

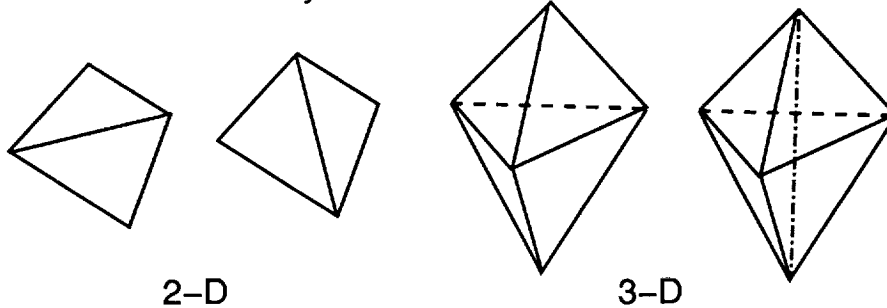
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# **3-D UNSTRUCTURED MESH GENERATION USING LOCAL TRANSFORMATIONS**

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### 3-D Combinatorial Edge Swapping

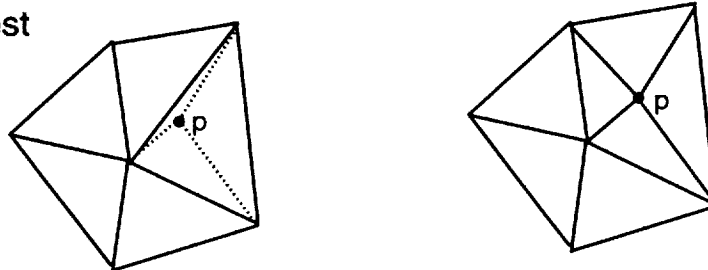
- Convex sets of  $n+2$  sites in  $\mathbb{R}^n$  can be configured in at most 2 ways



- This **local transformation** based on a Boolean decision serves as mechanism for local optimization

### 3-D Incremental Triangulation via Local Transformations

- Joe (1989) and Rajan (1991) showed that 3-D Delaunay triangulations can be constructed using local transformations based on the Boolean circumsphere test



2-D Example of Incremental Insertion and Optimization

- We have constructed triangulation algorithms in 3-D which locally optimize other mesh qualities: max-min dihedral angles, min-max dihedral angles, etc.

### Motivations

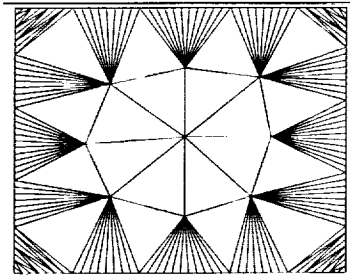
- Develop a mesh generation capability suitable for generating highly stretched meshes required for viscous flow computations at high Reynolds numbers
- Experience has shown that existing triangulation methods such as Delaunay triangulation are not suitable for the generation of highly stretched meshes
- Investigate triangulation algorithms which accommodate mesh generation and adaptation while maintaining high robustness

### Randomized $\triangle$ Algorithms Based on Local Transformations

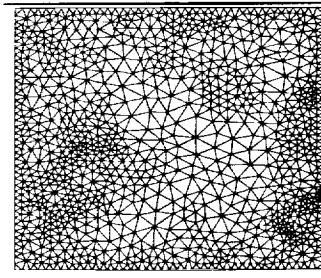
- Worst case optimal complexity can be achieved by *randomizing* the order in which sites are introduced into the triangulation (Guibas, Knuth, Sharir, 1992)
  - $n \log(n)$  expected performance in 2-D
  - $n^2$  expected worst case performance in 3-D
- Suggests a new "continuous" data structure which encodes a family of triangulations (coarsest to finest)
  - 2-D randomized theory predicts  $O(n)$  size of this structure
  - We have exploited this construction to produce a novel multigrid scheme and theory for solving differential eqns

## A New Approach to Multigrid for Unstructured Meshes

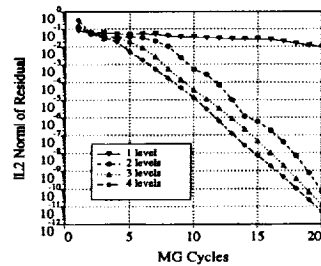
- Solution of Burgers' equation using continuous data structure



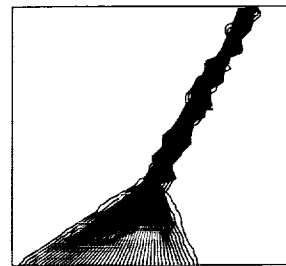
Coarsest Mesh



Finest Mesh



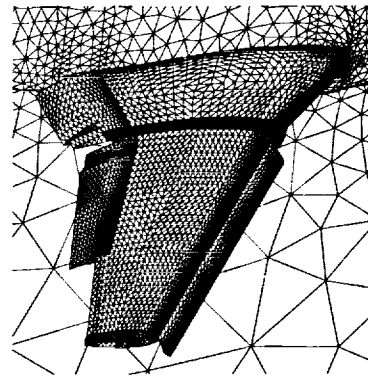
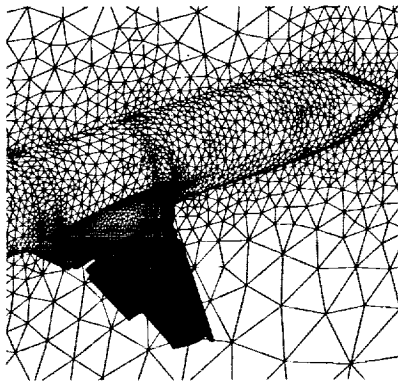
Convergence History



Solution Contours

## Surface Mesh Generation Using Local Transforms

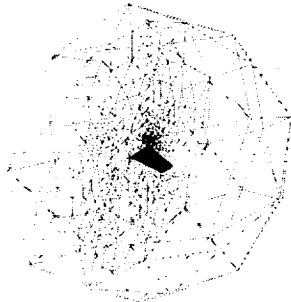
- Exploring new techniques capable of generation isotropic or stretched elements on tensor product spline patches
- Method supports adaptation based on geometrical or soln error
- Extension to manifold B-rep objects is being carried out by Code RFG (Maksymiuk, Chou)



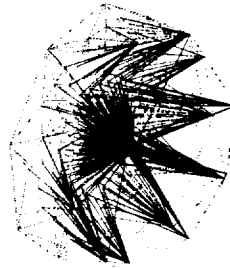
Mesh with isotropic and stretched elements

## Volume Triangulations

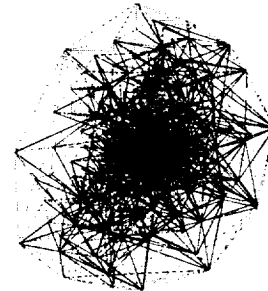
- (1) Initial Triangulation of Surface Data
- (2) Constrained/Conforming Triangulation to Preserve Body Integrity
- (3) Incremental Insertion and Optimization of Specified Sites



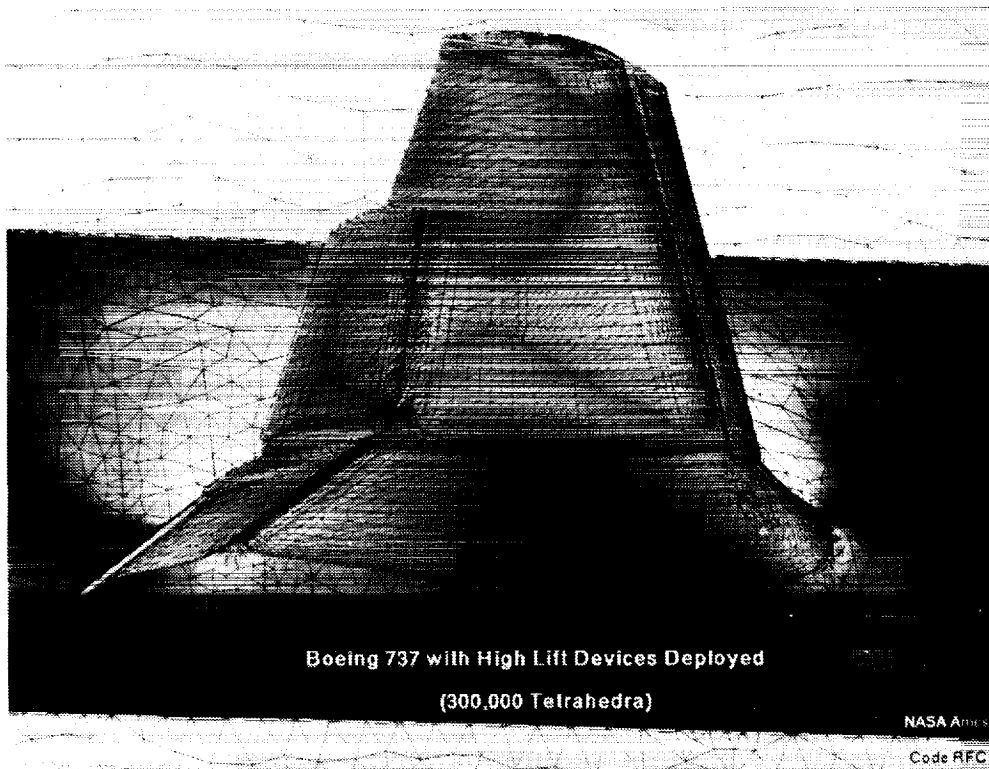
Surface Triangulation



Constrained/Conforming  
Triangulation of Boundary

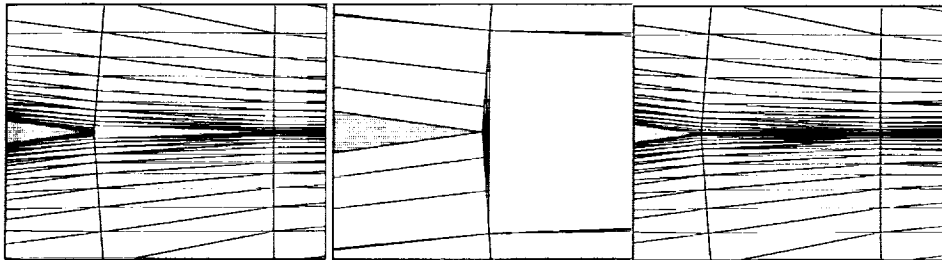


Final Volume Triangulation



## Why Some Standard Triangulation Methods Fail

- Delaunay triangulation has a well known characterization that it maximizes the minimum angle for triangle pairs
- Theoretical and practical considerations indicate that it may be more beneficial to minimize the maximum angle for triangle pairs
- Incremental insertion and local optimization can be used to produce locally optimal Min–Max triangulations



Delaunay triangulation      Extreme closeup of DT      Min-Max triangulation  
near an airfoil trailing edge      in trailing edge region      obtained by local optimization

## Viscous Mesh Generation

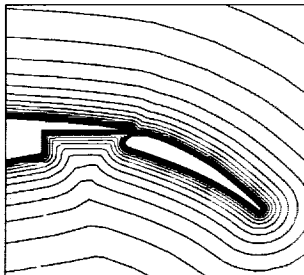
- Automatic generation of viscous meshes by adaptive placement of sites on level sets followed by Min–Max triangulation



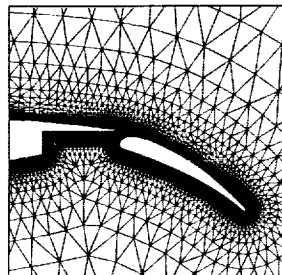
Point Selection ( $AR \gg 1$ )



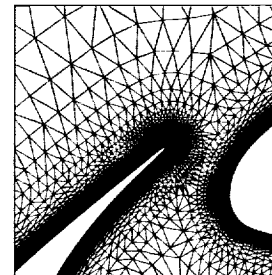
Point Selection and Adaptation ( $AR \approx 1$ )



Distance Function



Min-Max Triangulation



Closeup in Flap Region

## Future Directions

- Continue investigating optimization criteria for tetrahedral meshes
- Develop new strategies for site placement
  - Level set strategies
  - Steiner point strategies
- Solution adaptation based on *a priori* error estimates

