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THE EFFECT OF TURBULENCE AND MOLECULAR RELAXATION ON SONIC BOOM SIGNATURES

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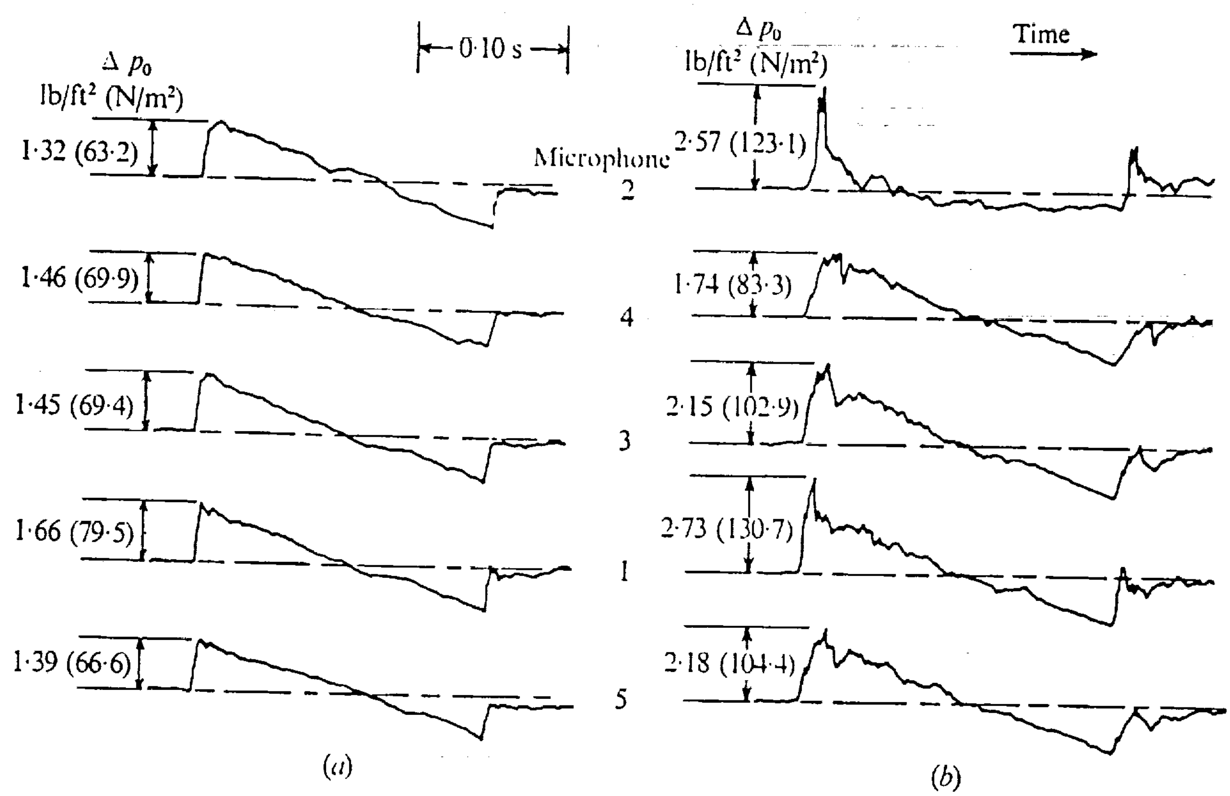
[redacted] Typical Flight Test Measurements for Two Different Meteorological Conditions. From Hilton, Huckel, & Maglieri (1966).

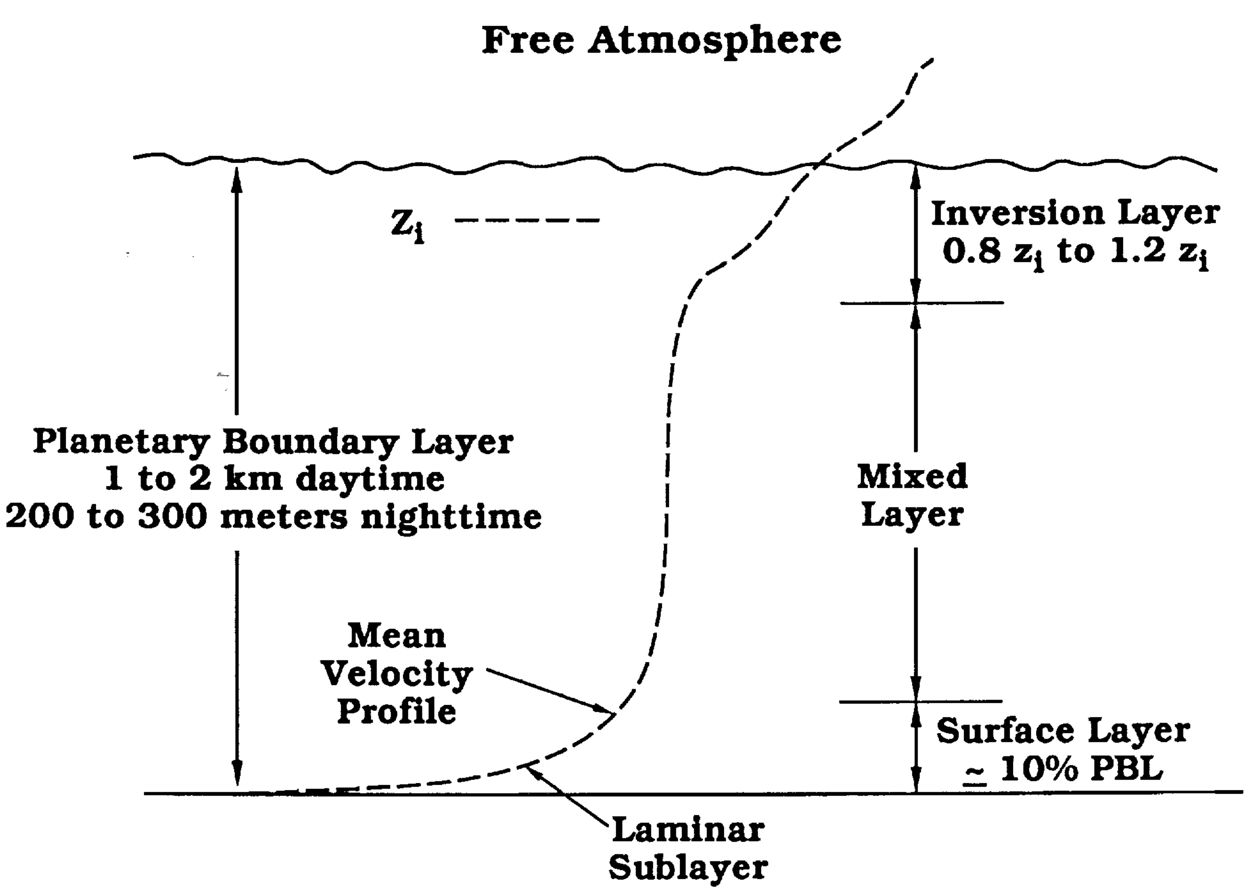
(a) Low wind velocity. (b) Strong gusty wind.

These are typical sonic boom measurements. Under turbulent conditions, signatures are distorted. Rise times are longer and are variable. Differences between booms such as (a) and (b) have been clearly demonstrated to be associated with atmospheric turbulence.



Planetary Boundary Layer

The pertinent turbulence is in the mixed layer of the planetary boundary layer. Sonic boom flight tests in the 1960s with microphones on towers and balloons have clearly demonstrated that. Most atmospheric models (e.g., Monin-Obukhov scaling, Turner classes, etc.) deal with the surface layer and do not relate to the mixed layer. Over the past couple of decades, there have been substantial advances in measuring and modeling mixed layer turbulence.



Objectives

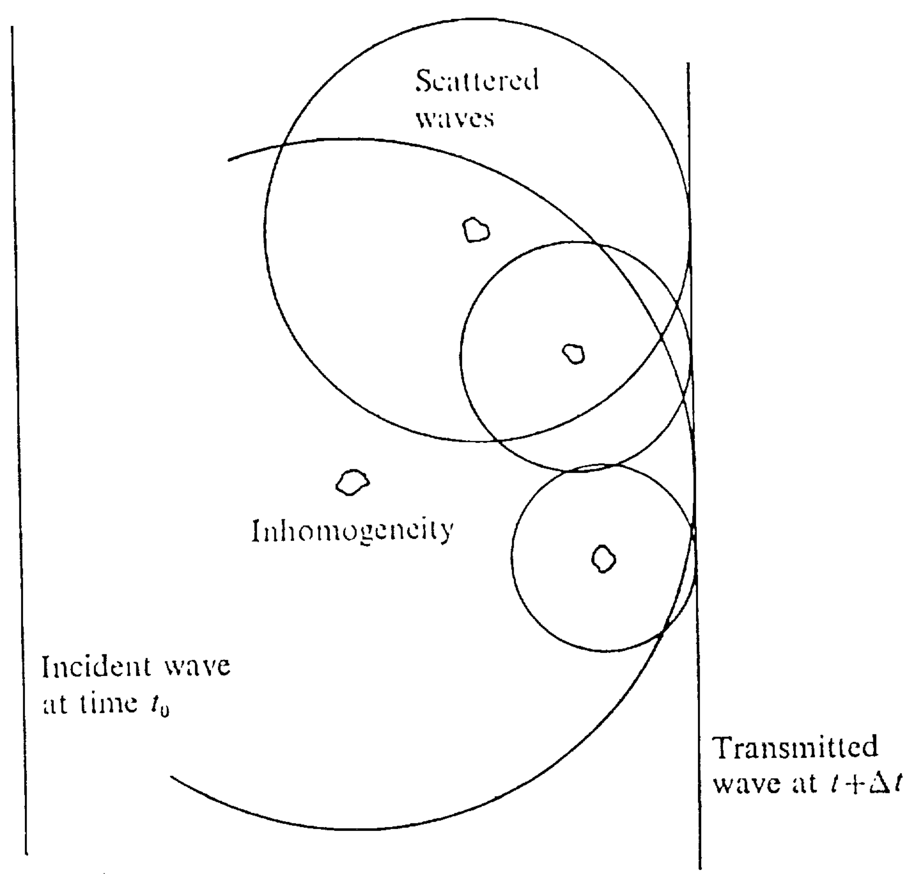
Our objectives are to assess the effect of turbulence and molecular absorption (which is now known to be a key factor in sonic boom shock structure) on shaped sonic booms. Today I will discuss the combination of physical mechanisms for idealized turbulence. In parallel, we are reviewing models for mixed layer turbulence, and these physical effects will eventually be generalized.

- **Identify Effects on Loudness of Shaped Booms.**
- **Combined Turbulence and Relaxation Effects.**
- **Realistic Turbulence Models – Including Variations of PBL Structure.**
- **ANSI S1.26-1978 Absorption.**
- **Current Status: Combine Physical Mechanisms For a Simple Shock in Homogeneous Turbulence.**

Schematic Representation of Scattering

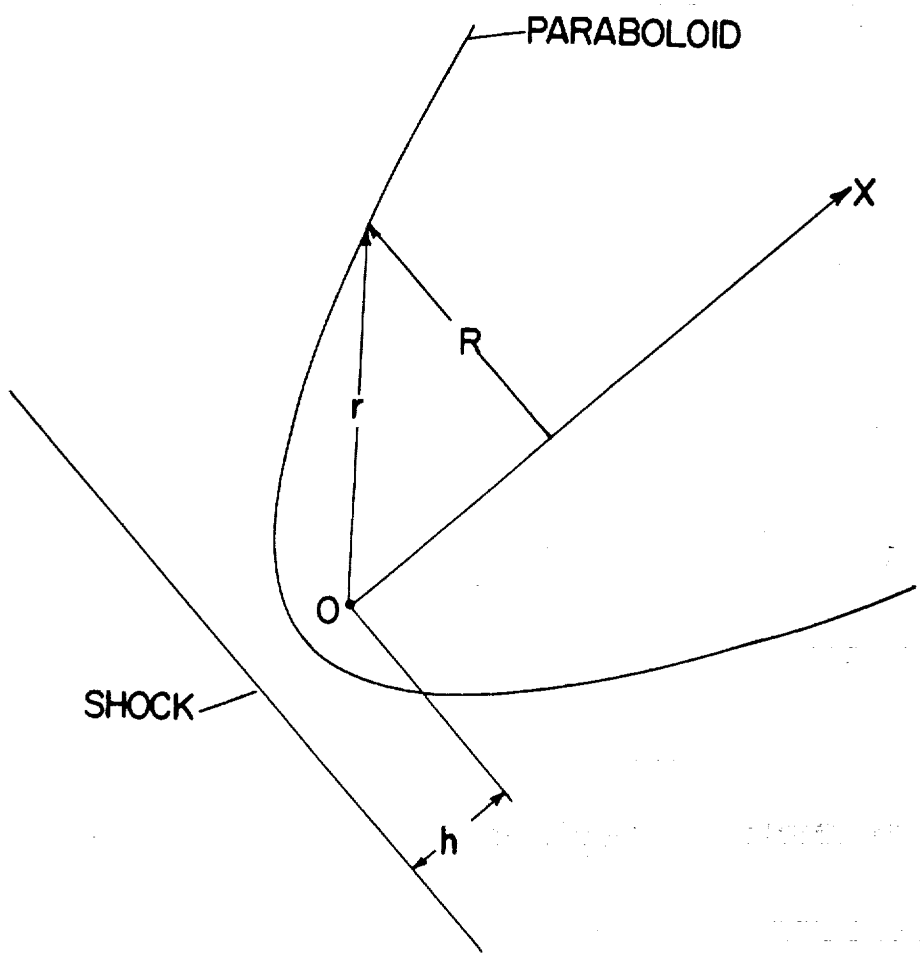
This is a schematic of turbulent scattering. When an incident wave interacts with a local inhomogeneity, a secondary scattered wave is emitted. These "first scattered" waves have long been considered to be the distortion on sonic booms. The energy in the first scattered waves is extracted from the shocks (scattering is strongest for high frequencies), causing the anomalous long rise times under turbulent conditions.

Classical analyses of scattering (as in the books by Chernov and Tatarskii) consider scattering of continuous harmonic waves, and scattering is considered to be associated with a 3-D scattering volume. Application of this formulation to sonic booms is very difficult due to the concentrated nature of a shock front.



Crow's Paraboloid of Dependence

Crow formulated scattering directly in the time domain, noting that the scattering volume reduces to a paraboloid which is equidistant between the receiver point (a distance h behind the shock) and the shock front. This formulation loses frequency information (which may be important for loudness), but exhibits very important physical characteristics. It also leads to a tractable solution.



Crow's Result

Crow's final result for mean square fluctuations (arrived at after a series of reasonable approximations) can be expressed as a simple integration of the turbulent dissipation function through the turbulent layer. This form corresponds to the paraboloid being within the Kolomogorov inertial subrange. Assuming the planetary boundary layer to be similar to a flat-plate wind tunnel boundary layer, Crow obtained a reasonable value for h_c. Kamali and Pierce have shown this to be in good agreement with flight test data, beyond the first few feet of the shock.

$$\left(\frac{p_1}{\Delta p}\right)^2 = \frac{1}{h^{7/6}} \int_0^{\infty} x^{5/6} \mathcal{AE}^{2/3}(x)\, dx$$

$$= (h_c / h)^{7/6}$$

$$h_c \approx 0.7 \text{ ft}$$

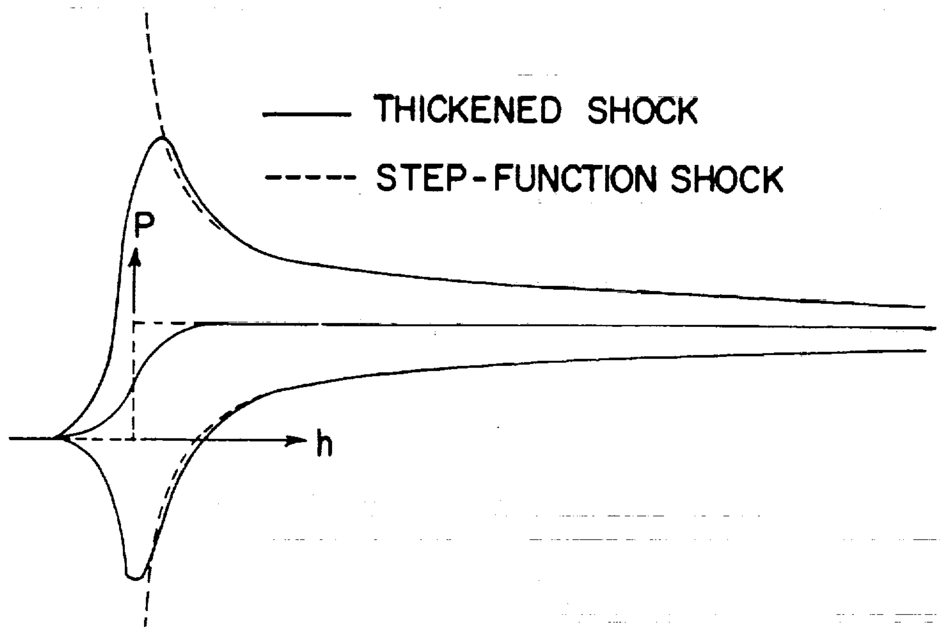
- **Agrees With Flight Test Data.**
- **Singular at $h = 0$.**
- **No Spectral Information or Structural Details.**
- **Somewhat Sensitive to Turbulence Model.**
- **Assumed Kolmogorov Inertial Subrange.**

Apply Crow Result to Thickened Shock

The singularity at $h = 0$ is due to a singularity in the scattering equations for very high frequencies. If we distribute his result over a finite shock structure, the singularity vanishes. This figure is for $T = h_c$. In most flight test data, T is at least several times bigger, in which case the RMS envelopes are smaller. Note that away from the shock itself, the simple step function result merges with the distributed form.

For the rest of today's talk, it is sufficient to use Crow's step function result.

$$\langle p_1^2 \rangle^{1/2} \leq \int_{-\infty}^{h} \left(\frac{h-\tau}{h_c}\right)^{-7/12} \frac{d\,p_o}{d\,\tau}\, d\,\tau$$



Root–Mean–Square Perturbations for a Thickened Shock of Thickness $T = h_c$ and $(h_c/h)^{7/12}$.

Extensions of Crow Model

Crow's model clearly captures the essence of shock wave scattering, and we would like to extend it. The first extension is to consider that the Kolmogorov subrange applies only up to some maximum eddy size. The second would be to allow a general turbulence model. Much of the simplification Crow obtained by assuming the Kolmogorov spectrum served to make some closed form integrals solvable in closed form. Today, we are not so shy about using numerical methods. It would also be nice to include loss processes, since we now know that molecular absorption can be important for the frequencies and distances involved.

We ultimately would like to recover the spectral characteristics of the scattered waves. The RMS envelopes by themselves may not tell an adequate story for loudness. Also, molecular relaxation is frequency dependent, and is difficult to estimate without spectra.

1. Paraboloid Larger than Eddy Size L_0

$$\left\langle \left| \frac{p_1}{\Delta p} \right|^2 \right\rangle = \frac{1}{h^{7/6}} \int_0^{L_0^2/8h} A\,E^{2/3}\,x^{5/6}\,dx$$

$$+ \frac{1}{h^{3/2}} \int_{L_0^2/8h}^{\infty} C\,x^{1/2}\,dx$$

2. General Turbulence Model and Attenuation

$$\left\langle \left| \frac{p_1}{\Delta p} \right|^2 \right\rangle = \frac{1}{h^{7/6}} \int_0^{\infty} G(x)\,e^{-\alpha x}\,dx$$

3. Include Spectral Characteristics

$$\left\langle \left| \frac{p_1}{\Delta p} \right|^2 \right\rangle = \frac{1}{h^{7/6}} \int_0^{\infty} G(x)\,\Phi(f)\,e^{-\alpha x}\,dx$$

Spectral Content of Scattered Sound

Classical harmonic scattering analysis provides spectra of scattered waves as a function of scattering angle and turbulence characteristics. This is a result for high frequencies. This type of formula has been well verified by experiments.

This is written in terms of wave number, which is easily converted to frequency. I have also introduced the macroscale length, which is a convenient quantity directly related to the eddy size.

$$\langle |p_1|^2 \rangle \propto k^4 E\left(2 k \sin \frac{\theta}{2}\right)$$

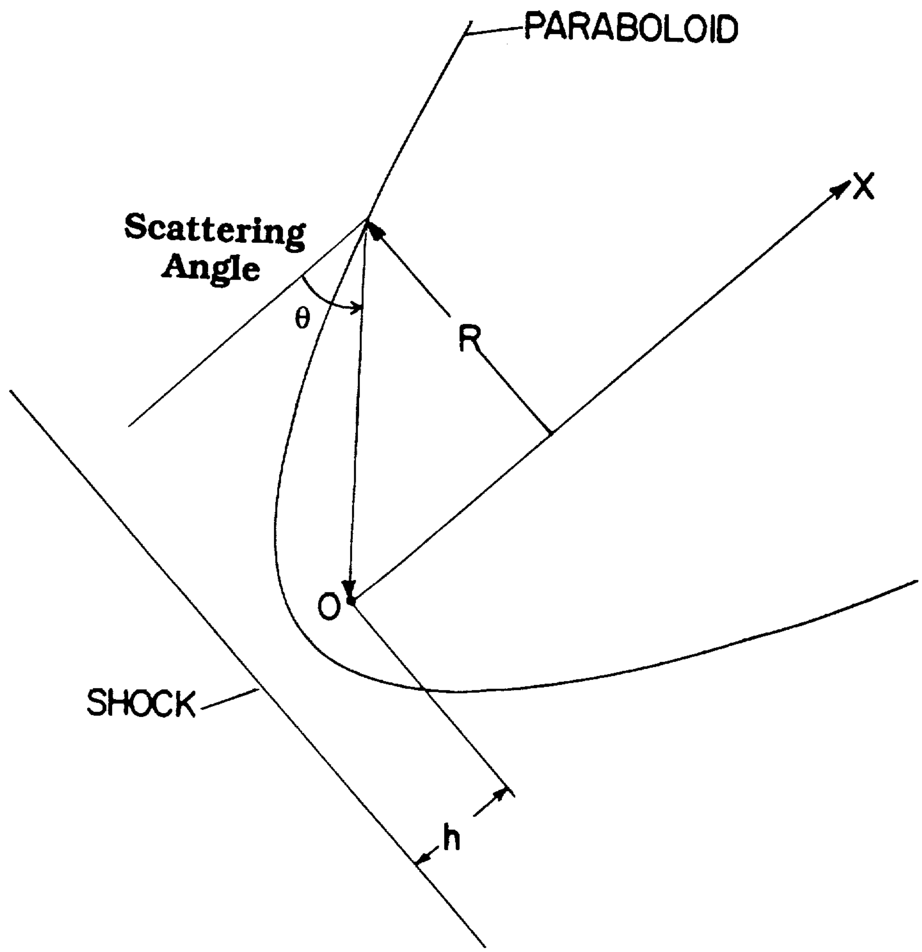
For a Shock With Power Spectrum $1/k^2$,

$$\langle |p_1|^2 \rangle \propto \begin{cases} k^2 & , \; k \leq \dfrac{\pi}{5 L_o \theta} \\ k^{-5/3} \; \theta^{-11/3} & , \; k > \dfrac{\pi}{5 L_o \theta} \end{cases}$$

$$\left(L_o \approx \frac{2}{5} \mathcal{L}_o\right)$$

Scattering Angle From Paraboloid

It turns out that the scattering angle is very simply obtained from the shape of the paraboloid.



$$R \approx (2\,hx)^{1/2}$$

$$\theta = (2\,h/x)^{1/2}$$

The spectrum can then very simply be applied to the general model, with all expressed in terms of x, h and turbulence parameters. This form includes the frequency content of the incident shock as well as the scattering dependence.

$$\Phi(f) = \frac{6}{11 f_o} \begin{cases} (f/f_o)^2 & f < f_o \\ (f/f_o)^{-5/3} & f > f_o \end{cases}$$

where $$f_o = \frac{a}{10 L_o} \left(\frac{x}{2h}\right)^{1/2} , \quad \int_0^\infty \Phi(f)\, df = 1$$

so that

$$\left\langle \left(\frac{p_1}{\Delta p}\right)^2 \right\rangle = \frac{1}{h^{7/6}} \int_0^\infty G(x)\ \Phi(f)\ e^{-\alpha(f)x}\, dx$$

Note that spectral contribution from distance x will have peak at f_o .

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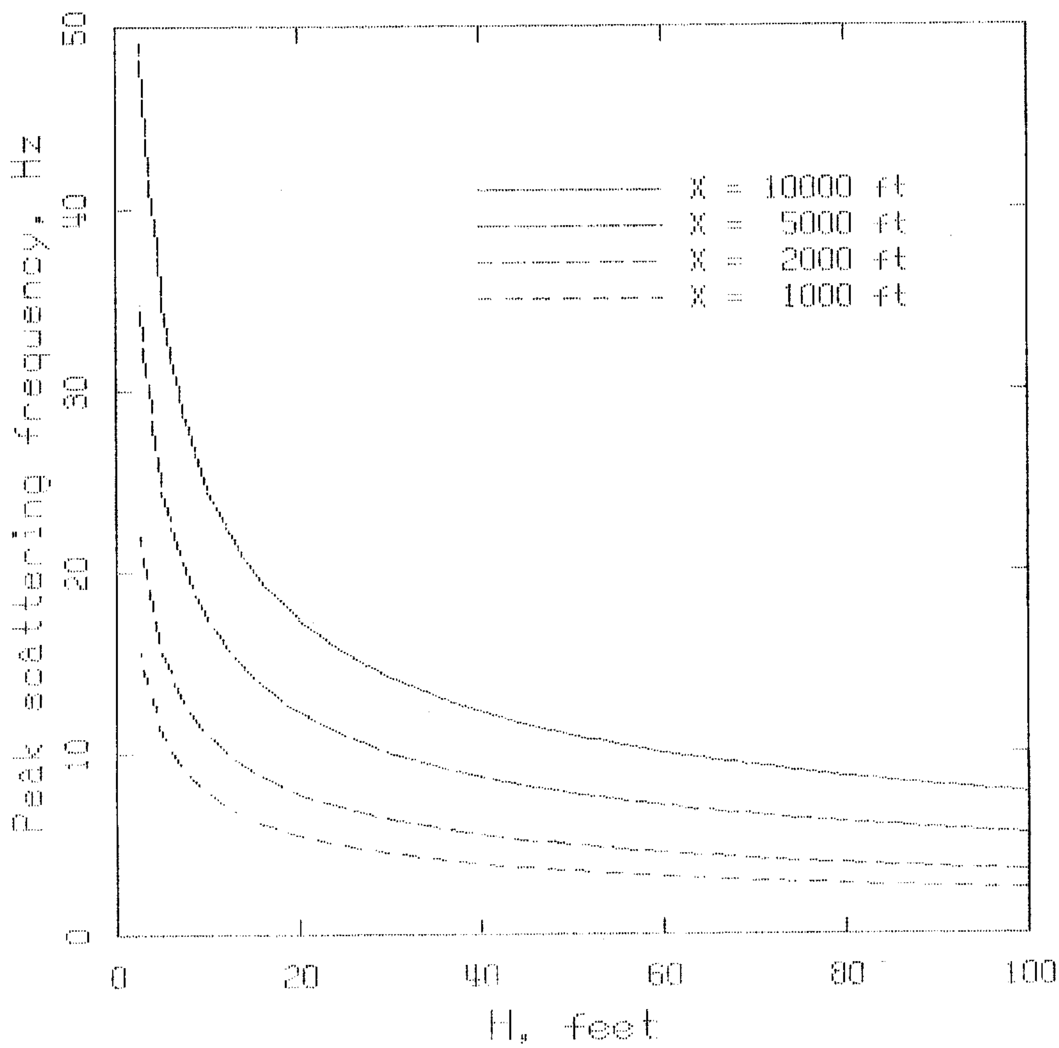
Peak Scattering Angles

This shows the peak frequency of the scattered sound as a function of h and x . Note that, while scattering is generally thought of as a high-frequency phenomenon, there is considerable scattered energy at low frequencies. This is consistent with the large-scale distortions seen in measured signatures.



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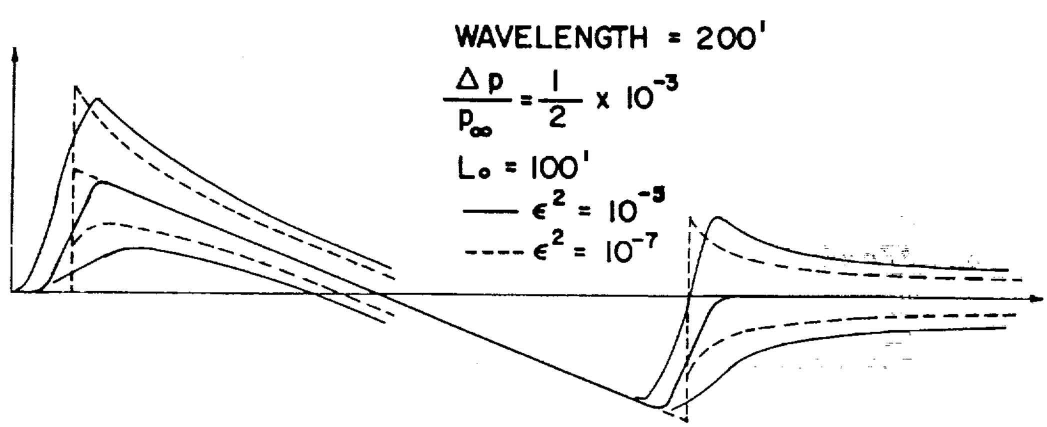
Attenuation by Multiple Scattering

One thought is that, since energy scattered from the shock can be treated as a dissipation mechanism (Plotkin/George theory for anomalous rise times), perhaps it can also attenuate scattered waves. This is the result of such a calculation. It is nice that there is an absolute cap on the perturbation envelopes. It is a little puzzling that the result is so insensitive to turbulence amplitude and to shock thickness. The fatal flaw with this model is that it does not say where the energy is dissipated to. Scattering can redirect sound, but it cannot destroy it. This is therefore a specious result.

$\alpha = 2\,\epsilon^2\,L_o\,k^2$ (Plotkin/George rise time theory)

Take α at f_o for each h, x

ROOT MEAN SQUARE PERTURBATIONS



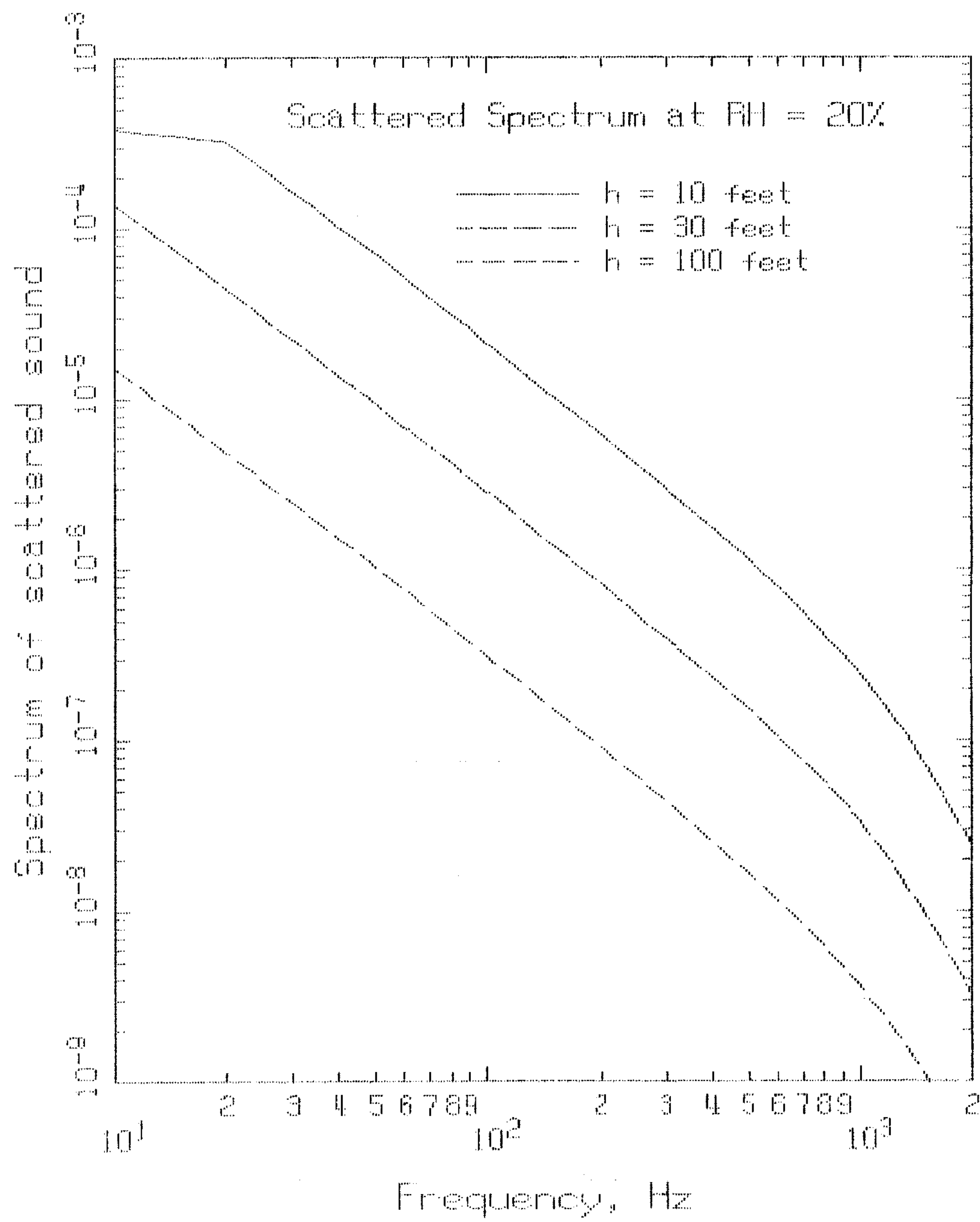
- **Relatively insensitive to turbulence.**
- **Question as to where the multiple scattered energy went.**

Attenuation by Molecular Absorption

The same formulation can easily handle molecular absorption, which is a genuine dissipation mechanism. The main result I have to show today is a calculation of the scattered spectrum including absorption.

- **ANSI S1.26-1978.**
- **Varies strongly with humidity.**
- **Current results: spectra of fluctuations.**
  - **Spectra at various h**
  - **Effect of humidity**

This shows the spectrum at three distances from the shock, one for humidity. Except at higher frequencies, where absorption kicks in, these spectra have a flatter shape than the f^{-2} shape of the incident shock. This is consistent with the high-frequency nature of scattering. This has the potential for a distorted boom to have greater high-frequency content than a clean boom. The high-frequency energy scattered out of the shock is regenerated by nonlinear steepening, but (as will be discussed later) the scattered waves are less susceptible to nonlinear distortion.



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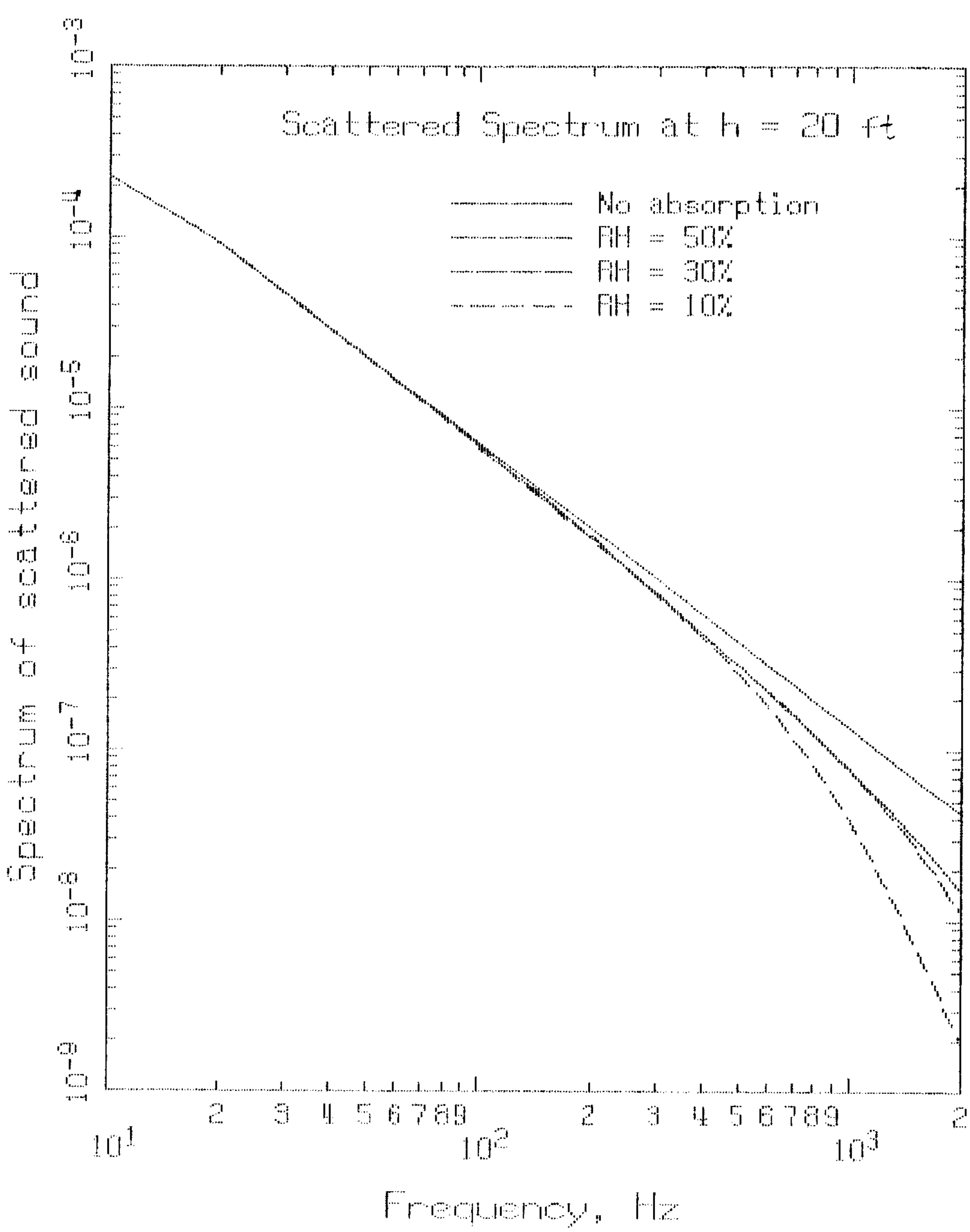
This shows the effect of humidity on the scattered spectrum. Absorption effects are important only at the higher frequency range of this figure, which corresponds to the range where absorption limits the frequency content of the shock itself. The potential "enhanced high frequency" content of a distorted boom involves frequencies which are lower than the frequencies associated with the basic relaxation-dominated shock structure.



Nonlinear Considerations

The nonlinear aspect of sonic boom must be considered. Unless the scattering angle is large enough for the scattered sound to fall behind the shock, it will not separate from the shock. This relation can be used *ad hoc* to justify leaving out very small scattering angles, which are singular, but is also a physical reality on what can be considered to be scattered.

Nonlinear wave propagates at speed

$$a_\infty\left(1 + \frac{\gamma+1}{2\gamma}\frac{\delta p}{p}\right)$$

Shock wave propagates at speed

$$a_\infty\left(1 + \frac{\gamma+1}{4\gamma}\frac{\Delta p}{p}\right)$$

For scattered wave to fall behind shock, require

$$\cos(\theta) < 1 - \frac{\gamma+1}{4\gamma}\frac{\Delta p}{p}$$

Nonlinear Attenuation

The existence of the shock (regardless of structure and mechanism) is what causes energy to be lost. A far-field N-wave decays as distance to the 3/4, rather than 1/2, entirely due to this. A short pulse will decay faster than a long one. This leads to the thought that perturbations may be susceptible to nonlinear decay. However, since they are smaller in magnitude, that is not likely to be the case. Scattering may actually cause more energy to get through – simply by removing it from the coherent front which is moving energy into the shock. A more complete analysis, examining the change in spectral content, is required. A psychoacoustic understanding is also needed of the effects of medium-frequency perturbations following a shock.

- **Steepening causes energy to flow into shock, where it is lost.**
  - **Total energy loss is governed by Rankine–Hugoniot relations, independent of actual dissipation mechanism.**
  - **Detailed structure of shock depends on mechanism.**
- **Scattered waves will steepen.**
  - **Rate of steepening proportional to local pressure jump.**
  - **Perturbations steepen slower than original wave.**

Conclusions

We are out to establish whether turbulent distortion has any effect (adverse or not) on sonic boom loudness. The material presented today is an indication of the approaches we are taking. The main new result is that scattering does not substantially enhance the highest frequencies (those associated with the shock), but does apparently enhance somewhat lower frequencies. Scattering does not appear to be a potential mechanism for increasing overall attenuation of sonic booms. As our analysis proceeds, we will be examining realistic atmospheric models and applying our analysis to minimized boom signatures.

- **Have estimated spectral content of scattered fluctuations.**
- **Medium frequencies are enhanced.**
- **Current model is being expanded to general turbulence.**
- **Seeking an understanding of interaction between various physical mechanisms.**