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J-INTEGRAL PATCH FOR FINITE ELEMENT ANALYSIS OF DYNAMIC FRACTURE DUE TO IMPACT OF PRESSURE VESSELS

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1. Introduction

Prediction of whether a pressurized cylinder will fail catastrophically when impacted by a projectile has important applications ranging from perforation of airplane's skin by a failed turbine blade to meteorite impact of a space station habitation module. This report summarizes the accomplishment of one task for a project, whose aim is to simulate numerically the outcome of a high velocity impact of pressure vessels. A finite element patch covering a vicinity of a growing crack has been constructed to estimate the J-integral (crack driving force) during the impact. Explicit expressions for the J-integral through the nodal values of displacement, strain, and stress have been written. The patch is to be used repeatedly to estimate the amount of crack growth during the the time of the impact. The resulting crack size is to be compared to an estimated critical crack size for the pressurized cylinder.

A literature search produced a number of papers dealing with evaluation of J-integral within finite element environment. Most of the research reports, however, present the shape of the finite element mesh only, with no detail on node locations. Such information was hard to utilize in the absence of an automated mesh generator. As a result, the simplest mesh was chosen for the patches, following (2). The same search turned up studies of the accuracy of finite element J-integral evaluations as well as the effect of the choice of the contour of integration. This provided a rational basis for the choices made in the present work.

A complementary literature search has been done to collect data on fracture toughness of 2219 aluminum alloys, since this material property enters the employed crack growth criterion.

The third literature search concerned reports on high- and hypervelocity impact studies (both experimental and theoretical) to form a basis for comparison with the numerical simulations produced by the entire project.

Complete computational details and the three literature reviews have been left with Rene Ortega.

2. Circumferential and Axial Patches

Both patches have the shape of a rectangle with an edge crack mapped onto a portion of the cylinder's surface as shown in Fig 1. The finite element mesh consists of 8-node isoparametric elements (1). Of these only the four which surround the crack tip are distorted, namely, the five nodes neighboring the crack tip are placed at the quarter distance from the tip instead of being half distance away (see Fig 2). Formulas shown in Fig 1 permit to find the 3D coordinates of any node.

3. J-integral expressions

J-integral is the following contour integral:

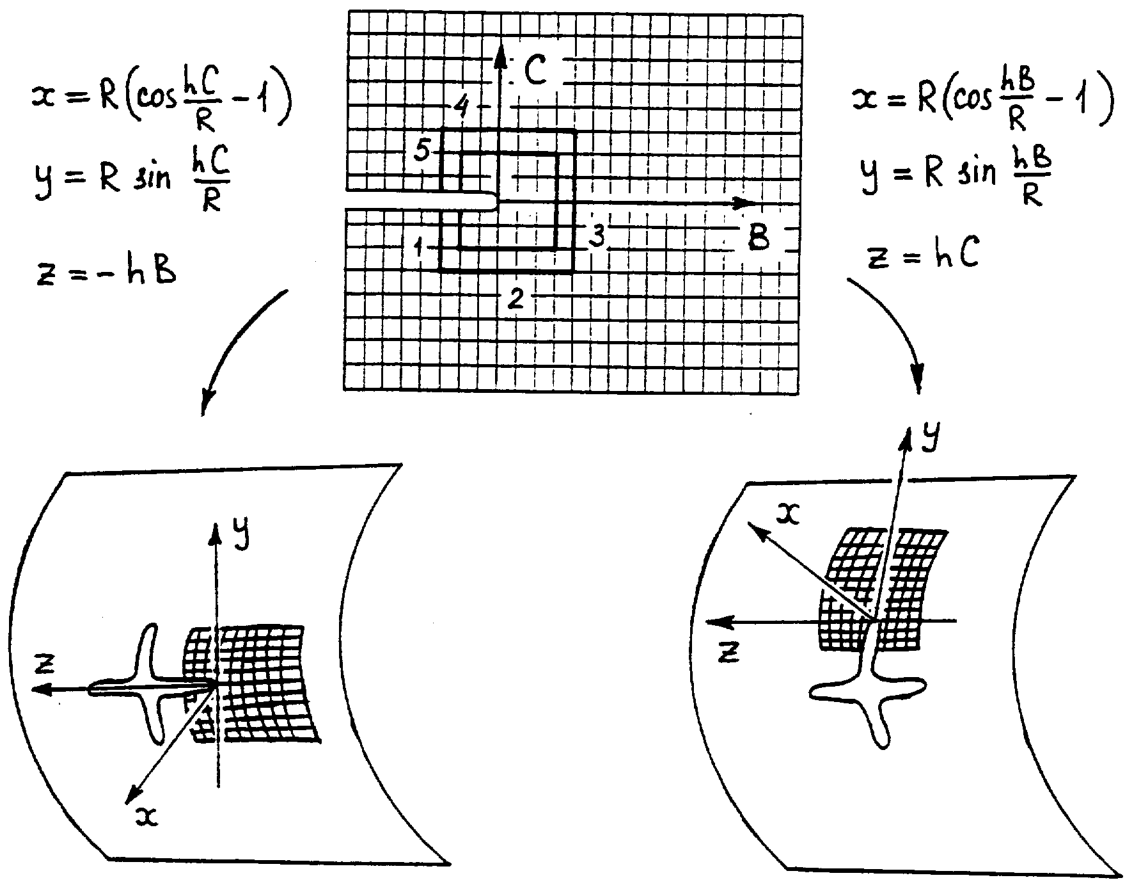

FIG. 1

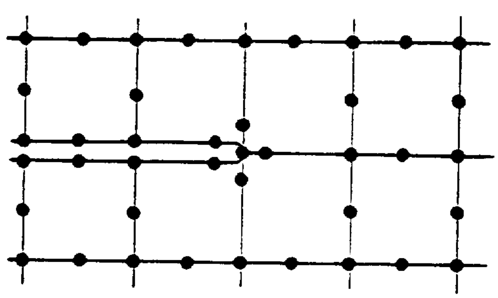

FIG. 2

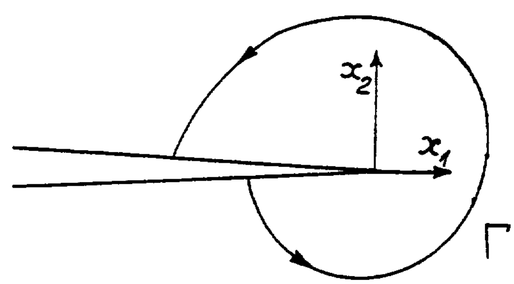

FIG. 3

$$J = \int_{\Gamma} (w\, n_1 - T_i \;\; \partial u_i / \partial x_1 \,) ds \qquad [1]$$

where w is the strain energy density, T_i is the traction, x_1 is the coordinate in the direction of the crack, and Γ is any contour that begins on one face of the crack and ends on the other (see Fig 3). The integral has the meaning of the potential energy release per unit crack advance (known as 'the energy release rate', or 'the crack driving force').

Explicit expressions for the J-integral through the nodal values of displacement, strain , and stress have been written for the two contours shown in Fig 1. The structure of those expressions is exemplified below for the inner contour.

Eq [1] is rewritten as

$$J = I_1 - I_2 \qquad [2]$$

where

$$I_1 = \int_{\Gamma} w \, dx_2 \qquad [3]$$

and

$$I_2 = \int_{\Gamma} \sigma_{ij} \, (\partial u_i / \partial x_1) \, n_j \, ds \qquad [4]$$

The contour is split into five paths $\Gamma_1, \ldots, \Gamma_5$ (see Fig 1), and the integrals [3,4] become the sums of the integrals over these paths:

$$I_k = I_{k1} + \ldots + I_{k5} \, , \quad k = 1,2. \qquad [5]$$

As examples, the expressions for I_{11} and I_{21} through he nodal values of u_2, ε_{ij}, and σ_{ij} are shown here:

$$I_{11} = - (h/6) \, (w^{229} + 4w^{244} + 2w^{255} + 4w^{270} + w^{281}) \qquad [6]$$

$$I_{21} = - (h/6) \, (f^{229} + 4f^{244} + 2f^{255} + 4f^{270} + f^{281}) \qquad [7]$$

where h is the mesh size, the upper indecies refer to node numbers,

$$w = \sigma_{ij} \, \varepsilon_{ij} / 2 \qquad [8]$$

$$f = \sigma_{11} \, \varepsilon_{11} + \sigma_{12} \, (\partial u_2 / \partial x_1) \qquad [9]$$

and, as a matter of example, the expression for $\partial u_2 / \partial x_1$ through the nodal values of u_2 is shown:

$$(\partial u_2 / \partial x_1)^{244} = (1/2h)(u_2^{257} + u_2^{231} - u_2^{227} - u_2^{253}) + (1/h)(u_2^{254} + u_2^{243} + u_2^{228} - u_2^{230} - u_2^{245} - u_2^{256}) \qquad [10]$$

3. Testing of the patch

To verify the numerical procedures, comparison has been proposed with an existing solution for a rectangular plate with an edge crack parallel to the clamped edges (4).

4. Discussion

The energy release rate and its J-integral representation employed in this study corresponded to static (or slowly growing) crack, whereas the crack under consideration is a fast growing one. However, it is known that the energy release rate for a moving crack is related to the static one as $G^{dyn} = g(v)G^{stat}$, where g(v) is a monotonically decreasing function of the crack velocity v which goes from 1 at v = 0 to 0 at v reaching the Rayleigh wave speed (3). Therefore employing G^{stat} overestimates the crack driving force and thus is conservative when a possibility of a catastrophic failure of the cylinder is considered.

If, nevertheless, the estimates will result in unrealistically large crack sizes at the end of the duration of the impact, expressions for dynamic J-integrals and their evaluation in finite element environment are available (see the literature review).

Finite element models of elastic-plastic crack growth in the presence of both small and large scale yielding are also available in the literature (see the literature review).

Acknowledgment

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