Fuzzy Control of Magnetic Bearings

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Abstract- **This paper considers the use of an adaptive fuzzy control algorithm implemented on a VLSI chip for the control of a magnetic bearing. The architecture of the adaptive fuzzy controller is similar to that of a neural network. The performance of the fuzzy controller is compared to that of a conventional controller by computer simulation.**

1   Introduction

Magnetic levitation is receiving increasing attention as a viable alternative to conventional methods of moving and positioning objects [1]. NASA, for example, has developed a cryogenic cooler that uses magnetic bearings and actuators exclusively [2]. One of the more difficult aspects of the application of magnetic bearings is the control of the position of the shaft in the bearing housing. Considerable attention has been given to this problem recently. Williams et. al. [3] reported on the digital control of active magnetic bearings and showed how the flexibility of digital control was extremely useful in implementing a number of control algorithms including second-derivative and integral feedback. Chen and Darlow [4] describe an analog control system for an active magnetic bearing that uses velocity and acceleration observers to improve damping and cancel imbalance and other disturbance forces to greatly improve the overall system performance. Keith et. al. [5] discuss the magnetic support of flexible shaft at speeds up to 14,000 RPM using a PC-based digital controller implementing a proportional-derivative control algorithm. A comparison with an earlier analog proportional-derivative controller is also made. Chen [6] describes an active magnetic bearing control scheme using three parallel feedback loops to achieve dynamic stiffness, static stiffness, and damping. He presents a closed-form solution for controller parameters in terms of desired stiffness and damping. Humphris et. al. [7] present a comprehensive treatment of the active magnetic bearing control problem and compare the relative performance of low bandwidth and high bandwidth controllers. Scudiere et. al. [8] used a Texas Instruments TMS32010 digital signal processor to implement a proportional-integral-derivative control algorithm to successfully control the position of a number of small spheres and rotors. Feeley et. al. [9] described root locus design of a double lead-lag controller mapped into an equivalent digital controller via the Tustin transformation. The resulting algorithm has been implemented on an Intel 80KC196C microprocessor and used to control an analog computer model of the NASA magnetic bearing.

The difficulty of the control problem stems from two basic causes. The first is due

to the physical nature of the magnetic bearing system itself. As shown in Section 2, the uncontrolled magnetic bearing system is unstable, uncertain, and highly nonlinear. The instability is due to the relentlessness of gravity in causing any suspended object to fall. The uncertainties arise from the difficulties in modeling viscous friction, eddy currents, leakage flux, and accounting for disturbance forces due to vehicle acceleration, motion of the shaft, and other random events. The nonlinearities arise in the square-law nature of magnetic forces, the nonlinear relationship between actuator current and magnetic flux, and the nonlinear properties of materials in the magnetic circuit. The second basic cause of difficulty in the control problem stems from the decision to use digital control. Sampling is inherent in digital control and it is reasonable to expect poorer performance from a digital control system using data samples than from its ideal analog equivalent using continuous data. This inevitable degradation in performance encountered in moving from analog to digital control must be compensated for by the use of more sophisticated digital control algorithms and the other advantages inherent in digital control.

A control scheme that is effective in overcoming these two basic causes of difficulty in the control problem is presented in this paper. The scheme is based on the theory of fuzzy systems. The modeling problem is addressed by substituting the imprecise linguistic model of fuzzy theory for the precise model of physical theory. The sampling problem is addressed by implementing the fuzzy algorithm in a parallel architecture suitable for VLSI implementation thereby reducing processing time and allowing high sampling rates.

The remainder of the paper is organized as follows. Section 2 describes the magnetic bearing system and presents a mathematical model developed by Feeley et. al. [10]. In Section 3 some essential elements of fuzzy control theory are presented and an adaptive fuzzy controller is developed. In Section 4 the performance of the fuzzy controller is analyzed using a computer simulation based on the nonlinear model of Section 2. A adaptive fuzzy control VLSI chip architecture is outlined in Section 5 and some conclusions and recommendations are given in Section 6

2 Magnetic Bearing System

A schematic cross-sectional side view of NASA's magnetic bearing is shown in Figure 1 supporting one end of a rigid shaft. An end view would show the circular cross-section shaft centered in the annular gap created by the bearing housing and the shaft. Figure 1 also shows the shaft magnetic material inlays that provide paths for the magnetic flux produced by the adjacent bearing actuators. The actuators are symmetrically located in the bearing housing and consist of magnetic material pole pieces and coils of copper wire. A position sensor is located close to each actuator to measure the position of the shaft. A total of four actuator and position sensor assemblies are located at 90° increments around the circumference of the housing. Coordinated control of opposing actuators permits positioning of the end of the shaft anywhere in the annular gap. An identical bearing assembly supports the other end of the shaft. For simplicity, rotational forces are not directly accounted for and half of the shaft mass is assumed to be concentrated at the point of action

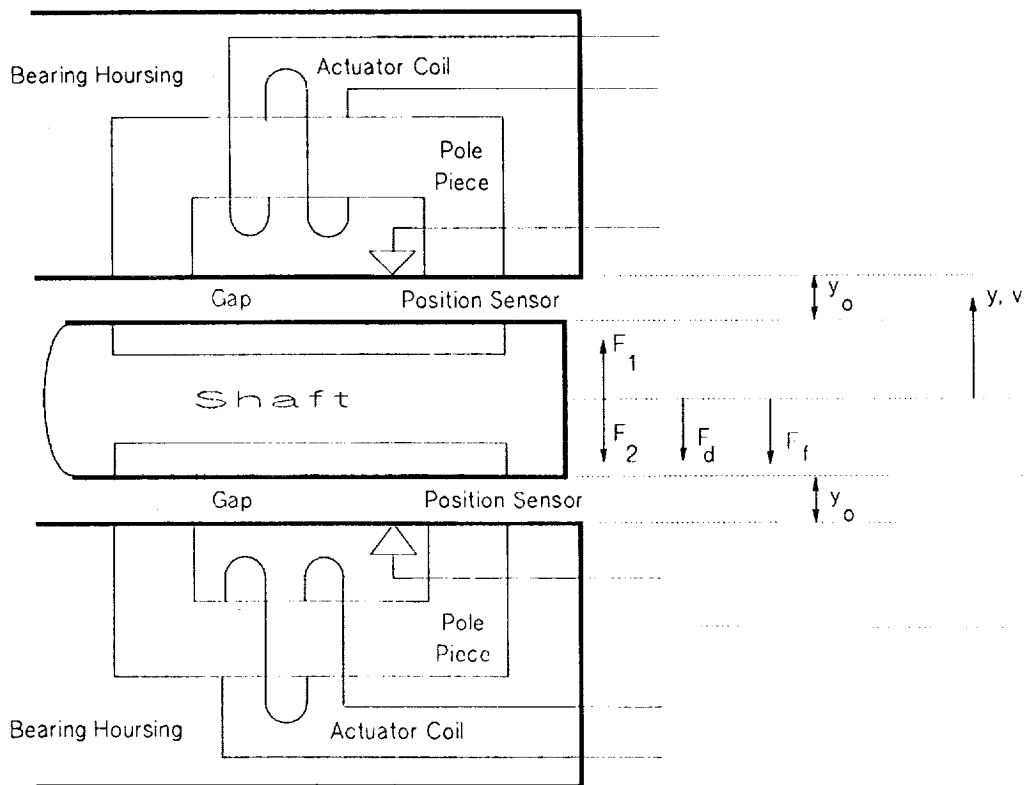of the magnetic forces of each bearing assembly.

Figure 1: Schematic cross-section of magnetic bearing assembly

Assuming motion in the one-dimensional coordinate system defined in Figure 1, application of Newton's second law yields

$$\frac{d^2y}{dt^2} = \frac{F_1 - F_2 - F_d - F_f}{M}$$

where y is the position of the shaft, F_1 is the magnetic force exerted by the upper actuator, F_2 is the magnetic force exerted by the lower actuator, F_d is a disturbance force, and F_f is a viscous friction force. F_1 and F_2 are, in turn, defined by

$$F_1 = \frac{\mu_0 A}{4} \left[\frac{N_c i_1}{y_0 - y} \right]^2$$

and

$$F_2 = \frac{\mu_0 A}{4} \left[\frac{N_c i_2}{y_0 - y} \right]^2$$

where μ is the magnetic permeability, A is the area of one pole face, N_c is the number of coil turns, i_1 and i_2 are the coil currents, and y_0 is the initial gap distance. The friction force is assumed proportional to the square of the shaft velocity and is modeled mathematically as $F_f = K_f v |v|$, and the disturbance force is taken as an exogenous input.

The electromagnetic of the actuator are modeled with the aid of the circuit diagram of Figure 2. The circuit model consists of two loops, one for the primary coil current i_c, and a second for the induced eddy current i_e. Applying Kirchoff's voltage law to each loop yields the circuit equations

$$v_c = Ri_c + N_c \frac{d\phi_c}{dt} + N_{ce} \frac{d\phi_e}{dt}$$

$$0 = R_e i_e + N_e \frac{d\phi_e}{dt} + N_{ec} \frac{d\phi_c}{dt}$$

where v_c is the voltage applied to the coil, N_c is the number of turns in the coil, ϕ_c is the flux produced by the coil current, N_{ce} is the number of turns of the coil linked by the flux produced by the eddy currents, ϕ_e is the flux produced by the eddy currents, R_e is the resistance of the eddy current paths, N_e is the number of turns in the equivalent eddy current coil, and N_{ec} is the number of turns in the equivalent eddy current coil linked by the flux produced by primary current. Assuming the entire mmf drop of the magnetic circuit is taken across the two air gaps, the fluxes can be expressed in terms of the currents as $\phi_c = \frac{\mu_0 A N_c i_c}{2y_{ag}}$ and $\phi_e = \frac{\mu_0 A N_e i_e}{2y_{ag}}$ where y_{ag} is the distance between the pole piece and the shaft, $y_0 - y$ for the upper gap and $y_0 + y$ for the lower gap. Solving these equations for the time derivatives of the currents leads to

$$
\begin{aligned}
\frac{di_{c1}}{dt} &= -\left[\frac{v}{y_0-y} + \frac{R_c}{N_c L_1}\right] i_{c1} & + & \frac{kR_e}{L_1} i_{e1} & + & \frac{1}{N_c L_1} v_{c1} \\
\frac{di_{e1}}{dt} &= \frac{kR_e}{L_1} i_{c1} & - & \left[\frac{v}{y_0-y} + \frac{N_c R_e}{L_1}\right] i_{e1} & - & \frac{k}{L_1} v_{c1} \\
\frac{di_{c2}}{dt} &= -\left[\frac{v}{y_0+y} + \frac{R_c}{N_c L_2}\right] i_{c2} & + & \frac{kR_e}{L_2} i_{e2} & + & \frac{1}{N_c L_2} v_{c2} \\
\frac{di_{e2}}{dt} &= \frac{kR_e}{L_2} i_{c2} & - & \left[\frac{v}{y_0+y} + \frac{N_c R_e}{L_2}\right] i_{e2} & - & \frac{k}{L_2} v_{c2}
\end{aligned}
$$

where $L_1 = \frac{\Delta \mu_0 A}{2(y_0-y)}$, $L_2 = \frac{\Delta \mu_0 A}{2(y_0+y)}$, $\Delta = N_c N_e - N_{ce} N_{ec}$, and $k = \frac{N_{ce}}{N_c} = \frac{N_{ec}}{N_e}$. The equations presented in this section constitute a consistent mathematical model relating the input voltages applied to the actuator coils, v_{c1} and v_{c2}, to the position of the shaft, y.

3 Fuzzy Control

Conventional feedback control systems measure, relatively precisely, certain process variables, operate on these measurements with a control algorithm to produce precise command signals, and apply these command signals to the process to control its behavior in some desired way. The control algorithm generally relies on an explicit mathematical model of the system to be controlled and some expression of desired system performance. A
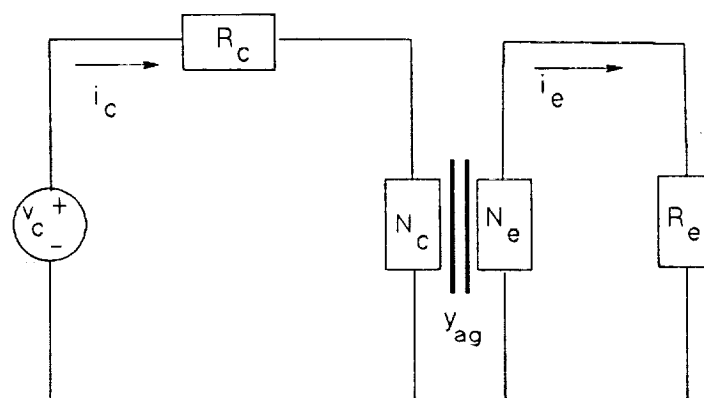
Figure 2: Circuit model of actuator.

crucial element in control algorithm design is the development of a suitable mathematical model of the system; in general, performance of the controlled system will be no better than the system model on which the control algorithm is based. The model should be neither too complicated, making the control algorithm too complex to implement, nor too simple, missing essential features of system behavior. Since most systems requiring automatic feedback control are dynamic and nonlinear, the development of a simple model that still captures the essence of important system performance characteristics is usually a time-consuming, and in some cases, impossible, task.

It is interesting to compare these automatic control systems with manual control systems where a human operator makes seemingly imprecise measurements, processes them rapidly in the brain, and produces the correct control command to, say, ride a bicycle. While it may not be impossible to build an automatic control system to control a bicycle (although we have never seen one), it would certainly be quite difficult. Yet, a young child can become a proficient rider after only a short training session with no knowledge whatsoever of the mathematics of bicycle dynamics. It is this paradox that led Zadeh [11] to the development of the theory of fuzzy sets, Mandami [12] to consider the linguistic synthesis of fuzzy control systems, and, most recently, Kosko [13] to explore its connections with neural networks in the adaptive control of dynamic systems.

As with neural network controllers, fuzzy controllers try to emulate the functions of the human brain. A fundamental difference between the two is that neural controllers assume no a' priori knowledge of system behavior, while fuzzy controllers start with a linguistic description of whatever is known about the system. There is, however, a striking similarity at the implementation level between neural network controllers and adaptive fuzzy controllers [13].

3.1 Fuzzy Variables and Fuzzy Values

The notions of fuzzy control are rooted in the theory of fuzzy sets [11]. The basic difference between conventional (crisp) set theory and fuzzy set theory lies in the values assigned to the variables. Consider, for example, a variable called position, y. In crisp theory y could take on values, say, from 0m to +10m. At any particular point in time, the position of an object could be given by the value, say, 4m. In fuzzy theory, however, the values assigned to the position variable, y, are of not the familiar, crisp, numerical type but, rather, an unfamiliar, fuzzy, linguistic type; e.g. "close", or "far", or "very far". This is consistent with the child bicyclist's assessment of position relative to an upcoming tree. Since one of the strengths of fuzzy theory is that it is basically quantitative in nature, it remains to relate the fuzzy values "close", etc. to appropriate numerical values in a fuzzy way consistent with our notion of the meanings of the corresponding linguistic values. In the example considered above, "close", "far", and "very far" may be characterized by the distributions shown in Figure 3 where the abscissa is the distance from the tree and the ordinate is the degree to which "close", etc. is an accurate representation of the distance to the tree. Certainly, if the cyclist is about to hit the tree it is "close" while if it is 10m away it is not. If, however, it is 4m away it is only "close" to a degree; more specifically "close" is an accurate description of the distance 4m with degree 0.21, while "far" is an accurate description of this same distance with degree 0.64, and "very far" is not at all accurate and, so, is descriptive with degree 0.0. This subjective assessment of "closeness", etc. is introduced by the designer in the development of these distributions, or as they are known in fuzzy theory, "membership functions". To summarize, it is correct to think of the the fuzzy values "close", etc. as "fuzzy numbers" whose relationship to "crisp numbers" is provided by a defining membership function.
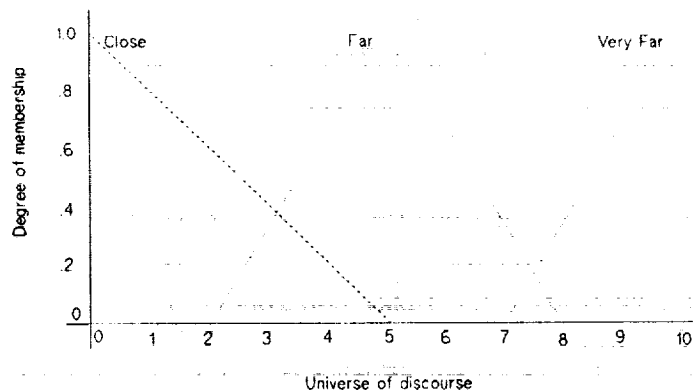

Figure 3: Membership functions for the fuzzy values "Close", "Far", and "Very Far" of the fuzzy variable "Position".

3.2   Fuzzy Functions

Analogous to the function of crisp mathematics that maps crisp input variables into crisp output variables, fuzzy mathematics uses a relational matrix to map fuzzy input variables into fuzzy output variables. The relational matrix is constructed from a linguistic rule base relating fuzzy input variables to fuzzy output variables. The linguistic rule base may be generated from a set of logical implications of the "IF-THEN" type. Consider, for example, a system with two fuzzy input position variables and , and one fuzzy output steering variable θ. Let the possible fuzzy values of be "left" (L), "center" (C), and "right" (R), let the possible fuzzy values of be "close" (C), "far" (F), and "very far" (VF), and the possible fuzzy values of be "left" (L), "center" (C), and "right" (R). A brief linguistic rule base might then consist of the following logical implications:

1. IF [x is L and y is C] THEN [θ should be R]

2. IF [x is R and y is C] THEN [θ should be L]

3. IF [x is C and y is V] THEN [θ should be C]

The relational matrix embodying these rules is shown in Figure 4, and is seen to be a concise display of the relationship between the pairs of fuzzy values of the fuzzy input variables and the fuzzy values of the fuzzy output variable. It is interesting to note that the relational matrix is not necessarily full. An important and powerful aspect of fuzzy control is that only those rules that are well known need be specified, the fuzzy calculations will "interpolate" or "extrapolate" to fill in missing rules. The fuzzy calculations will also resolve conflicting rules in an optimal way consistent with the specified linguistic rule base and defined fuzzy variables.

Figure 4: Relational matrix mapping fuzzy input variables x and y to fuzzy output variable θ.

3.3 Fuzzy Controller

Fuzzy control systems inevitably interact with the physical world of crisp measurements and actuators. On input to the controller, crisp values of crisp variables are converted to fuzzy values of fuzzy variables according to the membership function of the fuzzy variable. For example, in Figure 3 a crisp value of "4" of the crisp variable position would take on two fuzzy values "close" and "far" of the fuzzy position variable. The membership functions indicate that the crisp value "4" is the fuzzy value "close" with degree 0.21 and the fuzzy value "far" with degree 0.64. Thus, a single measurement of a crisp variable may activate a number of rules in linguistic rule base or, equivalently, the relational matrix. Each rule will operate on its fuzzy input variables, and their membership functions, to produce a modified membership function, or fuzzy value, for the the fuzzy output variable. The specific form of the output membership function may be determined either by the correlation-minimum or the correlation-product inferencing technique [13]. Since more than one rule may be activated by a single measurement it follows, then, that a number of fuzzy values of the output may also be generated. The output membership functions generated by the firing of several rules may be combined in a number of different ways to produce a single crisp output to activate a physical actuator. Two commonly used methods are the mean-of-maxima and the centroid methods[13].

The fuzzy controller under development for the magnetic bearing has two fuzzy input variables, position y and, change in position dy; and one fuzzy output variable, actuator voltage v. Each fuzzy variable may take on each of seven fuzzy values: "negative large" (NL), "negative medium" (NM), "negative small" (NS), "zero" (ZE), "positive small" (PS), "positive medium" (PM), and "positive large" (PL). The fuzzy values of the input variables are shown over their corresponding universe of discourse in Figure 5. The universe of discourse ranges from -5 volts (corresponding to a shaft position of -19μm) to +5 volts (corresponding to a position of +19μm). Fuzzy values are trapezoidal in shape with a maximum overlap of 25%, and are narrower near zero to provide finer control close to the desired value.
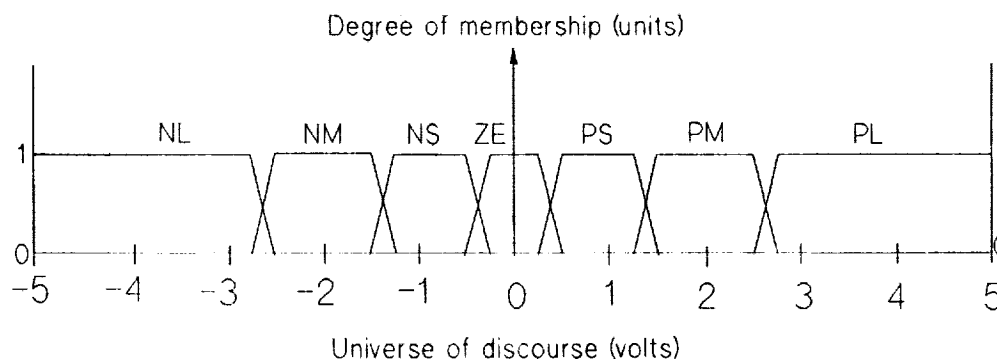

Figure 5: Fuzzy values of input variables y and dy.

Fuzzy values of the output variables are shown in Figure 6. They are triangular in

shape, have a maximum overlap of 25%, and are closer together near zero to provide finer control. The exact shapes and locations of the fuzzy input and output variables are design parameters whose optimal values are found by numerical experimentation.
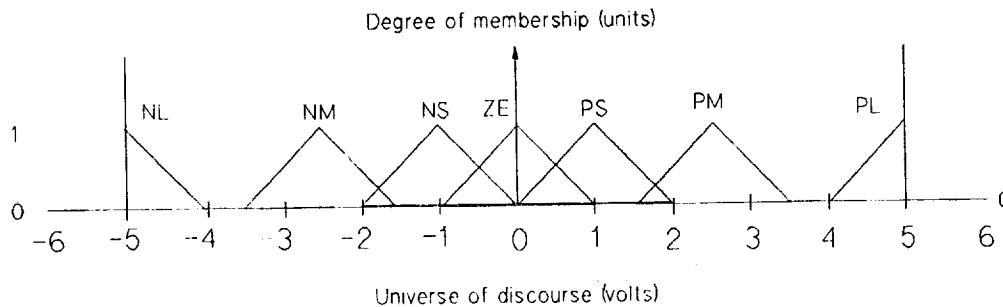

Figure 6: Fuzzy values of the output variable v.

The 7 x 7 relational matrix relating the fuzzy input pairs to fuzzy values of the output is shown in Figure 7. The relationship between the relational matrix and the corresponding set of forty nine IF-THEN implications is obvious.

The correlation-minimum inference procedure is used to process activated rules resulting in a truncation of the output membership function at the minimum value of the two input membership functions. Note that since a maximum of two input values overlap, a maximum of four (as opposed to a possible maximum of forty nine) rules can be activated at once. Combining of output fuzzy values and subsequent defuzzification is performed using the centroid method.

4 Performance of Magnetic Bearing with Fuzzy Controller

A linearized version of the nonlinear model presented in Section 2 was programmed using Matlab to test the performance of the fuzzy controller. Figure 8 shows the response to a $3.8\mu m$ (1 volt) step demand change in position. The figure shows that the fuzzy controller was successful in stabilizing the bearing and that response time is short. Sampling frequency was 10 K Hz. Oscillations are small and can be further reduced by reducing the size of the fuzzy sets representing zero error. Steady state error can be further reduced by adding an integral mode to the controller. These results are not surprising since the present fuzzy controller uses only position and velocity inputs and is essentially operating as a proportional-plus-derivative controller. Additional work is being conducted to correct these deficiencies. Several promising adaptive control policies are being investigated including modifying the input fuzzy set sizes and overlap, the output fuzzy set centroids, and the scaling gains K_e, K_c, and K_v. Best results were obtained with 25% set overlap and $K_e = 1$, $K_c = 18$, and $K_v = 5$.

Change in positon, dy

Position, y	NL	NM	NS	ZE	PS	PM	PL
NL	NL	NL	NM	NM	NS	NS	ZE
NM	NL	NM	NM	NS	NS	ZE	PS
NS	NM	NM	NS	NS	ZE	PS	PS
ZE	NM	NS	NS	ZE	PS	PS	PM
PS	NS	NS	ZE	PS	PS	PM	PM
PM	NS	ZE	PS	PS	PM	PM	PL
PL	ZE	PS	PS	PM	PM	PL	PL

Figure 7: Relational matrix for the magnetic bearing controller.

5 Architecture for a Fuzzy VLSI Chip

The architecture of a fuzzy VLSI chip is outlined in Figure 9. The basic fuzzy control algorithm is contained on a single chip. Rules are downloaded from a host computer at start-up and can be modified by the host computer later. The chip is of the all-digital type so off-chip A/D and D/A converters are required. The fuzzy control algorithm has four parts: 1) input calculations, 2) input membership determination, 3) rule evaluation, and 4) output defuzzification as described below.

5.1 Input Calculations

The single input to the chip is the position error in volts. The current error $er(n)$ and the previous error $er(n-1)$ are each stored in a separate registers. The current change in error $ch(n) = er(n) - er(n-1)$ is computed and stored in a third register. The variables ER and CH, used in membership function determination are found by multiplying $er(n)$ by K_e and $ch(n)$ by K_c, respectively. The scaling gains K_e and K_c are downloaded from the host computer and may be modified as required.

5.2 Input Membership Determination

The input membership determination is made by a table look-up. There are two look-up tables one for ER and one for CH. The output of the table look-up is the modified fit vector (A, m_A, m_B). Each look up table is of size 3 by m by n.
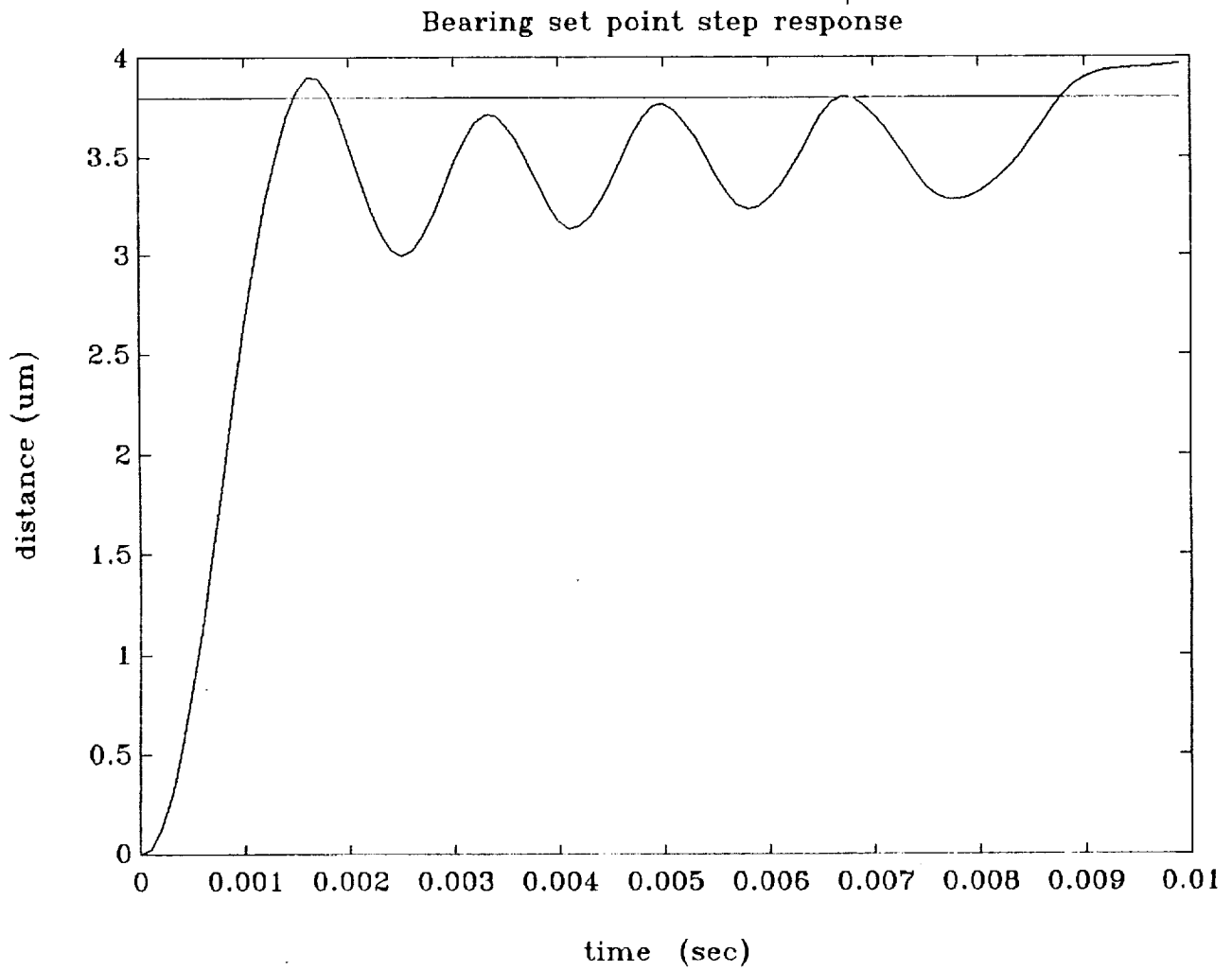
Figure 8: Response of fuzzily controlled magnetic bearing to 3.9μm step change in position.
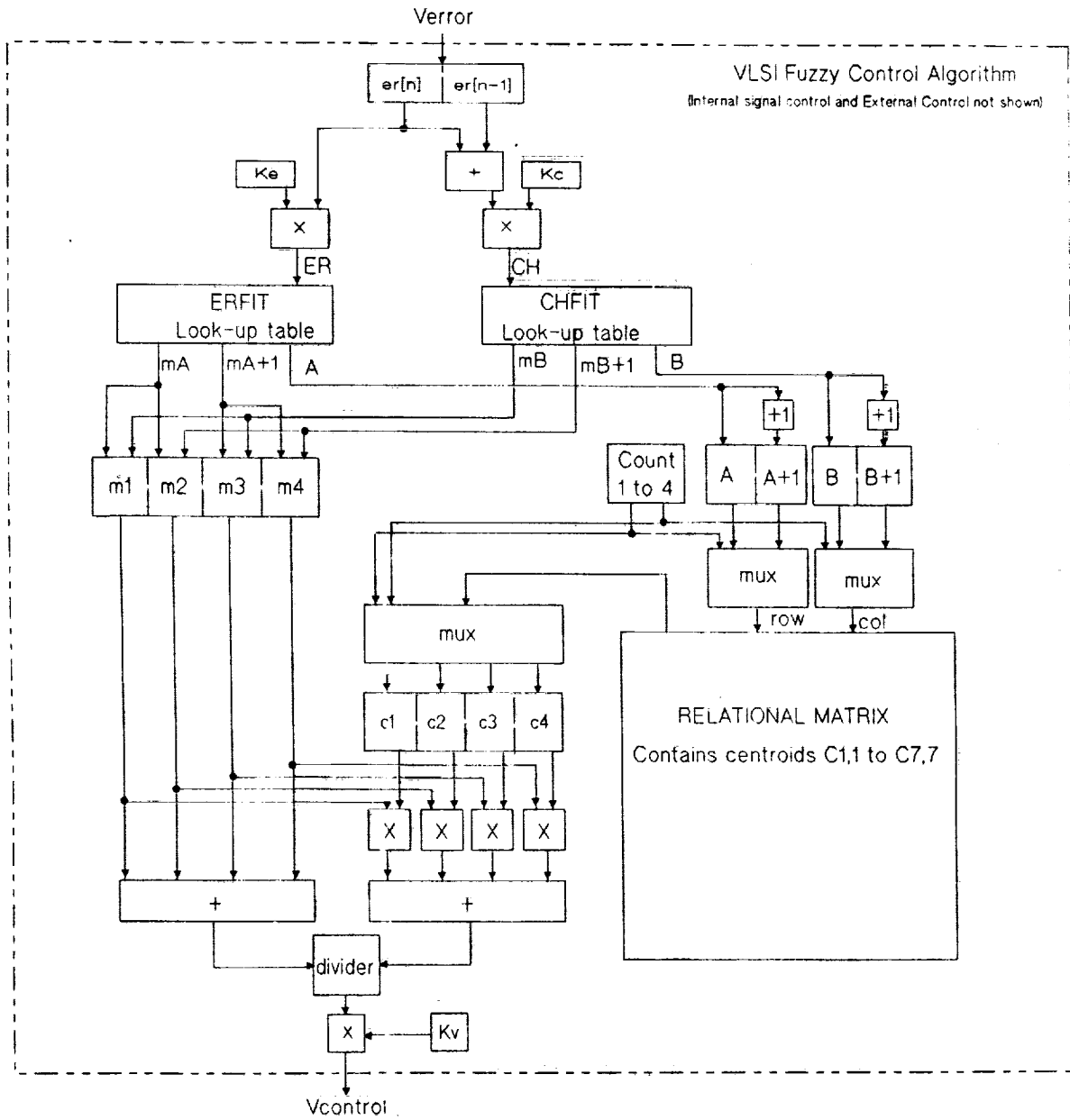
Figure 9: Architecture of a fuzzy VLSI chip.

5.3   Rule Evaluation

Four rules are evaluated for each input pair. Each evaluation finds the minimum of the input fuzzy sets and the centroid of the output fuzzy set. A 4 by n hold the minimum input membership values and a 4 by m register hold the corresponding centroids.

5.4   Output Defuzzification

Defuzzification is done in three steps. First, the minimum membership value is multiplied by the centroid for each of the rules activated. Second, each of these products is summed to produce; at the same time each of the minimum membership values is also summed. Finally, the sum of the minimum membership-centroid products is divided by the sum of the minimum memberships to produce the desired result. The result is then multiplied by an output voltage scaling gain K_v.

6   Summary and Conclusions

A mathematical model of a magnetic bearing was presented and was used to develop a computer simulation model to test alternative magnetic bearing control systems. A fuzzy control system for was developed and tested by computer simulation. Initial results show that the fuzzy controller stabilizes the magnetic bearing and produces acceptable steady-state and transient behavior. Further research is being conducted to optimize the fuzzy controller and to develop suitable adaptive algorithms. Particular emphasis is being placed on achieving zero steady-state error and rejecting acceleration disturbances. Performance comparisons between the fuzzy controller and a linear-quadratic-gaussian regulator are being conducted. A candidate VLSI chip architecture has been proposed to implement the fuzzy control algorithm and provide rapid sampling for real-time control. VLSI-based fuzzy control appears feasible for real-time control of uncertain nonlinear systems like an active magnetic bearing.

7   Acknowledgements

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