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COMPUTATIONAL GEOMETRY ISSUES

MARY-ANNE POSENAU
NASA LANGLEY RESEARCH CENTER

OUTLINE

Computational Geometry - how it fits in

Survey - recent work

A Computational Geometry Approach - current work

COMPUTATIONAL GEOMETRY

The design and analysis of algorithms and data structures for the solution of geometric problems.

WHY COMPUTATIONAL GEOMETRY

Complexity

Bounds

Robustness

"This program takes 2 minutes to generate a grid for model X on workstation Y."

Questions:

- Does the program always generate a grid?
- How does the number of grid cells affect execution time?
- What can be said about grid quality?

"O"-Notation

A function T(n) is O(f(n)) is there exist constants c and n_0 such that for all $n>n_0$, $T(n) \leq c\, f(n)$

Delaunay Triangulation - O(n log n)

- Shamos and Hoey - Divide and conquer
- Fortune - Sweepline
- Guibas, Knuth, Sharir - Randomized incremental

OPTIMALITY CRITERIA

The Constrained Delaunay Triangulation

- minimizes the largest circumcircle
- minimizes the largest min-containment circle
- maximizes minimum angle
- lexicographicaly maximizes list of angles, smallest to largest
- minimizes roughness as measured by Sobolev semi-norm
- guarantees a maximum principle
  for the discrete Laplacian approximation

OTHER OPTIMAL TRIANGULATIONS

Minimize max edge length - $O(n^2)$ Edelsbrunner, Tan

Greedy Triangulation - $O(n^2)$

Minimum weight triangulation

- not known to be NP-complete
- not known to be solvable in polynomial time
- variant is NP-complete
- approximations used

STEINER TRIANGULATION - RECENT RESULTS

Chew (89) - Range: [30°, 120°]

- size optimal among all uniform meshes

Baker, Grosse, Rafferty (88) - Range: [13°, 90°]

- aspect ratio < 4.6

Bern, Eppstein, Gilbert (90) - Range: [36°- 80°]

- aspect ratio < 5

Ruppert (93) - Range: [alpha, Pi-2 alpha]

- $\left|\frac{1}{\sin \alpha}\right| <$ aspect ratio $< \left|\frac{1}{\sin 2\alpha}\right|$
- size optimal within a constant C_{α}

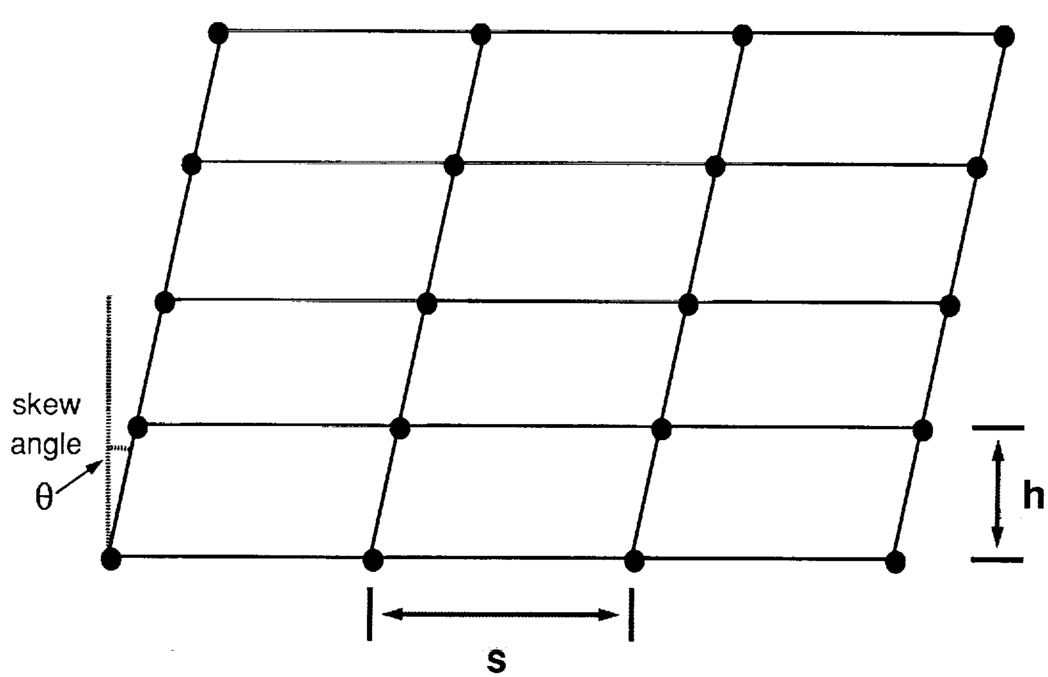
HIGH ASPECT RATIO TRIANGULATIONS

Delaunay triangulation can be unsuitable for high aspect ratio, body-conforming triangulations.

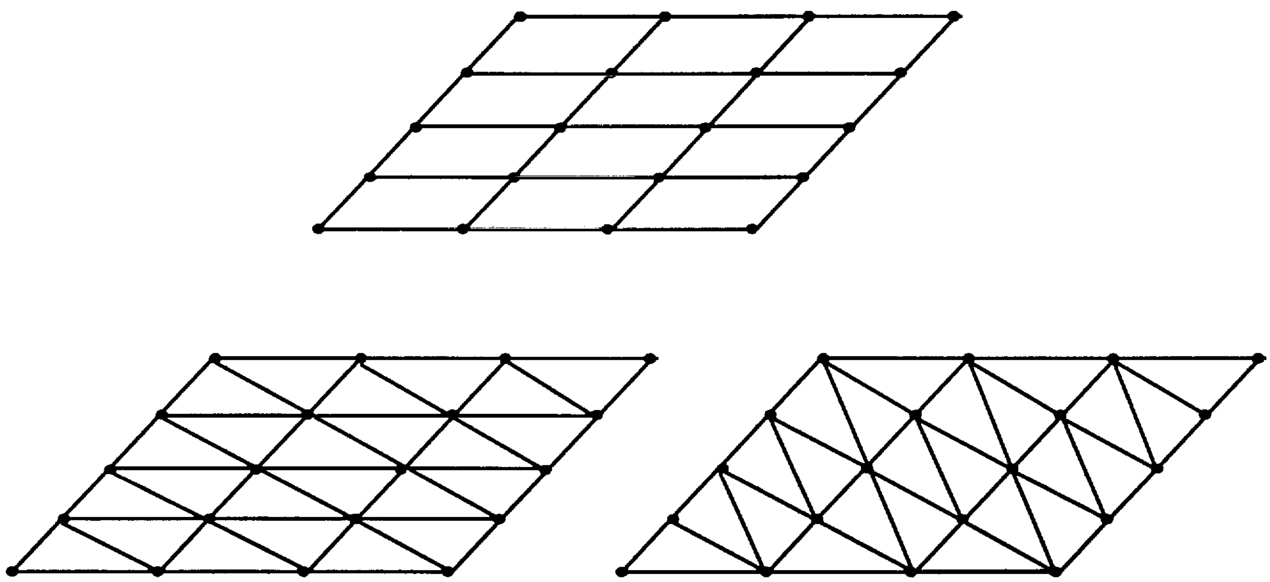
Robust, efficient, global algorithms are in need.

Computational geometers are not looking at this problem.

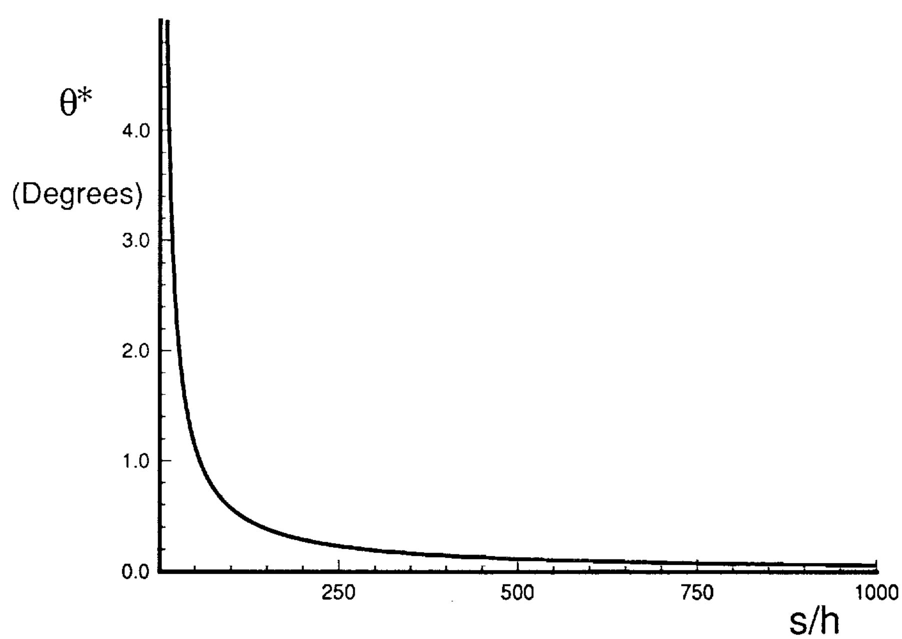
SKEWED STRUCTURED GRID



DELAUNAY REALIZABILITY

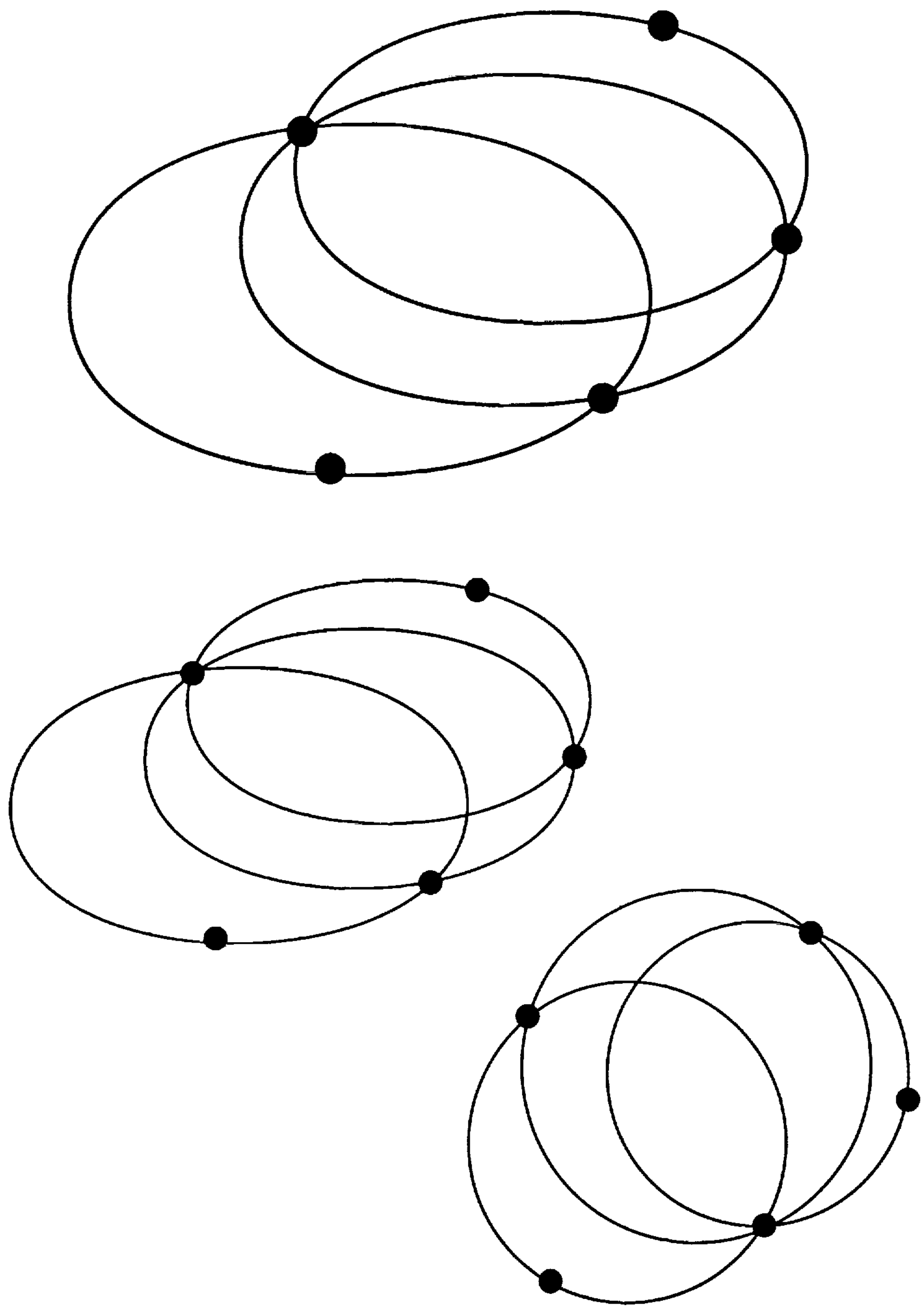


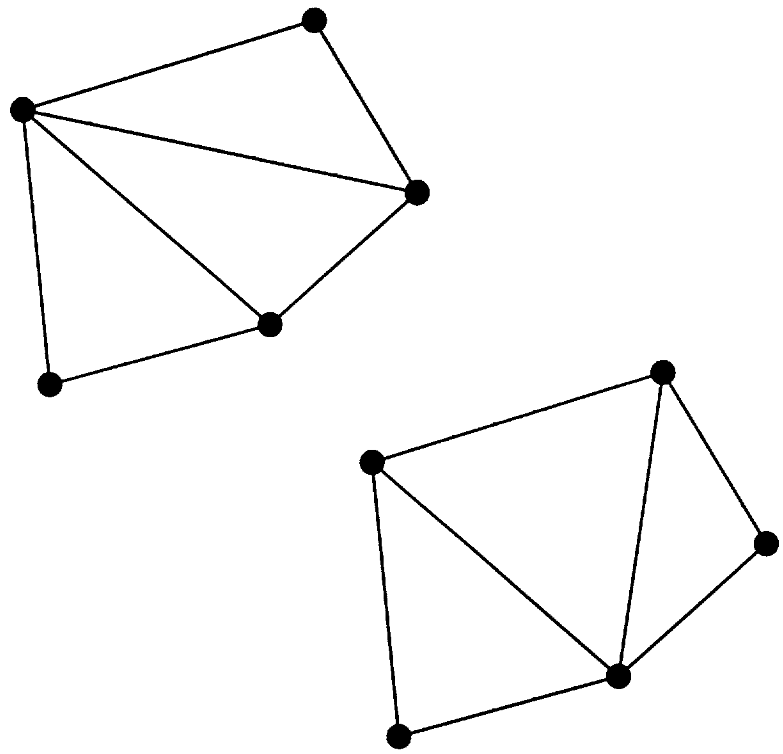
DELAUNAY ANGLE CUT-OFF vs. ASPECT RATIO



CONVEX DISTANCE FUNCTIONS
Chew, 1985

Change the concept of circumcircle to that of a convex distance function

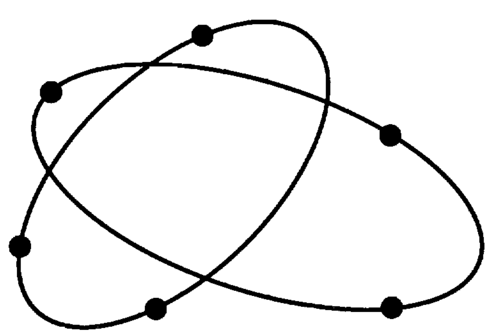


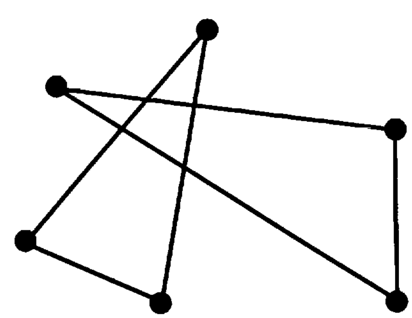


ISSUES

Generalize to a distance function which can vary throughout the plane.

Avoid ambiguous cases.





CONVEX BODY PROJECTION AND CONVEX HULL
Brown, 1979
Edelsbrunner, 1987

Project points from the plane to a paraboloid using parallel projection.

Find the convex hull of the 3D point set (all points will be on the convex hull).

The lower hull, projected back to the plane, will give the Delaunay triangulation of the point set in the plane.

Notes: One convex body handles entire domain.
Shifting the body to a new location gives the same result.

CONVEX BODY PROJECTION AND CONVEX HULL
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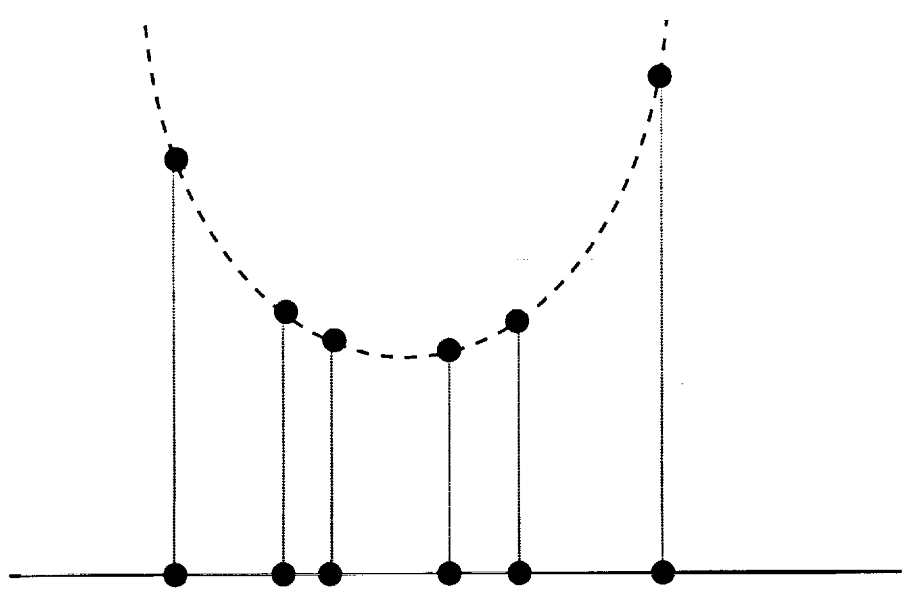
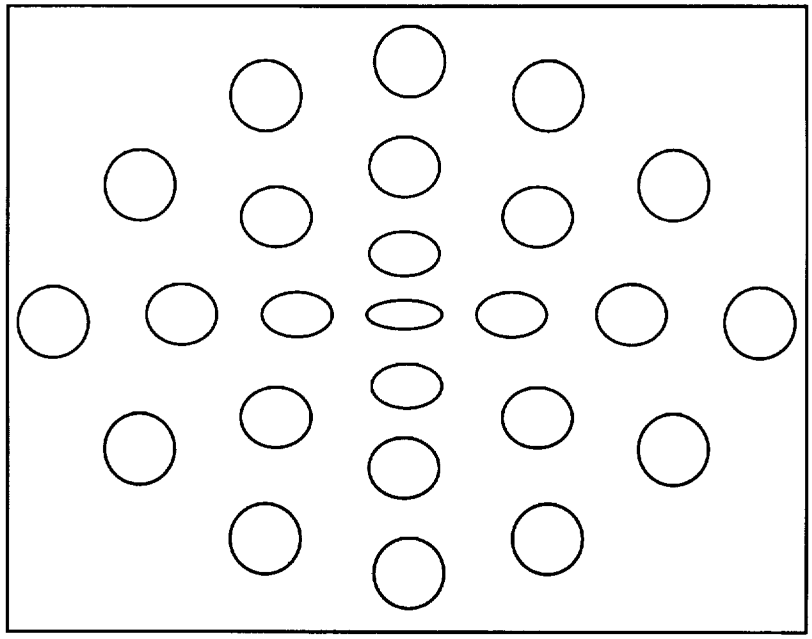


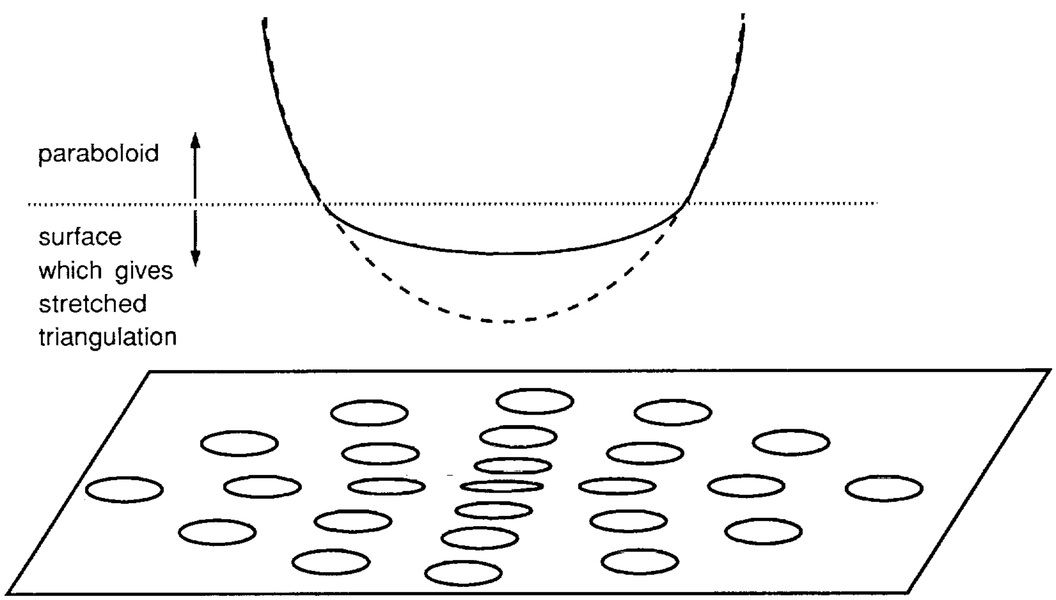
STRETCHED TRIANGULATIONS

Step 1a: Model simple stretching.



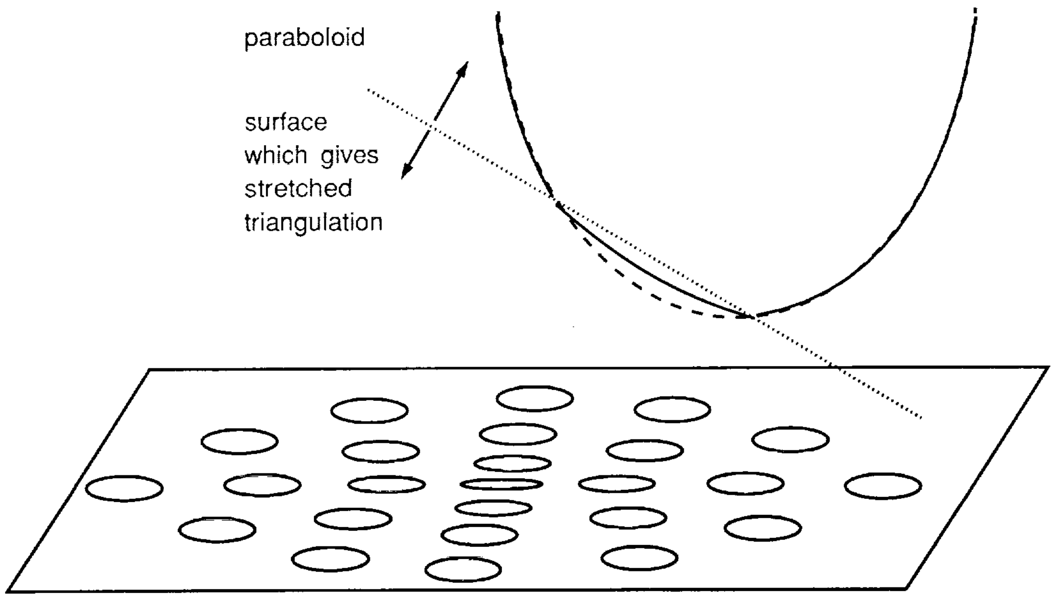
STRETCHED TRIANGULATIONS

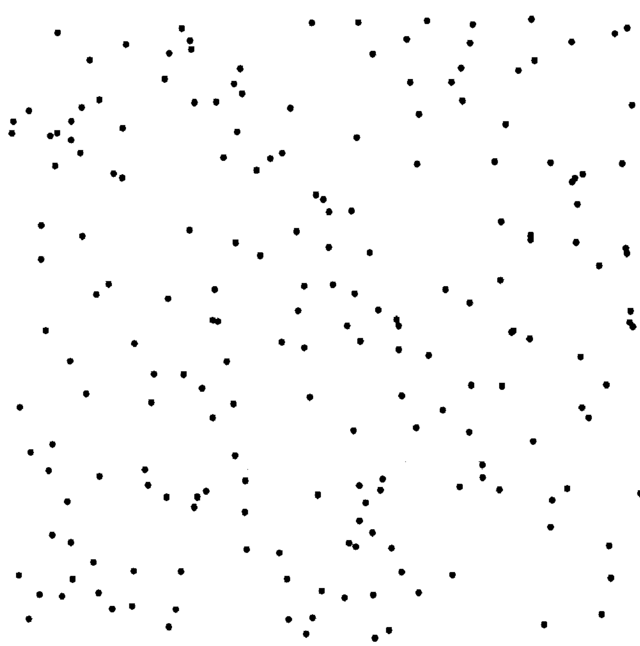
Step 1b: Design convex surface which will produce desired stretched triangulation.



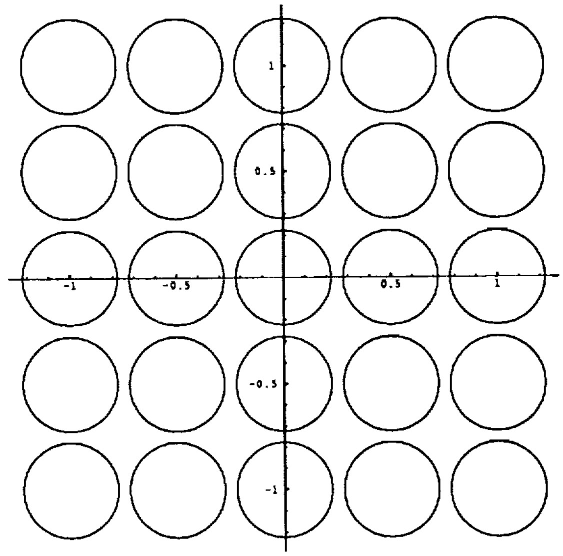
STRETCHED TRIANGULATIONS

Note: Body will not be "shift invariant".

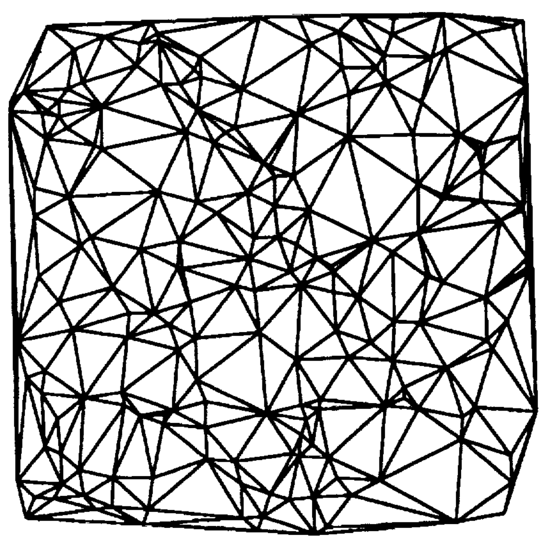




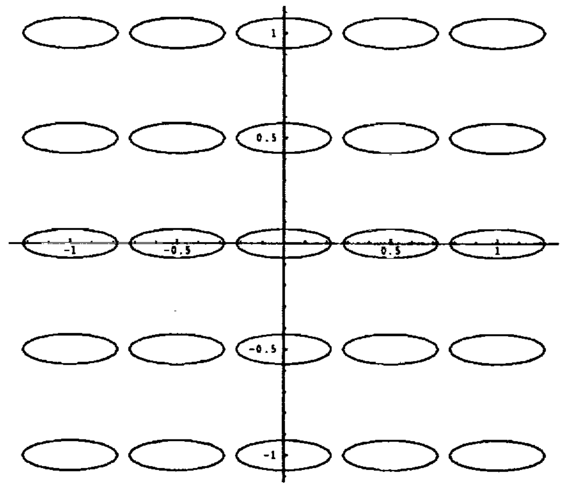
Test data used for all examples.



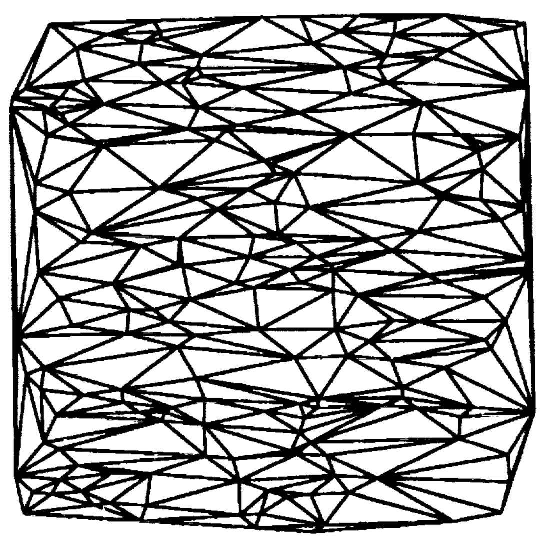
Circumshapes derived from paraboloid $x^2 + y^2$



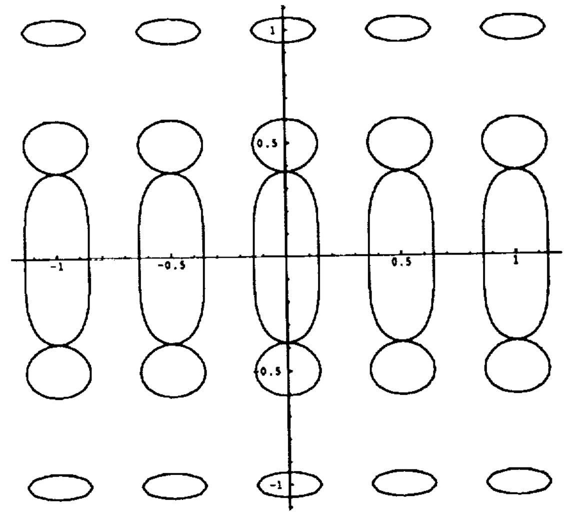
Triangulation derived from paraboloid $x^2 + y^2$
(Delaunay triangulation)



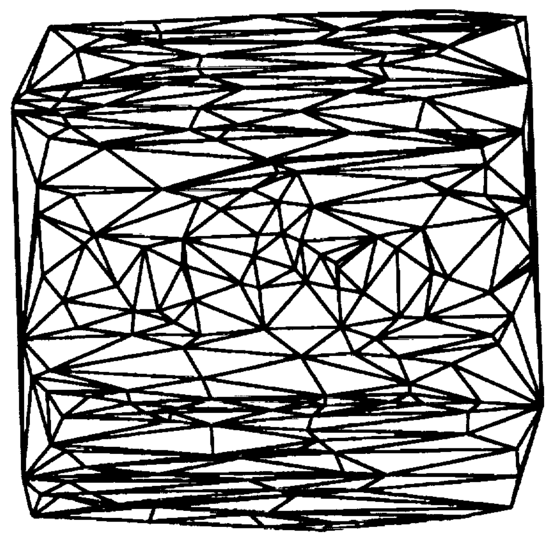
Circumshapes derived from $x^2 + 10y^2$, $\delta = 0.05$



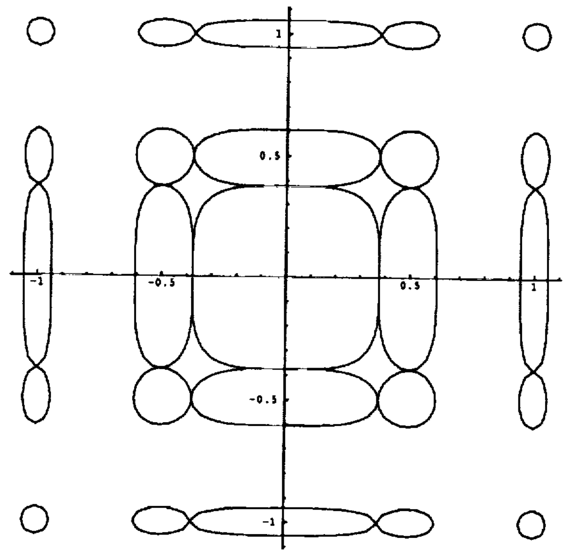
Triangulation derived from $x^2 + 10y^2$



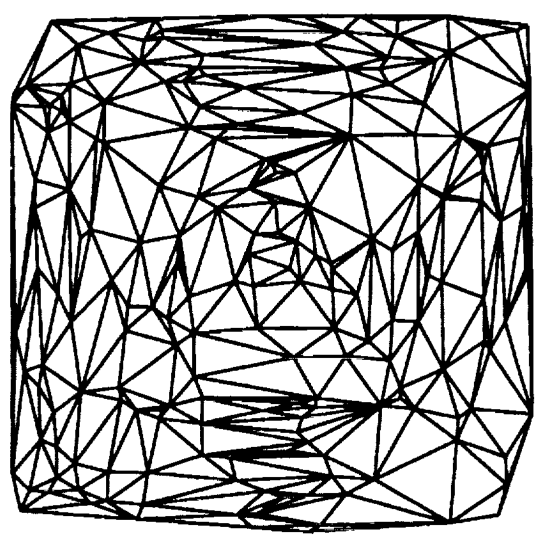
Circumshapes derived from $x^2 + y^4$, $\delta = 0.02$



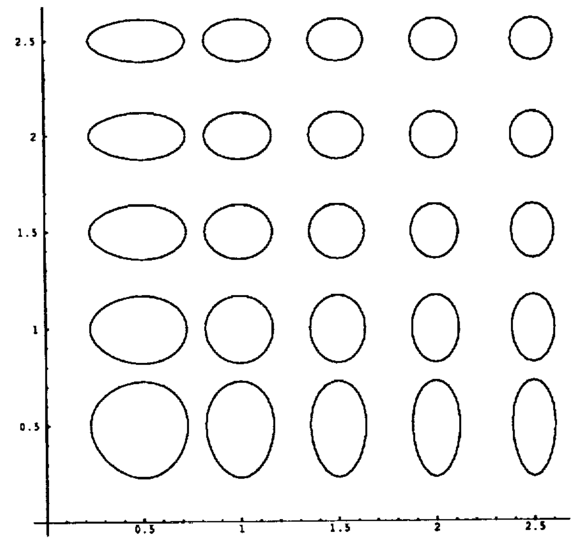
Triangulation derived from $x^2 + y^4$



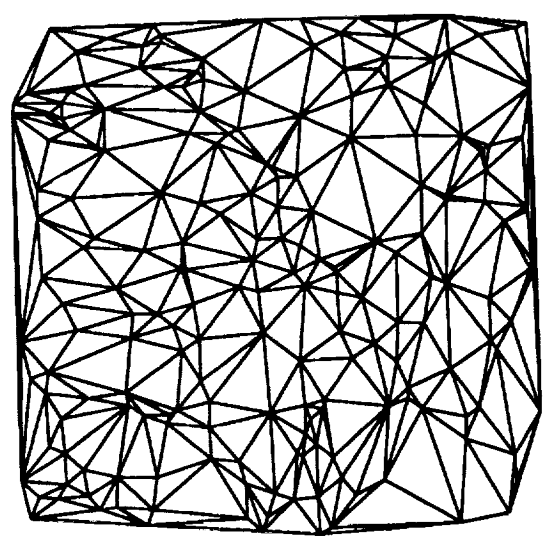
Circumshapes derived from $x^4 + y^4$, $\delta = 0.02$



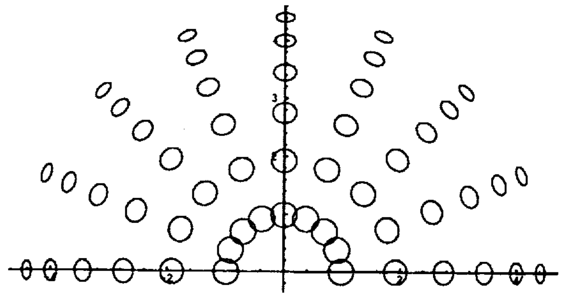
Triangulation derived from $x^4 + y^4$



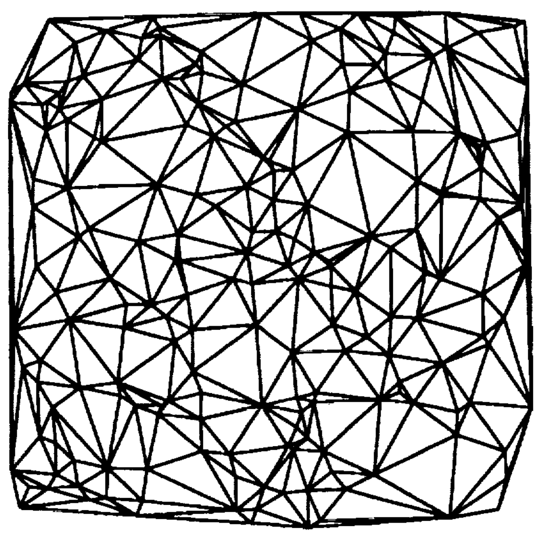
Circumshapes derived from $x^3 + y^3$, $\delta = 0.09$



Triangulation derived from $x^3 + y^3$



Circumshapes predicted from perspective projection, $z_{proj} = -100$, $\delta = 0.05$



Triangulation derived from perspective projection, $z_{proj} = -100$

CONCLUSIONS

Benefits of computational geometry - guarantees of
- grid quality
- efficient algorithms

Many efficient triangulation algorithms are available, but high aspect ratio triangulations are not among them.

Interdisciplinary cooperation will benefit grid generation and computational geometry.