AUTOMATIC ASET SELECTION FOR DYNAMICS ANALYSIS

Tom Allen

McDonnell Douglas Space Systems
Company - Huntsville Division

N94-17839
190532
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ABSTRACT

A method for selecting optimum NASTRAN analysis set degrees of freedom for the dynamic eigenvalue problem is described. Theoretical development of the Guyan reduction procedure on which the method is based is first summarized. The algorithm used to select the analysis set degrees of freedom is then developed. Two example problems are provided to demonstrate the accuracy of the algorithm.

1.0 INTRODUCTION

A NASTRAN user is faced with two major difficulties when solving a dynamic eigenvalue problem. First, an eigenvalue solution is expensive for most structural problems encountered in engineering applications. Second, many more degrees of freedom (DOF) are required to define a structure's elastic properties than are required to define its inertial properties, which tends to exacerbate the first difficulty.

A popular method for easing the severity of these difficulties is to reduce the problem size using Guyan reduction (Reference 1). This method allows the user to preserve the elastic properties of the reduced problem set while reducing the problem size to one more manageable for a dynamic eigenvalue solution. At the same time, the mass properties are also condensed with some penalty associated with the reduction of mass from the coordinates being eliminated. The present paper describes an approach for optimizing the partitioning process to minimize this penalty.

Theoretical development of the Guyan reduction method is presented in Section 2. Section 3 describes the algorithm used to select automatically the analysis set degrees of freedom. Verification of the method is presented in Section 4. Conclusions are presented in Section 5.

2.0 THE GUYAN REDUCTION METHOD

By way of introduction, the Guyan reduction method will first be reviewed.

The dynamic eigenvalue problem is given by the equation

$$([K] - \lambda[M])\{\phi\} = 0 \tag{1}$$

where K is the structural stiffness matrix, M is the structural mass matrix, λ is the eigenvalue, and ϕ is the eigenvector or modal displacements. The Guyan reduction method starts by partitioning Equation 1 into independent DOF, designated in NASTRAN as the A-set, and dependent DOF, designated as the O-(for OMIT) set. After performing this operation Equation 1 becomes

$$\left(\begin{bmatrix} \bar{K}_{aa} & K_{ao} \\ K_{ao}^T & K_{oo} \end{bmatrix} - \lambda \begin{bmatrix} \bar{M}_{aa} & M_{ao} \\ M_{ao}^T & M_{oo} \end{bmatrix}\right)\begin{Bmatrix} \phi_a \\ \phi_o \end{Bmatrix} = 0 \tag{2}$$

where the subscript "a" denotes A-set DOF and the subscript "o" denotes O-set DOF.

A set of constraints for the O-set displacements can be derived by solving for ϕ_o in terms of ϕ_a using statics, or

$$K_{ao}^T \phi_a + K_{oo}\phi_o = 0 \tag{3}$$

The O-set displacements now become

$$\phi_o = G_o \phi_a \tag{4}$$

where

$$G_o = -K_{oo}^{-1} K_{ao}^T \tag{5}$$

Equation 4 defines ϕ_o as the deflections at O-set DOF due to unit displacements at the A-set DOF. Stated another way, the O-set displacements, ϕ_o, are constrained to move in relation to A-set displacements, ϕ_a, as governed by the transformation matrix G_o. This relationship constitutes a Ritz transformation of the eigenvalue problem. The transformation written in terms of the full displacement set is

$$\{\phi\} = \begin{Bmatrix} \phi_a \\ \phi_o \end{Bmatrix} = [G]\{\phi_a\} = \begin{bmatrix} I \\ G_o \end{bmatrix}\{\phi_a\} \tag{6}$$

Using this Ritz transformation the reduced mass and stiffness matrices become

$$[M_{aa}] = [G]^T[M][G] \tag{7}$$

and

$$[K_{aa}] = [G]^T[K][G] \tag{8}$$

Performing these operations on the matrices in Equation 2 we get

$$[M_{aa}] = [\bar{M}_{aa}] + 2[M_{ao}][G_o] + [G_o]^T[M_{oo}][G_o] \tag{9}$$

and

$$[K_{aa}] = [\bar{K}_{aa}] + [K_{ao}][G_o] \tag{10}$$

The mass of the system will be redistributed based on the elastic connections between the O-set DOF and the A-set DOF as shown in Equation 9.

Note that Guyan reduction is exact when M_{oo} (and hence M_{ao}) is a null matrix and gives the best solution for any selected partition when it is not. It does not, however, address directly the problem of selecting most effectively the set of independent DOF that will best serve the aims of the user. For this, a means of removing terms from the mass matrix so as to minimize the impact on the solution accuracy must be determined.

3.0 ASET SELECTION ALGORITHM

As stated previously, Guyan reduction is exact when M_{oo} is null, or when the O-set mass to stiffness "ratio" is zero. As the mass to stiffness "ratio" between M_{oo} and K_{oo} increases, the accuracy of the Guyan reduction method decreases. This generalization forms the basis of the A-set selection method.

The six step method for determining the A-set DOF is as follows:

1. Execute NASTRAN to obtain an initial M_{aa}, K_{aa}, and A-set table. The mass and stiffness matrices can be reduced as desired in NASTRAN as long as the modal content over the frequency range of interest is retained. Note that no reduction need be performed at this stage but the initial constraint equation must be applied.
2. Define the number of DOF that will be in the final A-set. These DOF may also contain a "kernel" set of DOF that will remain in the A-set regardless of their mass to stiffness ratio.
3. Determine the minimum mass to stiffness "ratio" for the O-set DOF. Because M and K are diagonally dominant, this ratio is most easily approximated by stripping the diagonal from M and K and scanning for the minimum M_{ii}/K_{ii} which we will call min(M/K). The min(M/K) DOF is then partitioned from M and K and reduced from the system, provided it is not a member of the kernel set.
4. Repeat step 3 until the desired number of DOF remain in the A-set.
5. Write NASTRAN ASET bulk data cards for the retained DOF
6. Check the A-set to determine if desired modes are adequately defined.

To improve the efficiency of the check process, the mass and stiffness matrices may be saved during Step 5. These matrices can then be used in an eigenvalue analysis to determine if the selected A-set is adequate.

The user may, if desired, decide to refine the A-set further if it is concluded that more DOF can be reduced from the problem. To simplify this second reduction, the A-set listing and matrices from Step 5 can be used as input to Step 2. The process would then proceed as before.

Occasionally, too few DOF will be defined in the A-set. By keeping track of the DOF placed in the O-set during each iteration, the user may simply review DOF that were omitted during previous iterations to determine DOF that are required to define the mode or modes lost because of the Guyan reduction. He may then selectively include those DOF deemed necessary to the A-set by adding these DOF on his ASET bulk data cards. Alternatively, he may save intermediate ASET card images for convenience.

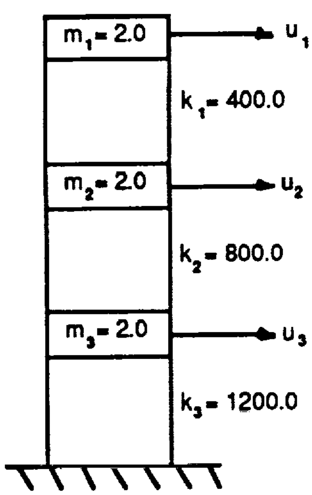
Because the algorithm currently works on one DOF at a time, the user should use NASTRAN to make the problem size as small as possible to decrease the solution time. Though reducing several DOF during each iteration is a desirable feature, no definitive method for including this feature in the algorithm has yet been developed. More information on this topic is presented in the conclusions.

The algorithm described above virtually guarantees that the smallest A-set will be obtained with minimal effort, provided that too severe a reduction is not specified. The general procedure for selecting the A-set automatically should be clear from the discussion above. The process is best illustrated, however, by performing sample calculations on a simplified model, as shown in the next section.

4.0 METHOD VERIFICATION

Two sample problems were developed to validate the A-set selection method. The first problem is a simplified model of a three story building. The reduction operations are performed by hand to clarify the algorithm. The second sample problem determines the A-set of a 3600 DOF NASTRAN model. The A-set for this problem was generated using a program developed by McDonnell Douglas Space Systems Company-Huntsville Division (MDSSC-HSV). The data from these sample problems verify the algorithm outlined in Section 3.

The simplified model of the three story structure is shown in Figure 1. The mass and stiffness matrices are also shown. The fundamental frequency of this system is 1.45 Hz. We want to reduce the problem to a one DOF system.



$$[M] = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \qquad \{\phi_1\} = \begin{Bmatrix} 1.0 \\ 0.585 \\ 0.255 \end{Bmatrix}$$

$$[K] = \begin{bmatrix} 400 & -400 & 0 \\ -400 & 1200 & -800 \\ 0 & -800 & 2000 \end{bmatrix}$$

$$f_1 = 1.45 \text{ Hz}$$

Figure 1. Simplified Three Story Building

First we find the min(M/K) for this system which is 2/2000 = 0.001 for displacement u_3. Partitioning this DOF from M and K yields

$$\bar{M}_{aa} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \qquad M_{ao} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad M_{oo} = 2$$

$$\bar{K}_{aa} = \begin{bmatrix} 400 & -400 \\ -400 & 1200 \end{bmatrix} \qquad K_{ao} = \begin{bmatrix} 0 \\ -800 \end{bmatrix} \qquad K_{oo} = 2000$$

The G_o matrix for this problem is

$$G_o = \frac{-1}{2000}[\,0 \quad -800\,] = [\,0.0 \quad 0.4\,]$$

The reduced mass and stiffness matrices are found using Equations 9 and 10 and are

$$M_{aa} = \begin{bmatrix} 2 & 0 \\ 0 & 2.32 \end{bmatrix}$$

$$K_{aa} = \begin{bmatrix} 400 & -400 \\ -400 & 880 \end{bmatrix}$$

We repeat the steps to determine the mass and stiffness of the one DOF system. Performing these steps produces M = 2.48 and K = 218.2. The frequency for this one DOF system is f_1 = 1.50 Hz which is 3.5 percent higher than the "exact" frequency of 1.45 Hz.

Though the frequencies show excellent agreement, correlation between the mode shapes should also be verified. Back transforming using G_o we get

$$\{\phi_1\} = \begin{Bmatrix} 1.0 \\ 0.455 \\ 0.182 \end{Bmatrix}$$

for the one DOF system. We will use the modal assurance criterion (MAC) described in Reference 2 to measure the correlation between this mode shape and the "exact" mode shape. The MAC between any two modes varies from zero, meaning no correlation, to unity, meaning perfect correlation. The MAC for these modes is 0.987 indicating that little modal accuracy was lost during the reduction.

The second sample problem involves finding an A-set for the model shown in Figure 2. The unreduced model has approximately 3600 DOF. Currently, the model A-set has 180 DOF which was used as a starting point for this problem. This A-set was further reduced to 50 DOF using the MDSSC-HSV developed program based on the selection algorithm described in Section 3. The final A-set size is approximately 25 percent of the original A-set size.

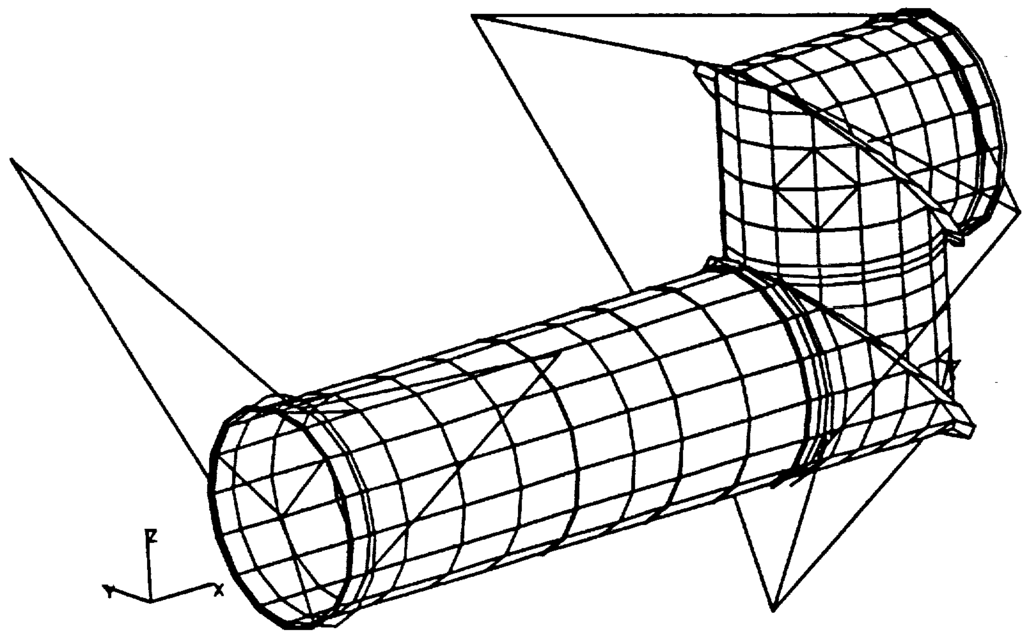


Figure 2. NASTRAN Model for Sample Problem 2

Table 1 shows a comparison between the frequencies and mode shapes of the 180 DOF model and the 50 DOF model. The frequencies show excellent agreement with a maximum difference of 1.4 percent for the sixth mode. The mode shapes are almost perfectly correlated between the the 180 DOF model and the 50 DOF model. Indeed, it may be possible to reduce the problem size even further.

Table 1. Frequency and Mode Shape Comparison Between 180 DOF Model and 50 DOF model

MODE	f_{180}	f_{50}	Δ%	MAC
1	11.9	11.9	0.0	>0.999
2	12.9	13.0	0.8	>0.999
3	24.1	24.2	0.4	0.998
4	24.9	25.0	0.4	0.996
5	33.1	33.3	0.6	0.998
6	62.3	63.2	1.4	0.992

5.0 CONCLUSIONS

A method for automatically selecting the NASTRAN A-set DOF was described. Theoretical development and an outline of the steps involved were provided. Two example problems were provided that demonstrate the use and the accuracy of the method. Some potential enhancements have been identified and will be briefly summarized here.

One potential enhancement noted earlier would be to reduce multiple DOF during each iteration. Because of the redistribution of the mass of the system, simply reducing a certain percentage of the DOF at each iteration is to be discouraged. The reason for this is best demonstrated with an example.

Consider the simply supported beam of Figure 3. Because all of the DOF have identical mass to stiffness ratios, the removal would begin with the first DOF with this min(M/K). If a 20 percent reduction rate were chosen then u_1 and u_2 would be removed in the first iteration, which could ultimately result in a poorly chosen A-set.

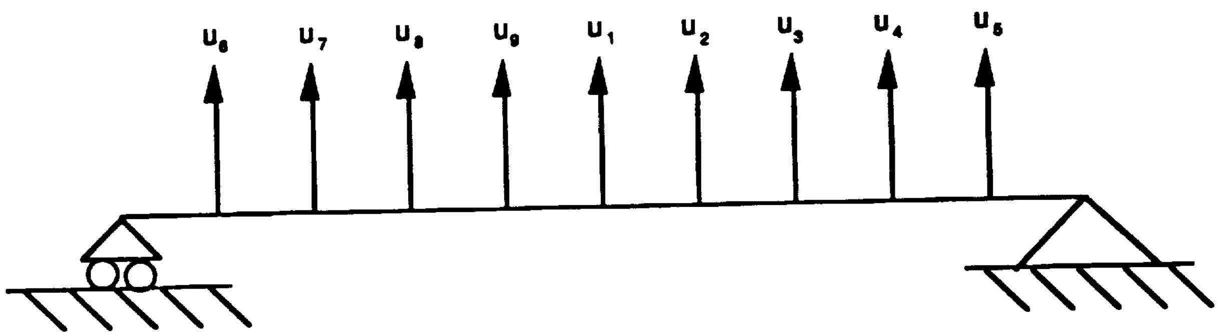


Figure 3. Simply Supported Beam

A second potential enhancement would be including a method in the algorithm that would determine the optimum number of A-set DOF based on a user defined upper bound frequency of interest. Because the algorithm removes terms with a high pseudo frequency, i.e. large K_{ii}/M_{ii}, an approach based on the pseudo frequencies of the reduced system could be used to predict the minimum required number of A-set DOF.

Even without these enhancements, the method has been successfully implemented at MDSSC-HSV. The often tedious, and sometimes error prone A-set selection process has been automated, saving engineering time while increasing A-set efficiency.

REFERENCES

1. Guyan, R.J.: "Reduction of Stiffness and Matrices," AIAA Journal, Volume 3, pg 380, 1965.

2. Ewins, D.J.: Modal Testing: Theory and Practice, John Wiley & Sons Inc., New York, June 1985.