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**Abstract**

The main research activities at the Center for Modeling of Turbulence and Transition (CMOTT) are described. The research objective of CMOTT is to improve and/or develop turbulence and transition models for propulsion systems. The flows of interest in propulsion systems can be both compressible and incompressible, three dimensional, bounded by complex wall geometries, chemically reacting, and involve "bypass" transition. The most relevant turbulence and transition models for the above flows are one- and two-equation eddy viscosity models, Reynolds stress algebraic- and transport-equation models, pdf models, and multiple-scale models. All these models are classified as one-point closure schemes since only one-point (in time and space) turbulent correlations, such as second moments (Reynolds stresses and turbulent heat fluxes) and third moments ( $\overline{u_i u_j u_k}$ ,  $\overline{u_i \theta^2}$ ), are involved. In computational fluid dynamics, all turbulent quantities are one-point correlations. Therefore, the study of one-point turbulent closure schemes is the focus of our turbulence research. However, other research, such as the renormalization group theory, the direct interaction approximation method and numerical simulations are also pursued to support the development of turbulence modeling.

## 1. Introduction

The center for modeling of turbulence and transition was established as a special focus group within the Institute for Computational Mechanics in Propulsion at NASA Lewis Research Center in 1990. Its objective is to improve and/or develop turbulence and transition models for computational fluid dynamics (CFD) applied in propulsion systems. With the advance of computer technology and algorithms, accurate turbulence and transition modeling becomes the pacing item for improving flow calculations used in propulsion system design in all its key elements. The flows of interest in propulsion systems are, in general, very complex since there are wall-bounded three-dimensional complex geometries, chemical reactions, compressibility and transition, etc. In order to accurately predict these flows one must correctly model the turbulent stresses and scalar fluxes which are one-point (in time and space) turbulent correlations. For flows with finite rate chemical reactions, accurate modeling of the production rate of species is crucial for turbulent flow calculations. Based on the above considerations, turbulence modeling activities at CMOTT are focused on one-point closure schemes, that is, using the moment closure schemes for the turbulent velocity field and the joint scalar pdf method for the reacting scalar field.

There are various moment closure schemes which have been developed for various engineering applications. However, in practice, one often finds that the existing models need to be improved and/or re-developed in order to reasonably simulate complex flow structures appearing in propulsion systems. For this purpose, CMOTT devotes itself to improving and/or re-developing these moment closure schemes which include eddy viscosity (one- and two-equation) models, second moment algebraic- and transport-equation models, non-equilibrium multiple-scale models, and bypass transition models. In addition, other studies supporting the development of one-point closure schemes have been also carried out (for example, studies on renormalization group theory (RNG), direct interaction approximation (DIA), direct numerical simulation (DNS) and large eddy simulation (LES)).

In this report, we first describe the general development of turbulent constitutive relations, turbulent mechanical and thermal dissipation and a new eddy viscosity equation. Second, we describe the detailed developments on each moment closure scheme and the pdf method. Then the RNG and DIA methods and finally, the numerical simulation of particular turbulence phenomena, such as rotation and bypass transition, etc., are considered.

Each research subject is the joint project of several CMOTT researchers and visitors. In describing research activities, the names of involved researchers will be mentioned for reference.

## 2. General Developments

### 2.1 Turbulent Constitutive Relations

#### Reynolds stress

Using the invariant theory in continuum mechanics and Generalized Cayley-Hamilton formulas for tensor products, a turbulent constitutive relation (or a general turbulence model) for any turbulent correlations can be obtained, in principle. Therefore, this theory provides an avenue to develop better turbulence models than those existing. For example, a commonly used constitutive relation for Reynolds stresses  $\overline{u_i u_j}$  (in terms of the mean deformation rate tensor  $U_{i,j}$  and the turbulent velocity and length scales characterized by the turbulent kinetic energy  $k$  and its dissipation rate  $\varepsilon$ ) is

$$-\overline{u_i u_j} = C_\mu \frac{k^2}{\varepsilon} (U_{i,j} + U_{j,i}) - \frac{2}{3} k \delta_{ij} \quad (2.1.1)$$

The effective eddy viscosity  $\nu_T$  defined as

$$\nu_T = \frac{-\overline{u_i u_j}}{U_{i,j} + U_{j,i}} = C_\mu \frac{k^2}{\varepsilon} \quad \text{for } i \neq j \quad (2.1.2)$$

is isotropic since  $\nu_T$  is a scalar quantity. However, the invariant theory enables us to formulate the following general model (Shih and Lumley<sup>1</sup>, Johansson<sup>2</sup>):

$$\begin{aligned} \overline{u_i u_j} = & \frac{2}{3} k \delta_{ij} + 2a_2 \frac{K^2}{\varepsilon} (U_{i,j} + U_{j,i} - \frac{2}{3} U_{i,i} \delta_{ij}) \\ & + 2a_4 \frac{K^3}{\varepsilon^2} (U_{i,j}^2 + U_{j,i}^2 - \frac{2}{3} \Pi_1 \delta_{ij}) \\ & + 2a_6 \frac{K^3}{\varepsilon^2} (U_{i,k} U_{j,k} - \frac{1}{3} \Pi_2 \delta_{ij}) \\ & + 2a_7 \frac{K^3}{\varepsilon^2} (U_{k,i} U_{k,j} - \frac{1}{3} \Pi_2 \delta_{ij}) \\ & + 2a_8 \frac{K^4}{\varepsilon^3} (U_{i,k} U_{j,k}^2 + U_{i,k}^2 U_{j,k} - \frac{2}{3} \Pi_3 \delta_{ij}) \\ & + 2a_{10} \frac{K^4}{\varepsilon^3} (U_{k,i} U_{k,j}^2 + U_{k,j}^2 U_{k,i} - \frac{2}{3} \Pi_3 \delta_{ij}) \\ & + 2a_{12} \frac{K^5}{\varepsilon^4} (U_{i,k}^2 U_{j,k}^2 - \frac{1}{3} \Pi_4 \delta_{ij}) \\ & + 2a_{13} \frac{K^5}{\varepsilon^4} (U_{k,i}^2 U_{k,j}^2 - \frac{1}{3} \Pi_4 \delta_{ij}) \\ & + 2a_{14} \frac{K^5}{\varepsilon^4} (U_{i,k} U_{l,k} U_{l,j}^2 + U_{j,k} U_{l,k} U_{l,i}^2 - \frac{2}{3} \Pi_5 \delta_{ij}) \\ & + 2a_{16} \frac{K^6}{\varepsilon^5} (U_{i,k} U_{l,k}^2 U_{l,j}^2 + U_{j,k} U_{l,k}^2 U_{l,i}^2 - \frac{2}{3} \Pi_6 \delta_{ij}) \\ & + 2a_{18} \frac{K^7}{\varepsilon^6} (U_{i,k} U_{l,k} U_{l,m}^2 U_{j,m}^2 + U_{j,k} U_{l,k} U_{l,m}^2 U_{i,m}^2 - \frac{2}{3} \Pi_7 \delta_{ij}) \end{aligned} \quad (2.1.3)$$

where

$$\begin{aligned}\Pi_1 &= U_{i,k}U_{k,i}, & \Pi_2 &= U_{i,k}U_{i,k}, & \Pi_3 &= U_{i,k}U_{i,k}^2, & \Pi_4 &= U_{i,k}^2U_{i,k}^2, \\ \Pi_5 &= U_{i,k}U_{l,k}U_{l,i}^2, & \Pi_6 &= U_{i,k}U_{l,k}^2U_{l,i}^2, & \Pi_7 &= U_{i,k}U_{l,k}U_{l,m}^2U_{i,m}^2\end{aligned}\quad (2.1.4)$$

From Eq.(2.1.3), the effective eddy viscosity

$$(\nu_T)_{ij} = \frac{-\overline{u_i u_j}}{U_{i,j} + U_{j,i}} \quad (2.1.5)$$

is no longer a scalar and, hence, is an anisotropic eddy viscosity. It is noticed that the first two terms on the right hand side of Eq.(2.1.3) represent the standard  $k$ - $\varepsilon$  eddy viscosity model (2.1.1) and that the first five terms of Eq.(2.1.3) are of the same form as the models derived from both the two-scale DIA approach (Yoshizawa<sup>3</sup>) and the RNG method (Rubinstein and Barton<sup>4</sup>).

Eq.(2.1.3) is a general model for  $\overline{u_i u_j}$ . It contains 11 undetermined coefficients which are, in general, scalar functions of various invariants of the tensors in question, such as  $S_{ij}S_{ij}$  (strain rate) and  $\Omega_{ij}\Omega_{ij}$  (rotation rate) which are  $(\Pi_2 + \Pi_1)/2$  and  $(\Pi_2 - \Pi_1)/2$  respectively. The detailed forms of these scalar functions must be determined by other model constraints, for example, realizability, and by experimental data. Eq.(2.1.3) contains 12 terms; however, its quadratic tensorial form may be sufficient for practical applications. We will see later in section 3.3 that the constitutive relation (2.1.3) has a significant impact on the development of Reynolds stress algebraic equation models.

### Turbulent scalar flux $\overline{\theta u_i}$

We assume the following functional form:

$$\overline{\theta u_i} = F_i(U_{i,j}, T_{i,i}, k, \varepsilon, \overline{\theta^2}, \varepsilon_\theta) \quad (2.1.6)$$

where  $\overline{\theta^2}$  is the variance of a fluctuating scalar and  $\varepsilon_\theta$  is its dissipation rate. Eq.(2.1.6) indicates that the scalar flux depends on not only the mean scalar gradient  $T_{i,i}$ , but also the mean velocity gradient  $U_{i,j}$  and the scales of both velocity and scalar fluctuations characterized by  $k$ ,  $\varepsilon$ ,  $\overline{\theta^2}$ ,  $\varepsilon_\theta$ .

Applying the invariant theory, we may obtain the following general constitutive

relation for  $\overline{\theta u_i}$ :

$$\begin{aligned}
 \overline{\theta u_i} = & a_1 k \left( \frac{k \overline{\theta^2}}{\varepsilon \varepsilon_\theta} \right)^{1/2} T_{,i} + \frac{k^2}{\varepsilon} \left( \frac{k \overline{\theta^2}}{\varepsilon \varepsilon_\theta} \right)^{1/2} (a_2 U_{i,j} + a_3 U_{j,i}) T_{,j} \\
 & + \frac{k^3}{\varepsilon^2} \left( \frac{k \overline{\theta^2}}{\varepsilon \varepsilon_\theta} \right)^{1/2} (a_4 U_{i,k} U_{k,j} + a_5 U_{j,k} U_{k,i} + a_6 U_{i,k} U_{j,k} + a_7 U_{k,i} U_{k,j}) T_{,j} \\
 & + \frac{k^4}{\varepsilon^3} \left( \frac{k \overline{\theta^2}}{\varepsilon \varepsilon_\theta} \right)^{1/2} (a_8 U_{i,k} U_{j,k}^2 + a_9 U_{i,k}^2 U_{j,k} + a_{10} U_{k,i} U_{k,j}^2 + a_{11} U_{k,i}^2 U_{k,j}) T_{,j} \\
 & + \frac{k^5}{\varepsilon^4} \left( \frac{k \overline{\theta^2}}{\varepsilon \varepsilon_\theta} \right)^{1/2} (a_{12} U_{i,k}^2 U_{j,k}^2 + a_{13} U_{k,i}^2 U_{k,j}^2 \\
 & \quad + a_{14} U_{i,k} U_{l,k} U_{l,j}^2 + a_{15} U_{j,k} U_{l,k} U_{l,i}^2) T_{,j} \\
 & + \frac{k^6}{\varepsilon^5} \left( \frac{k \overline{\theta^2}}{\varepsilon \varepsilon_\theta} \right)^{1/2} (a_{16} U_{i,k} U_{l,k}^2 U_{l,j}^2 + a_{17} U_{j,k} U_{l,k}^2 U_{l,i}^2) \\
 & + \frac{k^7}{\varepsilon^6} \left( \frac{k \overline{\theta^2}}{\varepsilon \varepsilon_\theta} \right)^{1/2} a_{18} U_{i,k} U_{l,k} U_{l,m}^2 U_{j,m}^2 T_{,j}
 \end{aligned} \tag{2.1.7}$$

The coefficients  $a_1 - a_{18}$  are, in general, functions of the time scale ratio  $\frac{k}{\varepsilon} / \frac{\theta^2}{\varepsilon_\theta}$  and the other invariants formed by the tensors in question, for example,  $T_{,k} T_{,k}$ ,  $T_{,i} U_{i,j} T_{,j}$ , etc.. Again, Eq.(2.1.7) implies that the effective eddy diffusivity

$$(\gamma_T)_i = \frac{-\overline{\theta u_i}}{T_{,i}}$$

is not isotropic. It is noticed that the first term on the right hand side of Eq.(2.1.7) is the standard eddy diffusion model, and the models derived from the two-scale DIA (Yoshizawa<sup>5</sup>) and the RNG method (Rubinstein and Barton<sup>6</sup>) are similar to the first two terms of Eq.(2.1.7). In practice, a form containing the first two terms on the right hand side of Eq.(2.1.7) may suffice. Further development of this model for turbulent heat transfer is described in Section 3.4.

The Researchers involved with the subject in this section are T.-H. Shih, J. Zhu, A. Shabbir, J.L. Lumley<sup>†</sup> and A. Johansson.<sup>‡</sup>

## 2.2 Mechanical and Scalar Dissipation Equation

### Mechanical dissipation $\varepsilon$

In turbulence modeling, we often need turbulent characteristic velocity and length scales. While the turbulent kinetic energy  $k$  is used to characterize the velocity scale, the mechanical dissipation rate  $\varepsilon$  and the scalar dissipation rate  $\varepsilon_\theta$  are used to characterize the length scales for mechanical and scalar fields, respectively. Comparing with the turbulent kinetic energy equation, the exact dissipation rate equation is

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very complicated. In this equation, all the terms which represent important turbulence physics (for example, turbulent diffusion, generation and destruction) are unknown and are of complex forms that are all related to small scales of turbulence. Therefore, in the literature, the exact dissipation equation is not considered as a useful equation to work with. Instead, one creates a model equation by assuming an analogy to the turbulent kinetic energy equation, i.e., one assumes that the model dissipation rate equation also has generation and destruction terms which are assumed to be proportional respectively to the production and dissipation terms in the turbulent kinetic energy equation over the period of large eddy turn-over time characterized by  $k/\varepsilon$ . The resulting model dissipation rate equation is written as

$$\begin{aligned} \varepsilon_{,t} + U_i \varepsilon_{,i} = \nu \varepsilon_{,ii} - (\overline{\varepsilon u_i})_{,i} \\ - C_{\varepsilon 1} \frac{\varepsilon}{k} \overline{u_i u_j} U_{i,j} - C_{\varepsilon 2} \frac{\varepsilon^2}{k} \end{aligned} \quad (2.2.1)$$

Recently, Lumley<sup>7</sup> proposed a dissipation rate equation based on the concept of spectral energy transfer caused by interactions between eddies of different sizes. This model equation mimics the physics of statistical energy transfer from large eddies to small eddies and is of a different form than equation (2.2.1).

In this study, we explore another rational way to obtain the model dissipation rate equation which contains certain important physics and hope it will work better than the existing one. The idea is that first, there is a relationship between the dissipation rate  $\varepsilon$  and the mean-square vorticity fluctuation  $\overline{\omega_i \omega_i}$  at high Reynolds numbers or in homogeneous turbulence:

$$\varepsilon = \nu \overline{\omega_i \omega_i}$$

and second, all the terms appearing in the  $\overline{\omega_i \omega_i}$  equation have more clear physical meanings than that in the  $\varepsilon$  equation so that the  $\overline{\omega_i \omega_i}$  equation is easier to model. Once the  $\overline{\omega_i \omega_i}$  equation is modeled, a model dissipation rate equation will be readily obtained.

The exact equation for  $\overline{\omega_i \omega_i}$  is

$$\begin{aligned} \left(\frac{\overline{\omega_i \omega_i}}{2}\right)_{,t} + U_j \left(\frac{\overline{\omega_i \omega_i}}{2}\right)_{,j} = \nu \left(\frac{\overline{\omega_i \omega_i}}{2}\right)_{,jj} - \frac{1}{2} (\overline{u_j \omega_i \omega_i})_{,j} + \overline{\omega_i u_{i,j}} \Omega_j \\ - \overline{u_j \omega_i} \Omega_{i,j} + \overline{\omega_i \omega_j} U_{i,j} + \overline{\omega_i \omega_j u_{i,j}} - \nu \overline{\omega_{i,j} \omega_{i,j}} \end{aligned} \quad (2.2.2)$$

where  $u_i$  and  $U_i$  are the fluctuating and mean velocities, and  $\omega_i$  and  $\Omega_i$  are the fluctuating and mean vorticity which are defined by

$$\omega_i = \epsilon_{ijk} u_{k,j} \quad \Omega_i = \epsilon_{ijk} U_{k,j} \quad (2.2.3)$$

Tennekes and Lumley<sup>8</sup> clearly described the physical meaning of each term in equation (2.2.2). Order of magnitude analysis shows that the first, third, fourth and fifth terms on the right hand side of Eq.(2.2.2) become small compared with all other

terms in the equation as the turbulent Reynolds number increases. The sixth and seventh terms are the production due to fluctuating vortex stretching and the dissipation due to the viscosity of the fluid. As the turbulent Reynolds number increases these last two terms become dominant and the balance between them determines the evolution of vorticity fluctuations. Neglecting terms  $\overline{\omega_i u_{i,j}} \Omega_j$ ,  $-\overline{u_j \omega_i} \Omega_{i,j}$ ,  $\overline{\omega_i \omega_j} U_{i,j}$  and  $\nu (\frac{\overline{\omega_i \omega_i}}{2})_{,jj}$ , the evolution of  $\overline{\omega_i \omega_i}$  at large Reynolds number will be described by the following equation,

$$\left(\frac{\overline{\omega_i \omega_i}}{2}\right)_{,t} + U_j \left(\frac{\overline{\omega_i \omega_i}}{2}\right)_{,j} = -\frac{1}{2} (\overline{u_j \omega_i \omega_i})_{,j} + \overline{\omega_i \omega_j u_{i,j}} - \nu \overline{\omega_{i,j} \omega_{i,j}} \quad (2.2.4)$$

To model  $\overline{\omega_i \omega_j u_{i,j}} - \nu \overline{\omega_{i,j} \omega_{i,j}}$ , let us first estimate  $\overline{\omega_i \omega_j u_{i,j}}$ . we define an anisotropic tensor  $b_{ij}^\omega$ :

$$b_{ij}^\omega = \frac{\omega_i \omega_j}{\omega_k^2} - \frac{1}{3} \delta_{ij} \quad (2.2.5)$$

then  $\overline{\omega_i \omega_j u_{i,j}}$  can be written as

$$\overline{\omega_i \omega_j u_{i,j}} = \overline{b_{ij}^\omega \omega_k^2 u_{i,j}} \quad (2.2.6)$$

We expect that the vortex stretching tends to align vortex lines with the strain rate so that the anisotropy  $b_{ij}^\omega$  would be proportional to the strain rate  $s_{ij}$ , i. e.,

$$b_{ij}^\omega \propto \frac{s_{ij}}{s}, \quad \text{where } s = (2s_{ij}s_{ij})^{1/2}, \quad s_{ij} = (u_{i,j} + u_{j,i})/2 \quad (2.2.7)$$

This leads to the following model:

$$\overline{\omega_i \omega_j u_{i,j}} \propto \overline{\omega_k^2 (2s_{ij}s_{ij})^{1/2}} \propto \overline{\omega_k^2} \sqrt{2s_{ij}s_{ij}} \quad (2.2.8)$$

where we have assumed that  $\omega_k^2$  and  $(2s_{ij}s_{ij})^{1/2}$  are well correlated.

Using the relation,  $\omega_i = \epsilon_{ijk} u_{k,j}$ , it is not difficult to show that at large turbulent Reynolds number,

$$\overline{\omega_i \omega_i} \approx 2\overline{s_{ij}s_{ij}} \quad (2.2.9)$$

and Eq.(2.2.8) can be also written as

$$\overline{\omega_i \omega_j u_{i,j}} \propto \overline{\omega_k^2} \sqrt{\overline{\omega_i^2}} = \frac{\overline{\omega_k^2} \overline{\omega_i^2}}{\sqrt{\overline{\omega_i^2}}} \quad (2.2.10)$$

Equation (2.2.10) indicates that this term is of the order  $(u^3/l^3)R_t^{3/2}$  as it should be. On the other hand, from eq.(2.2.4) the term  $\overline{\omega_i \omega_j u_{i,j}} - \nu \overline{\omega_{i,j} \omega_{i,j}}$  must be of the order  $(u^3/l^3)R_t$  which is the order of magnitude of all the other terms in Eq.(2.2.4), therefore the term  $-\nu \overline{\omega_{i,j} \omega_{i,j}}$  must cancel the term (2.2.10) or (2.2.8) such that the difference of these two terms is smaller than the term (2.2.10) or (2.2.8) by an order

of  $R_t^{1/2}$ . This suggests that the combination  $\overline{\omega_i \omega_j u_{i,j}} - \nu \overline{\omega_{i,j} \omega_{i,j}}$  can be modeled by the following two terms:

$$\frac{\overline{\omega_k^2 \omega_i^2}}{\frac{k}{\nu} + \sqrt{\overline{\omega_i^2}}}, \quad \overline{\omega_k^2} S \quad (2.2.11)$$

because the ratio of  $k/\nu$  to  $\sqrt{\overline{\omega_i^2}}$  and the ratio of  $s$  to  $S$  are of order  $R_t^{1/2}$ , where  $k \approx u^2$  is the turbulent kinetic energy and  $S$  is the mean strain rate  $(2S_{ij}S_{ij})^{1/2}$ . Equation (2.2.11) does give the right order of magnitude for  $\overline{\omega_i \omega_j u_{i,j}} - \nu \overline{\omega_{i,j} \omega_{i,j}}$ . Therefore, the dynamical equation for fluctuating vorticity (2.2.4) at large Reynolds number can be modeled as

$$\left(\frac{\overline{\omega_i \omega_i}}{2}\right)_{,t} + U_j \left(\frac{\overline{\omega_i \omega_i}}{2}\right)_{,j} = -\frac{1}{2}(\overline{u_j \omega_i \omega_i})_{,j} + C_{\omega 1} \overline{\omega_k^2} S - C_{\omega 2} \frac{\overline{\omega_k^2 \omega_i^2}}{\frac{k}{\nu} + \sqrt{\overline{\omega_i^2}}} \quad (2.2.12)$$

Using  $\varepsilon = \nu \overline{\omega_i \omega_i}$ , we readily obtain the following model dissipation rate equation,

$$\varepsilon_{,t} + U_j \varepsilon_{,j} = -(\overline{u_j \varepsilon})_{,j} + C_{\omega 1} S \varepsilon - C_{\omega 2} \frac{\varepsilon^2}{k + \sqrt{\nu \varepsilon}} \quad (2.2.13)$$

where  $C_{\omega 1}$  and  $C_{\omega 2}$  are the model coefficients which are expected to be constant at large Reynolds number.

It should be noticed that Eq.(2.2.13) is different from the standard  $\varepsilon$  equation (2.2.1) by both the generation and destruction terms. First, the Reynolds stresses do not appear in the generation term and the new form of the generation term is similar to that proposed by Lumley<sup>7</sup> which is based on the concept of spectral energy transfer. Second, the destruction term is well behaved so that equation (2.2.13) will not have a singularity anywhere in the flow field. We expect that equation (2.2.13) will be numerically much more robust than equation (2.2.1).

Equation (2.2.13) can be applied to any level of turbulence modeling including second order closure models; however the turbulent transport term  $(\overline{\varepsilon u_i})_{,i}$  needs to be modeled differently at different levels of turbulence modeling. In an eddy viscosity model, the term  $(\overline{\varepsilon u_i})_{,i}$  will be modeled as

$$(\overline{\varepsilon u_i})_{,i} = -\left(\frac{\nu_T}{\sigma_\varepsilon} \varepsilon_{,i}\right)_{,i} \quad (2.2.14)$$

The coefficients  $C_{\omega 1}$ ,  $C_{\omega 2}$ ,  $\sigma_\varepsilon$  and the eddy viscosity  $\nu_T$  must be calibrated using experimental data (Shih *et al.*<sup>9</sup>)

### Scalar dissipation $\varepsilon_\theta$

A similar analysis leads to the following model scalar dissipation rate equation:

$$\varepsilon_{\theta,t} + U_j \varepsilon_{\theta,j} = -(\overline{u_j \varepsilon'_\theta})_{,j} + C_{\theta 1} S \varepsilon_\theta + C_{\theta 2} Pr^{-1/2} \Phi \sqrt{\varepsilon \varepsilon_\theta} - C_{\theta 3} \frac{\varepsilon \varepsilon_\theta}{k + \sqrt{\nu \varepsilon}} \quad (2.2.15)$$

where  $\Phi = \sqrt{\overline{T_i T_i}}$  and  $T$  is the mean scalar quantity, such as, the mean temperature. Further development of heat transfer model is described in Section 3.4.

The Researchers involved with the subject in this section are T.-H. Shih, W. Liou, A. Shabbir and Z. Yang.



### 2.3 Eddy Viscosity Transport Equation

In eddy viscosity models, one accepts the following simple constitutive relation

$$\overline{u_i u_j} = -2\nu_T S_{ij} + \frac{2}{3} k \delta_{ij} \quad (2.3.1)$$

and assumes that the eddy viscosity is characterized by some kind of velocity and length scales  $u'$  and  $\ell$ :

$$\nu_T \propto u' \ell \quad (2.3.2)$$

In two-equation  $k$ - $\varepsilon$  eddy viscosity models, for example, one specifies that

$$u' \propto k^{\frac{1}{2}}, \quad \ell \propto \frac{k^{\frac{3}{2}}}{\varepsilon} \quad (2.3.3)$$

and, hence, the eddy viscosity is assumed as

$$\nu_T = C_\mu \frac{k^2}{\varepsilon} \quad (2.3.4)$$

The eddy viscosity assumption (2.3.4) is commonly adopted in two-equation models. Eqs.(2.3.1) and (2.3.4) together with appropriate  $k$  and  $\varepsilon$  equations have been widely used in engineering calculations. However, for cases where the mean flow changes quickly or has a strong mean stream-line curvature or rotation, etc., this kind of model does not work very well, since the assumption (2.3.4) is too simple to account for the effect of the above mean flow structure on eddy viscosity.

The main purpose of this study is to drop the assumption (2.3.4) and to derive an exact equation for  $\nu_T$  based on Eq.(2.3.1) and other exact turbulence equations (i.e. first principles). In this way, we hope that some important turbulent physics can be brought into the eddy viscosity and that a physically sound turbulence eddy viscosity can be calculated.

Using Eq.(2.3.1), we may write for incompressible flows

$$\overline{u_i u_j} \overline{u_i u_j} = 2\nu_T^2 S^2 + \frac{4}{3} k^2, \quad \text{where } S^2 = 2S_{ij} S_{ij} \quad (2.3.5)$$

Differentiating both sides, we obtain

$$\frac{D}{Dt} \nu_T = -\frac{S_{ij}}{S^2} \frac{D}{Dt} \overline{u_i u_j} - \frac{\nu_T}{2S^2} \frac{D}{Dt} S^2 \quad (2.3.6)$$

The equation for  $\overline{u_i u_j}$  can be written as

$$\frac{D}{Dt} \overline{u_i u_j} = D_{ij} + P_{ij} + \Pi_{ij} - \varepsilon_{ij} + C_{0ij} \quad (2.3.7)$$

where

$$\begin{aligned}
D_{ij} &= [\nu \overline{u_i u_j} - \overline{u_i u_j u_k}]_{,k} \\
P_{ij} &= -\overline{u_i u_k} U_{j,k} - \overline{u_j u_k} U_{i,k} \\
\Pi_{ij} &= -\frac{1}{\rho} \overline{p_{,i} u_j + p_{,j} u_i} \\
\varepsilon_{ij} &= 2\nu \overline{u_{i,k} u_{j,k}} \\
Co_{ij} &= -2\varepsilon_{imk} \Omega_m \overline{u_k u_j} - 2\varepsilon_{jmk} \Omega_m \overline{u_k u_i}
\end{aligned}$$

Inserting Eq.(2.3.7) into Eq.(2.3.6), we obtain an exact transport equation for eddy viscosity

$$\frac{D}{Dt} \nu_T = -\frac{S_{ij}}{S^2} (D_{ij} + P_{ij} + \Pi_{ij} - \varepsilon_{ij} + Co_{ij}) - \frac{\nu_T}{2S^2} \frac{D}{Dt} S^2 \quad (2.3.8)$$

In this equation, all the important turbulence physics in the Reynolds stress equation, such as Reynolds stress diffusion term  $D_{ij}$ , production term  $P_{ij}$ , pressure-velocity gradient correlation term  $\Pi_{ij}$  and dissipation tensor  $\varepsilon_{ij}$ , are involved. Comparing with the standard eddy viscosity assumption (2.3.4), this exact eddy viscosity equation (2.3.8) contains very rich turbulence physics. This equation also implies that a second order closure model will naturally lead to a corresponding eddy viscosity model.

Now, as an example, we use Launder Reece and Rodi's<sup>10</sup> model and a gradient transport model for the triple velocity correlation ( $-\overline{u_i u_j u_k} = \frac{\nu_T}{\sigma} \overline{u_i u_j}_{,k}$ ) to derive a model equation for  $\nu_T$ . The resulting equation is

$$\begin{aligned}
\frac{D}{Dt} \nu_T &= [(\nu + \frac{\nu_T}{\sigma}) \nu_{T,k}]_{,k} + (\nu + \frac{3}{2} \frac{\nu_T}{\sigma}) \frac{\nu_{T,k} S_{,k}^2}{S^2} + (\nu + \frac{\nu_T}{\sigma}) \frac{2\nu_T S_{ij} S_{ij,kk}}{S^2} \\
&+ \frac{4}{15} k - C_1 \frac{\varepsilon}{k} \nu_T + 2(C_2 - 2) \nu_T \frac{S_{ik} S_{kj} S_{ji}}{S^2} - \frac{\nu_T}{2S^2} \frac{D}{Dt} S^2
\end{aligned} \quad (2.3.9)$$

Note that the Coriolis terms do not explicitly appear in this equation; however the rotation effect on  $\nu_T$  could be carried over through the mean flow field. In addition, we also note that there are no extra model coefficients introduced in Eq.(2.3.9). All model coefficients ( $\sigma$ ,  $C_1$  and  $C_2$ ) are brought in from the second order closure model. The values of these model coefficients may need adjustment in model applications. Note that Eq.(2.3.9) is not a self-consistent equation since the turbulent kinetic energy  $k$  and its dissipation rate  $\varepsilon$  are also involved. Eq.(2.3.9) together with  $k$ - $\varepsilon$  transport equations will provide a new three-equation model which may better represent the effect of mean flow structure as well as mean flow history on the eddy viscosity.

The Researchers involved with the subject in this section are T.-H. Shih, Z Yang, and W. Liou.

### 3. One-Point Closure Schemes

In this section, we describe the developments on each of the moment closure scheme and the pdf method which are of concern at CMOTT. The first two sections

3.1 and 3.2 describe the one- and two-equation isotropic eddy viscosity models. Sections 3.3 and 3.4 describe the new developments on Reynolds stress and scalar flux algebraic equation models. Section 3.5 assesses Reynolds stress transport equation models. Section 3.6 describes a multiple-scale model for non-equilibrium turbulence. Section 3.7 is about transition models. Finally, in Section 3.8 the pdf method for turbulent chemical reaction is described.

### 3.1 One-equation eddy viscosity model

Recently developed one-equation eddy viscosity models are either based on the assumption (Baldwin and Barth<sup>11</sup>):

$$\nu_T = C_\mu \frac{k^2}{\varepsilon} \quad (3.1.1)$$

or created according to computational experience (Spalart and Allmaras<sup>12</sup>). Both of them are successful in some flow calculations. This scheme is quite attractive in CFD because one only needs to solve one scalar  $\nu_T$  equation without bothering about other turbulence quantities. However, comparing with  $k$ - $\varepsilon$  two equation models, the above mentioned one-equation  $\nu_T$  models do not contain any more turbulent physics. In fact, Baldwin and Barth's model is, basically, a change of dependent variable based on Eq.(3.1.1) plus some extra approximations. Therefore, in principle, we should not expect any superior performance over two-equation models. However, if we do not use the assumption (3.1.1), there is the possibility to improve and extend the capability of one-equation eddy viscosity models.

The objective of this study at CMOTT is to derive a physically sound eddy viscosity equation which contains rich turbulent physics and accounts for various effects from mean flow structures.

Note that in Section 2.3 we have already derived an exact equation for the eddy viscosity (2.3.8) and also a model equation (2.3.9) which is based on the Reynolds stress transport equation model of Launder, Reece and Rodi (LRR). All turbulent physics contained in the Reynolds stress equation can be brought into the eddy viscosity equation. Therefore, in principle, the transport equation (2.3.9) should be better than existing one-equation models based on Eq.(3.1.1). However, Eq.(2.3.9) is not self-consistent because  $k$  and  $\varepsilon$  are also involved. To make Eq.(2.3.9) self-consistent, we must model  $k$  and  $k/\varepsilon$  in terms of  $\nu_T$  and  $S$ . In most shear flows, the energy-containing eddy turn-over time  $k/\varepsilon$  is of the same order as the mean flow time scale  $S^{-1}$ , so that  $\varepsilon/k \propto S$  is a reasonable model. In addition, a crude dimensional analysis gives  $k \propto \nu_T S$  and this is, of course, reasonable only for shear flows. After the above considerations, the resulting self-consistent one-equation model is:

$$\begin{aligned} \frac{D}{Dt} \nu_T &= \left[ \left( \nu + \frac{\nu_T}{\sigma} \right) (\nu_T)_{,k} \right]_{,k} - \frac{C_{\nu 0}}{\sigma} (\nu_T)_{,k} (\nu_T)_{,k} + C_{\nu 1} S \nu_T \\ &+ 2(C_{\nu 2} - 2) \nu_T \frac{S_{ik} S_{kj} S_{ji}}{S^2} - \frac{\nu_T}{2S^2} \frac{D}{Dt} S^2 \end{aligned} \quad (3.1.2)$$

where the diffusion terms from the Reynolds stress equation (2.3.7) have been manipulated and approximated. Eq.(3.1.2) clearly exhibits the various effects of the mean flow on the eddy viscosity.

The model coefficients  $C_{\nu 1}$ ,  $C_{\nu 2}$  and  $\sigma$  can be determined by using the experimental data of homogeneous shear flows, free shear flows and boundary layer flows as well as the relations in the inertial sublayer. Extensive tests of this model in various flows are carrying out at the CMOTT.

The Researchers involved with the subject in this section are T.-H. Shih, W. Liou, Z. Yang and J. Zhu.

### 3.2 Galilean and tensorial invariant realizable $k-\varepsilon$ model

The two-equation  $k-\varepsilon$  eddy viscosity model is one of the most widely used turbulence models in engineering calculations. The  $k-\varepsilon$  model has versions for high Reynolds numbers and for low Reynolds numbers. For wall bounded turbulent flows, the high Reynolds number  $k-\varepsilon$  model (for example, Launder and Spalding<sup>13</sup>) must be applied together with a wall function as its boundary condition, while the low Reynolds number  $k-\varepsilon$  model (for example, Jones and Launder<sup>14</sup>) can be integrated to the wall. The high Reynolds number  $k-\varepsilon$  model of Launder and Spalding is considered as a standard  $k-\varepsilon$  model. We notice that even though the model dissipation rate equation is created by assuming an analogy with the turbulent kinetic energy, there was not much modification until Lumley<sup>7</sup> and Shih *et al.*<sup>9</sup> For near wall turbulence, in addition to Jones and Launder's model, there are many other versions of low Reynolds number  $k-\varepsilon$  models (such as Chien<sup>15</sup>, Shih and Lumley<sup>16</sup>, Yang and Shih<sup>17</sup>) which have made better performance over Jones and Launder's model.

There are, probably, four or five issues worth mentioning about existing low Reynolds number  $k-\varepsilon$  models: the model constants are not consistent with those in the high Reynolds number  $k-\varepsilon$  model; the wall correction terms and damping functions are related to the wall distance so that models are not tensorial invariant; a nonrealistic dissipation rate near the wall is introduced; they are not always realizable since normal stress could become negative; and finally, they do not work very well for boundary layer flows with various pressure gradients.

The objective of this study at CMOTT is to overcome the above mentioned problems. First, we propose a vorticity dynamics based dissipation rate equation as a part of high Reynolds number  $k-\varepsilon$  base model.<sup>9</sup> Second, based on the invariant theory, inhomogeneous terms for the dissipation rate equation are proposed which enable the model to better respond to the change of pressure gradients (Yang and Shih<sup>18</sup>). Third, the wall distance parameter is removed from the damping function so that the model is tensorially invariant (Yang and Shih<sup>19</sup>). The model constants are consistent with those in the high Reynolds number  $k-\varepsilon$  model. Finally, the non-negativity of normal Reynolds stresses, the realizability condition, is imposed.

The Researchers involved with the subject in this section are Z. Yang, T.-H. Shih and C. Steffen.

### 3.3 Reynolds stress algebraic equation model

All eddy viscosity models including one- and two-equation models are isotropic. For the flows where anisotropy is important, for example, the secondary flows driven by turbulent normal stresses in a square duct or curved duct, eddy viscosity models do not produce correct flow structures. To overcome this intrinsic deficiency of isotropic eddy viscosity models, one proposes a Reynolds stress algebraic equation model which will provide an effective anisotropic eddy viscosity. The first such a model was proposed by Rodi<sup>20</sup> and it achieved some success in the prediction of anisotropic related flow structure. However, Rodi's formulation is a set of algebraic non-linear system equations for Reynolds stresses and it often creates numerical difficulty in obtaining a converged solution. Recently, Taulbee<sup>21</sup> obtained an explicit algebraic expression for the Reynolds stress using Pope's<sup>22</sup> tensor expansion formulation and solved this numerical difficulty. However, in general, Rodi's formulation assumes that the ratio  $\overline{u_i u_j} / k$  is constant and, of course, this is not really true for most turbulent flows of interest. Therefore, sometimes, this Reynolds stress algebraic equation model produces even worse results than the isotropic eddy viscosity models for cases where eddy viscosity models are appropriate.

Alternative ways for obtaining effective anisotropic eddy viscosity models have been tried by a few researchers, for example, the DIA method by Yoshizawa<sup>3</sup>, the RNG method by Rubinstein and Barton<sup>4</sup> and invariant theory by Shih and Lumley.<sup>1</sup> It is interesting to point out that the RNG and DIA methods result in the same formulation and that this formulation is the first five terms of a general constitutive relation Eq.(2.1.3) except that the model coefficients are different.

One of our goals at CMOTT is to search for an effective anisotropic eddy viscosity model for complex turbulent flows where the nonequilibrium of turbulence is not very severe so that the constitutive relation (2.1.3) is more or less valid. We have explored the potential capability of Eq.(2.1.3) and found that a truncation of Eq.(2.1.3) up to the quadratic terms of the mean velocity gradients is sufficient for various flows of interest. The model coefficients are determined such that realizability for the normal stresses is ensured. The detailed analysis is described by Shih *et al.*<sup>23</sup>

The quadratic version of Eq.(2.1.3) together with the standard  $k$ - $\epsilon$  transport equations, successfully predicts many complex flows as well as simple flows which include backward-facing step flows; confined coflowing jets; confined swirling coaxial jets; flows in 180° curved duct; flows in a diffuser and a nozzle; boundary layer flows with pressure gradient and turbulent free shear flows. See references<sup>23-25</sup> for detailed results.

The Researchers involved with the subject in this section are J. Zhu and T.-H. Shih.

### 3.4 Scalar flux algebraic equation model

In parallel with Reynolds stress algebraic equation model, we have also tried to develop an effective anisotropic scalar eddy diffusivity model for scalar (heat) fluxes based on the new constitutive relation (2.1.7) and the new thermal dissipation rate

equation (2.2.16). We have determined that it seems sufficient to truncate Eq.(2.1.7) up to linear terms of the mean velocity gradient, i.e.,

$$\overline{\theta u_i} = a_1 k \left( \frac{k \overline{\theta^2}}{\varepsilon \varepsilon_\theta} \right)^{1/2} T_{,i} + \frac{k^2}{\varepsilon} \left( \frac{k \overline{\theta^2}}{\varepsilon \varepsilon_\theta} \right)^{1/2} (a_2 U_{i,j} + a_3 U_{j,i}) T_{,j} \quad (3.4.1)$$

This equation indicates that the heat flux and the mean temperature gradient are not necessarily in alignment due to the distortion of the flow field. This means that the effective scalar eddy diffusivity is anisotropic.

Eq.(3.4.1) together with the  $\overline{\theta^2}$  and  $\varepsilon_\theta$  equations will be a closed set of model equations for turbulent heat fluxes. The model coefficients are calibrated from homogeneous flows. Detailed analysis and a few model tests are described in this research briefs by A. Shabbir.

The Researchers involved with the subject in this section are A. Shabbir and T.-H. Shih.

### 3.5 Reynolds stress transport equation model

The Reynolds stress transport equation model is considered as a next generation of advanced turbulence modeling for engineering applications. In principle, the second moment equations describe various effects of the mean flow and external agencies on the evolution of turbulence, hence, are the most attractive way (also the simplest correct way) to study turbulent flows.

Various closure models for second moment equations have been developed. The success of these closures are marginal and vary with each flow. To identify the sources of their deficiencies, one often uses simple flows where the specific model term in the second moment equations can be isolated, hence, the corresponding model can be checked against experimental data or direct numerical simulation (DNS). For example, using pre-distorted anisotropic homogeneous relaxation flows, we may check the return-to-isotropy models with experimental data or DNS. However, for other flows, several model terms, such as, triple velocity correlations, rapid and slow pressure-strain correlations, etc., simultaneously exist and can not be isolated in the experiments. In these cases (for example, in a homogeneous shear flow or a channel flow) only DNS can provide all the information for simultaneously checking various models.

We have examined various existing closure models using experimental data as well as DNS data (Shih *et al.*<sup>26</sup> and Shih and Lumley<sup>27</sup>). Conclusive statements are difficult to draw at this time. However, the following remarks can be made about various closures for the second moment equations, i.e., the triple velocity correlation  $T_{ijk}$ , the rapid and slow pressure related correlations  $\Pi_{ij}^{rp}$ ,  $\Pi_{ij}^{sl}$ , and the dissipation rate tensor  $\varepsilon_{ij}$ :

a)  $T_{ijk}$ . All the existing models, such as Daly and Harlow<sup>28</sup>, LRR<sup>10</sup>, Lumley<sup>29</sup>, etc., are not very satisfactory for highly inhomogeneous flows, such as flow near the wall. However, for flows where the inhomogeneity is not very high, the above closure models become close to each other and also closer to the DNS data. In addition, the triple velocity correlations in these situations are usually small comparing with

other terms in the equation, so that modeling of this term is not as critical as other terms for the results of turbulent flow calculations, except for the flow near the wall. b)  $\Pi_{ij}^{rp}$ . It is very clear from all the available DNS data that nonlinear models, such as, Shih and Lumley<sup>30</sup> are much better than linear models, such as SSG<sup>31</sup>. It seems also that the following constitutive relation

$$\Pi_{ij}^{rp} = F(\overline{u_i u_j}, U_{i,j})$$

is quite appropriate, i.e., its dependence on turbulent Reynolds number and other parameters is quite weak and can be neglected. However, one deficiency of this form observed by Reynolds<sup>32</sup> is that it can not take the rotation effect into account.

c)  $\Pi_{ij}^{sl}$ . This term is usually modeled together with the dissipation tensor  $\varepsilon_{ij}$  and the combination of the two is called the return-to-isotropy term. All existing models are unsatisfactory at the present time. They are far from "universal", i.e., their performance varies from flow to flow. It is noticed that some strange behavior of return-to-isotropy (for example, for some pre-distorted flow relaxation, turbulence evolves toward anisotropy before it returns to isotropy) occurs and cannot be possibly modeled with the following constitutive relation:

$$\Pi_{ij}^{sl} = F(\overline{u_i u_j}, k, \varepsilon)$$

In addition, the behavior of return-to-isotropy was found to depend not only on the Reynolds stresses at the present time but also on their history according to DNS data (Lee<sup>33</sup>). It may be also necessary to include triple velocity correlations into the above constitutive relations from the definition of  $\Pi_{ij}^{sl}$ . The term  $\Pi_{ij}^{sl}$  seems highly dependent on the turbulent Reynolds number and slowly approaches to its asymptote as Reynolds number goes to infinity, so that, in general, one should not exclude its dependence on turbulent Reynolds number even for moderate Reynolds numbers. In addition,  $\Pi_{ij}^{sl}$  is also noticeably affected by the mean strain rate according to the DNS data<sup>34</sup>, so that, in general, the mean strain rate should be also considered in the constitutive relation. In short, much more research is needed for developing a better model of  $\Pi_{ij}^{sl}$ .

The Researchers involved with the subject in this section are T.-H. Shih and A. Shabbir.

### 3.6 Non-equilibrium multiple-scale model

To consider the effect of the nonequilibrium of energy spectrum on turbulent quantities, such as  $k$ ,  $\varepsilon$  and  $\overline{u_i u_j}$ , etc., Hanjelic et al.<sup>35</sup> are the first to propose a partition in the turbulent energy spectrum. Because of the nonequilibrium, the rate that energy enters the low wave number region,  $\varepsilon_p$ , does not equal to the energy transfer rate from low wave numbers to high wave numbers,  $\varepsilon_t$ . Therefore it is reasonable to describe the evolution of the energy contained in low wave number region,  $k_p$ , and high wave number region  $k_t$ , separately. As a result, the time scale or the length scale defined by different energy transfer rates will be different and this multiple-scale concept reflects the nonequilibrium effect of turbulence.

We think that this concept would be more appropriate for compressible flows because the compressibility often creates nonequilibrium interactions between large and small eddies. We first modify Hanjelic et al.'s model, test it in various free shear flows and boundary layer flows and then extend it to compressible flows by consideration of the effects of compressibility on the equations for  $k_p$  and  $\varepsilon_p$ . The proposed model is tested in both compressible free shear flows and boundary layer flows. For detailed analysis and flow calculations see the report by Duncan *et al.*<sup>36</sup> and Liou and Shih<sup>37</sup>.

The Researchers involved with the subject in this section are W. Liou, T.-H. Shih and B. Duncan.

### 3.7 Bypass transition model

The onset of turbulence transition in the propulsion system is often highly influenced by the free stream turbulence. This transition process does not go through the linear instability but is mainly controlled by nonlinear processes. Therefore, it is sometimes called "bypass" transition. Because of this highly nonlinear process of transition, turbulence models may be used to predict it. In fact, many two-equation models, for example,  $k$ - $\varepsilon$  eddy viscosity models of Launder and Sharma<sup>38</sup>, Chien<sup>15</sup>, etc., do mimic bypass transition on a flat plate when the free stream has a certain amount of turbulent intensity. However, to obtain an accurate prediction of bypass transition, the study of the bypass transition process and physics is needed. The conventional turbulence models must be modified to take into account the intermittent phenomena of transitional flows.

We have proposed transition models based on a two-equation turbulence model using an intermittency factor to modify either the eddy viscosity or modeled  $k$ - $\varepsilon$  equations. Successful results for a flat plate boundary layer under various free-stream turbulence intensities are obtained. For details see the report by Yang and Shih<sup>39</sup>.

The Researchers involved with the subject in this section are Z. Yang and T.-H. Shih.

### 3.8 Joint scalar PDF model

One of the critical problems in turbulent combustion is how to treat the interaction between the chemical reaction on the turbulence. The estimation of the production rate of compositions based on the mean flow temperature would be in a very large error for flows with finite rate chemical reactions. The reason is that the production rate of compositions depends not only on the mean values of temperature  $T$  and compositions  $C_i$ , but also very much depends on the detailed fluctuations of temperature  $\theta$  and compositions  $c_i$ . The moment closure scheme of modeling the production rate of compositions in terms of the mean flow temperature, the mean compositions and various correlations consisting of the fluctuating temperature and composition, such as  $\overline{\theta^2}$ ,  $\overline{\theta c_i}$ ,  $\overline{c_i c_j}$ , ..., has not been successful. However, the PDF method allows us to treat chemical reaction exactly without modeling (Pope<sup>40</sup>). Therefore, for the study of turbulent combustion problems, we use the joint scalar PDF transport equation for the scalar field and the moment closure schemes for



the velocity field and develop a hybrid solver consisting of a Monte Carlo scheme and a conventional CFD method. For detailed description of this procedure and its applications see Hsu<sup>41</sup> and Hsu *et al.*<sup>42</sup>

The Researchers involved with the subject in this section are A. Hsu, A.T. Norris and J.Y. Chen<sup>†</sup>

#### 4. RNG and DIA

In developing one point turbulence models, conventional modeling methods can be supplemented by “non-conventional” methods such as renormalization group theory (RNG) and the direct interaction approximation (DIA). These are two point theories formulated in wavevector or fourier space; one point models are derived by integration over wavevectors. This approach provides theoretical support for conventionally derived models and sometimes suggests theoretically derived forms for the empirical elements, whether constants or functions, which appear in these models.

We have applied RNG methods to both the eddy viscosity and Reynolds stress transport equation models. In addition to the  $k - \epsilon$  model proposed by Yakhot and Orszag<sup>43</sup>, it is possible to obtain constitutive relations for Reynolds stress and heat fluxes (Rubinstein and Barton<sup>4,6</sup> which are special cases of the general results Eqs.(2.1.3) and (2.1.7). By applying the perturbation theories of Yakhot and Orszag<sup>43</sup> to the relevant correlations, expansions in powers of the mean velocity gradient are obtained for the stresses and heat fluxes; quadratic truncation of the series leads to a stress model Eq.(2.1.3) with constant  $a_4, a_6, a_7$  and a heat flux model Eq.(2.1.7) with constant  $a_2, a_3$  in which the constants are in good agreement with empirically selected values. The forms derived are also consistent with the DIA analysis of Yoshizawa<sup>3,5</sup>.

The RNG method also provides a formulation for closing the Reynolds stress transport equation (Rubinstein and Barton<sup>44</sup>). Perturbative evaluation of the correlations  $\Pi_{ij}^{rp}$  and  $\Pi_{ij}^{sl}$  leads to series expansions in powers of the mean velocity gradient. These series can be consolidated, or “resumed” using the known perturbation series for the Reynolds stresses by methods analogous to Pade approximation. Systematic lowest order summation leads to a Reynolds stress transport equation with a form identical to the LRR model equation and with constants in reasonable agreement with empirically chosen values. Higher order resummation leading to nonlinear models of the type described in Sec. 3.5 remains an open possibility. The possibility of such resummation in the context of DIA has been discussed by Yoshizawa<sup>45,46</sup>.

Recent work has focussed on nonequilibrium time dependent relations between the Reynolds stress and the mean flow derived from a simplification of the DIA theory of shear turbulence. In this theory, shear turbulence is modeled by a non-Markovian eddy damping acting against the mean shear. The RNG and DIA Reynolds stress transport models and the LRR model all assume Markovian damping; as in the

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molecular theory of transport coefficients, Markovian damping describes long time behavior and is incorrect at short times. The most important consequence of non-Markovian damping is a *strong suppression of eddy damping at short times*. This leads to closer agreement between the present theory and rapid distortion theory at short time. This is important in modeling oscillating shear flows: recent work of Mankbadi<sup>47</sup> shows that RDT based models best predict such flows. In transient homogeneous shear flow at high strain rates, the LRR model predicts rapid onset of eddy damping leading to excessive growth of turbulence kinetic energy at short times. The suppression of eddy damping at short times in the present model should lead to improved predictions for this flow as well.

Another consequence of this theory is a stress model Eq.(2.1.3) in which the coefficients  $a_2, \dots$  are functions of the mean strain rate. This theory can be described as RDT with a modified total strain determined by the response function of the DIA theory of isotropic turbulence. The introduction of a phenomenological modified total strain has often been advocated in the RDT literature to improve the agreement between RDT and shear flow data; here the modified total strain is deduced as a consequence of the theory. In the special but important case of simple shear flow in which  $\partial U_i / \partial x_j = S \delta_{i1} \delta_{j2}$ , the result can be formulated in terms of Eq.(2.1.3) in which, for example,  $a_2 = a_2(Sk/\varepsilon)$  and the function  $a_3$  is found exactly from RDT. There are analogous results for the coefficients  $a_4, a_6, a_7$ ; in simple shear flow, the remaining terms in Eq.(2.1.3) identically vanish. Extension of this theory to other mean shear tensors depends on the tabulation of the corresponding RDT solution.

The researchers involved with the subject in this section are R. Rubinstein and A. Yoshizawa.<sup>†</sup>

## 5. Numerical Simulation

To obtain a better understanding of the effect of compressibility and rotation on turbulence, numerical simulations of compressible homogeneous shear flows and rotational flows are carried out. The effects of compressibility and rotation on the energy spectrum and energy cascade between turbulent eddies has been analyzed (Hsu and Shih<sup>48</sup>). These simulations support the idea of the multiple-scale model for nonequilibrium compressible turbulent flows (W. Liou and Shih<sup>37</sup>).

Another numerical simulation is the transition subjected to the free stream large disturbances. The objective of this simulation is to obtain some insight into the transition physics and to provide data base for bypass transition modeling. Based on the assumption that the transition process is mainly controlled by large scale motions, we use a high accuracy finite difference Navier-Stokes solver with course grids to simulate the large scale motions of transition. A preliminary calculation of bypass transition was carried out. Various statistics of the calculated flow field are under examination.

The Researchers involved with the subject in this section are A. Hsu, C. Liou<sup>‡</sup>, Z. Yang, A. Shabbir, T.-H. Shih.

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