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FINAL REPORT
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NASA NAG 5-818

Contributions from Geomagnetic Inverse Theory to the Study of Hydromagnetic Conditions Near the Core-Mantle Boundary

This is the final report for NASA grant NAG 5-818, "Contributions from Geomagnetic Inverse Theory to the Study of Hydromagnetic Conditions near the Core-Mantle Boundary (CMB)." The grant, originally for three years, was extended for reasons described in the sixth semiannual report.

The proposal was first submitted on 1/10/86, and was expanded as described in a letter of 7/7/87 from the PI to Dr. E. A. Flinn, Chief, Geodynamics Branch. The original proposal was to study five questions concerning what the surface and satellite magnetic data imply about hydromagnetic and electromagnetic conditions near the CMB. Those questions are

(1) What do the surface and satellite data imply about the geomagnetic field **B** near the surface of the earth (Benton et al. 1982; Langel, et al., 1982; Backus, 1986)?

(2) How does one extrapolate **B** down through the conducting mantle to the CMB (Ducruix et al., 1980; Benton and Whaler, 1983; Backus, 1983)?

(3) If **B** on the CMB is visible, how accurately does it satisfy the frozen-flux approximation (Roberts and Scott, 1965; Booker, 1969; Voorhies and Benton, 1982; Gubbins, 1985)?

(4) If frozen flux is a good approximation on the CMB, what can be inferred about the fluid velocity **v** in the upper core (Backus, 1968; Booker, 1969; Whaler, 1980; Voorhies and Backus, 1985; Backus and LeMouël, 1986)?

(5) If **v** at the CMB is visible, does it suggest any dynamical properties of the core, such as vertical advection (Whaler, 1982), Alfvén-inertial waves (Hide, 1966), kink instabilities (Hide, 1985), or mantle effects (Hide and Malin, 1970; Bloxham, 1985).

The research plan was revised when it was discovered (Backus, 1987; 1988a,b,c) that the Bayesian (or stochastic inversion) methods currently used to construct models of the magnetic field at the core-mantle boundary (CMB) suffer serious defects peculiar to inference in high-dimensional model spaces. These defects do not involve the classical but controversial objections to Bayesian methods, and the PI has found no way to overcome them with any modification of a Bayesian approach. Therefore it seemed justifiable to study other, non-Bayesian, inference techniques in modelling the magnetic field at the CMB. In fact, it turned out to be possible to find methods of inference which could give **B** at the bottom of the mantle with statistically reliable error estimates (Backus, 1989).

The discussion of dynamics at the CMB undertaken during the term of the grant began with joint work (Backus and LeMouël, 1986) which established that, if the fluid in the upper core is nearly geostrophic (a hypothesis proposed and defended by LeMouël, et al, 1985), then the radial component of the magnetic field and its time derivative at the top of the free stream in the core suffice to determine the fluid velocity there except in a few "ambiguous patches".

There remained the problem of how to extrapolate the magnetic field inferred at the bottom of the mantle to the magnetic field at the top of the core free stream. A theoretical study of the boundary layers at the CMB was carried out, written up and delivered as an invited paper at a symposium honoring John Miles' 70th birthday (Backus, 1991a). This study has led to a very simple mathematical argument for expecting the tangential field, $\mathbf{B}_S$,

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to have a jump discontinuity across the boundary layers at the CMB when the magnetic diffusivity is considerably larger than the kinematic viscosity of the core. Of course, the boundary layers at the CMB are an old subject, but the continuity of $\mathbf{B}_S$ across them is an unsettled question. Loper (1970) found a discontinuity in the steady case, but Hide and Stewartson (1972) give a proof of continuity. My argument is so much simpler and more general than Loper's that I am convinced of its correctness. The problem with the argument of Hide and Stewartson is their neglect of time dependence.

The boundary layers just below the CMB mean that from outside the core one cannot see the tangential component of the magnetic field at the top of the free stream just below the CMB. Work on the toroidal field in the mantle, including the derivation of the governing equation, shows that there is no physical basis for ruling out a toroidal field of as much as 5 gauss in the lower mantle. Such a field would, of course, be invisible from the surface of the earth, and so would its accompanying electric field (Backus, 1982) if, as appears likely (Filloux, 1982), the mantle conductivity has an interior minimum. Thus the tangential magnetic field in the core at the top of the free stream is obscured not only by a core boundary layers but by the possibility of an invisible toroidal field in the lower mantle. Recent use of tangential field estimates (Barraclough et al, 1989) to study the core must be viewed with skepticism.

The foregoing work on the viscous and magnetic boundary layers at the top of the core led the author to realize that such boundary layer analyses treat the non-magnetic, non-viscous problem as completely understood. Magnetic and viscous effects on the dynamics are added by a singular perturbation theory. This works if the non-viscous, non-magnetic fluid is also non-rotating, but the dynamics of a *rotating* non-viscous, non-magnetic fluid has had a serious theoretical gap for over 100 years. Noone ever proved that the Poincaré modes were complete, i.e. that they sufficed for a study of the non-viscous, non-magnetic problem when amplitudes of disturbances are small. The PI found a proof of this completeness, and has submitted it for publication. No referees' reports have come in yet, so the PI's belief that the proof is correct rests at this point only on his own analysis. The proof works for ellipsoids, for which the Poncaré modes are known, but leaves open the question of whether a non-ellipsoidal rotating core has a complete denumerable sequence of eigenmodes. The proof does, however, provide a way to do the core dynamics without such a sequence, for even the most general core shape.

The bibliography contains the items referred to in the foregoing report as well as all the other work produced under this grant. NAG 5-818 ran parallel with an equal grant from NSF (EAR 85-21543) with the same title and subject. All the work was part of one project, supported jointly by NSF and NASA, and all the work reported in the bibliography belongs to both grants simultaneously.

The work continues under a new NSF grant, and a new NASA grant application has been submitted.

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TALKS

(* denotes an invited talk. @ denotes a poster)

@Keeping phases of gauss coefficients, San Francisco AGU, Dec 1986.

*Bayesian Inference in Geomagnetism, Mathematics colloquium, UCLA, Feb 1987.

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*Objections to Stochastic Inversion and Bayesian Inference, AGU, San Francisco, Dec 1987.

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*When and How to Avoid Bayesian Inference and Stochastic Inversion, 17th IUGG Conference on Mathematical Geophysics, Blanes, Spain, June 1988.

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