Polarization Analysis of a Balloon-Borne Solar Magnetograph

Final Report for

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Daniel J. Reiley

Russell A. Chipman (PI)

Physics Department

School of Science

University of Alabama in Huntsville

Huntsville, AL 35899

(205)895-6417x318

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I. Abstract

The 10^{-5} polarization specification for the Balloon-borne Vector Magnetograph (BVM) can be met. The 10^{-5} specification is shown to be a limitation on the diattenuation and retardance along the chief ray path through the optical system, such that the magnitude of the polarization aberration piston term is constrained to be less than $.5 \cdot 10^{-5}$. Coating specification sheets are provided which will ensure that the polarization sensitivity of the BVM will be less than 10^{-5}. An optical design is provided for a vector magnetograph. Finally, to provide a concrete mathematical meaning for polarization sensitivity, the polarization aberration matrix is averaged of the exit pupil, showing that the coupling between circular and linear states depends only on the magnitude of the polarization aberration piston term.

II. Polarization

Since our last meeting, we have completely re-calculated the polarization analysis of the Balloon-borne Vector Magnetograph (BVM). This analysis reflects our deeper understanding of the 10^{-5} polarization specification, and includes improved polarization specification sheets, which should allow cost savings analogous to the savings we allowed by moving from custom lenses to catalog lenses.

The mirrors and lenses in optical systems can change light's polarization state. Since solar magnetographs operate by measuring the polarization state of the solar disc, the effect of the mirrors and lenses on the polarization state must be well-characterized.

One way to characterize mirrors' and lenses polarization properties is to use polarization aberration theory. Polarization aberration theory is applied to a previous solar magnetograph design in the paper "Polarization analysis of the SAMEX solar magnetograph," which is included in this report as Appendix I.

Since the polarimeter used in this system will be a rotating-retarder polarimeter, the polarization state exiting the polarimeter will be constant. Therefore, the only optical elements which will affect the accuracy of the solar magnetic field measurements will be the polarization that is caused be the optics in front of the polarimeter.

This report includes specification sheets for coatings which are to be deposited on the optical surfaces in front of the polarimeter. These specifications will not only ensure that the system's polarization sensitivity will be less than 10^{-5}, but will also ensure that these coatings can be procured economically. Because limits on the polarization are rarely specified on coatings, vendors might only quote very expensive coatings.

The improved coating specification sheets eliminate this possibility because they include example coatings which meet the polarization specification. These example coatings are very simple, and should be very similar to stock coating designs which vendors commonly use for applications which aren't polarization-critical.

Because the BVM must accurately measure the linear polarized components of the light from the sum, the optics in front of the polarimeter must not couple circular polarization states into linear polarization states. The 10^{-5} polarization specification then means that, for circularly polarized light input to the system, the degree of linear polarization at any image point must be less than 10^{-5}.

For the simple example coatings described in the coating specification sheets, the maximum degree of linear polarization in the light incident on the polarimeter is $0.78 \cdot 10^{-5}$. For these coatings, several other polarization errors which we could identify are also small. When unpolarized light is incident, the maximum degree of polarization is $0.60 \cdot 10^{-5}$. When perfectly polarized light is incident, the degree of polarization is reduced by less than $0.01 \cdot 10^{-5}$.

III. Calculating the polarization errors

The theory behind the calculations on this instrument is called "Polarization aberration theory," and is presented in detail in Appendix I. For this analysis, polarization aberration theory was expanded to include the polarization aberration expansion in the form of a Mueller matrix.

To understand polarization aberration theory, one must first understand how the angle of incidence behaves at an interface. Figure 1 illustrates this behavior. The first column represents the angle of incidence and the orientation of the plane of incidence. The second column shows the geometry of the spherical wavefront incident on a spherical surface. Figure 1c shows the behavior of the angle of incidence for an on-axis object point. The angle of incidence increases linearly from the center of the pupil, and the orientation of the plane of incidence rotates twice around the pupil.

For off-axis object points, the geometry is still that of a spherical wavefront intersecting a spherical interface. Therefore, the angle of incidence will have the same form as the on-axis pattern. However, because the angle of incidence at the center of the wavefront is not zero, the pattern will be displaced from the center of the pupil.

To understand polarization aberration theory, one must also understand coating behavior at small angles of incidence. For small angles of incidence, both diattenuation and retardance are well-approximated by a quadratic, and go to zero at normal incidence.

The easiest way to understand each of the terms of the polarization aberration expansion is to first understand polarization aberration defocus. Figure 2 shows the a retardance pupil map for linear retardance defocus. The length of the lines represents the magnitude of the retardance at that point in the pupil. The orientation of the lines represents the orientation of the fast axis of the retardance.

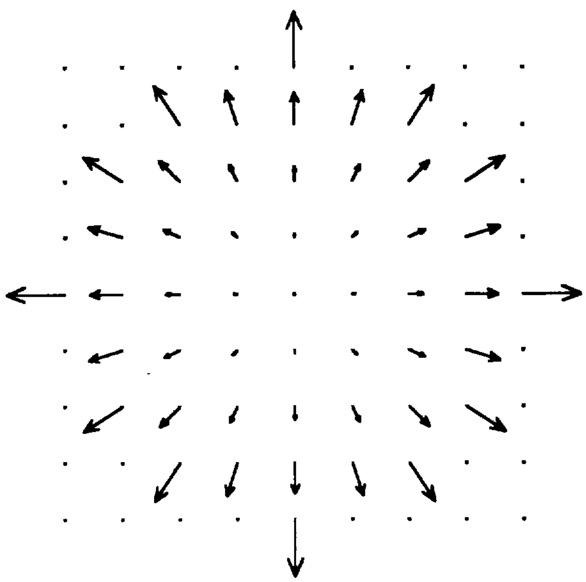


Figure 2

The magnitude of retardance increases quadratically with the radial pupil coordinate. This quadratic behavior is because:

1) angle of incidence increases linearly with radial pupil coordinate
2) retardance and diattenuation increase quadratically with angle of incidence

For off- axis object points, the retardance map has the same shape, but is no longer centered at the center of the pupil. This simple behavior is due to the simple form of the angle of incidence plots, shown in Figure 1. Figure 3 shows the retardance pupil map for an off-axis object point.

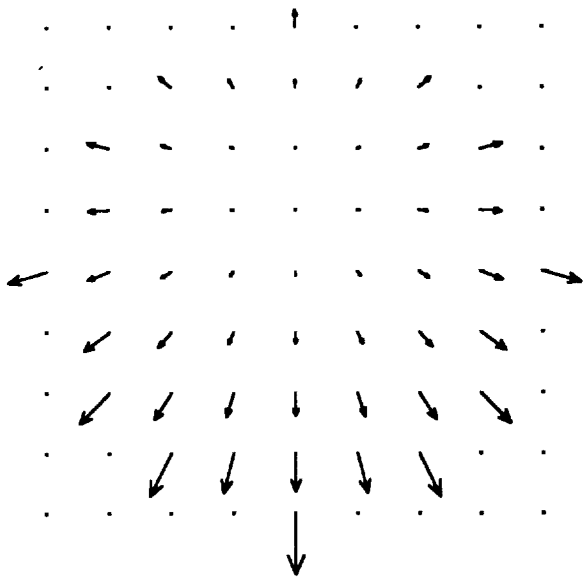


Figure 3

This off-axis retardance map can be decomposed into three components as follows: Because there is some retardance in the center of the pupil, we add piston, shown in Figure 4.

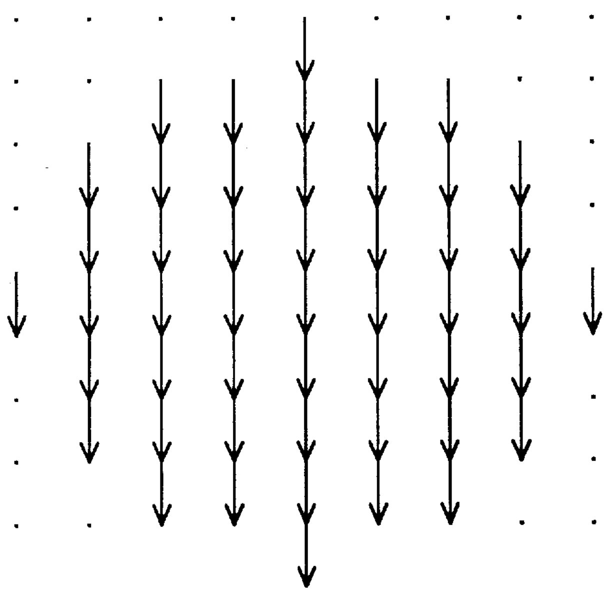


Figure 4

However, piston and defocus don't fully characterize the retardance map in Figure 3.; another term must be added to account for two characteristics of the retardance map. First, the top and the bottom of the pupil have different amounts of retardance. Second, the retardance along the x-axis in the pupil isn't parallel to the x-axis. Adding a single term, linear retardance tilt, approximates the retardance map shown in Figure 3. The linear retardance tilt term is shown graphically in Figure 5.

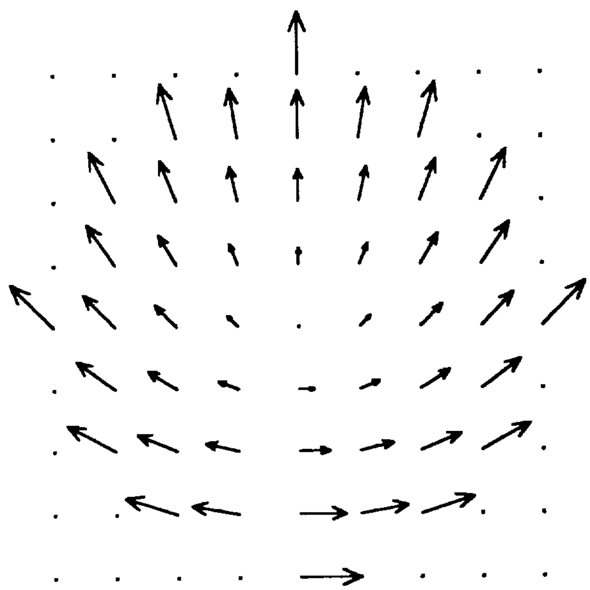


Figure 5

A mathematical description of these terms is given in Appendix I. Three complex coefficients, P1022, P1111, and P1200, are introduced to describe the magnitude of each of these terms. The real parts describe the magnitude of the diattenuation, and the imaginary parts describe the retardance.

The polarization aberration coefficients depend on the angles of incidence for the chief and marginal rays as well as the coatings on the various surfaces. With a tilted secondary mirror, the angles of incidence on the relevant surfaces are listed in Table 1.

Table 1

Angles of Incidence

(with secondary tilted .3°)

	marginal ray	chief ray
prefilter	0°	.16°
primary mirror	3.6°	.16°
secondary mirror	4.5°	.3°
front surface of lens	2.3°	2.2°
inner surface of lens	9.3°	4.7°
back surface of lens	6.5°	3.2°

For tilted and decentered optical systems, such as the BVM with an articulating secondary mirror, the meaning of the chief ray and the marginal ray are ambiguous. The find the chief ray angle of incidence on each surface, we traced rays through the center of the aperture stop to the center of the field of view and each edge of the field of view. We called the maximum angle of incidence on that surface the chief ray angle of incidence. This process was repeated for each of the relevant surfaces. A similar process was used to find the marginal ray angles of incidence.

Most coating design programs give intensity reflection or transmission coefficients, but the polarization aberration coefficients are defined in terms of amplitude reflection or transmission coefficients. The intensity reflection or transmission coefficients are related to the polarization aberration coefficients in the following way:

$$\begin{aligned} R_p(\theta_{c,i}) - R_s(\theta_{c,i}) &= r_p^2 - r_s^2 \\ &= (1-\eta_p)^2 - (1-\eta_s)^2 \\ &\cong 2(\eta_s - \eta_p) \\ &= 2(r_s(\theta_{c,i}) - r_p(\theta_{c,i})) \\ &= 4Re[P_{1200,i}], \end{aligned}$$

where R_s and R_p are the s- and p- intensity reflection coefficients, r_s and r_p are the s- and p- amplitude reflection coefficients, η_s and η_p are the small deviations of r_s and r_p from one,

$$\eta_s = 1 - r_s$$

$$\eta_p = 1 - r_p$$

$\theta_{c,i}$ is the chief ray angle of incidence on surface i, and $Re[P_{1200,i}]$ is contribution of surface i to the real part of polarization piston.

Similarly, most coating design programs give the s- and p- phase change on reflection or transmission in terms of degrees. The polarization aberration coefficients are given in terms of the difference in phase change in radians. The s- and p- phase change in degrees are related to the polarization aberration coefficients in the following way:

$$Im[P_{1200,i}] = \frac{1}{2}\frac{\pi}{180}(\delta_s(\theta_{c,i}) - \delta_p(\theta_{c,i})),$$

where δ_s and δ_p are the phase change on reflection or transmission in degrees, $\theta_{c,i}$ is the chief ray angle of incidence on surface i, and $Im[P_{1200,i}]$ is contribution of surface i to the imaginary part of polarization piston.

To find the total polarization piston for the system, use a coating program to calculate R_s, R_p, δ_s, and δ_p for each surface at the angle of incidence for the chief ray at that surface. Use the previous equations to find each surface's contribution to polarization piston, $P_{1200,i}$. Add the contributions from each surface to find the total polarization piston for the system:

$$P_{1200} = \sum_i P_{1200,i}.$$

IV. Specifying the coatings

To ensure that the polarization sensitivity for the BVM is within the 10^{-5} specification, we suggest the polarization budget listed in Table 2. This polarization budget, which is the basis for the coating specifications listed in Section VI, ensures that the degree of linear polarization is less than 10^{-5} when circularly polarized light is incident.

Table 2

Polarization Piston Budget

Prefilter		
filter side	$0.005\cdot 10^{-5} < Re\,P_{1200} < 0.04\cdot 10^{-5}$	$0 < Im\,P_{1200} < 0.12\cdot 10^{-5}$
other side	$0.005\cdot 10^{-5} < Re\,P_{1200} < 0.015\cdot 10^{-5}$	$0 < Im\,P_{1200} < 0.02\cdot 10^{-5}$
Primary mirror	$-0.01\cdot 10^{-5} < Re\,P_{1200} < 0$	$-0.07\cdot 10^{-5} < Im\,P_{1200} < -0.035\cdot 10^{-5}$
Secondary mirror	$-0.025\cdot 10^{-5} < Re\,P_{1200} < 0$	$-0.16\cdot 10^{-5} < Im\,P_{1200} < -0.1\cdot 10^{-5}$
Doub. (Front)	$0.35\cdot 10^{-5} < Re\,P_{1200} < 0.5\cdot 10^{-5}$	$0 < Im\,P_{1200} < 0.005\cdot 10^{-5}$
Doub. (int.) (fixed)	$Re\,P_{1200} = 0.08\cdot 10^{-5}$	$Im\,P_{1200} = 0$
Doub. (back)	$-0.83\cdot 10^{-5} < Re\,P_{1200} < -0.5\cdot 10^{-5}$	$0 < Im\,P_{1200} < 0.33\cdot 10^{-5}$

We also suggest that the mirrors have high reflectivity and the lens surfaces have high transmission, greater than 98% over the entire range of relevant incident angles and apertures.

Although we provide simple example coating designs which meet this specification, we choose not to recommend coating designs with this report because the most economical way to meet these specifications varies widely from vendor to vendor. Most vendors have stock coating designs which will meet these specifications, and will not need to charge for a coating design. To demonstrate the ease of designing coatings which meet these requirements,

Table 3

Polarization Piston Terms:

Simple coatings

Surface:	**Coating**	$Re P_{1200}$	$Im P_{1200}$
Pref. Front	$(LH)^4(HL)^4$	$0.03 \cdot 10^{-5}$	$0.10 \cdot 10^{-5}$
Pref. Back	QWOT MgF	$0.01 \cdot 10^{-5}$	$0.0004 \cdot 10^{-5}$
Primary	HLHLAl	$-0.005 \cdot 10^{-5}$	$-0.04 \cdot 10^{-5}$
Secondary	HLHLAl	$-0.02 \cdot 10^{-5}$	$-0.12 \cdot 10^{-5}$
Doub. Front	QWOT MgF	$0.39 \cdot 10^{-5}$	$0.002 \cdot 10^{-5}$
Doub.int. 1	none	$0.06 \cdot 10^{-5}$	0
Doub.Int. 2	none	$0.02 \cdot 10^{-5}$	0
Doub. Back	4 layer AR	$-0.79 \cdot 10^{-5}$	$0.30 \cdot 10^{-5}$
SUM		$-0.30 \cdot 10^{-5}$	$0.24 \cdot 10^{-5}$

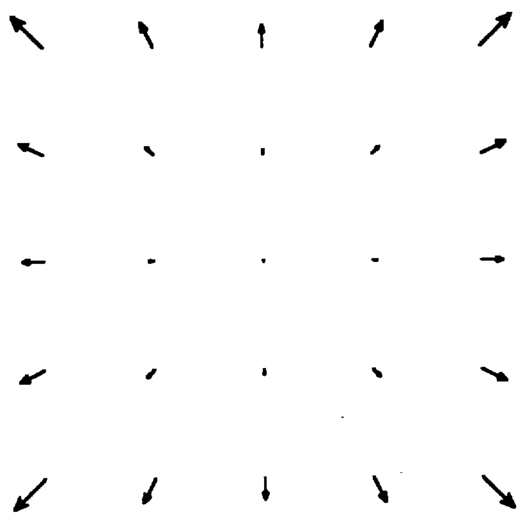
The coatings should be specified to degrade the surface figure by less than $\lambda/10$ RMS over the entire clear aperture, to prevent degradation of image quality. Surface quality should be specified to meet MIL-SPEC 60-40 scratch-dig specifications; this is a typical surface quality specification for scientific-grade optics. Surface durability should be specified to meet the MIL-SPEC scotch tape test and eraser test; this is a fairly stringent requirement, which is justified because this instrument will be used outdoors.

The following three pages describe the polarization errors graphically.

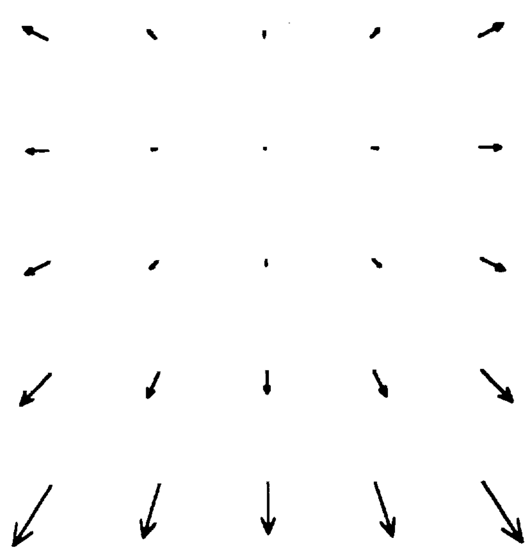
Diattenuation Vectors

In the image plane, the main polarization defect introduced by the coatings is the coupling between linear and circular states caused by P1200

For a centered system (no tilted secondary), the diattenuation and retardance vectors are radially oriented in the image plane, with quadratically increasing amplitude..

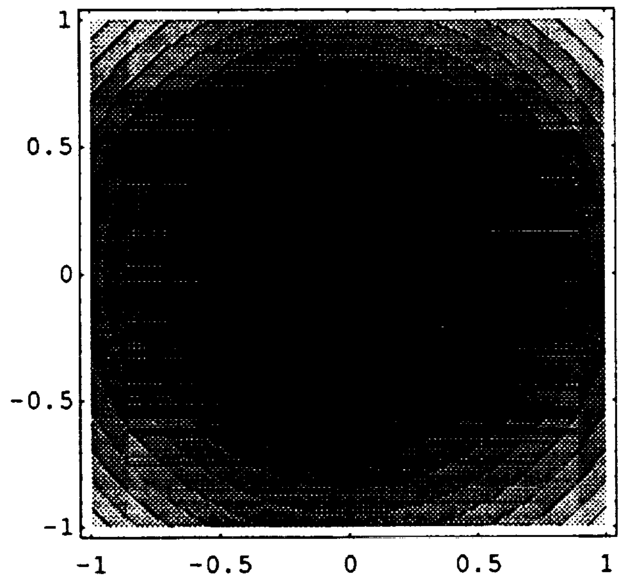


For a tilted secondary, the diattenuation and retardance vectors have the same pattern, but are not centered in the center of the field. The exact location of the center depends on the coatings and amouont of tilt, but this figure is reasonable. For the specified coatings, the magnitude of the largest line shown here is .8E-5.



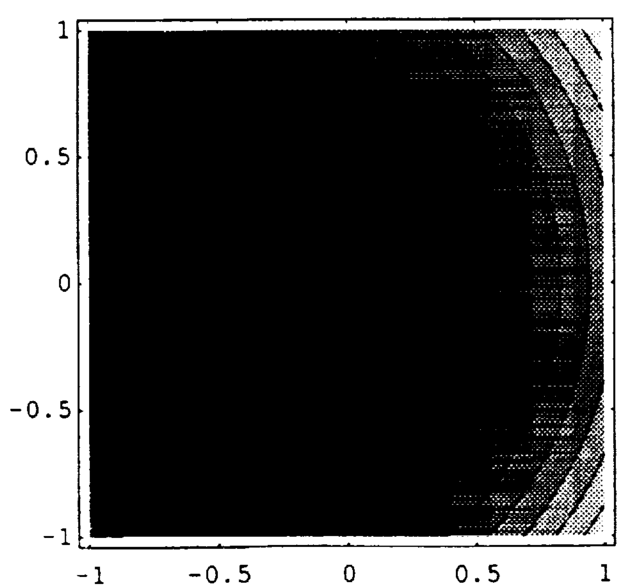
- **Effect of other polarization aberrations**

In the image plane, the effect of the other polarization aberrations is depolarization. For a radially symmetric system, this depolarization increases quadratically from the center of the field of view.



With a tilted secondary, this depolarization increases quadratically from some other point in the field of view.

The plot shown here is rotated 90 degrees with respect to the diattenuation plots.



For the specified coatings, the maximum contour line shown here would be less than 1E-7.

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V. Optical design of the EXVM

While working on the polarization analysis of the NASA-designed instrument, we noticed that it could be fabricated much more economically if catalog lenses were used instead of custom lenses. After discussing the possibility with Mona Hagyard, Allen Gary, and Ed West of NASA, we decided to redesign the optical system to take advantage of these lenses.

Catalog lenses are lenses which are made for common optical tasks such as collimating and focussing laser beams. Companies such as Melles-Griot, Spindler & Hoyer, JML, Newport, Edmund, and CVI have dedicated large amounts of capital to produce large numbers of commonly-requested focal lengths and diameters. Because these lenses are made on a regular basis, they are much less expensive than custom lenses of similar quality.

Table 4 lists the optical prescription for this system. Each of the lenses is a catalog lens. Figure 6 is a schematic of the system.

Table 4

Optical prescription for the EXVM

	RDY	THI	GLA	Clear Aperture
> OBJ:	INFINITY	INFINITY		
1:	INFINITY	0.001000	BK7_SCHOTT	306.653
2:	INFINITY	650.000000		306.653
STO:	-2394.73740 K: -1.000000	-877.239800	REFL	304.8
4:	-858.24060 K: -2.82538	1069.000000	REFL	83.9923
5:	INFINITY	7.000000	BK7_SCHOTT	24.1539
6:	INFINITY	5.000000		23.8961
7:	INFINITY	50.000000	BK7_SCHOTT	23.6162
8:	INFINITY	5.000000		21.7747
9:	INFINITY	6.000000	BK7_SCHOTT	21.4949
10:	INFINITY	5.000000		21.2739
11:	INFINITY	7.000000	BK7_SCHOTT	20.994
12:	INFINITY	5.000000		20.7362
13:	INFINITY	3.000000	BK7_SCHOTT	20.4563
14:	INFINITY	315.000000		20.3458
15:	718.39000	4.000000	SF8_SCHOTT	27.4918
16:	92.73000	6.600000	SSK4_SCHOTT	27.6017
17:	-128.08000	105.000000		27.8829

18:	INFINITY	300.000000	BK7_SCHOTT	20.9224
19:	INFINITY	33.500000		18.1876
20:	210.75000	5.000000	BAK4_SCHOTT	20.4624
21:	-81.29000	4.400000	F3_SCHOTT	20.5015
22:	-515.63000	75.000000		20.5668
23:	INFINITY	-128.880000	REFL	20.5172
	ADE: 45			
24:	141.25000	-4.800000	SF5_SCHOTT	20.4319
25:	47.31500	-3.000000	BK7_SCHOTT	20.7112
26:	-61.74800	-15.000000		20.7653
27:	INFINITY	0.010000	BK7_SCHOTT	23.7989
28:	INFINITY	-48.000000		23.7975
29:	INFINITY	-4.000000	BK7_SCHOTT	33.5049
30:	INFINITY	-48.000000		34.0372
31:	INFINITY	0.010000	BK7_SCHOTT	43.7446
32:	INFINITY	-112.017000		43.7432
33:	-188.36000	-12.500000	BK7_SCHOTT	66.3972
34:	139.24000	-6.000000	SF5_SCHOTT	66.5534
35:	415.67000	-55.000000		66.8955
36:	INFINITY	165.233309	REFL	66.142
	ADE: 45			
37:	INFINITY	4.000000	BK7_SCHOTT	63.8781
38:	INFINITY	12.000000		63.842
39:	INFINITY	21.000000	BK7_SCHOTT	63.6776
40:	INFINITY	0.000275		63.713
41:	INFINITY	18.000000	BK7_SCHOTT	63.713
42:	INFINITY	0.000000		63.8744
43:	INFINITY	21.000000	BK7_SCHOTT	63.8744
44:	INFINITY	12.000000		64.0626
45:	INFINITY	4.000000	BK7_SCHOTT	64.2261
46:	INFINITY	80.000000		64.2619
47:	673.17000	6.000000	SF5_SCHOTT	65.3518
48:	222.27000	10.000000	BK7_SCHOTT	65.1645
49:	-302.87000	439.000000		65.1296
50:	121.71195	3.800000	BK7_SCHOTT	13.7199
51:	-89.71796	2.500000	SF5_SCHOTT	13.2806
52:	-268.15906	131.029506		13.0547
53:	32.16000	4.460000	SK11_SCHOTT	17.9977
54:	-22.47000	1.500000	SF5_SCHOTT	17.4991
55:	-89.36000	34.925611		17.4214
56:	INFINITY	-120.845313	REFL	9.74839
	ADE: -15			
57:	-182.72000	-2.000000	SF5_SCHOTT	16.8009
58:	-44.88000	-4.800000	SK11_SCHOTT	16.9879
59:	64.04000	-72.000000		17.6003
60:	INFINITY	257.078523	REFL	20.7726
	ADE: 15			
61:	INFINITY	-72.000000	REFL	32.0994
	ADE: 45			
62:	INFINITY	108.000000	REFL	35.2717
	ADE: -45			
IMG:	INFINITY	0.000000		40.0302

SPECIFICATION DATA

EPD	304.80000		
DIM	MM		
WL	656.27	632.80	525.02
YAN	0.00000	0.05717	0.08167

REFRACTIVE INDICES

GLASS CODE	656.27	632.80	525.02
SSK4 SCHOTT	1.614266	1.615305	1.621923
SF8 SCHOTT	1.682505	1.684452	1.697362
BAK4 SCHOTT	1.565761	1.566704	1.572695
F3 SCHOTT	1.608063	1.609545	1.619244
BK7 SCHOTT	1.514323	1.515089	1.519867
SF5 SCHOTT	1.666612	1.668457	1.680666
SK11 SCHOTT	1.561011	1.561883	1.567374

INFINITE CONJUGATES
EFL -14063.1395
BFL 109.3923
FFL -0.3236E+06
FNO -46.1389
IMG DIS 108.0000
OAL 2301.9661
PARAXIAL IMAGE
HT 20.0458
ANG 0.0817
EXIT PUPIL
DIA 13.2190
THI -500.5187

Figure 6: Schematic of the EXVM

EXVM - Solar Magnetograph

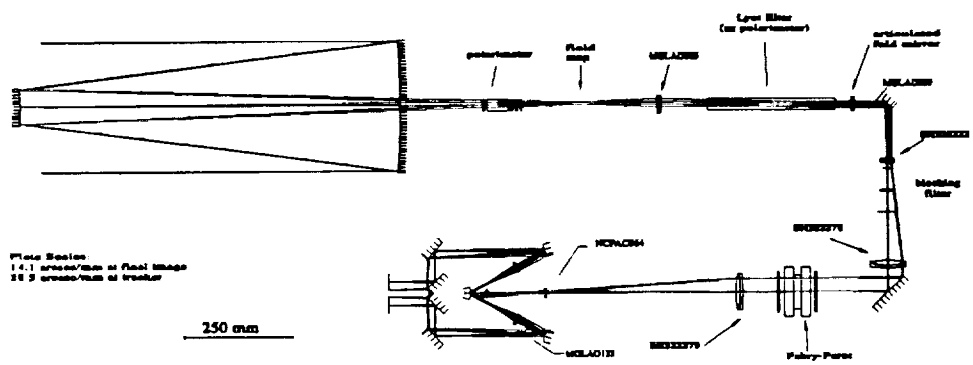


Table 5 lists the first-order properties of the system. HMY is the height of the marginal ray, UMY is the slope of the marginal ray, HCY is the height of the chief ray, and UCY is the slope of the chief ray. A second representation of the system's first-order properties is in Figure 7, the y-y bar diagram of the system.

Table 5

First-Order properties of the EXVM system

	HMY	UMY	HCY	UCY
EP	152.400000	0.000000	0.000000	0.001425
1	152.400000	0.000000	-0.926518	0.000938
2	152.400000	0.000000	-0.926517	0.001425
STO	152.400000	0.127279	0.000000	-0.001425
4	40.745715	-0.032327	1.250428	0.004339
5	6.187768	-0.021270	5.889186	0.002855
6	6.038879	-0.032327	5.909172	0.004339
7	5.877242	-0.021270	5.930868	0.002855
8	4.813749	-0.032327	6.073623	0.004339
9	4.652112	-0.021270	6.095319	0.002855
10	4.524493	-0.032327	6.112450	0.004339
11	4.362856	-0.021270	6.134146	0.002855
12	4.213967	-0.032327	6.154132	0.004339
13	4.052330	-0.021270	6.175829	0.002855
14	3.988520	-0.032327	6.184394	0.004339
15	-6.194598	-0.015503	7.551287	-0.001762
16	-6.256610	-0.019362	7.544239	0.001940
17	-6.384400	-0.000403	7.557043	-0.033548
18	-6.426724	-0.000265	4.034458	-0.022073
19	-6.506285	-0.000403	-2.587524	-0.033548
20	-6.519789	0.011009	-3.711396	-0.014919
21	-6.464743	0.008406	-3.785991	-0.015829
22	-6.427755	0.021331	-3.855639	-0.021001
23	-4.827902	-0.021331	-5.430683	0.021001
24	-2.078715	-0.006732	-8.137238	0.035827
25	-2.046401	-0.012020	-8.309207	0.021038
26	-2.010341	-0.001344	-8.372319	0.102462
27	-1.990187	-0.000884	-9.909252	0.067415
28	-1.990196	-0.001344	-9.908578	0.102462
29	-1.925704	-0.000884	-14.826762	0.067415
30	-1.922168	-0.001344	-15.096423	0.102462
31	-1.857677	-0.000884	-20.014606	0.067415
32	-1.857685	-0.001344	-20.013932	0.102462
33	-1.707182	-0.003984	-31.491437	0.010229
34	-1.657381	-0.002464	-31.619301	0.030977
35	-1.642596	-0.006831	-31.805163	-0.000019
36	-1.266885	0.006831	-31.804096	0.000019
37	-0.138158	0.004495	-31.800888	0.000013
38	-0.120180	0.006831	-31.800837	0.000019
39	-0.038206	0.004495	-31.800604	0.000013
40	0.056179	0.006831	-31.800336	0.000019
41	0.056181	0.004495	-31.800336	0.000013
42	0.137083	0.006831	-31.800106	0.000019
43	0.137083	0.004495	-31.800106	0.000013
44	0.231468	0.006831	-31.799838	0.000019
45	0.313442	0.004495	-31.799605	0.000013
46	0.331420	0.006831	-31.799554	0.000019
47	0.877909	0.003536	-31.798001	0.019142
48	0.899127	0.004338	-31.683148	0.006086
49	0.942511	0.004976	-31.622283	0.063529

50	3.127019	-0.005514	-3.732943	0.052290
51	3.106067	-0.001674	-3.534241	0.043518
52	3.101882	-0.010687	-3.425446	0.081834
53	1.701589	-0.025971	7.297245	-0.029926
54	1.585757	-0.019463	7.163775	-0.006418
55	1.556562	-0.044568	7.154148	-0.065280
56	0.000000	0.044568	4.874194	0.065280
57	-5.385824	0.014580	-3.014621	0.032160
58	-5.414985	0.024355	-3.078942	0.039443
59	-5.531890	-0.010837	-3.268270	0.032867
60	-4.751638	0.010837	-5.634672	-0.032867
61	-1.965719	-0.010837	-14.083994	0.032867
62	-1.185466	0.010837	-16.450397	-0.032867
IMG	-0.015088	0.010837	-20.000000	-0.032867

Figure 7: y- y bar diagram of the EXVM.

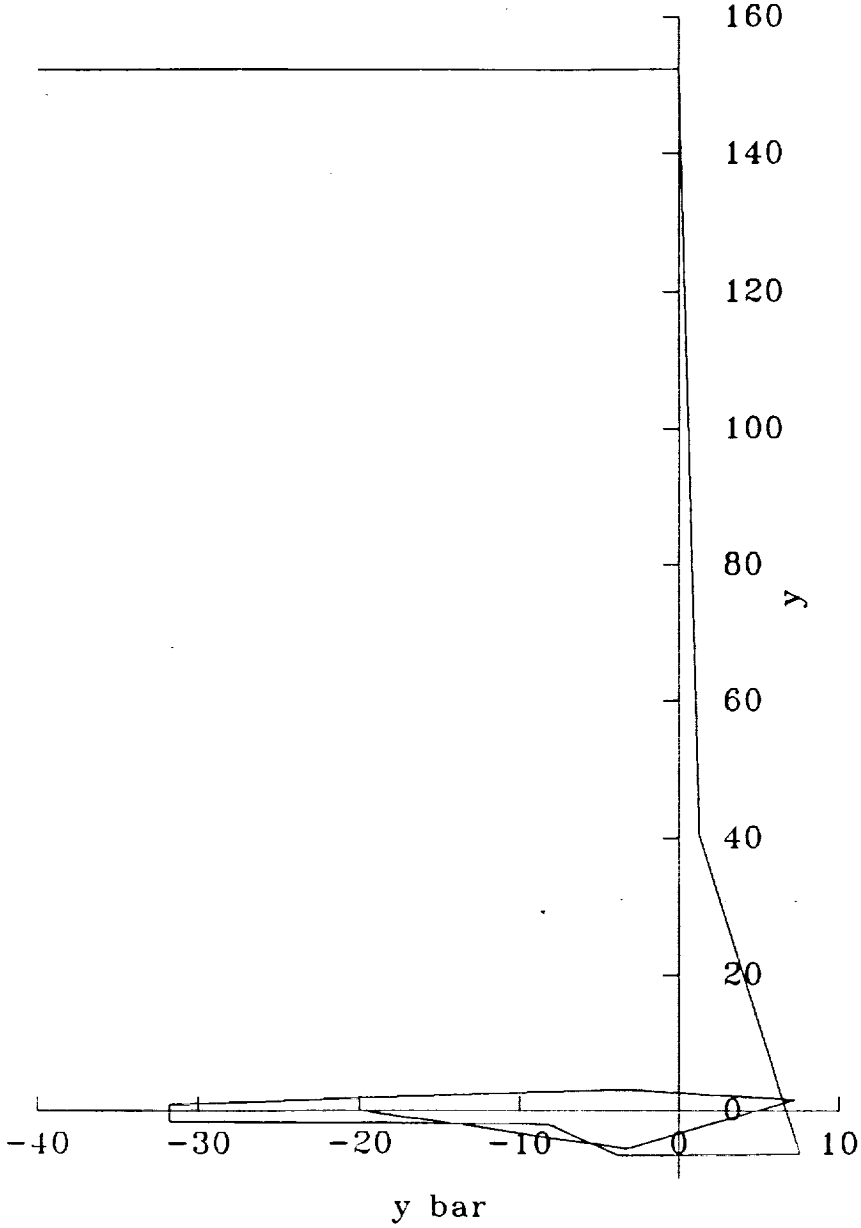


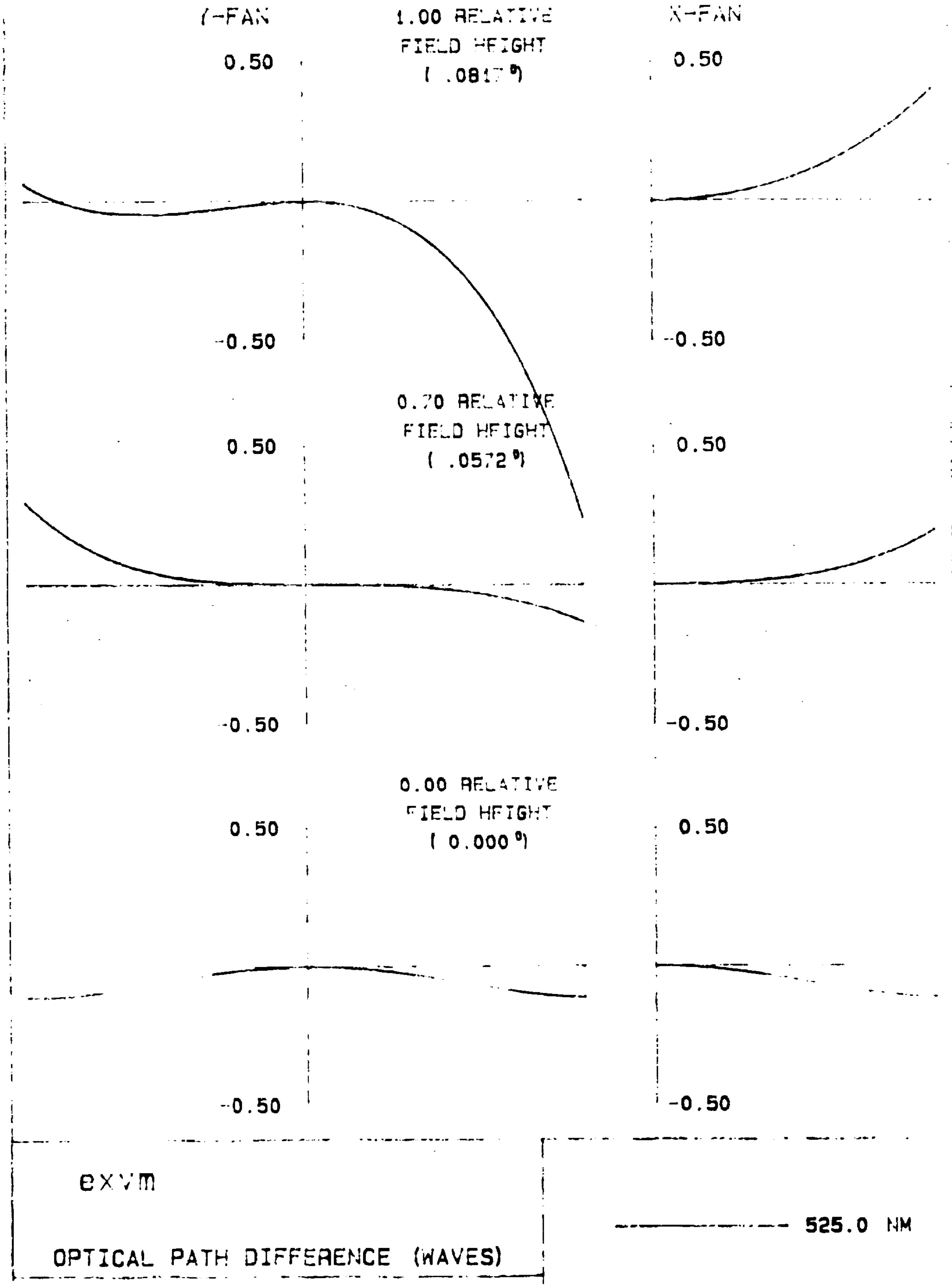
Table 6 lists the surface-by-surface third-order aberrations of the EXVM. SA stands for spherical aberration, TCO stands for tangential coma, TAS stands for tangential astigmatism, SAG stands for saggital astigmatism, PTB stands for Petzval blur, and DST stands for distortion. All aberrations are in millimeters.

Table 6
Third-order aberrations of the EXVM

	SA	TCO	TAS	SAG	PTB	DST
1	0.000000	0.000000	0.000000	0.000000	0.000000	0.000012
2	0.000000	0.000000	0.000000	0.000000	0.000000	-0.000012
STO	3.624628	-0.243555	0.003637	0.000000	-0.001818	0.000000
	-3.624628	0.000000	0.000000	0.000000		0.000000
4	-1.136824	0.123181	0.000625	0.003591	0.005074	-0.000130
	1.136776	0.104658	0.003212	0.001071		0.000033
5	-0.005470	0.002203	-0.000296	-0.000099	0.000000	0.000013
6	0.005338	-0.002150	0.000289	0.000096	0.000000	-0.000013
7	-0.005195	0.002092	-0.000281	-0.000094	0.000000	0.000013
8	0.004255	-0.001714	0.000230	0.000077	0.000000	-0.000010
9	-0.004112	0.001656	-0.000222	-0.000074	0.000000	0.000010
10	0.004000	-0.001611	0.000216	0.000072	0.000000	-0.000010
11	-0.003857	0.001553	-0.000208	-0.000069	0.000000	0.000009
12	0.003725	-0.001500	0.000201	0.000067	0.000000	-0.000009
13	-0.003582	0.001443	-0.000194	-0.000065	0.000000	0.000009
14	0.003526	-0.001420	0.000191	0.000064	0.000000	-0.000009
15	0.011116	-0.012094	0.005631	0.002707	0.001245	-0.000982
16	-0.016057	0.046209	-0.044970	-0.015419	-0.000643	0.014791
17	0.008307	-0.046646	0.093831	0.035623	0.006518	-0.066680
18	0.000000	0.000003	0.000229	0.000076	0.000000	0.006349
19	0.000000	-0.000003	-0.000232	-0.000077	0.000000	-0.006428
20	0.002187	0.010711	0.021248	0.009591	0.003762	0.015656
21	-0.010937	-0.011472	-0.004501	-0.001827	-0.000490	-0.000639
22	0.005467	-0.006563	0.004241	0.002490	0.001615	-0.000996
23	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
24	-0.002159	-0.006579	-0.012924	-0.008470	-0.006243	-0.008601
25	0.002601	0.021819	0.063919	0.023238	0.002897	0.064989
26	-0.000593	-0.013574	-0.115584	-0.046569	-0.012061	-0.355156
27	0.000000	0.000029	-0.002204	-0.000735	0.000000	0.056016
28	0.000000	-0.000029	0.002204	0.000735	0.000000	-0.056016
29	0.000000	0.000028	-0.002132	-0.000711	0.000000	0.054201
30	0.000000	-0.000028	0.002128	0.000709	0.000000	-0.054101
31	0.000000	0.000027	-0.002057	-0.000686	0.000000	0.052286
32	0.000000	-0.000027	0.002057	0.000686	0.000000	-0.052286
33	0.000006	0.000629	0.025908	0.011272	0.003954	0.393726
34	-0.000052	-0.002109	-0.029773	-0.010581	-0.000984	-0.144424
35	0.000047	0.001007	0.009266	0.004503	0.002121	0.031962
36	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
37	-0.000001	0.000000	0.000000	0.000000	0.000000	0.000000
38	0.000001	0.000000	0.000000	0.000000	0.000000	0.000000
39	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
40	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
41	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
42	-0.000001	0.000000	0.000000	0.000000	0.000000	0.000000
43	0.000001	0.000000	0.000000	0.000000	0.000000	0.000000
44	-0.000002	0.000000	0.000000	0.000000	0.000000	0.000000
45	0.000003	0.000000	0.000000	0.000000	0.000000	0.000000
46	-0.000003	0.000000	0.000000	0.000000	0.000000	0.000000
47	0.000013	-0.000221	0.002591	0.001737	0.001310	-0.010080
48	-0.000005	0.000247	-0.004633	-0.001956	-0.000617	0.031830

49	0.000000	-0.000087	-0.005347	-0.000143	0.002459	-0.012892
50	0.001168	0.003753	0.010140	0.007459	0.006119	0.007992
51	-0.001403	0.009618	-0.023498	-0.008851	-0.001528	0.020220
52	0.000687	-0.008760	0.040530	0.015702	0.003288	-0.066755
53	0.000823	0.018063	0.156585	0.068533	0.024507	0.501119
54	-0.008358	-0.090578	-0.331361	-0.113232	-0.004167	-0.409025
55	0.009103	0.064030	0.159999	0.059912	0.009868	0.140474
56	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
57	0.048899	0.162022	0.183774	0.064475	0.004826	0.071211
58	-0.088585	-0.198013	-0.149627	-0.051266	-0.002086	-0.038199
59	0.063628	0.035672	0.018974	0.014529	0.012307	0.002715
60	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
61	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
62	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
SUM	0.024479	-0.038080	0.081811	0.068093	0.061233	0.182185

Figure 8: Wavefront aberrations of the EXVM in the final image plane.



Section VI - Coating specification sheets

These optical elements will be used in a solar vector magnetograph, which measures the magnetic fields on the sun's surface by measuring the polarization state across the surface of the sun. This particular instrument is designed to make extraordinarily accurate magnetic field measurements, and therefore, must make extraordinarily accurate polarization measurements. To make these accurate polarization measurements, the coatings must be specified to maintain the image's polarization state very accurately.

The angles of incidence for this system are small, so the polarization associated with the surfaces will also tend to be small. The polarization specifications for these coatings are meant to ensure that the polarization of the system is small enough to meet its design goals.

The small values for the polarization specifications are justified for several reasons. First, the extreme accuracy of the final instrument would be able to detect any increase from these values. Second, since there are several optical elements in the system, the polarization induced by each element must be small. Finally, although the values for the polarization specification are small, simple coatings meet the specification.

The example coatings which are included with these specification sheets are meant only as a guide to vendors. We expect vendors to find stock coatings which are more appropriate than the example coatings.

Prefilter - filter side

diagram of blank

The coating design should meet specifications 1 and 2.

1. Average intensity transmittance - bandpass filter

    $\geq$ __% over entire diameter, incident angles 0° - .16°

    λ = 525nm

    $\leq$ __% outside of __ nm bandpass, centered on λ =525nm

    over entire diameter, incident angles 0° - .16°

2. Polarization

a) Retardance

Retardance is defined as:

$\Delta \equiv \delta_s - \delta_p$,

where δ_s and δ_s are the phase change on transmission for the s- and p- polarizations

The retardance specification is:

$0 < \Delta < 0.00014°$

over entire clear aperture, incident angles 0° - .16°

λ = 525nm

b) Intensity Difference

Intensity difference is defined as:

$\mathcal{D} \equiv R_p - R_s$,

where R_s and R_p are the intensity transmission coefficients for s- and p- polarized light.

The intensity difference specification is:

$0.0000002 < \mathcal{D} < 0.0000016$

over entire clear aperture, incident angles 0° - .16°

λ = 525nm

3. Coating thickness variations

The coating should be deposited with a process which is known to not degrade transmitted wavefronts by over λ /5 at 525 nm.

4. Surface quality
   must meet MIL-SPEC 60-40 scratch-dig specification over entire surface

5. Surface durability
   must meet MIL-SPEC scotch tape and eraser tests over entire surface

The following simple coating meets the polarization specifications. This coating design is meant only as a guideline for the vendors. The vendors should have stock coatings which are more appropriate.

Example coating design:
Bandpass filter 1LHLHLHLHHLHLHLHL1.5
where L represents a coating layer of MgF_2 (n = 1.3883) of a thickness equal to one-quarter wave optical path length at λ = 525nm, and where H represents a coating layer of ZnS (n = 2.375) of a thickness equal to one-quarter wave optical path length at λ = 525nm.

This example coating design gives a retardance value of $\Delta = 0.00011°$ and an intensity difference of $\mathcal{D} = 0.0000014$.

Prefilter - backing side

diagram of blank

The coating design should meet specifications 1 and 2.

1. Average intensity transmittance - backing side of filter plate
   >97% over entire diameter, incident angles 0° - .16°
   λ = 525nm

2. Polarization - must be met on each side of prefilter

a) Retardance

Retardance is defined as:

$\Delta \equiv \delta_s - \delta_p$,

where δ_s and δ_s are the phase change on transmission for the s- and p- polarizations

The retardance specification is:

$0 < \Delta < 0.00002°$

over entire clear aperture, incident angles 0° - .16°

λ = 525nm

b) Intensity Difference

Intensity difference is defined as:

$\mathcal{D} \equiv R_p - R_s$,

where R_s and R_p are the intensity transmission coefficients for s- and p- polarized light.

The intensity difference specification is:

$0.0000002 < \mathcal{D} < 0.0000005$

over entire clear aperture, incident angles 0° - .16°

λ = 525nm

3. Coating thickness variations

The coating should be deposited with a process which is known to not degrade transmitted wavefronts by over λ /5 at 525 nm.

4. Surface quality

must meet MIL-SPEC 60-40 scratch-dig specification over entire surface

5. Surface durability
   must meet MIL-SPEC scotch tape and eraser tests over entire surface

A single layer of quarter wave optical thickness at $\lambda = 525nm$ of MgF_2 ($n = 1.3883$) meets the polarization specification. This coating design is meant only as a guideline for the vendors. The vendors should have stock coatings which are more appropriate. The example coating design gives retardance of $\Delta = 0.0000004°$ and intensity difference of $\mathcal{D} = 0.0000004$.

Primary mirror

diagram of mirror

The coating design must meet specifications 1 and 2:

1. Average intensity reflectivity
   >98% over entire diameter, incident angles 0° - 3.8°
   $\lambda = 525nm$

2. Polarization - must be met on each side of prefilter

a) Retardance

Retardance is defined as:

$\Delta \equiv \delta_s - \delta_p$, where

δ_s and δ_s are the phase change on reflection for the s- and p- polarizations

The retardance specification is:

$-0.00008° < \Delta < -0.00003°$

over entire clear aperture, incident angles 0° - .16°

$\lambda = 525nm$

b) Intensity Difference

Intensity difference is defined as:

$\mathcal{D} \equiv R_p - R_s$,

where R_s and R_p are the intensity reflection coefficients for s- and p- polarized light.

The intensity difference specification is:

$-0.0000004 < \mathcal{D} < 0$

over entire clear aperture, incident angles 0° - .16°

$\lambda = 525nm$

3. Coating thickness variations

The coating should be deposited with a process which is known to not degrade transmitted wavefronts by over $\lambda/5$ at 525 nm.

4. Surface quality

must meet MIL-SPEC 60-40 scratch-dig specification over entire surface

5. Surface durability
   must meet MIL-SPEC scotch tape and eraser tests over entire surface

A simple four-layer reflection-enhancement coating (HLHL) meets this specification. L represents a coating layer of MgF_2 (n= 1.3883) of a thickness equal to one-quarter wave optical path length at λ =525nm, and H represents a coating layer of ZnS (n= 2.38) of a thickness equal to one-quarter wave optical path length at λ =525nm. This coating design is meant only as a guideline for the vendors. The vendors can probably design a coating which is more appropriate.

The example coating design gives retardance of Δ = -0.00004° and an intensity difference of $\mathcal{D}$ =-0.0000002.

Secondary mirror

diagram of mirror

The coating design must meet specifications 1 and 2:

1. Average intensity reflectivity

   >98% over entire diameter, incident angles 0° - 4.6°

   λ = 525nm

2. Polarization - must be met on each side of prefilter

a) Retardance

   Retardance is defined as:

   $\Delta \equiv \delta_s - \delta_p$, where

   δ_s and δ_s are the phase change on reflection for the s- and p- polarizations

The retardance specification is:

   $-0.00015° < \Delta < 0$

   over entire clear aperture, incident angles 0° - 0.3°

   λ = 525nm

b) Intensity Difference

   Intensity difference is defined as:

   $\mathcal{D} \equiv R_p - R_s$,

where R_s and R_p are the intensity reflection coefficients for s- and p- polarized light.

   The intensity difference specification is:

   $-0.000001 < \mathcal{D} < 0$

   over entire clear aperture, incident angles 0° - 0.3°

   λ = 525nm

3. Coating thickness variations

   The coating should be deposited with a process which is known to not degrade transmitted wavefronts by over $\lambda/5$ at 525 nm.

4. Surface quality

   must meet MIL-SPEC 60-40 scratch-dig specification over entire surface

5. Surface durability

must meet MIL-SPEC scotch tape and eraser tests

A simple four-layer reflection-enhancement coating (HLHL) meets this specification. L represents a coating layer of index 1.3883 of a thickness equal to one-quarter wave optical path length at λ =525nm, and H represents a coating layer of index 2.38 of a thickness equal to one-quarter wave optical path length at λ =525nm. This coating design is meant only as a guideline for the vendors. The vendors can probably design a coating which is more appropriate.

The example coating design gives retardance of Δ = -0.00014° and an intensity difference of $\mathcal{D}$ = -0.00000068.

Front doublet surface

diagram of lens

The coating designs for both the front and the back of the doublet must meet specifications 1 and 2:

1. Average intensity transmittance

   <98% over entire diameter, incident angles 0° -2.2_°

   λ = 525nm

2. Polarization - must be met on each side of prefilter

a) Retardance

   Retardance is defined as:

   $\Delta \equiv \delta_s - \delta_p$, where

   δ_s and δ_s are the phase change on transmission for the s- and p- polarizations

The retardance specification is:

   $0 < \Delta < 0.00005°$

   over entire clear aperture, incident angles 0° - 2.2°

   λ = 525nm

b) Intensity Difference

   Intensity difference is defined as:

   $\mathcal{D} \equiv R_p - R_s$,

where R_s and R_p are the intensity transmission coefficients for s- and p- polarized light.

   The intensity difference specification is:

   $0.000012 < \mathcal{D} < 0.000020$

   over entire clear aperture, incident angles 0° - 2.3°

   λ = 525nm

3. Coating thickness variations

   The coating should be deposited with a process which is known to not degrade transmitted wavefronts by over λ /5 at 525 nm.

4. Surface quality

must meet MIL-SPEC 60-40 scratch-dig specification over entire surface

5. Surface durability
   must meet MIL-SPEC scotch tape and eraser tests

A simple coating consisting of a single, quarter wave of MgF_2 optical thickness at λ =525nm and index = 1.3883 meets specifications 1 and 2. This coating design is meant only as a guideline for the vendors. The vendors can probably design a coating which is more appropriate.

The example coating design gives retardance of $\Delta = 0.00001°$ and an intensity difference of $\mathcal{D} = 0.000014$.

Back doublet surface

diagram of lens

The coating designs for both the front and the back of the doublet must meet specifications 1 and 2:

1. Average intensity transmittance

    $<98\%$ over entire diameter, incident angles 0° -$2.2\_^\circ$

    $\lambda = 525$nm

2. Polarization - must be met on each side of prefilter

a) Retardance

Retardance is defined as:

$\Delta \equiv \delta_s - \delta_p$, where

δ_s and δ_s are the phase change on transmission for the s- and p- polarizations

The retardance specification is:

$0 < \Delta < 0.00038^\circ$

over entire clear aperture, incident angles 0° - 3.2°

$\lambda = 525$nm

b) Intensity Difference

Intensity difference is defined as:

$\mathcal{D} \equiv R_p - R_s$,

where R_s and R_p are the intensity transmission coefficients for s- and p- polarized light.

The intensity difference specification is:

$-0.000033 < \mathcal{D} < -.000020$

over entire clear aperture, incident angles 0° - 3.2°

$\lambda = 525$nm

3. Coating thickness variations

The coating should be deposited with a process which is known to not degrade transmitted wavefronts by over $\lambda/5$ at 525 nm.

4. Surface quality

must meet MIL-SPEC 60-40 scratch-dig specification over entire surface

5. Surface durability
   must meet MIL-SPEC scotch tape and eraser tests

The following 4-layer antireflection coating meets the polarization specification:

Layer #	Optical Thickness (waves)	Refractive Index	
1			
0	.0000	1.6219	Glass
1	.2500	1.4569	SiO2
2	.2500	1.6299	CeF3
3	.5000	2.3750	ZnS
4	.2500	1.3881	MgF2
5	.0000	1.0000	Air

REF. WAVELENGTH = .5250 MICRONS
DESIGN ANGLE = .0000 DEGREES

The example coating design gives retardance of $R_p - R_s$ = -0.000032 and an intensity difference of $\delta_s - \delta_p$ = = 0.00034° .

Appendix I. The paper "Polarization Analysis of the SAMEX Solar Magnetograph" contains both a good summary of polarization aberration theory and a good demonstration of polarization aberration analysis.

Appendix II. Polarization Aberrations of unresolved point spread functions

This section explicitly provides the mathematics behind the polarization specification which was presented in the body of the report. First, we present the reasoning for the derivation of the Mueller matrix averaged over the point spread function. Next, we derive the average Mueller matrix for rotationally symmetric systems. Then, we use this Mueller matrix to explore three possible explanations of the 10^{-5} polarization specification in rotationally symmetric systems. Finally, we explain how the result from rotationally symmetric systems can be generalized into a result for slightly decentered systems, such as a vector magnetograph with an actuating secondary mirror.

In many imaging situations, the point spread function that is formed by the system's optics is smaller than the resolution of the system's readout. The EXVM is a good example of this type of system; the pixels on the CCD array are about the same size as the point spread function.

For this situation, the Stokes vector averaged over the point spread function is equal to the pupil-averaged Stokes vector. This equality can be understood by recognizing that the intensity in the pupil of any two orthogonal polarization states is the same as the intensity of those polarization states in the image. For example, if a 90% of the intensity in the pupil is y-polarized and 10% is x-polarized, the intensity distribution in the image plane will also be 90% y-polarized and 10% x-polarized. The polarization state changes across the pupil and also changes across the point spread function, but the average Stokes vector will be the same.

This equality can also be derived. The Jones vector in the exit pupil is

$$\vec{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix}.$$

The Stokes vector in the exit pupil is defined in terms of sums and differences in intensity measurements,

$$\vec{S} = \begin{bmatrix} I \\ Q \\ U \\ V \end{bmatrix} = \begin{bmatrix} I_h + I_v \\ I_h - I_v \\ I_{45^\circ} - I_{135^\circ} \\ I_R - I_L \end{bmatrix} = \begin{bmatrix} E_x^* E_x + E_y^* E_y \\ E_x^* E_x - E_y^* E_y \\ 2Re(E_x^* E_y) \\ 2Im(E_x^* E_y) \end{bmatrix},$$

where I, Q, U, V are the Stokes vector elements, I_h is the intensity of the light which would pass through a horizontal polarizer, I_v is the intensity of the light which would pass through a vertical polarizer, I_{45° is the intensity of the light which would pass through a 45° polarizer, I_{135° is the intensity of the light which would pass through a 135° polarizer, I_R is the intensity of light which would pass through a right circular polarizer, and I_L is the intensity which would pass through a left circular polarizer.

The pupil-averaged stokes vector is

$$<\vec{S}>_{pupil} = \left\langle \begin{pmatrix} E_x^* E_x + E_y^* E_y \\ E_x^* E_x - E_y^* E_y \\ 2Re(E_x^* E_y) \\ 2Im(E_x^* E_y) \end{pmatrix} \right\rangle_{pupil} = \begin{pmatrix} <E_x^* E_x + E_y^* E_y>_{pupil} \\ <E_x^* E_x - E_y^* E_y>_{pupil} \\ <2Re(E_x^* E_y)>_{pupil} \\ <2Im(E_x^* E_y)>_{pupil} \end{pmatrix} = \begin{pmatrix} <E_x^* E_x>_{pupil} + <E_y^* E_y>_{pupil} \\ <E_x^* E_x>_{pupil} - <E_y^* E_y>_{pupil} \\ 2Re(<E_x^* E_y>_{pupil}) \\ 2Im(<E_x^* E_y>_{pupil}) \end{pmatrix}.$$

where the brackets represent

$$<f>_{pupil} = \int f d\vec{\rho},$$

where $\vec{\rho}$ is the pupil coordinate. The units are chosen such that the area of the pupil is unity. $E_x^* E_x$ Is the intensity of the x-polarized light in the exit pupil. Since there are no optical elements between the exit pupil and the image, there will be the same amount of x-polarized light in the image plane. Therefore,

$$<E_x^* E_x>_{pupil} = <E'^*_x E'_x>_{psf},$$

where the primed quantities refer to the quantities in the image plane. Clearly, the same relationship holds true for the y-polarized light:

$$<E_y^* E_y>_{pupil} = <E'^*_y E'_y>_{psf}.$$

Since there is the same amount of light polarized both horizontally and vertically in the exit pupil and in the image, the average of the first two elements in the Stokes vector are equal in the pupil and the point spread function,

$$<I>_{pupil} = <I>_{psf}$$

$$<Q>_{pupil} = <Q>_{psf}$$

Since the choice of horizontal and vertical is arbitrary, the same relationship holds true for the third element in the Stokes vector,

$$<U>_{pupil} = <U>_{psf}.$$

Since the first three elements of the Stokes vector are equal, the fourth must be equal as well,

$$<V>_{pupil} = <V>_{psf}.$$

These equalities can also be derived rigorously using properties of the Fourier transform. The relationship between the Jones vector in the pupil plane, $\vec{E}$, and the Jones vector in the image plane, $\vec{E}'$, is a Fourier transform,

$$\vec{E}' = \int \vec{E} e^{i2\pi\vec{k}\cdot\vec{\rho}} d\vec{\rho}$$

Instead of explicitly using the x- and y- components of the electric field, we will derive the more general case, for $E_\alpha^* E_\beta$, where both α and β can be either x or y. Multiplying the Fourier transform for both components of the electric field yields

$$E'^{*}_{\alpha}E'_{\beta} = \int E^{*}_{\alpha}e^{-i2\pi\vec{h}\cdot\vec{\rho}}d\vec{\rho}\int E_{\beta}e^{i2\pi\vec{h}\cdot\vec{\rho}'}d\vec{\rho}'$$
$$= \int\int E^{*}_{\alpha}E_{\beta}e^{i2\pi\vec{h}\cdot(\vec{\rho}'-\vec{\rho})}d\vec{\rho}d\vec{\rho}'$$

Averaging over the point spread function yields

$$<E'^{*}_{\alpha}E'_{\beta}>_{psf} = \int E'^{*}_{\alpha}E'_{\beta}d\vec{h}$$
$$= \int\int\int E^{*}_{\alpha}E_{\beta}e^{i2\pi\vec{h}\cdot(\vec{\rho}'-\vec{\rho})}d\vec{\rho}d\vec{\rho}'d\vec{h}$$

Rearranging the order of integration yields

$$<E'^{*}_{\alpha}E'_{\beta}>_{psf} = \int\int\int E^{*}_{\alpha}E_{\beta}e^{i2\pi\vec{h}\cdot\vec{\rho}'}d\vec{\rho}'e^{-i2\pi\vec{h}\cdot\vec{\rho}}d\vec{h}d\vec{\rho}$$
$$= \int\mathcal{F}^{-1}\{\mathcal{F}\{E^{*}_{\alpha}E_{\beta}\}\}d\vec{\rho}$$
$$= \int E^{*}_{\alpha}E_{\beta}d\vec{\rho}$$
$$= <E^{*}_{\alpha}E_{\beta}>_{pupil}.$$

Therefore,

$$<E^{*}_{\alpha}E_{\beta}>_{pupil} = <E'^{*}_{\alpha}E'_{\beta}>_{psf}$$

Since α and β can both be either x or y, this proof explicitly shows that the Stokes vector averaged over the exit pupil is equal to the Stokes vector averaged over the point spread function.

Because the average Stokes vector in the point spread function is the same as the average Stokes vector in the image, the average Mueller matrix in the point spread function is the same as the average point spread function in the pupil. Qualitatively, this equality is even easier to understand than the equality for the Stokes vector. Since there are no polarizers, retarders, or depolarizers between the exit pupil and the image, the Mueller matrix for propagation from the exit pupil to the image must be the identity matrix.

This relationship can also be proven rigorously. Assuming a uniform input polarization state, we can distribute the averages among the matrices in the following way when we average over the point spread function.

$$< S >_{PSF} = < M\, S_{in} >_{PSF}$$
$$= < M >_{PSF} S_{in}$$

A similar relationship holds when we average over the pupil.

$$< S >_{pupil} = < M\, S_{in} >_{pupil}$$
$$= < M >_{pupil} S_{in}$$

Since we have already proven that the Stokes vector averaged over the pupil is equal to the Stokes vector over the point spread function ($< S >_{pupil} = < S >_{PSF}$), we can equate the previous two equations

$$< M >_{PSF} S_{in} = < S >_{PSF} = < S >_{pupil} = < M >_{pupil} S_{in} .$$

Therefore, the Mueller matrix averaged over the point spread function is equal to the Mueller matrix averaged over the exit pupil,

$$< M >_{PSF} = < M >_{pupil} .$$

The easiest way to find the pupil-averaged Mueller matrix, $\overline{M}(h)$, is to begin with the polarization aberration Jones matrix, $J(\rho, \phi, h)$. This matrix will have different forms depending on the symmetries of the system.

The polarization aberration expansion which we will begin with is the Jones matrix for rotationally symmetric systems:

$$J(\rho, \phi, h) = P_{0000}\sigma_0 + P_{1200}\sigma_1 h^2$$
$$P_{1111} h\rho(\cos\phi\sigma_1 - \sin\phi\sigma_2)$$
$$P_{1022}\rho^2(\cos 2\phi\sigma_1 - \sin 2\phi\sigma_2),$$

The polarization aberration terms have both real and imaginary parts

$$P_{1xxx} = r_{1xxx} + iq_{1xxx}$$

To convert this Jones matrix into a Mueller matrix, use the following relationships:

$$2M_{11}(\rho,\phi,h) = J^*_{11}J_{11} + J^*_{21}J_{21} + J^*_{12}J_{12} + J^*_{22}J_{22}$$

$$2M_{12}(\rho,\phi,h) = J^*_{11}J_{11} + J^*_{21}J_{21} - J^*_{12}J_{12} - J^*_{22}J_{22}$$

$$2M_{13}(\rho,\phi,h) = J^*_{11}J_{12} + J^*_{21}J_{22} + J^*_{12}J_{11} + J^*_{22}J_{21}$$

$$2M_{14}(\rho,\phi,h) = j(J^*_{11}J_{12} + J^*_{21}J_{22} - J^*_{12}J_{11} - J^*_{22}J_{21})$$

$$2M_{21}(\rho,\phi,h) = J^*_{11}J_{11} + J^*_{12}J_{12} - J^*_{21}J_{21} - J^*_{22}J_{22}$$

$$2M_{22}(\rho,\phi,h) = J^*_{11}J_{11} + J^*_{22}J_{22} - J^*_{21}J_{21} - J^*_{12}J_{12}$$

$$2M_{23}(\rho,\phi,h) = J^*_{12}J_{11} + J^*_{11}J_{12} - J^*_{22}J_{21} - J^*_{21}J_{22}$$

$$2M_{24}(\rho,\phi,h) = j(J^*_{11}J_{12} + J^*_{22}J_{21} - J^*_{21}J_{22} - J^*_{11}J_{12})$$

$$2M_{31}(\rho,\phi,h) = J^*_{11}J_{21} + J^*_{21}J_{11} + J^*_{12}J_{22} + J^*_{22}J_{12}$$

$$2M_{32}(\rho,\phi,h) = J^*_{11}J_{21} + J^*_{21}J_{11} - J^*_{12}J_{22} - J^*_{22}J_{12}$$

$$2M_{33}(\rho,\phi,h) = J^*_{11}J_{22} + J^*_{21}J_{12} + J^*_{12}J_{21} + J^*_{22}J_{11}$$

$$2M_{34}(\rho,\phi,h) = j(J^*_{11}J_{22} + J^*_{21}J_{12} - J^*_{12}J_{21} - J^*_{22}J_{11})$$

$$2M_{41}(\rho,\phi,h) = j(J^*_{21}J_{11} + J^*_{22}J_{12} - J^*_{11}J_{21} - J^*_{12}J_{22})$$

$$2M_{42}(\rho,\phi,h) = j(J^*_{21}J_{11} + J^*_{12}J_{22} - J^*_{11}J_{21} - J^*_{22}J_{12})$$

$$2M_{43}(\rho,\phi,h) = j(J^*_{21}J_{12} + J^*_{22}J_{11} - J^*_{11}J_{22} - J^*_{12}J_{21})$$

$$2M_{44}(\rho,\phi,h) = j(J^*_{22}J_{11} + J^*_{11}J_{22} - J^*_{12}J_{21} - J^*_{21}J_{12})$$

J_{rc} represents the element from row r and column c in the Jones matrix. M_{rc} represents the element from row r and column c in the Mueller matrix. This transformation from the Jones calculus into the Mueller calculus can be found in several references: E.L. O'Neil, Introduction to Statistical Optics, (Addison Wesley, 1963), A. Gerrard and J.M. Burch, Introduction to Matrix Methods in Optics, (Wiley, 1975).

A second, more succinct, method of transforming a Jones matrix into a Mueller matrix uses the Kroneker product:

$$M = U(J \otimes J^{*})U^{-1}.$$

U is defined as

$$U = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & i & -i & 0 \end{pmatrix}.$$

The Kroneker product of two 2X2 matrices is

$$\begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \otimes \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} = \begin{pmatrix} a_1 a_2 & a_1 b_2 & b_1 a_2 & b_1 b_2 \\ a_1 c_2 & a_1 d_2 & b_1 c_2 & b_1 d_2 \\ c_1 a_2 & c_1 b_2 & d_1 a_2 & d_1 b_2 \\ c_1 c_2 & c_1 d_2 & d_1 c_2 & d_1 d_2 \end{pmatrix}.$$

This formalism can be found in "Obtainment of the polarizing and retardation parameters of a non-depolarizing optical system from the polar decomposition of its Mueller matrix," J. Gil, E. Bernabeu, Optik, vol. 76, no. 2, 1987.

Converting the polarization aberration Jones matrix into a Mueller matrix yields a matrix which is so complicated that its form is not evident. However, by looking at the resulting Mueller matrix for the various aberration terms, we can see the form of the entire Mueller matrix. In the following analysis, we will use the polarization aberration expansion for a rotationally symmetric system. The results from other types of systems, such as a Cassegrain telescope with an articulating secondary mirror, can be inferred from our results. We choose not to work with the polarization aberration expansion for a decentered system because the math becomes so unwieldy that the benefits of the added generality are more than outweighed by the difficulties of the added complexity.

The Mueller matrix for a system with only polarization defocus is

$$\begin{pmatrix} P_{0000}^2 + P_{1022}^2\rho^4 & 2P_{0000}r_{1022}\rho^2\cos 2\phi & -2P_{0000}r_{1022}\rho^2\sin 2\phi & 0 \\ 2P_{0000}r_{1022}\rho^2\cos 2\phi & P_{0000}^2 + P_{1022}^2\rho^4\cos 4\phi & P_{1022}^2\rho^4\sin 4\phi & -2P_{0000}q_{1022}\rho^2\sin 2\phi \\ -2P_{0000}r_{1022}\rho^2\sin 2\phi & -P_{1022}^2\rho^4\sin 4\phi & P_{0000}^2 - P_{1022}^2\rho^4\cos 4\phi & -2P_{0000}q_{1022}\rho^2\cos 2\phi \\ 0 & 2P_{0000}q_{1022}\rho^2\sin 2\phi & 2P_{0000}q_{1022}\rho^2\cos 2\phi & P_{0000}^2 - P_{1022}^2\rho^4 \end{pmatrix}.$$

As expected, this is the Mueller matrix for a retarder and diattenuator with orientation that rotates at twice the angular pupil coordinate. The Mueller matrix for a system with only polarization tilt is

$$\begin{pmatrix} P_{0000}^2 + P_{1111}h^2\rho^2 & 2P_{0000}r_{1111}h\rho\cos\phi & -2P_{0000}r_{1111}h\rho\sin\phi & 0 \\ 2P_{0000}r_{1111}h\rho\cos\phi & P_{0000} + P_{1111}h^2\rho^2\cos 2\phi & -P_{1111}^2h^2\rho^2\sin 2\phi & -2P_{0000}q_{1111}h\rho\sin\phi \\ -2P_{0000}r_{1111}h\rho\sin\phi & -P_{1111}^2h^2\rho^2\sin 2\phi & P_{0000}^2 - P_{1111}^2h^2\rho^2\cos 2\phi & -2P_{0000}q_{1111}h\rho\cos\phi \\ 0 & 2P_{0000}q_{1111}h\rho\sin\phi & 2P_{0000}q_{1111}h\rho\cos\phi & P_{0000}^2 + P_{1111}h^2\rho^2 \end{pmatrix}.$$

As expected, this is the Mueller matrix for a retarder and diattenuator oriented radially. The Mueller matrix for a system with only polarization piston is

$$M_{piston} = \begin{pmatrix} P_{0000}^2 + h^4 P_{1200}^2 & 2h^2 P_{0000} r_{1200} & 0 & 0 \\ 2h^2 P_{0000} r_{1200} & P_{0000}^2 + h^4 P_{1200}^2 & 0 & 0 \\ 0 & 0 & P_{0000}^2 - h^4 P_{1200}^2 & -2h^2 P_{0000} q_{1200} \\ 0 & 0 & 2h^2 P_{0000} q_{1200} & P_{0000}^2 - h^4 P_{1200}^2 \end{pmatrix}.$$

As expected, this is the Mueller matrix for a vertical or horizontal diattenuator and retarder.

Integrate over ρ and ϕ to obtain the pupil-averaged Mueller matrix.

$$< M >_{pupil}(h) = \int_0^1 \int_0^{2\pi} M(\rho, \phi, H) \rho \, d\rho \, d\phi$$

For rotationally symmetric systems, the pupil-averaged Mueller matrix is

$$= \begin{pmatrix} P_{0000}^2 + \frac{1}{3} P_{1022}^2 + \frac{1}{2} h^2 P_{1111}^2 + h^4 P_{1200}^2 & 2h^2 P_{0000} r_{1200} & 0 & 0 \\ 2h^2 P_{0000} r_{1200} & P_{0000}^2 + h^4 P_{1200}^2 & 0 & 0 \\ 0 & 0 & P_{0000}^2 - h^4 P_{1200}^2 & -2h^2 P_{0000} q_{1200} \\ 0 & 0 & 2h^2 P_{0000} q_{1200} & P_{0000}^2 - \frac{1}{3} P_{1022}^2 - \frac{1}{2} h^2 P_{1111}^2 - h^4 P_{1200}^2 \end{pmatrix} \quad 1$$

$$\cong \begin{pmatrix} P_{0000}^2 & 2h^2 P_{0000} r_{1200} & 0 & 0 \\ 2h^2 P_{0000} r_{1200} & P_{0000}^2 & 0 & 0 \\ 0 & 0 & P_{0000}^2 & -2h^2 P_{0000} q_{1200} \\ 0 & 0 & 2h^2 P_{0000} q_{1200} & P_{0000}^2 \end{pmatrix} \quad 2$$

This is the Mueller matrix which would be measured in the image plane if the system were placed in a polarimeter. The approximation in Equation 2 is the usual approximation used in aberration theory; terms of high order in h and terms on the order P_{1xxx}^2 are neglected. Making this approximation has a slight drawback; the resulting Mueller matrix is slightly unphysical. If linearly polarized light at 45° is incident on the Mueller matrix,

$$S_{in} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix},$$

the pupil-average stokes vector is

$$S_{out} = \begin{pmatrix} P_{0000}^2 \\ 2h^2 P_{0000} r_{1200} \\ P_{0000}^2 \\ 2h^2 P_{0000} q_{1200} \end{pmatrix}.$$

The degree of polarization for this Stokes vector is

$$\mathrm{DOP} = \sqrt{Q^2 + U^2 + V^2}/I$$
$$\cong 1 + 2h^4 P_{1200}^2$$

This Stokes vector is unphysical because the degree of polarization is greater than one. This is not a serious problem, however, because, to be consistent with our previous approximations, we should drop terms of order P_{1xxx}^2. In this case, the degree of polarization for this Stokes vector is unity,

$$\mathrm{DOP} \cong 1 .$$

Now that we have a pupil-averaged Mueller matrix, we can rigorously define three polarization errors.

Polarization error #1 is that unpolarized light should not couple into polarized light. The first effect of this polarization error is that magnetic field is measured when no magnetic field is present. The second effect of this polarization error is that the orientation of linearly polarized light is measured incorrectly, causing the orientation of the magnetic field to be measured incorrectly. The symptom of this polarization error is a spurious, radially oriented linear polarization with magnitude that increases quadratically with field coordinate. Mathematically, this polarization error is $2\,|\,Re(P_{1200})\,| < 10^{-5}$.

We can investigate this polarization error more thoroughly using the Mueller calculus. The Stokes vector for unpolarized light is

$$S = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}.$$

When this unpolarized light is incident on the system, she Stokes vector averaged over the exit pupil is

$$S = \begin{pmatrix} P_{0000}^{2} \\ 2h^{2} P_{0000} r_{1200} \\ 0 \\ 0 \end{pmatrix}.$$

This specification is a limit on the diattenuation along the chief ray. Numerically, this polarization specification is

$2 P_{0000} r_{1200} < 10^{-5}$.

For the BVM with an articulating secondary mirror and a 30 cm. primary mirror with the simple coatings listed in Table 3, the value for polarization error #1 is $0.60 \cdot 10^{-5}$

Polarization error #2 is that polarized light should maintain its polarization state. This specification would limit all off-diagonal elements in the pupil-averaged Mueller matrix. One way to quantify this specification would be

$$2 \mid P_{1200} \mid < 10^{-5}.$$

This specification is essentially a limit on the diattenuation and retardance along the chief ray path. This polarization error is the most important polarization error for the BVM because it causes circularly polarized light to couple into linearlly polarized light. Circularly polarized light has a Stokes vector of

$$S = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

If this light is incident on an optical system, the pupil-averaged stokes vector in the image plane is

$$S = \begin{bmatrix} P_{0000}^2 \\ 2h^2 P_{0000} r_{1200} \\ -2h^2 P_{0000} q_{1200} \\ P_{0000}^2 \end{bmatrix}.$$

The degree of linear polarization for this Stokes vector is

$DOLP = 2h \mid P_{1200} \mid$

We can use this degree of linear polarization as a definition of polarization sensitivity. The polarization specification would then be

$2 \mid P_{1200} \mid < 10^{-5}$.

For the BVM with an articulating secondary mirror and a 30 cm. primary mirror with the simple coatings listed in Table 3, the value for polarization error #2 is $0.78 \cdot 10^{-5}$.

Polarization error #3 is that polarized light should not couple into unpolarized light. Determining the amount of depolarization in a Mueller matrix is an active area of research, but we can determine some properties of the pupil-averaged Mueller matrix by examining the Mueller matrix in eq. 1 on a term-by-term basis. For a system with only polarization defocus, the pupil-averaged Mueller matrix is

$$M_{defocus} = \begin{pmatrix} P_{0000}^2 + \frac{1}{3}P_{1022}^2 & 0 & 0 & 0 \\ 0 & P_{0000}^2 & 0 & 0 \\ 0 & 0 & P_{0000}^2 & 0 \\ 0 & 0 & 0 & P_{0000}^2 - \frac{1}{3}P_{1022}^2 \end{pmatrix}.$$

For a system with only polarization tilt, the pupil-averaged Mueller matrix is

$$M_{tilt} = \begin{pmatrix} P_{0000}^2 + \frac{1}{2}h^2 P_{1111}^2 & 0 & 0 & 0 \\ 0 & P_{0000}^2 & 0 & 0 \\ 0 & 0 & P_{0000}^2 & 0 \\ 0 & 0 & 0 & P_{0000}^2 - \frac{1}{2}h^2 P_{1111}^2 \end{pmatrix}.$$

The pupil-averaged Mueller matrices for a system with only tilt or with only defocus is the Mueller matrix for a depolarizer. The first element in the Stokes vector will be increased, but the other three will remain the same or decrease. For a system with only polarization piston, the pupil-averaged Mueller matrix is

$$M_{piston} = \begin{pmatrix} P_{0000}^2 + h^4 P_{1200}^2 & 2h^2 P_{0000} r_{1200} & 0 & 0 \\ 2h^2 P_{0000} r_{1200} & P_{0000}^2 + h^4 P_{1200}^2 & 0 & 0 \\ 0 & 0 & P_{0000}^2 - h^4 P_{1200}^2 & 2h^2 P_{0000} q_{1200} \\ 0 & 0 & -2h^2 P_{0000} q_{1200} & P_{0000}^2 - h^4 P_{1200}^2 \end{pmatrix}.$$

The defocus term and the tilt term are clearly depolarizers because the matrices only have terms along the diagonal and because the M_{11} term is larger than the other terms. The piston term, perhaps not so clearly, is a diattenuator and a retarder. The depolarization of the entire Mueller matrix is

$$D = \tfrac{1}{3} P_{1022}^2 + \tfrac{1}{2} h^2 P_{1111}^2$$

This depolarization is clearly a small effect; all of the aberration terms are on the order of P_{1xxx}^2. For the BVM with an articulating secondary mirror and a 30 cm. primary mirror with the simple coatings listed in Table 3, the value for polarization error #1 is $0.01 \cdot 10^{-5}$

For slightly decentered systems, such as a Cassegrain telescope with a tilted secondary mirror, the polarization aberration expansion in Jones matrix form becomes much longer, and the corresponding pupil-averaged Mueller matrix becomes correspondingly larger. Rather than writing the equations, it is more instructive to think through the problem.

The Mueller matrix in eq. 2 is the Mueller matrix which represents a diattenuator. The magnitude of the diattenuation increases quadratically from the center of the field to the edge. For the Mueller matrix for the decentered system will look much more complicated, but the meaning will be very similar. The orientation of the diattenuation will be oriented radially about some non-axial image point, and the magnitude of the diattenuation will increase quadratically from the same, non axial image point. Therefore, we specified the coatings to meet the 10^{-5} polarization specification at the most extreme point in the image plane with a tilted secondary mirror.