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UNCERTAINTY ANALYSIS OF DIFFUSE-GRAY RADIATION ENCLOSURE PROBLEMS --A Hypersensitive Case Study

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ABSTRACT

An uncertainty analysis of diffuse-gray enclosure problems is presented. The genesis of this study was a diffuse-gray enclosure problem which proved to be hypersensitive to the specification of view factors. This genesis is discussed in some detail. The uncertainty analysis is presented for the general diffuse-gray enclosure problem and applied to the hypersensitive case study. It was found that the hypersensitivity could be greatly reduced by enforcing both closure and reciprocity for the view factors. The effects of uncertainties in the surface emissivities and temperatures are also investigated.

INTRODUCTION

All thermal analysis computations involve uncertainties. Geometries are imprecisely specified, thermal physical properties are not known exactly, and process data (boundary conditions) such as temperatures, pressures, and velocities are to some degree uncertain. Some of these uncertainties are a natural part of the process being modeled. The thermal-physical properties will naturally vary from point to point in physical space. The thermal conductivity will depend on such local conditions as impurity concentrations, grain structure, and voids in all but the purest and most carefully handled materials. Thermal radiation properties can vary considerably over a surface depending on factors such as roughness and oxidation. Also, the boundary conditions will not be precisely applied in the actual process. Other uncertainties result from a lack of input data. In the early design computation stages, field data may not have been collected and previous project experiences or handbook data must be used to estimate certain process conditions. Finally, all thermal analysis models rely ultimately on experimental measurements for material properties, boundary conditions, or design data bases and correlations. Experimental uncertainty is always present.

The treatment of experimental uncertainties is well developed. National and international standards for the treatment of measurement uncertainty have been published. The ANSI/ASME (1986) standard is one example. The book by Coleman and Steele (1989) gives a good review of current practices for experimental uncertainties. The treatment of thermal analysis uncertainties is not philosophically different from the treatment of measurement uncertainties. A set of basic rules (thermal analysis model/data reduction equation) is applied to a set of data (physical properties and boundary conditions/basic measurements) to produce a result. The goal of the uncertainty analysis is to follow the estimated or measured variances in the data through the rules into uncertainties in the result.

The nuclear engineering community routinely incorporates uncertainty analysis in reactor certification and design calculations and has developed a considerable body of literature on this subject. A recent series of articles in Nuclear Engineering and Design (Boyack et al., 1990, Wilson et al., 1990,

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and Wulff et al., 1990) are representative of activity in nuclear engineering. The book edited by Ronen (1988) is also a good source. The fields of sensitivity analysis in control theory (Cruz, 1973) and reliability based mechanical design (Rao, 1992) are closely related to uncertainty analysis.

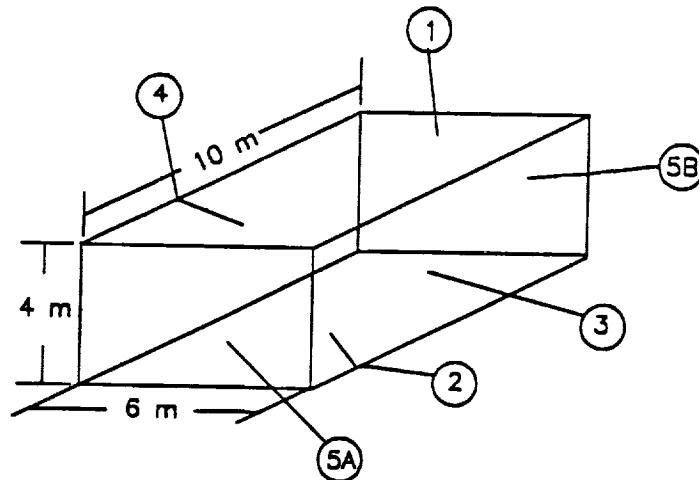
The use of uncertainty analysis in the mechanical and aerospace engineering thermal analysis community is still rather rare. Emery and Fadale (1990) and Fadale and Emery (1992) present analyses of uncertainties in finite element conduction heat transfer computations. Mehta (1991) discusses aspects of uncertainty in computational fluid dynamics.

This paper presents an uncertainty analysis of diffuse-gray radiation enclosures. Such problems contain uncertainties in the view factor matrix which arise from the geometric specification, in the material properties through the emissivities, and in the process specifications through the surface temperatures. Under the right (or wrong) conditions these uncertainties can have a profound effect on the computed heat flux results. The genesis of this study was a homework problem in the second heat transfer course at Mississippi State University. This genesis is discussed below. This is followed by the development and application of the uncertainty analysis and discussion.

GENESIS

The following problem from the heat transfer text by Incropera and Dewitt (1985) was assigned in the second heat transfer course at Mississippi State University during the Fall 1992 term.

- 13.62 A room is represented by the following enclosure, where the ceiling (1) has an emissivity of 0.8 and is maintained at 40°C by embedded electrical heating elements. Heaters are also used to maintain the floor (2) of emissivity 0.9 at 50°C . The right wall (3) of emissivity 0.7 reaches a temperature of 15°C on a cold, winter day. The left wall (4) and end walls (5A, 5B) are very well insulated. To simplify the analysis, treat the two end walls as a single surface (5). Assuming the surfaces are diffuse-gray, find the net radiation heat transfer from each surface.



Two students, Miguel and Simon, ignored the simplification and worked the problem as a six-sided enclosure. Miguel computed the view factors from the formulae for opposed parallel plates and perpendicular plates with a common edge and obtained the following view factor matrix:

$$\begin{bmatrix} 0.0 & 0.394 & 0.1921 & 0.1921 & 0.1109 & 0.1109 \\ 0.394 & 0.0 & 0.1921 & 0.1921 & 0.1109 & 0.1109 \\ 0.2881 & 0.2881 & 0.0 & 0.196 & 0.1139 & 0.1139 \\ 0.2881 & 0.2881 & 0.196 & 0.0 & 0.1139 & 0.1139 \\ 0.2774 & 0.2774 & 0.1898 & 0.1898 & 0.0 & 0.066 \\ 0.2774 & 0.2774 & 0.1898 & 0.1898 & 0.066 & 0.0 \end{bmatrix}$$

Simon, on the other hand, obtained values for the view factors from plots provided in the text and obtained the following view factor matrix:

$$\begin{bmatrix} 0.0 & 0.38 & 0.19 & 0.19 & 0.11 & 0.11 \\ 0.38 & 0.0 & 0.19 & 0.19 & 0.11 & 0.11 \\ 0.28 & 0.28 & 0.0 & 0.19 & 0.11 & 0.11 \\ 0.28 & 0.28 & 0.19 & 0.0 & 0.11 & 0.11 \\ 0.28 & 0.28 & 0.19 & 0.19 & 0.0 & 0.07 \\ 0.28 & 0.28 & 0.19 & 0.19 & 0.07 & 0.0 \end{bmatrix}$$

Both students used a diffuse-gray enclosure computer program to find the net radiation heat flux at each surface. This program was based on the net radiation method. This is a two step process. First the following equation is solved for the net radiosity vector, q_0 .

$$[I - (I - D_e)F]q_0 = \sigma D_e D_i i \quad (1)$$

where I is the identity matrix, D_e is a diagonal matrix with the surface emissivities as members, F is the view factor matrix, D_i is a diagonal matrix with surface temperatures for members, and i is a vector of 1's. The heat flux vector is then calculated from the net radiation energy balance.

$$q = (I - F)q_0 \quad (2)$$

Both students modeled the adiabatic surfaces as perfect reflections ($\epsilon = 0$). The results are summarized in Table 1.

Simon has slight errors in his view factors, but all in all they look very reasonable. All of the values are within a percent or two. The row sums of view factors are 0.98, 0.98, 0.97, 0.97, 1.01, and 1.01; so, the closure requirement is reasonably met. His radiosities are not seriously in error. The maximum error is 3.5%. However, his heat flux results, which are off by 376%, 18% and 13% for surfaces 1, 2, and 3, respectively, are profoundly in error. Also, his answers are in gross violation of global conservation of energy. For a steady-state analysis such as this one, the net energy stored in the enclosure should be zero. Miguel only has 10 w out of 5000 w left over which is a reasonable error. On the other hand, Simon has 2400 w out of 6000 w left over. Clearly something is terribly wrong. A quick independent check revealed that Simon had executed the program correctly. His radiosity results are indeed solutions of equation (1), and the problem is not numerical. At least not numerical in so far

Table 1. Comparison of Miguel's and Simon's Solutions

Surface	$q_b(w/m^2)$		$q(w/m^2)$		$Q(m)$	
	Miguel	Simon	Miguel	Simon	Miguel	Simon
1	546.2	542.7	-3.69	10.21	-221.4	612.4
2	609.0	607.3	83.87	99.32	5032.2	5959.2
3	442.6	435.9	-120.53	-105.00	-4821.2	-4200.0
4	543.3	524.4	0.0	0.0	0.0	0.0
5A	543.4	542.4	0.0	0.0	0.0	0.0
5B	543.4	542.4	0.0	0.0	0.0	0.0
				Total	-10.40	2371.6

as the evaluation of the inverse of the matrix in equation (1) goes. Simon's results are the proper solution to the problem as Simon posed it.

Simon's view factor matrix did not strictly enforce closure¹. It is common practice to force closure by only considering N-1 elements on each row to be independent and computing the remaining element from the closure rule. In fact, Brewster (1992) insists that not only closure but also reciprocity ($a_{ij}f_j = a_{ji}f_i$) must be enforced. He quotes avoidance of singular or poorly conditioned matrixes which cannot be inverted as the reason. As discussed above, inversion is not a problem in Simon's case. In fact, the matrix of coefficients, $[I - (I - D_o)F]$, which results with Simon's view factors, is well behaved with a condition number of 2.83, compared to a condition number of 2.85 using Miguel's view factors.

Closure is important philosophically and physically; so, we naively adjusted the diagonal elements in Simon's view factor matrix to force closure. The resulting view factor matrix was

$$\begin{bmatrix} 0.02 & 0.38 & 0.19 & 0.19 & 0.11 & 0.11 \\ 0.38 & 0.02 & 0.19 & 0.19 & 0.11 & 0.11 \\ 0.28 & 0.28 & 0.03 & 0.19 & 0.11 & 0.11 \\ 0.28 & 0.28 & 0.19 & 0.03 & 0.11 & 0.11 \\ 0.28 & 0.28 & 0.19 & 0.19 & -0.01 & 0.07 \\ 0.28 & 0.28 & 0.19 & 0.19 & 0.07 & -0.01 \end{bmatrix}$$

The physically unrealistic negative view factors were not corrected. Table 2 shows the revised results which are vastly improved. The heat flux errors are now 21%, 1%, and 2% for surfaces 1, 2, and 3, respectively. This result is an order of magnitude improvement. This result is somewhat surprising since we have enforced a closure where the individual view factors are even more in error. We have forced plane surfaces to see themselves and have forced physically unrealistic negative view factors. However, on the other hand, we have enforced an important physical constraint.

Anecdotically, we can surmise from this experience that this problem is hypersensitive to errors in the view factor specification when all NxN view factors are independent. However, a rather naive enforcement of closure greatly reduces this sensitivity.

¹As is well known, since a ray emitted from a surface must either strike that surface or one of the other surfaces in the enclosure, the rows of the view factor matrix must sum to unity, $\sum_{j=1}^N f_{ij} = 1$.

Table 2. Simon's revised results

Surface	$q_o(w/m^2)$		$q(w/m^2)$		Q(m)	
	Miguel	Simon	Miguel	Simon	Miguel	Simon
1	546.2	546.0	-3.69	-2.92	-221.4	-174.6
2	609.0	601.9	83.87	82.92	5032.2	4975.2
3	442.6	441.4	-120.53	-117.89	-4821.2	-4715.6
4	543.3	543.0	0.0	0.0	0.0	0.0
5A	543.4	543.0	0.0	0.0	0.0	0.0
5B	543.4	543.0	0.0	0.0	0.0	0.0
				Total	-10.40	85.6

The computations in equations (1) and (2) are readily amenable to an uncertainty analysis. The sensitivity of the radiosities and heat fluxes to each view factor, emissivity, and temperature can be computed and used with estimates of the uncertainties in the input data to determine estimates of the uncertainties in the computed values for radiosity and heat flux. Such an analysis provides a systematic way to investigate the problems that were apparent in the above discussion, provides a way of determining the source of the hypersensitivity, and provides a means to determine the fidelity of the input data required for a desired model accuracy. This uncertainty analysis is developed below.

UNCERTAINTY ANALYSIS

In the following, we discuss the propagation of uncertainties from the input into the result, the definition of the sensitivity coefficients, and the development of the relations needed to compute the sensitivity coefficients for this problem. In this investigation, uncertainties in view factors, emissivity, and temperatures are considered.

Uncertainty Propagation

The development of the first-order general uncertainty analysis is discussed in detail by Coleman and Steele (1988), and only the result is given here. If all of the uncertainties in the data are taken to be independent (no common or correlated sources of uncertainty), the uncertainties in the results are obtained by taking the root-sum-square of the product of the sensitivity coefficient and the input variable uncertainty.

$$\begin{aligned}
 U_{r_k}^2 = & \sum_{i=1}^N \sum_{j=1}^N (\Theta_{k,f_j})^2 U_{f_j}^2 \\
 & + \sum_{i=1}^N (\Theta_{k,e_i})^2 U_{e_i}^2 + \sum_{i=1}^N (\Theta_{k,t_i})^2 U_{t_i}^2
 \end{aligned} \tag{3}$$

Here the result, r_k , is either the heat flux, q_k , or the radiosity, $q_{o,k}$. The sensitivity coefficients are the first partial derivatives of the result with respect to each input variable. For the view factors

$$\theta_{k, f_j} = \frac{\partial r_k}{\partial f_j} \quad (4)$$

for the emissivities

$$\theta_{k, \epsilon_i} = \frac{\partial r_k}{\partial \epsilon_i} \quad (5)$$

and for the temperatures

$$\theta_{k, t_i} = \frac{\partial r_k}{\partial t_i} \quad (6)$$

The terms can be computed efficiently for small problems (considering algebra, programming, and computation time) by direct brute-force finite differences. The forward finite difference formula gives

$$\theta_{k, x} = \frac{r_k(x + \delta x) - r_k(x)}{\delta x} \quad (7)$$

For the six-surface enclosure considered by Miguel and Simon, this approach would require 49 complete solutions of the enclosure problem to compute the derivatives in equations (4), (5), and (6). For large problems this can become onerous. Also, simple forward differences can be troublesome if the scales are not considered. Fortunately, the direct computation of the sensitivity coefficients is straight forward.

Sensitivity Analysis

The sensitivity computations can be reduced to a series of matrix multiplications by direct expansion of equations (1) and (2). But first a brief consideration of their origin is in order. The radiosity on a surface can be written as the sum of the emitted and reflected radiation

$$q_s = \sigma D_s D_s^A t + (I - D_s) q_i \quad (8)$$

where q_i is the irradiation. For a diffuse enclosure the irradiation can be written in terms of the radiosities as

$$D_s q_i = F^T D_s q_s \quad (9)$$

where D_s is the diagonal matrix with the surface areas as members. Solving equation (9) for q_i , substituting into equation (8), and rearranging gives

$$[I - (I - D_s) D_s^{-1} F^T D_s] q_s = \sigma D_s D_s^A t \quad (10)$$

The net heat rate on the surfaces is given by the difference between the radiosity and irradiation as

$$D_s q = D_s q_s - D_s q_i \quad (11)$$

Using equation (9) and rearranging yields

$$q = (I - D_s^{-1} F^T D_s) q_s \quad (12)$$

Usually at this stage of the development the view factor reciprocity relationship

$$F^T D_s = D_s F \quad (13)$$

is substituted into equations (10) and (12) to give equations (1) and (2). However, in this investigation, we are interested in the sensitivity of this analysis to perturbations in the view factors where reciprocity is not strictly enforced. In this case, it is more appropriate to work with equations (10) and (12) directly so that the sensitivities are properly weighted.

A term-by-term differentiation of equation (10) with respect to f_{ij} gives

$$-(I - D_s) D_s^{-1} \frac{\partial F^T}{\partial f_{ij}} D_s q_s + [I - (I - D_s) D_s^{-1} F^T D_s] \frac{\partial q_s}{\partial f_{ij}} = 0 \quad (14)$$

which can be solved for the radiosity sensitivities using the matrix inverse

$$\frac{\partial q_s}{\partial f_{ij}} = [I - (I - D_s) D_s^{-1} F^T D_s]^{-1} [(I - D_s) D_s^{-1} \frac{\partial F^T}{\partial f_{ij}} D_s q_s] \quad (15)$$

A term-by-term differentiation of equation (12) results in

$$\frac{\partial q}{\partial f_{ij}} = -D_s^{-1} \frac{\partial F^T}{\partial f_{ij}} D_s q_s + (I - D_s^{-1} F^T D_s) \frac{\partial q_s}{\partial f_{ij}} \quad (16)$$

Likewise, for the sensitivities with respect to emissivity, a term-by-term differentiation of equation (10) gives

$$\frac{\partial D_s}{\partial \epsilon_i} D_s^{-1} F^T D_s q_s + [I - (I - D_s) D_s^{-1} F^T D_s] \frac{\partial q_s}{\partial \epsilon_i} = \sigma \frac{\partial D_s}{\partial \epsilon_i} D_s^A i \quad (17)$$

Solving for the radiosity sensitivities yields

$$\frac{\partial q_s}{\partial \epsilon_i} = [I - (I - D_s) D_s^{-1} F^T D_s]^{-1} [\sigma \frac{\partial D_s}{\partial \epsilon_i} D_s^A i - \frac{\partial D_s}{\partial \epsilon_i} D_s^{-1} F^T D_s q_s] \quad (18)$$

A term-by-term differentiation of equation (12) results in

$$\frac{\partial q}{\partial \epsilon_i} = (I - D_s^{-1} F^T D_s) \frac{\partial q_s}{\partial \epsilon_i} \quad (19)$$

Finally, for the sensitivities with respect to temperature, differentiation of equation (10) gives

$$[I - (I - D_s) D_s^{-1} F^T D_s] \frac{\partial q_s}{\partial \alpha_i} = \sigma D_s A D_s^{-1} \mathbf{1}_i \quad (20)$$

where $\mathbf{1}_i$ is a vector with 1 at location i and zeros otherwise. Solving for the sensitivities

$$\frac{\partial q_s}{\partial \alpha_i} = [I - (I - D_s) D_s^{-1} F^T D_s]^{-1} [\sigma D_s A D_s^{-1} \mathbf{1}_i] \quad (21)$$

Equation (12) yields as before

$$\frac{\partial q}{\partial \alpha_i} = (I - D_s^{-1} F^T D_s) \frac{\partial q_s}{\partial \alpha_i} \quad (22)$$

The calculation procedure for heat flux and uncertainties is as follows:

- 1) Invert $[I - (I - D_s) D_s^{-1} F^T D_s]^{-1}$
- 2) Compute q_s by multiplication with $\sigma D_s A D_s^{-1} \mathbf{1}_i$
- 3) Compute q using equation (12)
- 4) Compute the radiosity derivatives using equations (15), (18), and (21).
- 5) Compute the heat flux derivatives using equations (16), (19), and (22).
- 6) Compute the uncertainties using equation (3).

In this procedure, only one matrix inversion is required. All of the other computations involve only matrix multiplication.

APPLICATION FOR MIGUEL AND SIMON'S PROBLEM

The uncertainty analysis discussed above was added to the diffuse-gray enclosure computer program, and the analysis was carried out using Miguel's input data. For sensitivities with respect to the view factors, four cases are considered: a) all view factors are independently specified, 2) closure is enforced alone, 3) reciprocity is enforced alone, and 4) both closure and reciprocity are enforced simultaneously. That discussion is followed by an examination of the sensitivities with respect to emissivity and temperature and the overall uncertainty problem.

The difference in the four cases for view factor results from the formation of the view factor matrix transpose F^T . If all view factors are independent (closure and reciprocity not enforced) the derivative, $\partial F^T / \partial f_{ij}$, only has one nonzero element, a 1 at place (j,i) . Table 3 shows the normalized sensitivities with respect to the view factors for the heat flux on the three active surfaces. Considering that the view factors are of order 1, the heat flux on surface 1 is seen to be hypersensitive to uncertainties in view factors from the first column of F^T and strongly sensitive to the other view factor uncertainties. The heat flux on the second surface is much better behaved but still has a strong sensitivity to uncertainty

Table 3: Normalized sensitivities with respect to view factors for the active surfaces in Miguel and Simon's problem with neither closure nor reciprocity enforced.

Surface	$\theta_{k,j}/q_k$					
k = 1	124.17	7.47	18.21	53.73	53.72	53.72
	138.46	8.32	20.30	59.91	59.89	59.89
	67.08	4.03	9.84	29.02	29.02	29.02
	82.35	4.95	12.08	35.63	35.62	35.62
	49.41	2.97	7.24	21.38	21.37	21.37
	49.41	2.97	7.24	21.38	21.37	21.37
k = 2	-0.74	-6.02	-0.94	-2.76	-2.76	-2.76
	-0.82	-6.71	-1.04	-3.08	-3.08	-3.08
	-0.40	-3.25	-0.51	-1.49	-1.49	-1.49
	-0.49	-3.99	-0.62	-1.83	-1.83	-1.83
	-0.29	-2.40	-0.37	-1.10	-1.10	-1.10
	-0.29	-2.40	-0.37	-1.10	-1.10	-1.10
k = 3	0.33	0.17	4.99	1.45	1.45	1.45
	0.36	0.19	5.56	1.62	1.62	1.62
	0.18	0.09	2.69	0.78	0.79	0.79
	0.22	0.11	3.31	0.96	0.96	0.96
	0.13	0.07	1.98	0.58	0.58	0.58
	0.13	0.07	1.98	0.58	0.58	0.58

in the second column of F^T . The third surface is relatively less sensitive but is by no means insensitive. The third column has normalized sensitivities of order 3, and the other view factors have normalized sensitivities of order 0.1 to 1.0.

This clearly shows the origin of Simon's difficulty. If all view-factor uncertainties are assumed to be equal ($U_{f_{ij}} = \text{const}$) in equation (3) and all uncertainties in emissivity and temperature are ignored, the uncertainty in heat flux is given by

$$\frac{U_{q_k}}{q_k} = U_{f_{ij}} \sqrt{\sum_{i=1}^N \sum_{j=1}^N \frac{\theta_{k,j}^2}{q_k}} \quad (23)$$

which gives for each active surface

$$\frac{U_{q_1}}{q_1} = 283.74 U_{f_{ij}} \quad (24)$$

$$\frac{U_{q_2}}{q_2} = 14.12 U_{f_{ij}} \quad (25)$$

$$\frac{U_{q_3}}{q_3} = 10.20 U_{f_{ij}} \quad (26)$$

Table 4: Normalized sensitivities with respect to view factors for the active surfaces in Miguel and Simon's problem with closure enforced.

Surface	$\Theta_{k,j}/q_k$					
k = 1	----	-118.73	-107.81	-71.67	-71.71	-71.71
	132.39	----	12.18	52.48	52.44	52.44
	58.23	-5.90	----	19.52	19.50	19.50
	47.53	-31.21	-23.96	----	-0.02	-0.02
	28.52	-18.70	-14.36	0.01	----	0.00
	28.52	-18.70	-14.36	0.01	0.00	----
k = 2	----	-5.28	-0.20	-2.02	-2.02	-2.02
	5.89	----	5.67	3.63	3.64	3.64
	0.11	-2.75	----	-0.98	-0.98	-0.98
	1.34	-2.16	1.21	----	0.00	0.00
	0.80	-1.30	0.72	0.00	----	0.00
	0.80	-1.30	0.72	0.00	0.00	----
k = 3	----	-0.16	4.66	1.13	1.13	1.13
	0.17	----	5.37	1.43	1.43	1.43
	-2.52	-2.60	----	-1.91	-1.91	-1.91
	-0.75	-0.85	2.34	----	0.00	0.00
	-0.45	-0.51	1.41	0.00	----	0.00
	-0.45	-0.51	1.41	0.00	0.00	----

To obtain 5% accuracy in q_1 , U_{f_y} must be less than 0.0001546, or the view factors must be known with approximately 4 digit accuracy. On the other hand, to obtain 5% accuracy in q_3 requires U_{f_y} less than 0.0057 or about 2 digit accuracy. This is in line with Simon's experience. His maximum error was 0.014 on F_{12} and F_{21} . This resulted in 376% error for q_1 but only 13% error for q_3 .

If closure is enforced by computing the diagonal elements from

$$f_{ii} = 1 - \sum_{\substack{j=1 \\ j \neq i}}^N f_{ij} \quad (27)$$

the derivative $\partial F^T / \partial f_{ij}$ contains two nonzero terms—a -1 at place (i,i) and a 1 at place (j,i). Table 4 shows the results of the sensitivity analysis for heat flux when closure is enforced. The table shows that closure alone does not reduce the overall sensitivity. Normalized sensitivity coefficients of 100 are still found. In our previous work with Simon's solution it appeared that enforcing closure greatly improved the results. However, equation 1 implicitly assumes that reciprocity exists and Simon's view factors reasonably meet that reciprocity requirement.

When reciprocity is imposed cross-diagonal terms in F are related by

$$a_i f_{ij} = a_j f_{ji} \quad (28)$$

In this case, view factors in the lower-left triangle of F are computed from those in the upper-right triangle by equation (28). Table 5 shows the results of the sensitivity analysis when only reciprocity is enforced. No improvement is seen. In fact, for this case the overall uncertainty in heat flux would be higher than the case where reciprocity is not enforced.

Table 5: Normalized sensitivities with respect to view factors for the active surfaces in Miguel and Simon's problem with reciprocity enforced.

Surface	$\theta_{k,f_{ij}}/q_k$					
k = 1	124.91	146.78	119.53	178.32	178.26	178.26
	-----	8.37	26.51	67.74	67.70	67.70
	-----	-----	9.89	41.35	41.33	41.33
	-----	-----	-----	35.85	71.67	71.67
	-----	-----	-----	-----	21.49	42.98
	-----	-----	-----	-----	-----	21.49
k = 2	-0.74	-6.84	-1.53	-3.49	-3.49	-3.49
	-----	-6.71	-5.92	-9.07	-9.06	-9.06
	-----	-----	-0.51	-2.11	-2.11	-2.11
	-----	-----	-----	-1.83	-3.66	-3.66
	-----	-----	-----	-----	-1.10	-2.19
	-----	-----	-----	-----	-----	-1.10
k = 3	0.33	0.53	5.25	1.77	1.78	1.78
	-----	0.19	5.69	1.79	1.79	1.79
	-----	-----	2.69	4.09	4.09	4.09
	-----	-----	-----	0.96	1.93	1.93
	-----	-----	-----	-----	0.58	1.16
	-----	-----	-----	-----	-----	0.58

Table 6 shows the results when both reciprocity and closure are simultaneously enforced using equations (27) and (28). An order of magnitude decrease in the sensitivities is observed. If the uncertainties in f_{ij} are again taken to be constant and the uncertainties in emissivity and temperature are ignored, the uncertainties for the active surfaces are given by

$$\frac{U_{q_1}}{q_1} = 27.78 U_{f_{ij}} \quad (29)$$

$$\frac{U_{q_2}}{q_2} = 1.83 U_{f_{ij}} \quad (30)$$

$$\frac{U_{q_3}}{q_3} = 1.89 U_{f_{ij}} \quad (31)$$

Now, to obtain 5% accuracy in q_1 , the uncertainty in f_{ij} must be less than 0.0018 which is between 2 and 3 digit accuracy. Recall that, when Simon's view factors were revised to enforce closure, reciprocity was implicitly included in equation (1), and the error for q_1 was 21%, which is in line with equation (29).

The sensitivity analysis has given us a great deal of insight into the hypersensitivity of Simon's problem. It has improved our understanding of why enforcing closure in Simon's case so greatly improved the problem and has shown that this improvement actually requires the simultaneous enforcement of both closure and reciprocity for the view factors. We can draw the conclusion that both closure and reciprocity should be strictly enforced to minimize the sensitivity of the diffuse-gray enclosure analysis to errors in view factors.

For simple geometries such as this one, view factor determination can be made with whatever accuracy is necessary. However, the material properties, emissivities, process specifications, and

Table 6: Normalized sensitivities with respect to view factors for the active surfaces in Miguel and Simon's problem with both closure and reciprocity enforced.

Surface	$\theta_{k,U}/q_k$					
k = 1	----	13.66	-20.46	-0.38	-0.39	-0.39
	----	----	3.33	5.67	5.67	5.67
	----	----	----	-4.45	-4.44	-4.44
	----	----	----	----	0.00	0.00
	----	----	----	----	----	0.00
k = 2	----	0.61	-0.04	-0.01	-0.01	-0.01
	----	----	1.55	0.39	0.39	0.39
	----	----	----	0.22	0.22	0.22
	----	----	----	----	0.00	0.00
	----	----	----	----	----	0.00
k = 3	----	0.02	0.88	0.01	0.01	0.01
	----	----	1.47	0.15	0.15	0.15
	----	----	----	0.43	0.43	0.43
	----	----	----	----	0.00	0.00
	----	----	----	----	----	0.00

temperatures will always contain uncertainties. Table 7 shows the normalized sensitivities for this problem with respect to emissivity and temperature for the three active surfaces. The normalized sensitivities to errors in emissivity are on the order of 10 which is about the same as the view factor sensitivities with both closure and reciprocity enforced. The normalized sensitivity to errors in temperature are on the order of 1. Care must be taken when comparing these normalized sensitivities if the variables have vastly different scales. Emissivity is on the order of 1 while temperature in degrees K is on the order of 100-1000. An absolute error of 5°K will cause much more error in the heat flux result in this problem than a 0.05 error in emissivity.

Using the uncertainty values ($U_{f_{ij}} = 0.0001$; $U_{\epsilon_i} = 0.1$, $i = 1,2,3$; $U_{\epsilon_i} = 0$, $i = 4,5A,5B$; $U_{T_i} = 1^\circ\text{K}$, $i = 1,2,3$; and $U_{T_i} = 0$, $i = 4,5A,5B$) and Miguel's view factors with both reciprocity and closure enforced, gives the heat flux and uncertainty results shown in Table 8. The table shows that these very reasonable uncertainties result in significant uncertainties in the heat transfer result. The percentage uncertainty on the nearly adiabatic surface 1 is very large. These uncertainties are mainly caused by the uncertainties in emissivity and temperature since Miguel's very precise view factors were used.

CONCLUSIONS

Uncertainty analysis was used to propagate the uncertainties in the view factors, emissivities, and temperatures into uncertainties in the computed heat flux. This analysis allowed us to determine the nature and source of the hypersensitivity to view factor in Simon's case and to find a way to reduce this hypersensitivity. It was found that to avoid hypersensitivity to view factor specification both closure and reciprocity must be simultaneously enforced. Even when the view factors are precisely specified considerable uncertainty remains because of uncertainties in emissivity and temperature specification.

The sensitivity analysis and associated uncertainty analysis are very enlightening. The computational overhead is small since only one matrix is inverted for both the diffuse-gray enclosure solution and the sensitivity analysis. Therefore, it is strongly recommended that all diffuse-gray enclosure solutions be coupled with an uncertainty analysis.

Table 7: Normalized sensitivities with respect to emissivity and temperature for the active surfaces in Miguel and Simon's problem with both closure and reciprocity enforced.

Surface	$\theta_{k, \epsilon_i} / q_k$					
	1	2	3	4	5A	5B
	emissivity					
k = 1	1.01	12.97	-12.97	0.01	0.01	0.01
k = 2	0.03	0.84	0.65	0.00	0.00	0.00
k = 3	-0.01	0.29	1.27	0.00	0.00	0.00
	temperature					
k = 1	-1.24	0.96	0.29	0.00	0.00	0.00
k = 2	-0.04	0.06	-0.01	0.00	0.00	0.00
k = 3	0.02	0.02	-0.03	0.00	0.00	0.00

Table 8: Overall uncertainty in Miguel and Simon's Problem: $U_{f_u} = 0.0001$, $U_{\epsilon_i} = 0.1$, $U_{\epsilon_i} = 1^\circ\text{K}$.

	Surface		
	1	2	3
q(w)	-3.63	83.9	-120.5
$U_q(w)$	± 8.82	± 10.9	± 16.4
U_q/q	$\pm 270\%$	$\pm 13\%$	$\pm 14\%$

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