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RESPONSE-COEFFICIENT METHOD FOR HEAT-CONDUCTION TRANSIENTS WITH TIME-DEPENDENT INPUTS

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SUMMARY

A theoretical overview of the response-coefficient method for heat-conduction transients with time-dependent input forcing functions is presented with a number of illustrative applications. The method may be the most convenient and economical if the same problem is to be solved many times with different input-time histories or if the solution time is relatively long. The method is applicable to a wide variety of problems (including irregular geometries, position-dependent boundary conditions, position-dependent physical properties, and nonperiodic irregular input histories). Nonuniform internal energy generation rates within the structure can also be handled by the method. The area of interest is long-time solutions (in which initial condition is unimportant) and not the early transient period. The method can be applied to one-dimensional problems in cartesian, cylindrical, and spherical coordinates as well as to two-dimensional problems in cartesian and cylindrical coordinates.

THEORETICAL OVERVIEW

The analytical formulation of the heat-conduction problems (in cartesian coordinates) covered by this paper is given in the appendix A (ref. 3). A problem may have any combination of the following four time-dependent inputs: specified boundary temperature, specified boundary heat flux or heat flow, specified ambient temperature in a convective boundary condition, and specified internal energy generation rate within the structure. One may be required to determine outputs such as selected surface temperatures or boundary heat fluxes as a function of time in response to the time-dependent input forcing functions in the problem.

After the problem is discretized spatially by using either finite differences or finite elements, the resulting system of first-order ordinary differential equations in time with constant coefficients can be expressed as (ref. 2)

$$\underline{H}\,\underline{T}' + \underline{S}\,\underline{T} = \underline{F} \qquad (1)$$

where $\underline{H}$ is the capacitance matrix, $\underline{T}$ is the temperature vector containing nodal temperatures, $\underline{S}$ is the conductance matrix, and $\underline{F}$ is the input vector containing time-dependent and constant input values. The vector $\underline{T}'$ contains time derivatives of nodal temperatures. The matrices $\underline{H}$ and $\underline{S}$ are constant but the vector $\underline{F}$

is time dependent.

The response-coefficient method utilizes the exact solution to equation (1) rather than following the standard approach used by the finite-difference or finite-element method. This involves using an integrating factor or Duhamel's theorem , and solving a generalized eigenvalue problem. Since this paper will also discuss a number of response-coefficient applications, see ref. 3 for the details of theoretical development.

After expressing the input vector $\underline{F}$ in equation (1) as a function of time-dependent and constant inputs, the integration in the exact solution can be carried out. If a time interval is selected and if each time-dependent input is assumed to be linear within the time interval, it is possible to obtain current values of nodal temperatures as a function of current and previous values of time-dependent inputs as well as constant input values. The initial condition will not be important for long-time solutions because only the most recent input history should be relevant in finding current values of the selected outputs.

Typically one is interested only in a selected temperature or heat flow in the system. There are two ways of writing the output equations. It is possible to express current values of the desired outputs in terms of current and previous values of time-dependent inputs as well as constant input values. It is also possible to incorporate previously-computed outputs into the computations of the current outputs to facilitate calculations. Equations (B1) and (B2) for these two options are given in the appendix B. Equation (B2) is preferred because it involves much fewer calculations compared to equation (B1). The coefficients in equation (B2) (the $\underline{B}$, the C, and $\underline{D}$) are called "response coefficients".

Once all of the response coefficients have been determined in a problem, it is possible to keep track of the desired outputs as a function of time by the repeated use of equation (B2). It should be noted that equation (B2) can handle multiple inputs and multiple outputs. If there is only one input and one output, then the $\underline{B}$ will become scalar numbers.

ONE-DIMENSIONAL EXAMPLE (CARTESIAN COORDINATES)

The response-coefficient method can be used to handle one-dimensional composite wall structures with nonuniform internal energy generation within the structure. Consider a homogeneous wall with convective boundaries on both sides given in the appendix C. The ambient temperature on the left-hand side is the time-dependent input and the ambient temperature on the right-hand side is constant. Let the desired output be the heat flux at the right-hand side. We have one time-dependent input, one constant input, and one output in this problem. Therefore, the response coefficients will all be scalars, as can be seen in the appendix C. A detailed discussion of this example problem and the computation of the output based on a given input history can be found in ref. (3).

ONE-DIMENSIONAL EXAMPLE (CYLINDRICAL COORDINATES)

The response-coefficient method can be used to handle one-dimensional composite cylinders (solid or hollow) with nonuniform internal energy generation within the structure. Consider a long circular cylinder made of steel (Appendix D). Let the surface temperature be time dependent. The output is the surface heat flow per unit length. We have one time-dependent input and one output in this problem. Therefore, the response coefficients will all be scalars, as can be seen in the appendix D. More details of this example problem and the computation of the output based on a given nonperiodic input history can be found in ref. (4).

ONE-DIMENSIONAL PROBLEM (SPHERICAL COORDINATES)

The response-coefficient method can be used to handle one-dimensional composite spheres (solid or hollow) with nonuniform internal energy generation within the structure. Ref. (5) describes a hollow steel sphere covered with asbestos. The inner surface of this composite structure has a time-dependent specified temperature and the outer surface is exposed to a time-dependent ambient temperature (convective heat-transfer coefficient known). Assuming that the input-time histories are given on an hourly basis, it is desired to find the hourly variation of the heat flux at the outer surface. There are two time-dependent inputs and one output in this problem. More details of this example problem and the response coefficients are given in ref. (5).

TWO-DIMENSIONAL EXAMPLE PROBLEM (CYLINDRICAL COORDINATES)

The response-coefficient method can be used to handle two-dimensional composite structures in cylindrical coordinates (r,z) with nonuniform internal energy generation within the structure. Ref. (6) determines the annual heat loss through the walls and floor of a buried solar energy storage tank with water as the storage medium. In this example, a vertical cylindrical tank has a water level at the ground surface. The insulation and the earth surrounding the tank is the conduction system. The temperature variation is in the r- and z-directions in this axially-symmetric problem. Some assumptions need to be made to find the solution. A time interval of two weeks is used to approximate the yearly variation of the ambient temperature. The data for the problem, the response coefficients, and the variation of the instantaneous heat loss (from which annual heat loss is determined) are given in ref. (6).

DISCUSSION

For a given problem and time interval, the response coefficients need to be found only once. They do not depend on any particular input-time history. For this reason, if the same problem is to be solved many times with different input-time histories, the response-coefficient method may be advantageous. If the solution is needed for a very long period of time, this would also make the response-coefficient method advantageous because arithmetical operations required by equation (B2) can be carried out easily once the coefficients are found. Moreover, there is no stability problem in this method since the exact solution to equation (1) is utilized.

The response-coefficient method can be used to handle three-dimensional problems as well; however, currently it has not been developed to do that. The method has not been developed to handle time- or temperature-dependent thermal conductivity or convective heat-transfer coefficient. Another limitation is that radiation boundary condition cannot be handled at the present time.

REFERENCES

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APPENDIX A

This paper is concerned with heat-conduction transients described by the partial differential equation:

$$\frac{\partial}{\partial x}\left(k\frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial t}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial t}{\partial z}\right) + g''' = \rho c \frac{\partial t}{\partial \theta} \qquad (A1)$$

where k, ρ, and c can be position dependent and g''' may depend upon both position and time. There will be no restriction to simple geometrical shapes. The conditions on the boundaries of the region may be combinations of convection, specified heat flux, or specified temperature as shown by equations (A2-A4).

$$\textit{Convection:} \quad -k\frac{\partial t}{\partial n} = h(t_\infty - t) \qquad (A2)$$

$$\textit{Specified heat flux:} \quad -k\frac{\partial t}{\partial n} = q_s'' \qquad (A3)$$

$$\textit{Specified temperature:} \quad t = t_s \qquad (A4)$$

The convective heat-transfer coefficient may depend upon position but not upon time. The quantities t_∞, q_s'', and t_s may depend upon both position and time. In fact, the primary purpose of this paper is to discuss problems in which t_∞, q_s'', t_s, and g''' are prescribed functions of time. Since long-time solutions rather than initial transients are of interest, the initial condition will be unimportant.

APPENDIX B

The output vector for current values ($\underline{u}^{(0)}$) can be expressed as a function of the input vectors for current($\underline{s}^{(0)}$ and previous values ($\underline{s}^{(\mu)}$) as well as constant input vector ($\underline{p}$) as follows:

$$\underline{u}^{(0)} = \underline{A}_0\underline{s}^{(0)} + \sum_{\mu=1} \underline{A}_\mu\underline{s}^{(\mu)} + \underline{E}\underline{p} \qquad (B1)$$

In this equation, μ is a time-step index that is zero at the present time and increases by one for each time-step backward in time. See ref.(1) for the expressions for the coefficient matrices (the $\underline{A}$ and $\underline{E}$).

After incorporating previously-computed outputs into the computation of the current outputs, equation (B1) takes the form:

$$\underline{u}^{(0)} = \underline{B}_0\underline{s}^{(0)} + \sum_{\mu=1} \underline{B}_\mu\underline{s}^{(\mu)} + \sum_{\mu=1} C_\mu\underline{u}^{(\mu)} + \underline{D}\underline{p} \quad (B2)$$

In this equation $\underline{u}^{(\mu)}$ represents the previous output vector at the time index μ. See ref.(1) for the expressions for the $\underline{B}$, the C, and $\underline{D}$; which are named as "response coefficients". The summations have a finite number of terms because the $\underline{B}$ and the C get smaller and smaller as μ increases. The total number of the $\underline{B}$ and the C in equation (B2) is much smaller than the total number of the $\underline{A}$ in equation (B1). The number of rows in the $\underline{B}$ will be equal to the number of different outputs desired and the number of columns in $\underline{B}$ will be equal to the number of different time-dependent inputs.

APPENDIX C

Length and physical properties of the homogeneous wall

$$L = 8 \text{ in.}$$
$$k = 0.6 \text{ Btu/hr-ft-F}$$
$$\rho = 61 \ \ell bm/ft^3$$
$$c = 0.2 \text{ Btu}/\ell\text{bm-F}$$

Convective heat-transfer coefficient on each side

$$h = 1.46 \text{ Btu/hr-ft}^2\text{-F}$$

Time interval

$$\Delta\theta = 1 \text{ hr}$$

Response coefficients

$$B_0 = 0.004$$
$$B_1 = 0.044$$
$$B_2 = 0.031$$
$$B_3 = 0.002$$
$$C_1 = 0.944$$
$$C_2 = -0.144$$
$$C_3 = 0.001$$
$$D = -0.080$$

APPENDIX D

Radius and physical properities of long circular cylinder

$$R = 10 \text{ cm}$$
$$k = 0.03 \text{ kW/m-C}$$
$$\rho = 8000 \text{ kg/m}^3$$
$$c = 0.5 \text{ kJ/kg-C}$$

Time interval

$$\Delta\theta = 5 \text{ minutes}$$

Response coefficients

$$B_0 = -0.340$$
$$B_1 = 0.376$$
$$B_2 = -0.036$$
$$C_1 = 0.274$$