..} A^{i-1}BC^i \end{bmatrix} \tag{14}$$

By applying the relation between $\mathbf{u}$ and $\mathbf{u}_a$ in Eq. (13), the new improved matrices from series reduction can be obtained as,

$$\overline{K} = T^TKT \quad \text{and} \quad \overline{M} = T^TMT \tag{15}$$

It is interesting to note that M_{oa} vanishes for lumped formulations of the mass matrix. Taking the value of i to be unity, the transformation in Eq. (14) reduces to

$$T = \begin{bmatrix} I \\ -K_{oo}^{-1}K_{oa} + BC \end{bmatrix} \tag{16}$$

which is the improved reduced system (IRS) proposed by O'Callahan [2].

3. DMAP Alter: A rigid format alter for dynamic analysis in NASTRAN has been developed to incorporate the improved Guyan reduction with the series terms. A parameter called GOPT is used to choose the number of correction terms. The alter listing is also provided in this section.

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$        CSA/NASTRAN ALTER FOR IMPROVED GUYAN REDUCTION
$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$$
$
RFINSERT SMP2 $
PARAM //C,N,NOP/V,Y,GOPT=-1 $
PARAM //C,N,SUB/V,N,GOUT/V,Y,GOPT/C,N,2 $
COND LGOPT,GOPT $
UPARTN USET,MFF/MAAB,MOA,,MOO/*F*/*A*/*O* $
FBS LOO,,MOO/AMAT/1 $
FBS LOO,,MOA/BMAT1/1 $
MPYAD AMAT,GO,BMAT1/BMAT $
SOLVE MAA,KAA/CMAT/1 $
$
MPYAD BMAT,CMAT,/SUM $
COND OUT,GOUT $
MATMOD SUM,,,,,/PRDT,/13 $
LABEL LOOPTOP $
EQUIV SUM,SUM1/NEVER $
EQUIV PRDT,PRDTX/NEVER $
SMYPAD AMAT,PRDT,CMAT,,,/PRDTX/3 $
ADD SUM,PRDTX/SUM1 $
EQUIV SUM1,SUM/ALWAYS $
EQUIV PRDTX,PRDT/ALWAYS $
REPT LOOPTOP,GOUT $
LABEL OUT $
ADD GO,SUM/GONE $
SMP2 USET,GONE,MFF/MAA $
SMP2 USET,GONE,KFF/KAA $
LABEL LGOPT $
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4. Validity of Guyan Reduction: The inverse of the matrix $\left[I - \lambda K_{oo}^{-1} M_{oo}\right]$ in Eq. (4) can be expanded as a converging Neumann series only if all the eigenvalues of $\lambda K_{oo}^{-1} M_{oo}$ are less than unity. In other words, the Guyan reduction is valid only for those frequencies less than the smallest frequency of the eigenvalue problem formed out of the omitted degrees of freedom. The effect of violating this condition will be scrutinized in the next section.

5. Demonstration Examples:

5.1 Uniform Cantilever: The first example is concerned with a cantilevered bar clamped at one end. The relevent structural parameters are taken to be the modulus of elasticity (E) being equal to 30×10^6 *psi*, weight density (ρg) as 0.2839 lb/in^3, area of cross section as 1 in^2 and the length of the bar (L) as 72 in.

The characteristic equation of this cantilever is $\cos \sqrt{\rho/E}\omega L = 0$ from which the theoretical natural frequencies can be computed. The cantilever is divided into twenty finite elements. The retained degrees of freedom for Guyan reduction are the axial displacements at the free end and at two successive nodes. The reduction transformation includes n as the number of additional series terms. The natural frequencies from improved Guyan reduction for different values of n are listed in Tables 1 through 5.

Table 1. Cantilever Frequency Comparisons ($n = 0$)

Standard Guyan Reduction

Mode Number	Theoretical Frequency (Hz)	Computed Frequency (Hz)	Error %
1	7.012E2	7.428E2	5.926E0
2	2.104E3	7.562E3	2.595E2
3	3.506E3	1.655E4	3.722E2

Table 2. Cantilever Frequency Comparisons ($n = 1$)

Mode Number	Theoretical Frequency (Hz)	Computed Frequency (Hz)	Error %
1	7.012E2	7.012E2	8.014E-2
2	2.104E3	2.583E3	2.279E1
3	3.506E3	1.390E4	2.964E2

Table 3. Cantilever Frequency Comparisons ($n = 2$)

Mode Number	Theoretical Frequency (Hz)	Computed Frequency (Hz)	Error %
1	7.012E2	7.011E2	-2.169E-2
2	2.104E3	2.239E3	6.440E0
3	3.506E3	7.043E3	1.009E2

Table 4. Cantilever Frequency Comparisons ($n = 3$)

Mode Number	Theoretical Frequency (Hz)	Computed Frequency (Hz)	Error %
1	7.012E2	7.012E2	-2.170E-2
2	2.104E3	2.167E3	3.003E0
3	3.506E3	3.879E3	1.063E1

Table 5. Cantilever Frequency Comparisons ($n = 4$)

Mode Number	Theoretical Frequency (Hz)	Computed Frequency (Hz)	Error %
1	7.012E2	7.010E2	-3.235E-2
2	2.104E3	2.120E3	7.937E-1
3	3.506E3	3.680E3	4.950E0

The accuracy of the computed frequencies is improved by taking into account the higher order correction terms. However, when $n \geq 6$, the reduced mass matrix is no longer positive definite and the eigenvalue solution process breaks down. This limitation of adding a finite number of correction terms can be explained by the fact that the third frequency of the overall structure exceeds the lowest frequency of the omit set (O-set) system thus violating the convergence criterion for Guyan reduction.

Another cantilever example is constructed by assuming that the three elements near the free end are made up of a material with $E = 30 \times 10^4$ *psi* instead of steel. By retaining the same degrees of freedom as in the previous example of all steel construction, it becomes possible to add an almost limitless number of correction terms. This is because there is no overlap between the frequency spectrum of the first three modes of the full system and that of the O-set system.

5.2 Plama Motor Generator (PMG): This example comes from the modal testing and finite element analysis of the PMG Far End Package (Figure 1). The PMG experiment is a payload on a Delta II 7925 launch vehicle. The mission is scheduled to take place in July 1993.

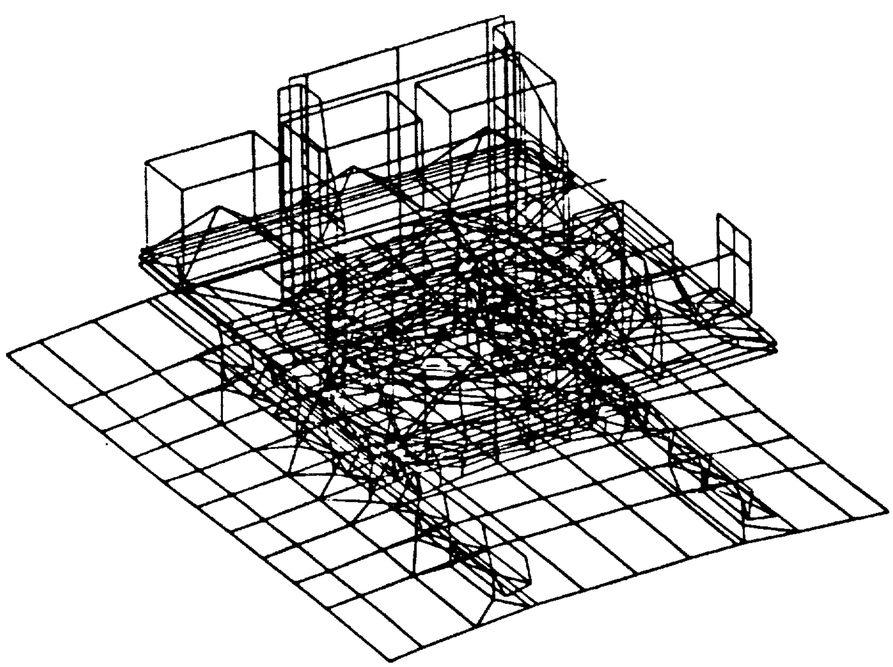

Figure 1. PMG Far End Package

The analysis set degrees of freedom correspond to the accelerometer locations used in the modal survey test. The improved Guyan reduction is performed with different n on the PMG Far End Package model. The computed frequencies are compared with those of the full model which are taken as the reference solution and the results are listed in Tables 6 through 8. Several frequencies that were not found by the standard Guyan reduction start to reappear by adding the correction terms.

Table 6. PMG Frequency Comparisons ($n = 0$)

Mode Number	Reference Frequency (Hz)	Computed Frequency (Hz)	Error %
1	56.32	56.39	0.13
2	84.46	84.66	0.23
3	100.66	101.20	0.53
4	118.19	119.58	1.17
5	159.46	160.48	0.63
6	170.06	—	—
7	185.19	—	—
8	215.16	220.65	2.55
9	217.65	224.09	2.95
10	228.36	236.73	3.56
11	234.52	—	—
12	243.43	256.55	5.38
13	264.53	—	—
14	299.03	—	—
15	305.16	330.53	8.31

Table 7. PMG Frequency Comparisons ($n = 1$)

Mode Number	Reference Frequency (Hz)	Computed Frequency (Hz)	Error %
1	56.32	56.32	0.00
2	84.46	84.46	0.45E-4
3	100.66	100.66	0.59E-3
4	118.19	118.20	0.46E-2
5	159.46	159.46	0.16E-2
6	170.06	171.65	0.92

Table 8. PMG Frequency Comparisons ($n = 2$)

Mode Number	Reference Frequency (Hz)	Computed Frequency (Hz)	Error %
1	56.32	56.82	0.89
2	84.46	85.52	1.24
3	100.66	101.12	0.45
4	118.19	118.31	0.09
5	159.46	160.32	0.53
6	170.06	170.18	0.064
7	185.19	185.29	0.054
8	215.16	215.19	0.016
9	217.65	217.71	0.026
10	228.36	228.61	0.10
11	234.52	234.60	0.035
12	243.43	242.79	-0.26
13	264.53	264.65	0.043
14	299.03	299.59	0.17
15	305.16	305.60	0.14

6. Conclusion:

A noniterative procedure to enhance the standard Guyan reduction with a series of terms has been presented. In practice, it may be possible to add only a finite number of the correction terms as demonstrated by the NASTRAN examples.

7. References:

1. Guyan, R. J., Reduction of Stiffness and Mass Matrices, AIAA Journal, Vol. 3, 1965.

2. O'Callahan, J. C., A Procedure for an Improved Reduced System Model, Procedings of the Seventh International Modal Analysis Conference, 1989.