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Simplified, Inverse, Ejector Design Tool

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Abstract

A simple lumped parameter based inverse design tool has been developed which provides flow path geometry and entrainment estimates subject to operational, acoustic and design constraints. These constraints are manifested through specification of primary mass flow rate or ejector thrust, fully-mixed exit velocity and static pressure matching. Fundamentally, integral forms of the conservation equations coupled with the specified design constraints are combined to yield an easily invertible linear system in terms of the flow path cross-sectional areas. Entrainment is computed by back substitution. Initial comparison with experimental and analogous one-dimensional methods show good agreement. Thus, this simple inverse design code provides an analytically based, preliminary design tool with direct application to High Speed Civil Transport (HSCT) design studies.

Nomenclature

A= cross-sectional area

c= speed of sound

d= nozzle diameter

F= thrust

K= empirical constant

m= mass flow rate

M= Mach number

P,p= Total pressure, static pressure

R= gas constant

T= temperature

S= source term

u,V= velocity

Greek Symbols

ρ= density

γ= specific heat ratio

ϵ= small parameter

χ= acceleration potential

Subscripts

1,2,3= primary, secondary, exit (downstream) locations, respectively.

e= exit location, "e" equivalent to "3".

∞= ambient conditions.

0= total conditions.

Introduction

Due to the challenges associated with the High Speed Civil Transport Program and the attendant noise suppression issues, preliminary design of ejector/mixer nozzles is currently of considerable interest. This interest concerning ejector nozzles and their noise suppression potential stems from the physical mechanisms associated with jet engine aircraft operation. In its most fundamental form, jet exhaust noise may be described as a high speed "free" jet effluxing into a quiescent atmosphere. Lighthill (1961) described the fluid mechanics and acoustics of this flow in a series of extraordinary papers. Summarizing results from his work and others, we may write the rather instructive relationship for acoustic power, P_a, a measure of "noise":

$$P_a = K \frac{\rho^2 d^2 V^8}{\rho_\infty c_\infty^5} \qquad (I.1)$$

where:

  ρ=density of the jet
  d=nozzle diameter
  ρ_∞=density of the ambient air
  c_∞=velocity of sound
  V=velocity of the jet relative to the surrounding air.
  K=empirical "constant".

Equation (1) leaves very little doubt that the velocity, V, of the jet has a strong influence upon noise generation. Thus, one noise "suppression" technique involves minimizing the velocity of the jet.

This strategy of noise suppression must be balanced, though, by the thrust requirements of a practical flight vehicle. To emphasize this constraint, we consider the ideal nozzle thrust relationship (ideally expanded):

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$$F = \dot{m} V \qquad \text{(I.2)}$$

with, m=nozzle mass flow rate. Clearly, to reduce jet velocity for fixed thrust, we must increase the nozzle mass flow rate, m. This requirement is precisely the situation where a mass augmenting device, such as an ejector nozzle, provides a sensible choice.

Figure (1) presents a representative nozzle attached to a turbojet engine and Figure (2) depicts a schematic representation of this nozzle for analysis. Fundamentally, an ejector nozzle system is merely a mixing chamber in which a high speed primary (core) is used to entrain fluid from a secondary flow. The fluid dynamic mechanisms associated with this entrainment include both viscous and "pressure" components. The actual process involved is extraordinarily complex, involving highly compressible, turbulent flow phenomena. More specifically, the entrainment is directly related to the local shear layer vorticity (see Townsend, 1976). The so called pressure entrainment is manifested through the global momentum conservation statements and dominates long, well mixed ejectors. Fortunately, though, rather coarse analyses may still be used to provide basic information by integrating between "known" locations and "avoiding" complex regions.

The clear practical application of devices such as the ejector nozzle make a hierarchy of predictive analysis of considerable interest. This hierarchy may range from simple design correlations to state of the art Navier-Stokes simulations. Concentrating upon the inverse design problem for which geometry is unknown and operating constraints are imposed (as opposed to the analysis problem for which geometry is specified and the operating conditions are unknown) we propose to develop an simplified, mathematical analysis based upon first principles.

The basic premise of any inverse design analysis, is to define a set of design constraints, literally desirable operational conditions, and then design a system which meets these constraints. From the previous paragraphs two constraints upon the operation of

the ejector nozzle system are immediately obvious:

> (1) specification of the nozzle exit velocity, V, which we will denote from now on as u_e, ($V=u_e$).
>
> (2) specification of the nozzle thrust, F_e.

Specification of the ejector thrust, though, may be at times somewhat inconvenient for the cycle analyst who chooses an engine of a certain size (primary stream mass flow rate) and iteratively looks (via optimization methods) to see if the choice has satisfied the thrust requirements. Thus, it will be desirable to include a variation of the analysis which is mass flow constrained versus thrust constrained.

These two constraints, exit velocity and thrust or core flow rate, are not sufficient to permit computation of the associated geometry, but reasonable <u>operational</u> design constraints may be imposed. The relationship between constraints and unknown variables is most readily discussed by considering the governing equations describing the ejector flow. This discussion is the basis of the next section. To help gain confidence in the validity of this methodology, the report will then summarize available theoretical and experimental results. It is hoped, that this report will demonstrate the use of this simple analytical in the area of preliminary design.

Analysis

To provide a design tool of reasonable simplicity while retaining sufficient physics to adequately model the flow, the inverse design problem is described by a relatively simple set of algebraic equations. These relationships comprise both thermodynamic definitions and conservation statements with the simplifying assumption of quasi-one-dimensional (Q-1-d) flow, namely, that all quantities are functions of the streamwise coordinate only. Further, the streamwise varying conservation equations are integrated (in the streamwise direction) to yield the algebraic governing equations. Derivation of these relationships may be found in any gas dynamics text, for example, Anderson (1982). These statements imply that the "downstream" conditions are fully mixed (cross-stream velocity small and streamwise velocity constant, at "mix out") an assumption asymptotically valid only. We remark that this is probably not an overwhelming restriction, in that, we would always strive to design an ejector that achieves adequate mixing at the exit.

Although our inverse design approach has several variations (the variations are determined by which quantities we assume to be specified and which we assume to be unknown), most of the governing relationships are common to the overall formulation. We will proceed to list the governing equations common to this problem and then the equations required for the variations. The conservation equations (with reference to Figure 2):

mass:

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3 \tag{1}$$

momentum:

$$\dot{m}_1 u_1 + p_1 A_1 + \dot{m}_2 u_2 + p_2 A_2 + \int p(A) \, dA = \dot{m}_3 u_3 + p_3 A_3 \qquad (2)$$

and energy:

$$\dot{m}_1 T_{01} + \dot{m}_2 T_{02} = \dot{m}_3 T_{03} \qquad (3)$$

where we employ the following definitions for mass flow at the subscripted locations:

$$\dot{m}_1 \equiv \rho_1 u_1 A_1 \qquad (4)$$

$$\dot{m}_2 \equiv \rho_2 u_2 A_2 \qquad (5)$$

$$\dot{m}_3 \equiv \rho_3 u_3 A_3 \qquad (6)$$

Note, that the formulation of the energy equation has implicitly stated that the specific heat is constant.

The local ideal gas state relationships written at the three locations are:

$$p_1 = \rho_1 R T_1 \qquad (7)$$

$$p_2 = \rho_2 R T_2 \qquad (8)$$

and:

$$p_3 = \rho_3 R T_3 \qquad (9)$$

Further, the definitions of total pressure and total temperature at locations (1) and (2):

$$\frac{p_{01}}{p_1} = (1 + \frac{\gamma-1}{2}M_1^2)^{\frac{\gamma}{\gamma-1}} \qquad (10)$$

$$\frac{p_{02}}{p_2} = (1 + \frac{\gamma-1}{2}M_2^2)^{\frac{\gamma}{\gamma-1}} \qquad (11)$$

and the total temperatures:

$$\frac{T_{01}}{T_1} = 1 + \frac{\gamma-1}{2}M_1^2 \qquad (12)$$

$$\frac{T_{02}}{T_2} = 1 + \frac{\gamma-1}{2}M_2^2 \qquad (13)$$

The Mach number relationships:

$$M_1^2 = \frac{u_1^2}{\gamma R T_1} \qquad (14)$$

$$M_2^2 = \frac{u_2^2}{\gamma R T_2} \qquad (15)$$

and

$$M_3^2 = \frac{u_3^2}{\gamma R T_3} \qquad (16)$$

In the preceding paragraphs, we have developed (16)

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independent relationships which describe an ejector flow. We now proceed to describe the variations of the inverse ejector design analysis. By far the most instructive way to proceed is to define our unknown quantities as compared to what is known or specified. We start with the quantities that are known versus those that are unknown regardless of what formulation variation we are concerned with. The <u>known</u> quantities include:

Primary stream (1):

- Total Pressure, P_{01}
- Total Temperature, T_{01}

Secondary Stream (2):

- Total Pressure, P_{02}
- Total Temperature, T_{02}
- Mach Number, M_2 (typically choked, $M_2=1.$)

and the downstream quantities:

- exit static pressure $p_3=p_e$
- exit velocity, $u_3=u_e$

where we recognize the exit velocity , u_e, specification as a manifestation of the noise constraint discussed previously. The exit pressure specification is a reasonable design requirement, namely, to design the nozzle to achieve ideal expansion. Specification of the secondary Mach number is justified by our desire to demand as much flow as possible through the secondary stream and yet still satisfy the downstream pressure constraint without the necessity of a strong terminal shock, thus forcing $M_2=1.0$.

The <u>unknown</u> quantities include:

Primary stream:

- Mach number, M_1
- density, ρ_1
- temperature, T_1
- static pressure, p_1
- velocity, u_1
- Cross-sectional area, A_1

Secondary Stream:

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- density, ρ_2
- temperature, T_2
- static pressure, p_2
- velocity, u_2
- Cross-sectional Area, A_2
- Secondary mass flow rate m_2

and the downstream quantities:

- density, ρ_3
- Mach number, M_3
- temperature, T_3
- Cross-sectional Area, A_3
- Exit mass flow rate m_3

Summarizing, we see we have (17) unknowns. Obviously, we have more unknown quantities, (17), than equations, (16). To proceed, we must either reduce the number of unknown quantities or increase the number of equations. The choice of what is unknown, depends upon what information is available (or conveniently attainable) and what quantities we would like to compute. Now, regardless of how we constrain the ejector, we must specify the upstream "flow and thermodynamic" quantities (such as velocity, Mach number, pressure etc.). This is performed in two possible ways:

(1) Primary Mach number, M_1, is specified, bringing our equation versus unknown tally to (16) and (16). This is not an easy to obtain quantity and, thus, not normally employed.

or, alternatively:

(2) Static pressure matching is demanded between the primary and secondary streams. This requirement is desirable to reduce "shock" generation due to pressure field imbalances. This design constraint takes the form of the simple relationship:

$$p_1 = p_2 \tag{17}$$

Bringing our equation versus unknown tally to (17) versus (17). This is the preferable procedure.

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Now since the number of equations versus unknowns is equal, to demand any more information, we must add both a constraint (equation) <u>and</u> an unknown for a unique solution to exist. There are two computational problems of interest:

(1) Specification of the ejector net thrust, which introduces the relationship:

$$F_e \equiv F_N = \dot{m}_3 u_3 - [\dot{m}_1 + \dot{m}_2] u_\infty \qquad (18)$$

<u>and</u> a correspondingly unknown primary mass flow rate, m_1.

or,

(2) Specification of the primary flow rate, m_1, <u>and</u> the correspondingly, unknown net thrust F_N.

Thus, this rather large set of relationships, (1) through (18), provides a consistent set of equations describing the ejector problem. Although this system is well posed, a detail that has been omitted involves closure for the integral within the momentum equation. In Appendix (I) we justify the closure to the integral which we merely repeat:

$$\int p(A) dA \approx \frac{1}{2} [p_e + \frac{1}{2} (p_1 + p_2)] [A_3 - A_1 - A_2] \qquad (19)$$

Notice that all terms in this approximation are immediately available within the scope of our integral (control volume) equations.

With specification of this term we may now proceed to solve, (1) through (19), for the unknown quantities. Now, clearly, specification of the thermodynamic/fluid dynamic, $(\rho, p, T, u, M \ldots)$, quantities in the primary and secondary stream is relatively straightforward. Essentially, whenever we have access to the total quantities and any single static quantity (p, T, M ...) we may use the "local" definitions of the quantities and state equation to

compute all of the static quantities. Thus, employing this strategy it is convenient to consider all of the primary and secondary stream thermodynamic/fluid dynamic quantities as known. This computation is not quite as transparent for problems in which the closure, equation (17), is obtained via static pressure matching at the confluence and, therefore, bears slightly more discussion. Basically, we combine equations (12), (13) and (17) to yield:

$$\frac{P_{01}}{P_{02}} = \left[\frac{1 + \frac{\gamma - 1}{2} M_1^2}{1 + \frac{\gamma - 1}{2} M_2^2} \right]^{\frac{\gamma}{\gamma - 1}} \qquad (20)$$

which may be solved for the primary stream Mach number:

$$M_1^2 = \frac{2}{\gamma - 1} \left[(1 + \frac{\gamma - 1}{2} M_2^2) \left(\frac{P_{01}}{P_{02}} \right)^{\frac{\gamma - 1}{\gamma}} - 1 \right] \qquad (21)$$

thus permitting us to compute the primary stream quantities.

The computation of the geometry dependent unknowns which is what we are really after (such as, the cross-sectional areas, and the mass flow rates) along with the downstream conditions (ρ_3, T_3, M_3) must now be performed. We relegate detailed derivation of this reduction to Appendix (II) and merely quote the results. Although explicit formulas are available for the above quantities it is probably most instructive to write the governing system for the cross-sectional areas (see any good linear algebra text for solution method, for example, Noble and Daniel, 1977). The two systems are, of course, different depending upon the problem. Writing the system for the "thrust constrained problem":

$$\begin{bmatrix} (1-\frac{u_0}{u_e})\psi_1 & (1-\frac{u_0}{u_e})\psi_2 & 0 \\ \bar{M}_1-p_w-\psi_1 u_0 & \bar{M}_2-p_w-\psi_2 u_0 & p_w-p_e \\ \psi_1\bar{H}_1-\frac{1}{2}\psi_1 u_0 u_e & \psi_2\bar{H}_2-\frac{1}{2}\psi_2 u_0 u_e & -\frac{\gamma}{\gamma-1}p_e u_e \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} \frac{F_e}{u_e}+S_c \\ F_e+S_m \\ \frac{1}{2}u_e F_e+S_e \end{bmatrix} \qquad (22)$$

where the terms are defined (subscripts suppressed):

$$\psi \equiv \rho u \qquad (23)$$

$$\bar{M} \equiv \rho u^2 + p \qquad (24)$$

$$\bar{H} \equiv c_p T_{01} = \frac{\gamma R}{\gamma-1} T_{01} \qquad (25)$$

The source terms (S_c, S_m, S_e) found in the RHS column vector are introduced to add extra flexibility, for example, skin friction losses, S_m; heat transfer, S_e; or mass bleed, S_c. Obviously, these terms must be specified functions. Throughout the analysis, we will assume all source terms to be zero. Further, P_w, is defined:

$$p_w \equiv \frac{1}{2}[p_e+\frac{1}{2}(p_1+p_2)]$$

A few comments about the structure of the system would be appropriate. The most "striking" feature of the above system is that it is <u>linear</u>. The inverse ejector problem is linear in terms of the cross-sectional areas (at least to our level of approximation) which is direct contrast to the analysis problem which is inherently non-linear in terms of the thermodynamic/fluid dynamic quantities. This fact obviously considerably simplifies the solution of the problem to the trivial inversion of a (3x3) system.

In a similar way, the problem may be posed when the primary mass flow is specified (and the net thrust is to be computed).

In this case the governing system (also linear) may be written:

$$\begin{bmatrix} \bar{M} - p_w - 2\bar{H}_2 \dfrac{\psi_2}{u_e} & \dfrac{\gamma+1}{\gamma-1} p_e + p_w \\[2ex] \psi_2 (1 - 2 \dfrac{\bar{H}_2}{u_e^2}) & \dfrac{2\gamma}{\gamma-1} \dfrac{p_e}{u_e} \end{bmatrix} \begin{bmatrix} A_2 \\[2ex] A_3 \end{bmatrix} = \begin{bmatrix} 2\dot{m}_1 \dfrac{\bar{H}_1}{u_e} - \dot{m}_1 u_1 + (p_w - p_1) \dfrac{\dot{m}_1}{\psi_1} \\[2ex] \dot{m}_1 (\dfrac{2\bar{H}_1}{u_e} - 1) \end{bmatrix} \quad (26)$$

with the "auxiliary" equation:

$$A_1 = \frac{\dot{m}_1}{\psi_1} \quad (27)$$

The matrix system for the primary mass flow rate constrained problem is, of course, trivial to invert. We note, also, that the "source term" constants have been eliminated for convenience.

In summary, then, this section has sought to describe the theoretical basis of the inverse design methodology. Rather simple algebraic relationships of adequate flexibility combined with a series of design constraints and assumptions have been used to derive a method with the capability of predicting geometric parameters (cross-sectional area and mass flow rates) at several discrete locations within the ejector. In the next section, we will explore the validity of this analysis by comparing to both analogous theoretical problems and experimental data.

Results and Discussion

The previous section has sought to outline an elementary mathematical analysis describing an inverse design methodology. Before this methodology can be applied in any practical manner, the range of applicability must be verified. This verification process may be conveniently divided into two categories: (1) internal consistency checks (such as, simple mathematical constraints and cross referencing between analogous, inverse design and simple analysis methods); and (2) comparison to available experimental data (which is critical since these comparisons provide the only available independent verification of the method). We will begin with, the internal consistency of the method since it is fundamental and leaves no room for interpretation.

Due to the rather simple mathematical structure of the problem at hand several consistency checks are immediately available to us. These might include:

(1) Conservation statements: the quantities mass, momentum and energy are readily shown to be properly conserved.

(2) Mathematical inversion consistency: to guard against algebraic "blunders" in the matrix inversions, matrix residuals:

$$R = [A]\bar{x} - \bar{b} \tag{28}$$

are printed and shown to be virtually zero. Although, this is certainly a minimal requirement it is a useful test for code development/modification procedures.

(3) "Cross referencing" between variations of the inverse design methodology provides another comparison. Since the inverse design code analysis has formulation variation that are either "thrust constrained" or "mass flow rate constrained" opportunities for comparison immediately exist.

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By supplying these two code variations information from its counterpart we have shown these two codes to be self consistent in their formulations.

and, finally, a more stringent test:

(4) Comparison was made to a forward or analysis integral method (see Appendix (III) for a summary description of this analysis). A forward or analysis method may be distinguished from the inverse methods described here in that for an analysis method, the cross-sectional areas are specified and not computed. This comparison was performed for a simple constant area case and exhibited adequate correlation.

As noted, the above consistency requirements, although useful and mandatory, are really no more than reflections of the self consistency of the mathematics and algebraic manipulation. They do not provide any _independent_ information concerning the validity of the analysis. To obtain this information, we need to compare to either experimental information which, of course, has the greatest potential for providing realistic information or comparison to higher order numerical analyses (CFD).

Limited experimental information is available for a practical HSCT ejector nozzle, which is essentially a 2-d, lobed or forced ejector nozzle. The model was run in the "thrust constrained mode" with thrust set at 52631 lbf (free stream velocity assumed equal to zero). It is probably worth noting, that this rather odd thrust value, 52631 lbf, is a manifestation of the iterative modeling approach applied by the standard cycle analysis codes. In this approach, the thrust is not specified, but iteratively optimized, thus, yielding these close to design, 52600 lbf, (but not exact) thrust values. Thermodynamic information includes the ideally expanded jet velocity (isentropic expansion to ambient pressure), $V_j=3125$ ft/s. The design exit velocity was specified at $V_{mix}=1450$ ft/s. This velocity is chosen on the basis of meeting FAR36 Stage III noise requirements estimated using semi-empirical noise estimation methods (similar to Lighthill's relationship). This

relationship is shown graphically in figure (3). Referring to figure (4) excellent correlation is shown between the inverse design code and the single experimental mass entrainment value. This excellent comparison should be tempered by the fact, that only a single experimental value was available which limits our ability to more completely verify the method. Further, sensitivity studies indicate a strong dependence upon the exit velocity, V_{mix}, specification. Unfortunately, direct measurements of average exit jet velocity are not available and we are forced to infer the previously stated values. This dependence bears further discussion.

A sensitivity study for a mass flow constrained, generic ejector nozzle, using this methodology, is presented in figure (5). These curves exhibit the expected result that more entrainment is required to achieve a lower exit velocity. Further, the _linear_ relationship between geometry dependent variables, such as, entrainment and the thermodynamic relationship is clearly shown. We note, that the matrix equations (24) and (28) essentially state the same thing.

Although the previous discussion has centered upon mass entrainment (pumping), the inverse design analysis also has access to the required flow path cross-sectional areas at the primary, secondary and exit locations. These cross-sectional areas are provided for the nozzle for various primary ideal velocities. Referring to figure (6), several trends are immediately apparent. First, although the size of the primary and secondary flow paths change considerably, the overall ejector cross-sectional area remains virtually constant. Secondly, streamwise cross-sectional area variation is apparently small (relative to length). This confirms that the approximately "constant" cross-sectional area ejector is probably a viable design. Both of these trends indicate, that although the Mach number of the primary stream is relatively high, on the order of Mach 2.0, the ejector geometry is dominated by conservation effects. As an example, mass conservation is not strongly (in a relative sense) dependent upon

Mach number. Consider for example the classical Mach number area relationship which is merely a mass conservation statement:

$$\left(\frac{A}{A^*}\right)^2 = \frac{1}{M^2}\left[\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{\gamma-1}} \qquad (29)$$

Consulting any gas dynamics text, for example, Anderson, 1982 we would see that the area ratio, A/A*, varies no more than 1.0 to 2.0 while the Mach number ranges from 0.3 to 2.25. This insensitivity to Mach number and, thus, compressibility causes the cross-sectional area trends seen in the previous example. Mathematically this means:

$$\int \phi \, dA \approx \phi \int dA \; ; \qquad \phi = \begin{array}{c} \rho u \\ \rho u^2 + p \\ \rho u H \end{array} \qquad (30)$$

Which is the set of mathematical conditions needed for the approximately constant area ejector.

In any case, estimation of the flow path cross-sectional areas represents the first step in the prediction of the ejector geometry, and further, an estimate of the cross-section of the nacelle itself. Obviously, the actual nacelle cross-section is a function of much larger (and more complex) design requirements. Work, though, is underway to address these design requirements in a rational manner (at a level of fidelity consistent with our preliminary design philosophy). Along these lines, an approximate analysis is underway which estimates the streamwise length scale, a quantity obviously dominated by the mixing process (we note that our integral formulation has drawn its boundaries or control surfaces around this complex region and, thus, avoids such complexity at the cost of no streamwise length scale information).

A further validation case is available from the fully subsonic

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ejector experiment performed by (Gilbert and Hill, 1973). In this case a simple, 2-dimensional, subsonic primary ejector has been modeled. Primary mass flow rates are available, therefore, the mass flow constrained variation of the analysis is appropriate. Necessary, input information includes (for Gilbert and Hill's, run number 3):

P_{01}=4551.84 (psfa)

P_{02}=2103.8 (psfa)

T_{01}=706.0 (R)

T_{02}=533.0 (R)

u_e=350.0 (ft/s)

p_e=2113.0 (psfa)

Additionally, the primary and secondary Mach numbers were estimated:

M_1=1.0 (choked flow)

M_2=.25 (from experimental data)

Using the inverse design code, estimates of pumping and the exit area were obtained and compared to experimental values:

Given the simplicity of the model, these comparisons may be considered quite satisfactory. To be realistic, it is necessary to point out some of the uncertainty that enters our modeling of this flow. First, since complete mixing is

	W_2/W_1	A_3 ft^2
Exper.	3.76	.1278
Anal.	4.08	.1224
R. Err.	8.51%	4.00%

possible only asymptotically, estimation of the exit velocity is difficult. A related difficulty is found in estimating the primary and secondary Mach numbers. Note that the experimentally measured secondary and primary stream pressure difference: $(p_1-p_2)/p_1$=14.9%, is small but not zero. This difference was reproduced in our model since we did not demand static pressure matching at the inlet but merely imposed the experimentally measured Mach number. Large cross-stream pressure gradients would certainly invalidate the one-

dimensionality of our method. In this case, the cross-stream pressure gradient appears to have a negligible effect on the predicted parameters. Finally, estimation of secondary inlet recoveries (primary nozzle recovery was given) may be difficult. In spite of these limitations, this example from (Gilbert and Hill 1973), helps indicate that the inverse design methodology provides a useful mathematical model for preliminary design work.

Finally, since relatively little independent data is available to verify the accuracy of this analysis, a comparison to equivalent, 1-dimensional, code predictions from other sources is desirable. A comparison is shown for a class of generic ejector nozzles studied by industry. Although specific information about industry methodologies is necessarily incomplete due to the proprietary nature of these tools, plausible operating conditions, such as, primary and secondary conditions may be inferred. The exit velocity constraint, u_e, is especially difficult to estimate. When the proper exit velocity is applied, comparison is good. This comparison is presented in figure (7) for two operating points. The comparison is good, but again, this may be a matter of consistency between the mathematics of the industry study and our method.

Thus, although data for direct verification is rather sparse (hence the need for the analysis), initial comparisons are encouraging. Further, the method has been shown to be both internally self consistent (a minimal requirement) and consistent with 1-dimensional predictions from industry. These results are probably sufficient to cautiously use the analysis in its intended preliminary design role.

Conclusions

A simple integral based inverse design model has been developed to predict flow path cross-sectional areas and entrainment given a physically justifiable complement of design

constraints. These constraints include specification of primary mass flow rate or ejector net thrust, exit velocity and static pressure matching (ideal expansion). A simple closure to the secondary stream static conditions is provided by a primary and secondary stream static pressure matching. The resulting systems in terms of the flow path cross-sectional areas are shown to be linear and, therefore, easily invertible. Comparison to available experimental and analogous one-dimensional methods show good correlation. Further, the method is easily shown to be internally self consistent. Thus, this code provides a preliminary design tool for High Speed Civil Transport design issues; and, additionally, may provide the basis for more complete design methodologies.

Although this analysis provides an efficient preliminary design tool, it is incapable of providing higher information. This type of information includes streamwise "mixing length", pressure recovery, flow field information and pumping rates for non-ideal mixing. All of these quantities involve the physics of the actual mixing process and, thus, may not be modeled by the control volume technique employed in this report. A series of higher order models based upon ordinary and partial differential equation methods (but still much less complex than Navier-Stokes simulations) are under development. It is expected that the combination of the simple, inverse design model described in this report and these higher order models will provide the designer with a simple, yet powerful suite of design and analysis codes.

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Appendix I
Wall Pressure Integral Closure

The system of governing integral relationships used to describe the ejector flow provides a simple, yet physically realistic tool for preliminary design computations. This simplicity is directly related to the control surface chosen. Unfortunately, the momentum equation, since it is conserving a vector quantity (linear momentum), must retain terms that are not particularly convenient. This term is the wall pressure or "pdA" integral. Closure of the "pdA" integral, in terms of available quantities, requires some modeling.

Instead of merely assuming a convenient form for this term, we will briefly introduce a somewhat more sophisticated model, make assumptions, and draw what conclusions it permits. First, we consider the two dimensional pressure field as described by the small disturbance, acceleration potential (Robertson, 1965):

$$(1-M_{\infty}^2) \frac{\partial^2 \chi}{\partial x^2} + \frac{\partial^2 \chi}{\partial y^2} = 0 \qquad \text{(A1.1)}$$

with the attendant "wall" boundary conditions:

$$\frac{\partial \chi(x,0)}{\partial y} = \frac{\partial \chi(x,H)}{\partial y} = \pm \epsilon \equiv \mp (\gamma-1) M_{\infty}^2 \frac{d^2 f}{dx^2} \qquad \text{(A1.2)}$$

where:

$$\chi(x,y) \equiv (\frac{p(x,y)}{p_{\infty}})^{\frac{\gamma-1}{\gamma}} \qquad \text{(A1.3)}$$

and, $f(x)$, is the function describing the wall boundaries.

Now, we may choose to either solve equation (A1.1), (with two more boundary conditions, of course) or alternatively simplify it

directly. Since our simple integral analysis will be unable to
"see" two dimensional effects anyhow, we proceed to average by
introducing the integral operator (essentially integrating (A1.1))
in the cross-stream direction):

$$(1-M_\infty^2) \frac{\partial^2}{\partial x^2} (\int_0^H \chi dy) + \frac{\partial \chi(x,H)}{\partial y} - \frac{\partial \chi(x,0)}{\partial y} = 0 \qquad (A1.4)$$

Assuming that epsilon is small in equation (A1.2), (which is
consistent with our inverse design experiences), simplifying
equation (A1.4), and recognizing the definition of a cross-stream
averaged quantity, we may immediately write:

$$\frac{d^2\bar{p}}{dx^2} = 0 \quad \rightarrow \quad \bar{p} = c_1 x + c_0 \qquad (A1.5)$$

Note, that this solution has obviously assumed that the flow has
definite subsonic regions and is therefore, elliptic.

The purpose of this somewhat lengthy discussion has been to
"justify" under proper restrictions, the assumption that the
average streamwise pressure field may be described by the linear
function as in (A1.5). With this assumption and the convenient
specification that the cross-sectional area varies linearly we may
write the two relationships:

$$A(x) = (A_e - A_i) (\frac{x}{L}) + A_i \qquad (A1.6)$$

and

$$p(x) = (p_e - p_i) (\frac{x}{L}) + p_i \qquad (A1.7)$$

Now digressing briefly, the linear geometry relationship (A1.6)
bears further discussion for axi-symmetric flow. Considering a
conical section, (figure 8), we may write the relationship for the

radius as a function of the streamwise coordinate "x":

$$r(x) = (r_2 - r_1)\frac{x}{L} + r_1 \rightarrow r(x) = r_1(\epsilon\frac{x}{L} + 1) \; ; \quad \epsilon \equiv \frac{r_2 - r_1}{r_1}$$

Thus,

$$A(x) = \pi r_1^2[1 + \epsilon\frac{x}{L} + \epsilon^2(\frac{x}{L})^2] = \pi r_1^2[1 + \epsilon\frac{x}{L} + O(\epsilon^2)]$$

Which justifies, at least to first order, the linear cross-sectional area relationship, (A1.6).
Returning to (A1.6) and (A1.7), and eliminating the "x" dependence from them we write:

$$p(A) = (p_e - p_i)\frac{(A - A_i)}{(A_e - A_i)} + p_i \qquad (A1.8)$$

Thus, substituting into the wall pressure integral (and performing the indicated operation), we may easily write:

$$\int p dA = \frac{1}{2}[p_e + p_i][A_e - A_i] \qquad (A1.9)$$

To complete this analysis, we eliminate p_i, by merely assuming:

$$p_i = \frac{1}{2}(p_1 + p_2) \qquad (A1.10)$$

which is an exact statement when we apply the static pressure matching closure assumption.

Thus, we have obtained a simple algebraic closure relationship, which is linear in terms of the cross-sectional areas. Although this closure is approximate (viable for small divergence angles), experience shows that this is by no means a

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severe limitation, since the "solution" cross-sectional area variation is small (which might be considered fortunate, in that, for large divergence angles, the one-dimensional basis of the entire method must fail, due to large cross-stream gradient effects).

Appendix II
Reduction of Governing Equations

Previously, governing equations and mathematical results (in the form of two simple linear systems) were stated for both "thrust constrained" and "mass flow rate constrained" problems. It is certainly desirable to fully describe the necessary reduction to this point. Due to the formulation, as stated previously, the thermodynamic/fluid dynamic quantities (ρ, u, p, T, M, ...) in the primary and secondary stream entrances are essentially known quantities. Starting from this "point of view", we quote the conservation equations:

$$\rho_1 u_1 A_1 + \rho_2 u_2 A_2 - \rho_3 u_3 A_3 = 0 \qquad (A2.1)$$

$$(\rho_1 u_1^2 + p_1) A_1 + (\rho_2 u_2^2 + p_2) A_2 + \frac{1}{2} (3 p_e - \frac{1}{2} (p_1 + p_2)) (A_3 - A_1 - A_2) = p_3 A_3 + \rho_3 u_3^2 A_3$$

and

$$\rho_1 u_1 A_1 c_p T_{01} + \rho_2 u_2 A_2 c_p T_{02} = \rho_3 u_3 A_3 c_p T_{03} \qquad (A2.3)$$

We note, that $u_3 = u_e$ and $p_3 = p_e$ (for any of the inverse design variations) are both specified quantities.

We now proceed to consider the "thrust constrained" problem. For this variation, the required net thrust of the ejector is assumed specified:

$$F_e = F_N = \rho_3 u_3^2 A_3 - u_\infty (\rho_1 u_1 A_1 + \rho_2 u_2 A_2) = \text{const.} \qquad (A2.4)$$

Thus, we may simply eliminate the term on the right hand side of momentum:

$$\rho_3 u_3^2 A_3 = F_e + u_\infty (\rho_1 u_1 A_1 + \rho_2 u_2 A_2) \qquad (A2.5)$$

and, analogously the right hand side of the mass conservation relationship becomes:

$$\rho_3 u_3 A_3 = \frac{F_e}{u_e} + \frac{u_\infty}{u_e} (\rho_1 u_1 A_1 + \rho_2 u_2 A_2) \qquad (A2.6)$$

Now, reduction of the right hand side of the energy equation into either known or unknown cross-sectional area terms, requires somewhat more effort. We begin by solving (A2.5) for the exit density, ρ_3:

$$\rho_3 = \frac{F_e}{A_3 u_e^2} + u_\infty \left(\frac{\rho_1 u_1 A_1}{u_e^2 A_3} + \frac{\rho_2 u_2 A_2}{u_e^2 A_3} \right) \qquad (A2.7)$$

and by state:

$$T_3 = \frac{P_e}{\rho_3 R} \qquad (A2.8)$$

and, finally, by the Mach number relationship:

$$M_3^2 = \frac{1}{\gamma P_e A_3} [F_e + u_\infty (\rho_1 u_1 A_1 + \rho_2 U_2 A_2)] \qquad (A2.9)$$

Thus, we write (with some algebraic simplification):

$$\rho_3 u_3 A_3 C_p T_{03} = \frac{\gamma}{\gamma - 1} P_e u_e A_3 + \frac{1}{2} u_e F_e + \frac{1}{2} \rho_1 u_1 u_\infty A_1 u_e + \frac{1}{2} \rho_2 u_2 u_\infty A_2 u_e \qquad (A2.10)$$

At this point, it is clear, that the above relationships are all in terms of either known quantities or the unknown cross-sectional

areas. Collecting terms, the linear system (24) is recovered.

The alternate formulation, "mass flow rate constrained problem" is derived analogously. We proceed by computing the exit location density via state:

$$\rho_3 = \frac{\gamma P_e}{\gamma R T_3} \qquad\qquad (A2.11)$$

and substituting into the momentum equation:

$$\dot{m}_1 u_1 \rho_1 A_1 + \rho_1 u_2{}^2 A_2 + p_2 A_2 = p_e A_3 + \frac{\gamma P_e}{\gamma R T_3} u_e{}^2 A_3 \qquad\qquad (A2.12)$$

The next step is to solve for the speed of sound using the energy equation placed in the form:

$$\dot{m}_1 T_{01} + \rho_2 u_2 A_2 T_{02} = \frac{P_e}{R} u_e A_3 \left(1 + \frac{\gamma-1}{2} \frac{u_e{}^2}{\gamma R T_3}\right) \qquad\qquad (A2.13)$$

to yield:

$$\gamma R T_3 = \frac{\dfrac{\gamma-1}{2} u_e{}^2}{\dfrac{(\dot{m}_1 T_{01} + \rho_2 u_2 A_2 T_{02}) R}{p_e u_e A_3} - 1} \qquad\qquad (A2.14)$$

Back substitution into energy and substitution into momentum, (collecting terms) yield the matrix system, (28). Thus, we have arrived at the desired linear systems in terms of the cross-sectional areas. These systems represent the "fundamental" mathematical result of our inverse design analysis.

Appendix III
Summary Derivation of an "Analysis" Mixing Problem

As a simple test of consistency the "inverse" formulation was compared to the "forward" or analysis formulation. Although the analysis presented is restricted to constant area mixing (recognizing that this could be easily relaxed using the pressure closure relationship derived in Appendix I), it provides a useful test of the inverse design methodology. Further, it also illustrates the contrast in solution structure (notably non-linear versus linear) exhibited by the analysis method. This appendix briefly summarizes this formulation.

The governing equations(quasi-one-dimensional, integral relationships) are:

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3 \equiv W \tag{A3.1}$$

$$\dot{m}_1 u_1 + \dot{m}_2 u_2 + p_1 A_1 + p_2 A_2 = \dot{m}_3 u_3 + p_3 A_3 \equiv P_{mom} \tag{A3.2}$$

and

$$\dot{m}_1 T_{01} + \dot{m}_2 T_{02} = \dot{m}_3 T_{03} \equiv E \tag{A3.3}$$

Now, recall that for the analysis mixing problem geometry cross-sectional areas and the both upstream quantities are given (hence the definition of W,E and P). Our task in this problem is to compute the "fully" mixed conditions (subscript 3). Accordingly, we eliminate pressure and density through the state and mass conservation relationships:

$$p_3 = \rho_3 u_3 R \frac{T_3}{u_3} = W \frac{(\gamma R T_3)^{\frac{1}{2}}}{\gamma A_3} \frac{1}{M_3} \qquad (A3.4)$$

and noting:

$$\dot{m}_3 u_3 = W(\gamma R T_3)^{\frac{1}{2}} M_3 \qquad (A3.5)$$

With these quantities specified, we may substitute into the momentum equation:

$$P_{mom} = \frac{W(\gamma R T_3)^{\frac{1}{2}}}{\gamma} \frac{1}{M_3} + W(\gamma R T_3)^{\frac{1}{2}} M_3 \qquad (A3.6)$$

rewriting and squaring both sides:

$$M_3{}^2 \left(\frac{P_{mom}}{W}\right)^2 \gamma^2 = \gamma R T_3 [1 + \gamma M_3{}^2]^2 \qquad (A3.7)$$

Now, eliminating the speed of sound through the energy equation:

$$\gamma R T_3 = \frac{\gamma R \frac{E}{W}}{1 + \frac{\gamma - 1}{2} M_3{}^2} \qquad (A3.8)$$

we may write (after collecting terms) the non-linear (4th order relationship) for the exit Mach number:

$$\left[\gamma^2 \left(1 - \frac{G}{2}\right) + \frac{G\gamma}{2}\right] M_3{}^4 + \gamma(2 - G) M_3{}^2 + 1 = 0 \qquad (A3.9)$$

where "G" is the non-dimensional grouping:

$$G \equiv \frac{P^2_{mom}}{REW}$$
(A3.10)

The solution of the "4th" order non-linear equation is really quite simple when we recognize that the substitution $M_3^2 = Z$, immediately reduces the equation to a quadratic with two roots. These two roots denote the supersonic and subsonic solutions of the mixing process (the negative roots are obviously trivial). It would go well beyond the scope of this Appendix to discuss this solution further. The other quantities, velocity, temperature, etc are available by back substitution. Although this analysis did not play a direct role in the inverse design problem, it did provide a useful check, and point of reference for the inverse methodologies.
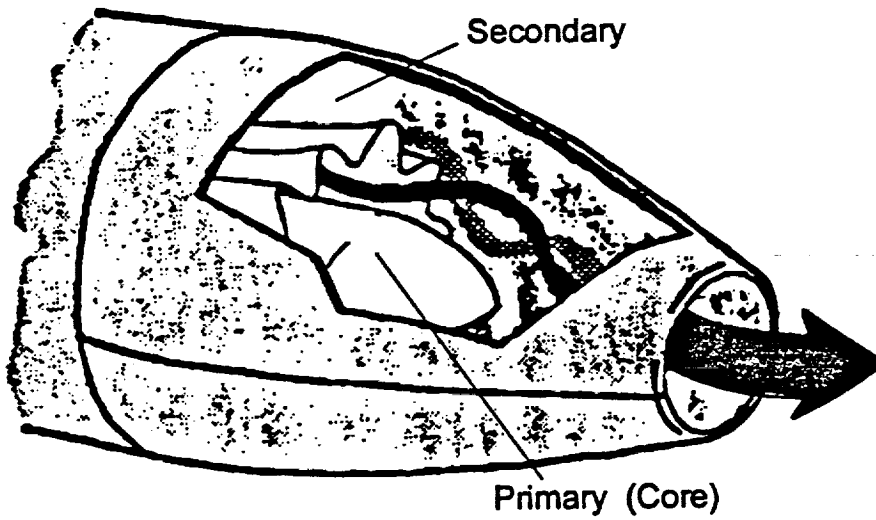
Figure 1. Forced Mixer on a Turbofan Engine.
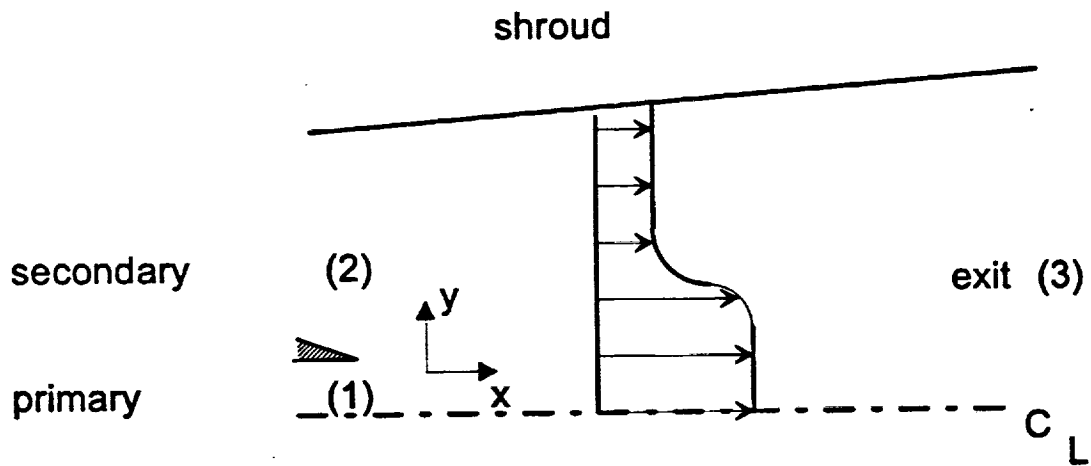
Figure 2. Ejector Schematic.
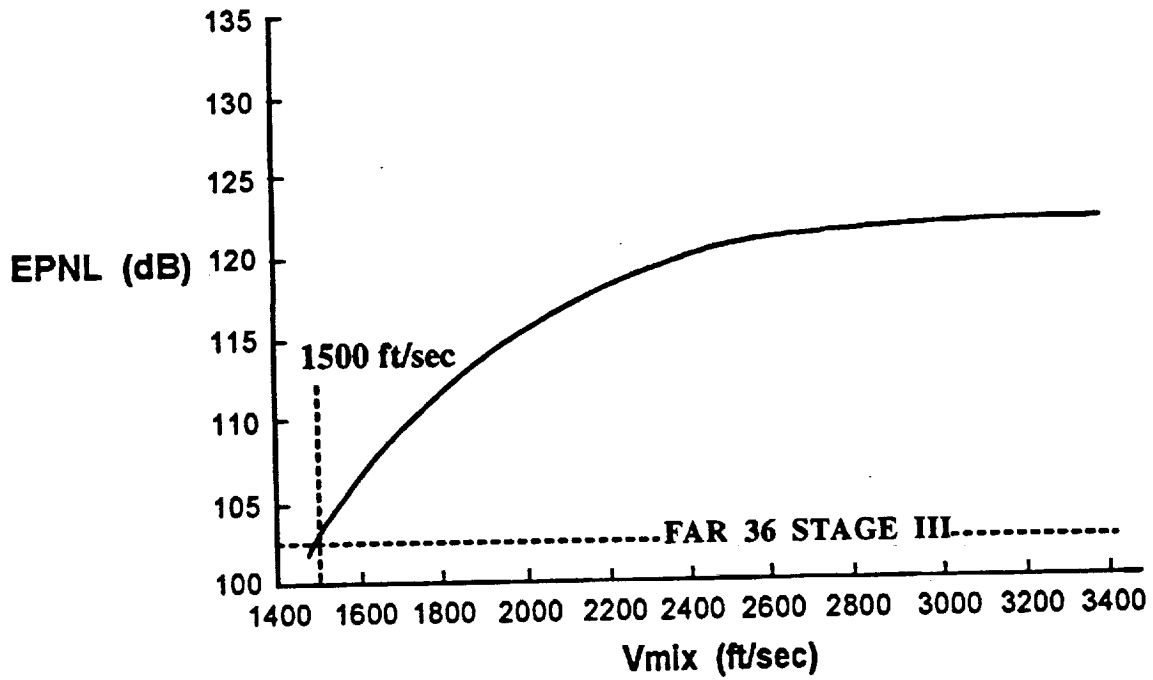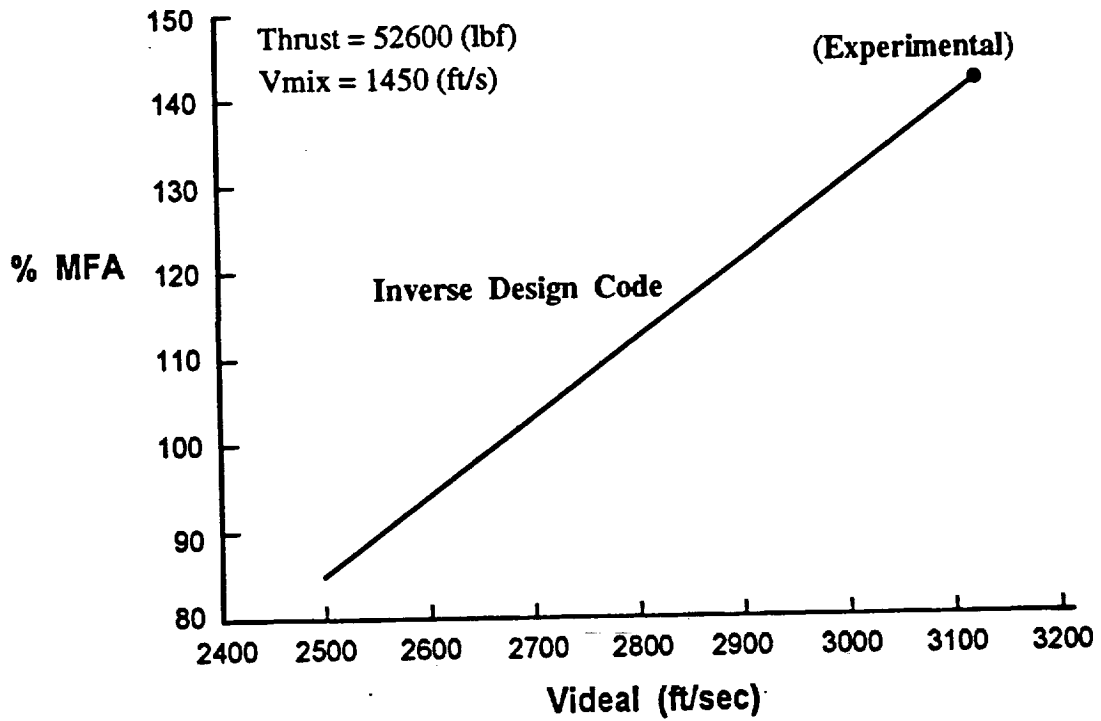
Figure 3. FAR36 Stage III Requirements.

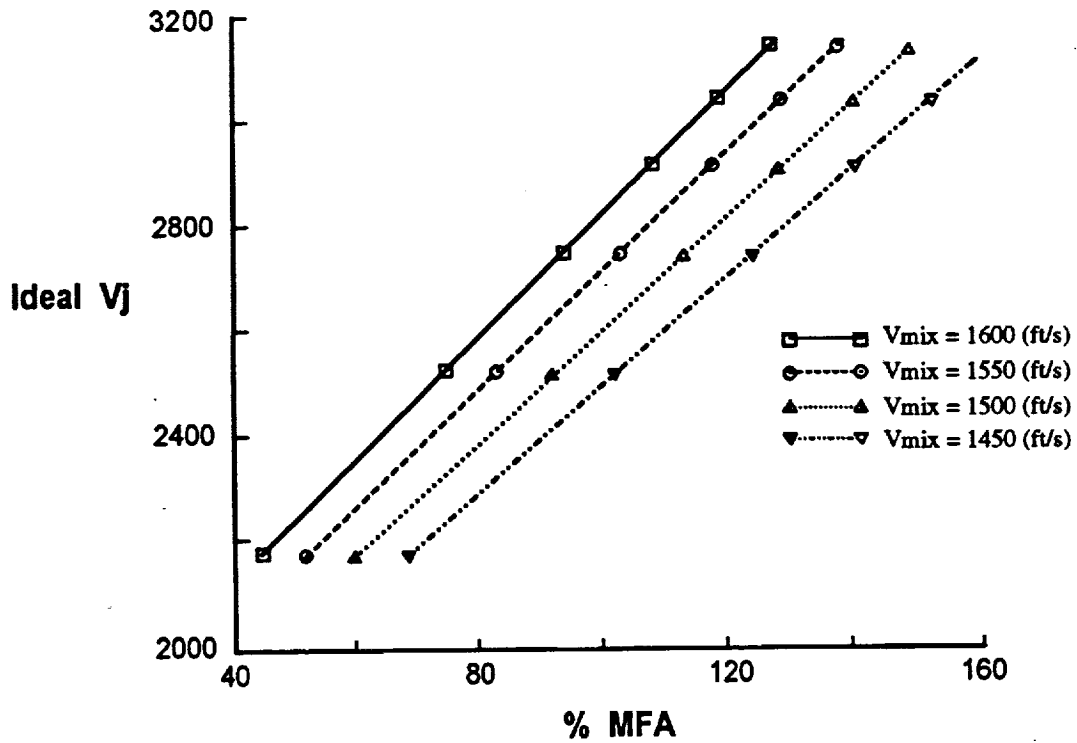Figure 4. Inverse Design Analysis/Experimental Comparison.

Figure 5. Exit Velocity Constraint Sensitivity,
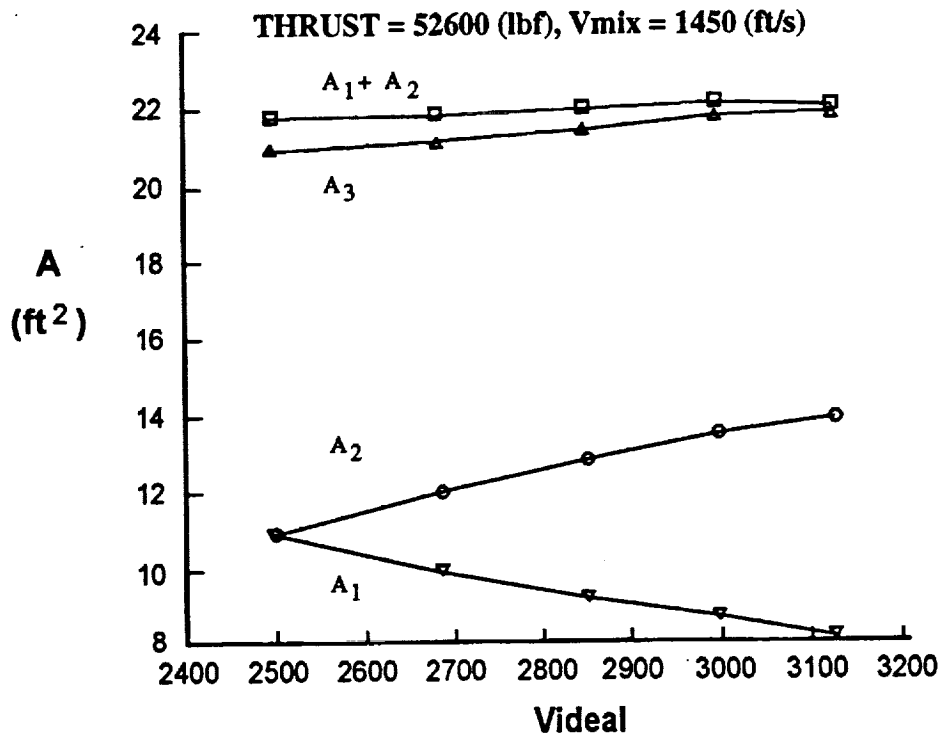Generic Nozzle, (J. Seidel, 1993).
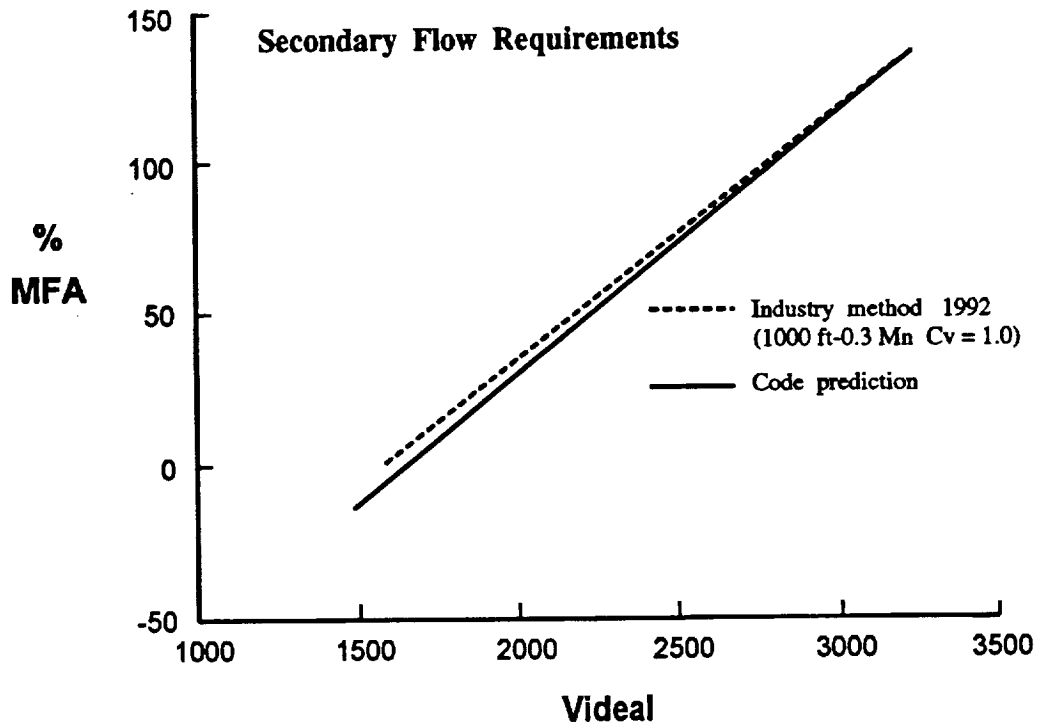

Figure 6. Geometry Prediction.

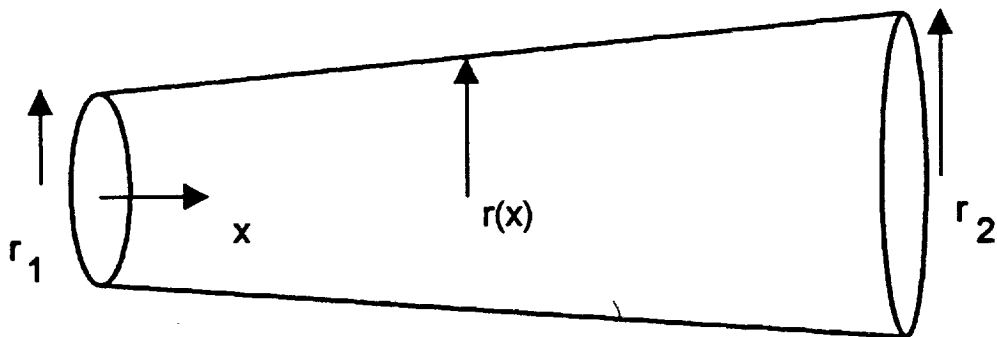Figure 7. Comparison to Analogous, Industry
Prediction Methods.

Figure 8. Linear Axi-symmetric (Conical) Geometry.

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13. ABSTRACT (Maximum 200 words)

A simple lumped parameter based inverse design tool has been developed which provides flow path geometry and entrainment estimates subject to operational, acoustic and design constraints. These constraints are manifested through specification of primary mass flow rate or ejector thrust, fully-mixed exit velocity and static pressure matching. Fundamentally, integral forms of the conservation equations coupled with the specified design constraints are combined to yield an easily invertible linear system in terms of the flow path cross-sectional areas. Entrainment is computed by back substitution. Initial comparison with experimental and analogous one-dimensional methods show good agreement. Thus, this simple inverse design code provides an analytically based, preliminary design tool with direct application to High Speed Civil Transport (HSCT) design studies.

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