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NASA Technical Memorandum 106641

Theoretical, Experimental, and Computational Evaluation of Several Vane-Type Slow-Wave Structures

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June 1994

National Aeronautics and Space Administration

THEORETICAL, EXPERIMENTAL, AND COMPUTATIONAL EVALUATION OF SEVERAL VANE-TYPE SLOW-WAVE STRUCTURES

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SUMMARY

Several types of periodic vane slow-wave structures were fabricated. The dispersion characteristics were found by theoretical analysis, experimental testing, and computer simulation using the MAFIA code. Computer-generated characteristics agreed to approximately within 2% of the experimental characteristics for all structures. The theoretical characteristics, however, deviated increasingly as the width to height ratio became smaller. Interaction impedances were also computed based on the experimental and computer-generated resonance frequency shifts due to the introduction of a perturbing dielectric rod.

INTRODUCTION

The vane type structures are frequently used as slow-wave structures for linear magnetron tube devices but there is no evidence in the literature of their use in traveling-wave tubes. These investigations were undertaken to assess the suitability of periodic vanes as the slow-wave structure for application in traveling-wave tubes in terms of their interaction impedances. These structures are easily fabricated. This allows us to experimentally determine the characteristics of scaled models of various vane structures and compare them to those obtained by general theoretical analysis and computer generated results.

THEORETICAL EVALUATION

A simplified TM mode analysis of a general vane structure with a roof has been carried out by Collin [1] and Watkins [2]. Assuming the vane width is much larger than the roof height, the dispersion relation can be written as

$$\frac{1}{k_0 L \, \tan(k_0 h)} = \frac{s}{L} \sum_{n=\infty}^{\infty} \left[\frac{\sin(\beta_n s/2)}{\beta_n s/2} \right]^2 \cdot \frac{1}{\tau_n L \, \tanh(\tau_n (w-h))} \tag{1}$$

where h is the vane height, w is the vane width, L is the period, s is the slot spacing, $k_0 = 2\pi f$ is the free space wave number and f is the frequency, $\beta_n = \beta_0 + 2n\pi/L$ is the nth axial wavenumber, and $\tau_n^2 = {\beta_n}^2 - {k_0}^2$. Hutter [3] has shown some typical dispersion results for vane structures.

For slow waves, $\beta_n > k_0$ and $\tau_n \approx \beta_n$. The theoretical dispersion characteristics for a general vane structure without a roof based on the assumption of infinitely wide ($w \to \infty$) rectangular vanes can then be found from the relation

$$\frac{1}{k_0 L \, \tan(k_0 h)} = \frac{s}{L} \sum_{n=\infty}^{\infty} \left[\frac{\sin(\beta_n s/2)}{\beta_n s/2} \right]^2 \cdot \frac{1}{\beta_n L}. \tag{2}$$

The theoretical dispersion characteristics (k_0 vs β_0) of four vane structures (Figure 1) were calculated for $s = 0.15$ inch, $L = 0.225$ inch, and the corresponding height h (Figure 2).

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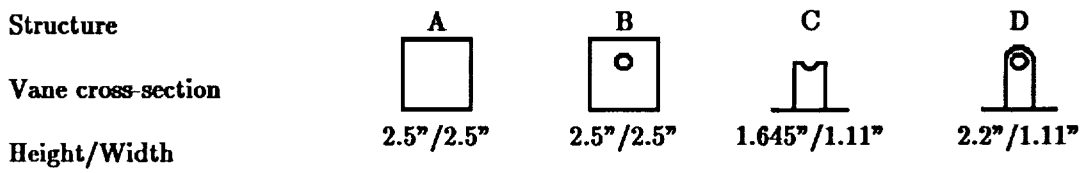


Figure 1 - Profiles of four vane structures.

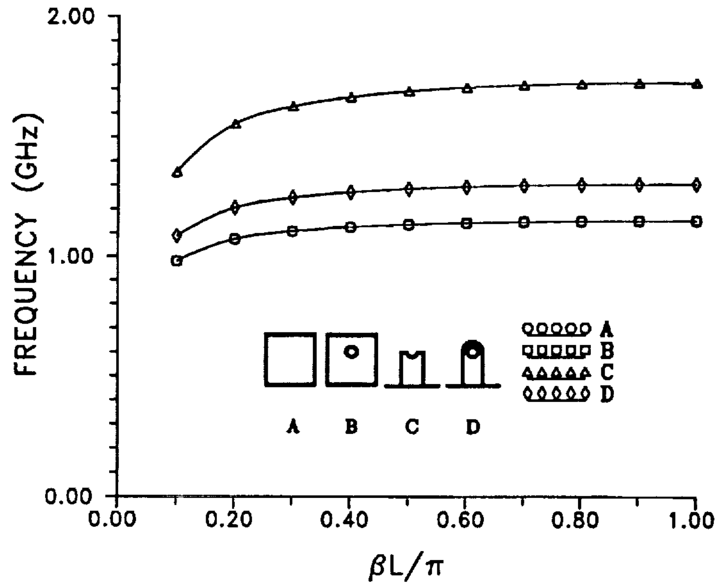


Figure 2 - Theoretical dispersion characteristics of the four vane structures.

EXPERIMENTAL EVALUATION

Vanes for each of the four types of structures were machined out of brass with a 1 inch extended base having alignment holes. Each combined vane and base had a thickness of 0.075 inch. Slot spacers 0.075 inch thick having the same shape and alignment holes as the extended base of the vanes were also machined out of brass. The vanes for structures *A* and *B* had heights and widths of 2.5 inches. The heights for structures *C* and *D* were 1.645 inches and 2.2 inches, respectively, while both structures had widths of 1.11 inches. The holes for structures *B*,*C*, and *D* are centered 1.645 inches above the base with a diameter of 0.59 inch.

The vanes and spacers for each structure were stacked between two end plates with two spacers between vanes and one spacer between vane and end plate to create a structure with a period of 0.225 inch and shorted by the end plates at mirror symmetry points. Magnetic signal coupling from coaxial cables was achieved using wire loops through drilled holes in the end plates. Coaxial electric probes were used initially, however resonances were difficult to observe.

Resonance frequency measurements near 1 GHz were taken for ten periods of all four structures with and without an alumina dielectric rod of relative permittivity $\epsilon_r = 9.2$ and radius $r = 0.125$ inch using a Hewlett-Packard 8510C network analyzer (Figures 3, 4, 5, and 6). For the perturbation measurement, the alumina rod was placed at the top center of structure *A* and at the center of the holes for the other structures. Initially one period of each structure was measured to identify the proper resonances with the proper wavenumber. Each structure was then lengthened one period at a time and measured until a maximum of ten periods was reached.

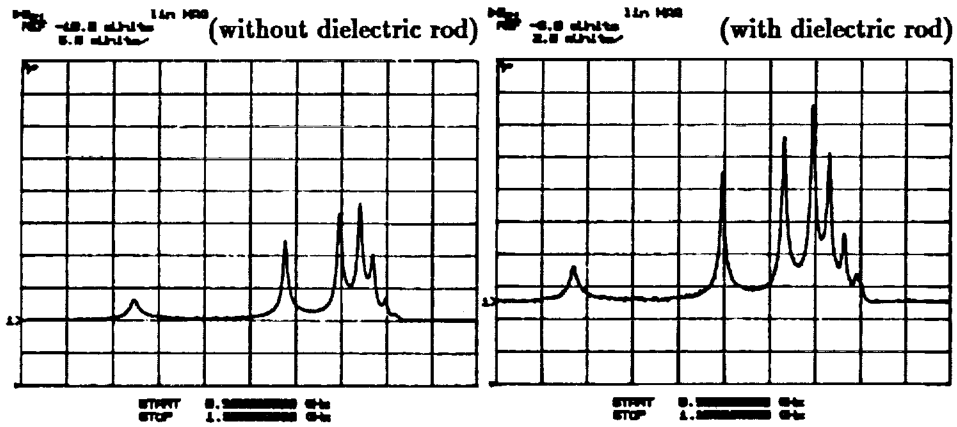


Figure 3 - Resonant frequencies for structure A without and with the dielectric rod.

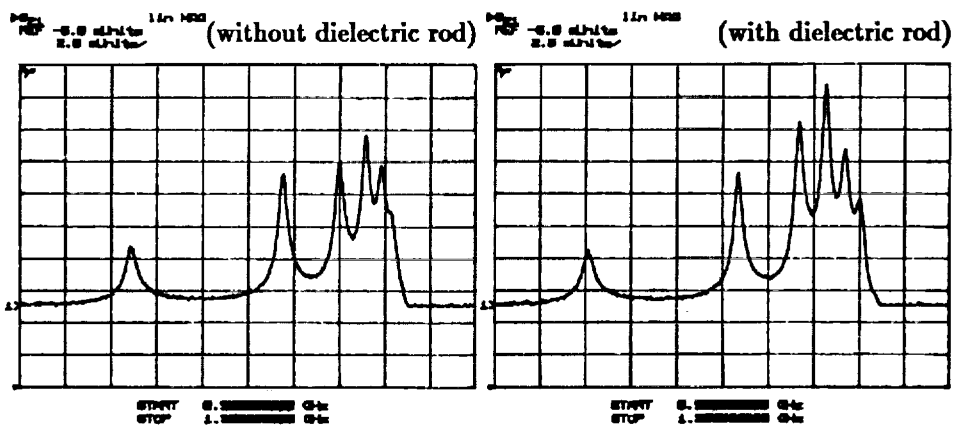


Figure 4 - Resonant frequencies for structure B without and with the dielectric rod.

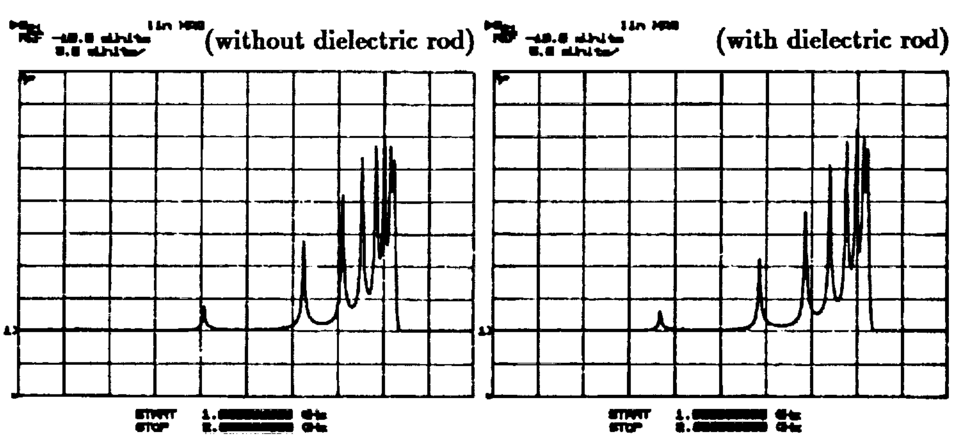


Figure 5 - Resonant frequencies for structure C without and with the dielectric rod.

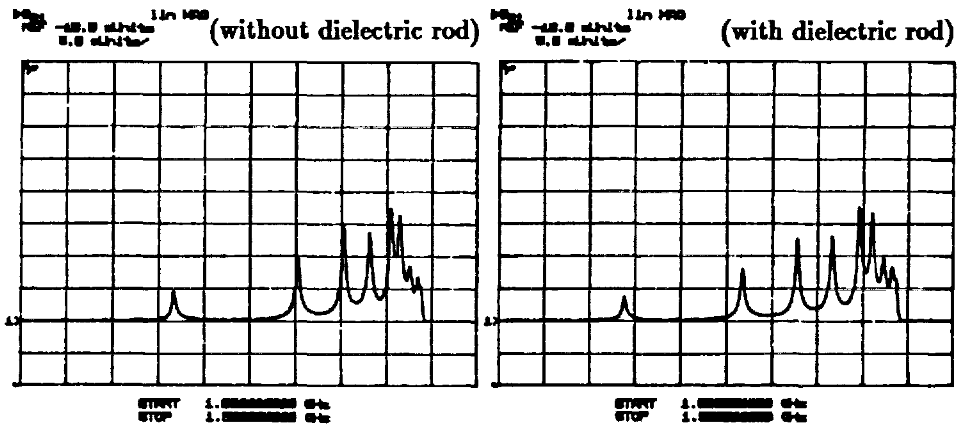


Figure 6 - Resonant frequencies for structure D without and with the dielectric rod.

The dispersion characteristics measured using ten periods for all four structures were fitted to a third degree polynomial in $\ln(\beta L\pi)$ (Figure 7).

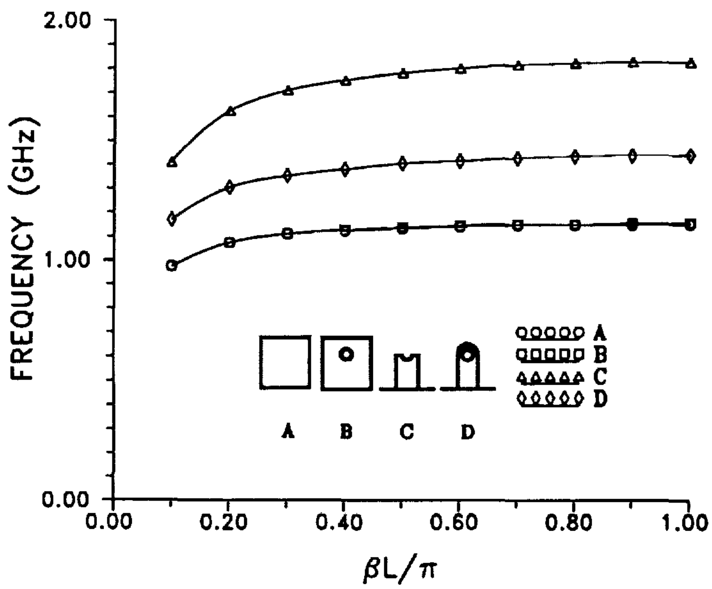


Figure 7 - Experimental dispersion characteristics of the four vane structures.

Group velocities were calculated using

$$v_g = 2\pi \cdot \frac{\mathrm{d}f}{\mathrm{d}(\ln(\beta L/\pi))} \cdot \frac{\mathrm{d}(\ln(\beta L/\pi))}{\mathrm{d}\beta}. \tag{3}$$

From the measured frequency shifts δf due to the introduction of the dielectric rod and the corresponding group velocities, the interaction impedances were calculated using

$$\overline{Z} = -\frac{1}{\beta^2 v_g} \frac{\delta f}{f} \frac{1}{(\epsilon_r - 1)\epsilon_0 \pi r^2}. \tag{4}$$

These data were fitted to a cubic spline (Figure 8).

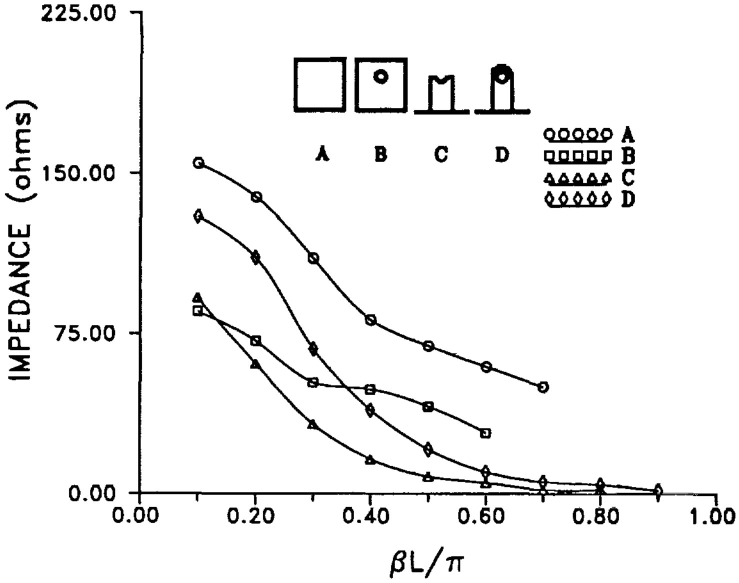


Figure 8 - Experimental interaction impedance characteristics of the four vane structures.

COMPUTATIONAL EVALUATION

The four vane structures were modeled and analyzed using the computer code MAFIA. MAFIA is an acronym for the solution of MAxwell's equations using the Finite Integration Algorithm. The MAFIA system [4] is a set of codes, written entirely in pure FORTRAN 77, used for the computer-aided design of fully three-dimensional and two-dimensional electromagnetic devices such as electrostatic devices, magnets, RF cavities, wave guides, antennas, and microelectronic circuits.

The Finite Integration Technique (FIT) algorithm [5] produces a set of finite-difference matrix equations for the electric and magnetic field vectors in the structure under study. The solution of these equations yields a static solution, frequency-domain solution, or the time-domain solutions of Maxwell's equations [6,7,8,9].

MAFIA is a complete and universal electromagnetic field problem analysis environment that solves Maxwell's equations with very few restrictive specializations, and is applicable over almost the same range of problems as are Maxwell's equations themselves [10]. A distinct advantage of the MAFIA code is the ability to find the resonance frequency corresponding to a particular wavenumber or phase shift per period by modeling only one period of the structure. This reduces the mesh, memory, and time required to find the resonance and alleviates the problem of having to check the mode pattern to make sure that the particular resonance is the proper one. The code resides on an IBM RISC 6000 computer at NASA Lewis Research Center.

One period of each structure was modeled using MAFIA. The phase shifts per period were varied from 18° to 180° in increments of 18° for each computer run to correspond to the experimental dispersion points. The resonance frequencies of the simulated structures were found with and without a simulated perturbing dielectric rod at the appropriate location having a cross-sectional area equal to that of the actual alumina rod. The computer-generated dispersion characteristics were also fitted to a third degree polynomial in $\ln(\beta L/\pi)$ with the group velocities calculated in similar fashion (Figure 9). Interaction impedances were then calculated for all four structures (Figure 10).

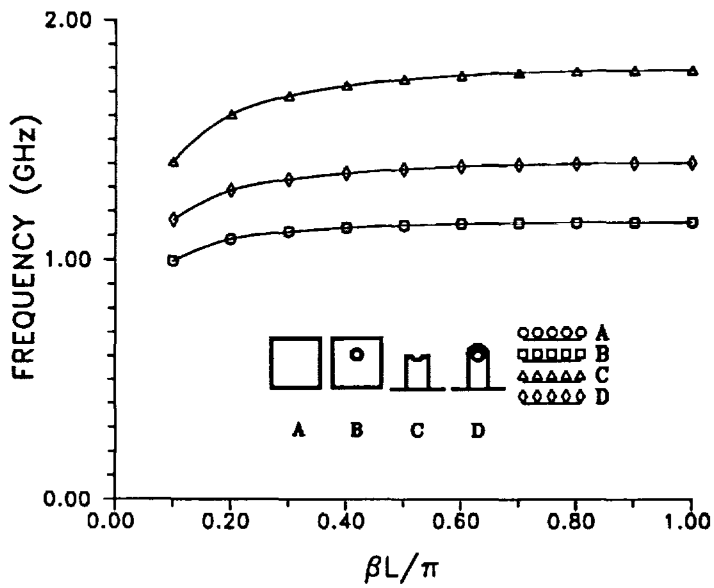


Figure 9 - Computer-generated dispersion characteristics of the four vane structures.

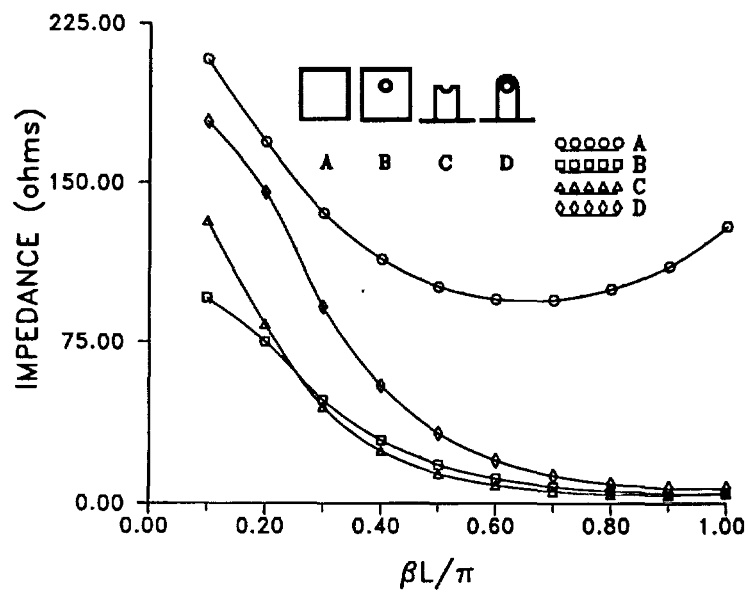


Figure 10 - Computer-generated interaction impedance characteristics of the four vane structures.

COMPARISON OF RESULTS

The theoretical dispersion characteristics for the infinitely wide, rectangular vane structures were within 0.69%, 0.73%, 5.64%, and 9.45% of the experimental dispersion characteristics for structures *A*, *B*, *C*, and *D*, respectively. This shows that the wider structures yield excellent results but as the structures narrow, the deviation between theoretical and experimental values becomes appreciable. The parameter which controls the high frequency cutoff of the particular vane structure is the vane height. The cutoff frequency is inversely proportional to the height of the vane.

The computer-generated dispersion characteristics were in very good agreement with the experimentally measured values for all structures. They were within 2.08%, 2.07%, 2.01%, and 2.44% for structures A, B, C, and D, respectively. Any discrepancies can be accounted for in the mesh approximation of the computer-generated structures.

The values for interaction impedance that were calculated from the experimental and computer-generated dispersion characteristics follow the same decreasing trend with wave number for all structures with the values obtained from computer results being slightly higher.

The impedances for structure A exhibit much greater values than structure B. This is due to the higher electric fields near the top of the structure than in the hole area. Also the impedances for structure D are greater than those of structure C over the frequency band.

Initially, the general theoretical approach to the solution of the dispersion characteristics for the vane type structures will yield fairly accurate results in order to estimate the usefulness of the structure. This will save time and resources (computer software and hardware, machining costs, etc.). If further analysis is needed, the computer-aided design approach is more cost effective than building an experimental cold test structure. The time involved to modify and test the structure in software is far less than the time to physically modify and test it. Once the proper dimensions of the design are found then experimental testing is the final step.

ACKNOWLEDGEMENT

The authors would like to gratefully acknowledge Karl R. Vaden, Jennifer E. Coy, and Lisa M. Skinner for their help in the experimental and computational effort.

APPENDIX A

Mathcad program to calculate the theoretical dispersion characteristics for an infinitely wide vane structure.

vane spacing	$\delta := .15 \cdot .0254$	**speed of light**	$c := 2.99792 \cdot 10^{8}$
vane period	$d := (.15 + .075) \cdot .0254$	**number of points**	$i := 1 .. 10$
vane height	$h := 2.5 \cdot .0254$	**number of harmonics**	$n := -20 .. 20$

Calculate the wave numbers and harmonics.

$$\beta(i,n) := .1 \cdot i \cdot \frac{\pi}{d} + \frac{2 \cdot \pi \cdot n}{d}$$

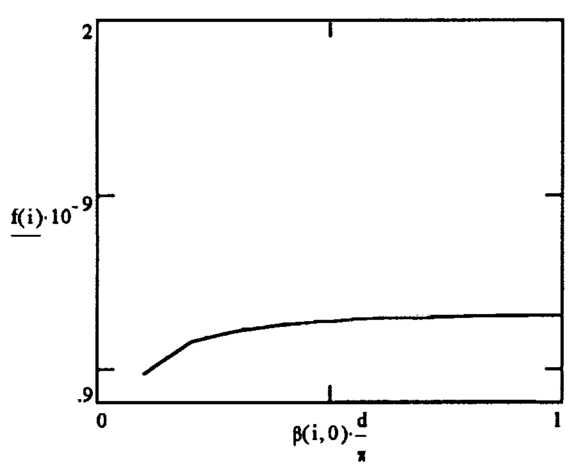
Calculate the sum of harmonic components.

$$\mathrm{sum}(i,k) := \sum_{n} \left[\frac{1}{\sqrt{\left| k^2 - \beta(i,n)^2 \right|} \cdot d} \cdot \frac{\sin\left(\beta(i,n) \cdot \frac{\delta}{2}\right)^2}{\left(\beta(i,n) \cdot \frac{\delta}{2}\right)^2} \right]$$

Solve the dispersion equation for *k*.

$$k := 30 \qquad k(i) := \mathrm{root}\left(-\frac{1}{k \cdot d \cdot \tan(k \cdot h)} - \frac{\delta}{d} \cdot \mathrm{sum}(i,k), k \right) \qquad f(i) := \frac{k(i)}{2 \cdot \pi} \cdot c$$

Plot of the dispersion characteristics



i	$f(i) \cdot 10^{-9}$
1	0.980266863
2	1.07335671
3	1.106659567
4	1.123767316
5	1.134027052
6	1.140644643
7	1.145005443
8	1.147788951
9	1.149346131
10	1.149848043

APPENDIX B

Mathcad program to calculate the theoretical dispersion characteristics and interaction impedances for vane structures using experimental or computational data.

vane thickness $t := .075 \cdot .0254$

vane spacing $s := .15 \cdot .0254$

period $L := s + t$

dielectric rod radius $r := .125 \cdot .0254$

dielectric constant $\varepsilon r := 9.2$

speed of light $c := 2.99792 \cdot 10^{8}$

electronic charge $q := 1.60218 \cdot 10^{-19}$

electronic mass $m := 9.1095 \cdot 10^{-31}$

permeability $\mu := 4 \cdot \pi \cdot 10^{-7}$

permittivity $\varepsilon := 8.854215 \cdot 10^{-12}$

number of points $N := \text{length}(f)$

Frequency shifts $\delta f(i) := f_i - fd_i$

Read frequency data file

$M := \text{READPRN}(\text{vnaexp})$

Unperturbed frequencies

$f := M^{<0>} \cdot 10^{9}$

$f := \vec{f}$

Perturbed frequencies

$fd := M^{<1>} \cdot 10^{9}$

$fd := \vec{fd}$

$i := 0..N-1$

Least squares cubic fit in ln(β)

$D := 3 \quad d := 0..D \quad \beta_i := .1 \cdot (i+1) \cdot \frac{\pi}{L} \quad \beta := \vec{\beta} \quad u := \overrightarrow{(\ln(\beta))} \quad du := \overrightarrow{\left(\frac{1}{\beta}\right)} \quad X^{<d>} := \overrightarrow{\left(u^{d}\right)}$

Calculate the coefficients

$\omega := 2 \cdot \pi \cdot f \qquad \text{coeff} := \left(X^{T} \cdot X\right)^{-1} \cdot \left(X^{T} \cdot \omega\right) \qquad \Omega(t) := \sum_{d} \text{coeff}_d \cdot t^{d} \qquad \Omega'(t) := \sum_{d} \text{coeff}_d \cdot d \cdot t^{d-1}$

Calculate the phase velocity, group velocity, and equivalent beam voltage

$$vp(i) := \frac{\omega_i}{\beta_i} \qquad vg(i) := \Omega'\left(u_i\right) \cdot du_i \qquad V(v) := \left[\sqrt{\left(1 - \frac{v^2}{c^2}\right)^{-1}} - 1\right] \cdot m \cdot \frac{c^2}{q}$$

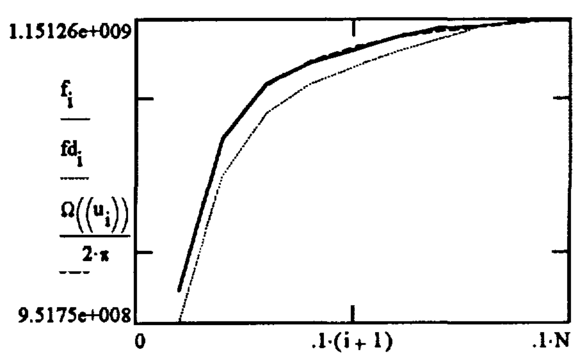
Calculate the interaction impedance

$$Z(i) := \frac{1}{\left(\beta_i\right)^2 \cdot vg(i)} \cdot \frac{\delta f(i)}{(\varepsilon r - 1) \cdot f_i \cdot \varepsilon \cdot \pi \cdot r^2}$$

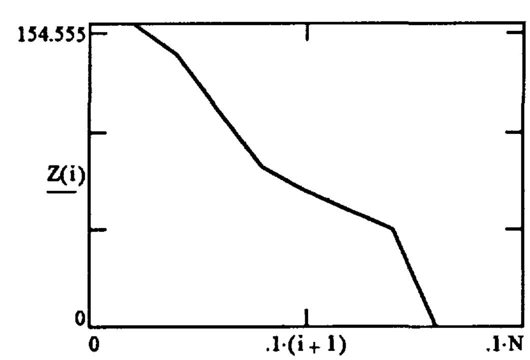
Values of various quantities

$\frac{i+1}{10}$	$f_i \cdot 10^{-9}$	$\Omega\left(u_i\right) \cdot \frac{10^{-9}}{2 \cdot \pi}$	$V(vp(i))$	$Z(i)$
0.1	0.974	0.974	39377.367	154.555
0.2	1.074	1.074	11051.065	138.777
0.3	1.109	1.108	5154.599	110.089
0.4	1.123	1.124	2951.494	81.567
0.5	1.131	1.133	1911.005	69.119
0.6	1.139	1.139	1344.288	59.182
0.7	1.146	1.143	998.367	49.798
0.8	1.147	1.147	765.186	0
0.9	1.15	1.149	607.213	0
1	1.15	1.151	491.676	0

Plot of frequency, fit, and perturbed frequency



Plot of interaction impedance



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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (*Leave blank*)

2. REPORT DATE
June 1994

3. REPORT TYPE AND DATES COVERED
Technical Memorandum

4. TITLE AND SUBTITLE
Theoretical, Experimental, and Computational Evaluation of Several Vane-Type Slow-Wave Structures

5. FUNDING NUMBERS
WU-235-01-1C

6. AUTHOR(S)
Thomas M. Wallett and A. Haq Qureshi

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135-3191

8. PERFORMING ORGANIZATION REPORT NUMBER
E-8945

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
National Aeronautics and Space Administration
Washington, D.C. 20546-0001

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
NASA TM-106641

11. SUPPLEMENTARY NOTES
Thomas M. Wallett, NASA Lewis Research Center, and A. Haq Qureshi, NASA Resident Research Associate at Lewis Research Center. Responsible person, Thomas M. Wallett, organization code 5620, (216) 433-3673.

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Unclassified - Unlimited
Subject Category 33

12b. DISTRIBUTION CODE

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14. SUBJECT TERMS
Vane structures; Dispersion; Interaction impedance

15. NUMBER OF PAGES
12

16. PRICE CODE
A03

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	