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COLUMBIA UNIVERSITY
IN THE CITY OF NEW YORK
NEW YORK, N.Y. 10027

HEYMAN CENTER
BOX 600, CENTRAL MAIL ROOM

(212) 854-8300
FAX: (212) 662-7289

CENTER FOR THE STUDY OF GLOBAL HABITABILITY

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PRINCIPAL INVESTIGATOR:	Ruth A. Levenson Ruth A. Levenson Senior Director for Special Projects and Director, Center for the Study of Global Habitability
APPROVED BY:	Beth H Israel Beth Israel Associate Director Office of Projects and Grants

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A TWO–POINT CLOSURE MODEL OF TURBULENT COMPRESSIBLE CONVECTION AND APPLICATION TO STELLAR INTERIORS

Turbulent thermal convection is of considerable importance in fluid dynamical transport phenomena occurring, for example, in the planetary boundary layer of the Earth, the interiors of stars, and accretion disks. In particular, during a significant portion of the evolutionary phase of many stars having convectively unstable cores or outer envelopes, a substantial fraction of energy is transported from the central layers to the outer layers by thermal convection. Moreover, as much of the interior of a star is in highly turbulent motion, a complete theory of stellar structure and evolution requires the explicit consideration of turbulence in order to have expressions for the turbulent quantities arising in the stellar structure equations, and particularly, the turbulent fluxes that appear in the total flux conservation equation, such as the convective flux, kinetic energy flux, etc. A reliable quantification of these fluxes continues to present a challenge in astrophysical fluid dynamics, primarily because astrophysical turbulence is almost always fully–developed and nearly inviscid, and therefore governed by strong nonlinear interactions that distribute the energy among a very wide spectrum of eddies with scales ranging from the characteristic dimension of the flow (i.e., the depth of the convective layer) to those sufficiently small to be affected by viscous dissipation. Furthermore, astrophysical flows are invariably compressible, anisotropic, and inhomogeneous, which requires the consideration of the dynamics of longitudinal modes and their interaction with the transverse modes, as well as complicated boundary conditions.

In the absence of a satisfactory theoretical treatment of turbulent convection (or even incompressible, homogeneous, isotropic turbulence), it has been customary to use the phenomenological mixing-length theory (MLT) to provide an expression for the turbulent convective flux in stellar structure and evolution models. However, the MLT has a number of significant limitations: 1) it assumes that only one isotropic eddy of size $\Lambda = \alpha H_p$, rather than the entire spectrum of eddies, contributes to the turbulence dynamics, where H_P is the pressure scale height and α is the mixing–length parameter; 2) it does not provide the mixing–length parameter α, which must be calibrated for each star so as to fit observational data; 3) it does not explicitly incorporate compressibility, rotation, and magnetic fields; 4) it is not compatible with inhomogeneity, and; 5) it yields predictions which are not in accord with experimental results at large Rayleigh numbers. It is clear that the MLT fails to account for basic physics, and as such, stellar models that parameterize convection using the MLT cannot be truly predictive. In view of this, considerable effort has been devoted to the direct numerical simulation of turbulent convection. Unfortunately, these simulations are relegated to Reynolds numbers (and other parameters) many orders of magnitude smaller (or larger) than those found in true astrophysical flows, so that their predictions may not be directly relevant to stellar structure modeling. The use of large-eddy simulations permits the simulation of larger Reynolds number flows, but they are based on subgrid-scale models which are not appropriate for astrophysical conditions, because they do not account for either buoyancy or compressibility.

In order to reach a compromise between analytical and numerical tractability, and the basic physics of turbulent convection, we have constructed a model of stationary turbulent convection that yields various turbulence statistics, including the convective flux, that are required in stellar evolution models. The basic physics that is accounted for includes: 1) the full spectrum of turbulent eddies that transport heat in a star contribute to the total convective flux, and; 2) the star is highly compressible. The model is based on a fully-deductive, two-point, spectral closure theory of turbulence—the direct interaction approximation (DIA)—which models the nonlinear interactions in the Navier-Stokes equations, and the energy input required to sustain the turbulence is modeled via a source function derived from the associated linear stability problem for compressible convection. The stability problem has been solved for the source function, and the turbulence model equations for compressible convection have been derived and coded numerically. The evaluation of the time–history integrals in the nonlinear terms in the evolution equations has been simplified considerably by implementing Padé approximants. Tests of the numerical code have been performed for the case of freely-decaying compressible turbulence, and the results have been found to be in good agreement with those of direct numerical simulations. The numerical solution of the turbulence model equations yields the compressible and incompressible turbulence energy spectra, as well as other statistics such as the convective flux, which will be parameterized in a form that can be readily implemented in a stellar evolution model. As the model developed yields the entire turbulence energy spectrum corresponding to the range of eddies existing in the convective region of a star, the convective flux is expected to be larger than the corresponding MLT value. However, the explicit inclusion of compressibility in the model is expected to lower the convective flux, as a portion of the energy will contribute to the compressible, longitudinal modes (acoustical waves) rather than to convection. The numerical computations quantify the net increase or decrease of the convective flux relative to the MLT value in different parameter regions.

The numerical solution of the set of four, coupled, nonlinear integro-differential DIA model equations has been completed for a range of parameter values appropriate to a red giant star, and the resulting parameterized convective fluxes are currently being implemented in a stellar structure model to assess the combined effects of fully-developed turbulence and compressibility in the convectively unstable regions of red giant stars.