PARAMETER ESTIMATION OF THE VIBRATIONAL MODEL
FOR THE SCOLE EXPERIMENTAL FACILITY

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The objective of this study is to experimentally determine an empirical model of the vibrational dynamics of the Spacecraft COntrol Laboratory Experiment (SCOLE) facility. The first two flexible modes of this test article are identified using a linear least-square identification procedure and the data utilized for this procedure are obtained by exciting the structure from a quiescent state with torque wheels. The time history data of rate gyro sensors and accelerometers due to excitation and after excitation in terms of free-decay are used in the parameter estimation of the vibrational model.

The free-decay portion of the data is analyzed using the Discrete Fourier transform to determine the optimal model order to use in modelling the response. Linear least-square analysis is then used to select the parameters that best fit the output of an Autoregressive (AR) model to the data. The control effectiveness of the torque wheels is then determined using the excitation portion of the test data, again using linear least squares.

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INTRODUCTION

Future NASA space missions may involve very large and highly flexible spacecraft that require active structural dynamics control. Large space structures would require very stringent pointing and vibration suppression requirements. The active controller that can achieve these objectives will have to be designed with very accurate knowledge of the dynamic behavior of the structure to ensure performance robustness to a variety of disturbances and uncertainties. It is recognized by control engineers that there are certain inherent problems in the design of active controllers for this class of large flexible spacecraft. Because of these concerns and of the desire to offer a means of comparing technical approaches directly, a NASA/IEEE Design Challenge [1] was being offered to the technical community. In 1983, the Spacecraft Control Branch at NASA Langley Research Center in Hampton, VA. initiated the Spacecraft Control Laboratory Experiment (SCOLE) program and the NASA/IEEE Design Challenge to promote direct comparison and a realistic test of different approaches to control design against a common open to the public laboratory test article. This facility provides researchers with a highly flexible test article, sensors, actuators, and digital control processing capability. The test article resembles a large space antenna attached to the Space Shuttle Orbiter by a long flexible mast, similar to proposed space flight experiments and various space-based antenna systems. The proposed model is shown in Figure 1. Using SCOLE, control laws for a multi-input output structural dynamics system can be implemented in real time from any remote site that has a computer terminal and modem communications capability. Much interest has been expressed by the research community concerning SCOLE. This is reflected in the technical output of five workshops held since the conception of SCOLE in 1983.

SCOLE APPARATUS

The SCOLE hardware and software support is described in detail in Refs. 2 and 3 and in this paper. For this work, SCOLE contains two major structural elements of interest: a planar, hexagonal tubular structure representing an antenna reflector, and a single tubular flexible mast connecting the antenna to the platform, as shown in Figure 2. The platform is fixed to ground and only the mast and reflector portions are dealt with in this experiment. The system actuators consist of three mast-end mounted reaction wheels that produce torque in three mutually orthogonal directions. The system sensors are comprised of a three-axis reflector-system mounted rotational rate sensor and both mast-mounted and reflector-mounted x and y-axis accelerometers. The experiments are run on SCOLE using a digital M68000-based

computer that has a UNIX-like operating system version called UNOS. Programming is accomplished in a combination of C and FORTRAN 77 programs. The computer has analog-to-digital (A/D) converters used for sampling the rate sensor data, digital-to-analog (D/A) converters used to command the reaction torque wheels, and a process timer which achieves precise internal timing of the data sampling process.

SYSTEM MODELS

The model we seek for SCOLE should incorporate the actual natural frequencies, damping ratios, and control effectiveness coefficients of the system. The viscous damping can be modeled in terms of ζ_i, the damping ratio of the ith mode. To this end, each mode of the vibrational dynamics of SCOLE is modeled as a single-input, single-output system [3] described by the state-space equation

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$

where

$$x = [\eta \dot{\eta}]^T \tag{2}$$

$$A = \begin{matrix} 0 & 1 \\ -\omega^2 & -2\zeta\omega \end{matrix} \tag{3}$$

$$B = \begin{matrix} 0 \\ b \end{matrix}. \tag{4}$$

In this equation,

$$\omega^2 = \frac{k}{m}, \tag{5}$$

$$2\zeta\omega = \frac{c}{m}, \tag{6}$$

where

$\underline{x}$ - modal state vector
u - control input of reflector end reaction torque wheels
b - control effectiveness parameter of actuator location
ω - natural frequency of mode

ζ - damping ratio of mode.

The output is of the form

$$y = H\underline{X} \tag{7}$$

for a rate sensor,

$$H = [0, c], \tag{8}$$

where c is the mode slope at the sensor location.

To obtain a difference equation model for digital computer control, the control input is assumed constant over the computer sample time interval of T seconds and the continuous-time model is converted to its discrete-time equivalent by integration over the interval.

Thus, the difference equation describing the motion appears as

$$\underline{x}_{K+1} = \phi\underline{x}_K + \Gamma\underline{u}_K, \tag{9}$$

where

$$\phi = e^{AT} \tag{10}$$

$$\Gamma = \int_0^T e^{At} dt \, B = (\phi - I) A^{-1} B, \tag{11}$$

since A is nonsingular and I is the 2x2 identity matrix.

The model used in this work is a linear, constant-coefficient, difference equation. To accomplish such a model, an auto-regressive form of the discrete time model is found by taking the z-transform of the last equation and solving for the sampled sensor output, y_k, in terms of the input actuator. The auto-regressive moving average (ARMA) model appears as

$$y_K = a_1 y_{K-1} + a_2 y_{K-2} + (b_1 u_{K-1} + b_2 u_{K-2}) b_{TW}, \tag{12}$$

where

$$a_1 = \phi_{11} + \phi_{22} \tag{13}$$

$$a_2 = \phi_{12}\phi_{21} - \phi_{11}\phi_{22} \tag{14}$$

$$b_1 = \Gamma_{12} \tag{15}$$

$$b_2 = \phi_{12}\Gamma_{22} - \phi_{22}\Gamma_{12} \qquad (16)$$

$$b_{TW} = c \cdot b. \qquad (17)$$

Identification of the ARMA model parameters is performed using the linear least-square estimation (LSE) algorithm. This method was selected because of its computation efficiency and implementation simplicity. The error equation used in the estimation is defined to be

$$e_K = y_K - [a_1 y_{K-1} + a_2 y_{K-2} + (b_1 u_{K-1} + b_2 u_{K-2}) b_{TW}]. \qquad (18)$$

The sum of the squared error,

$$J = \frac{1}{2} \sum_{K=0}^{N} e_K^2, \qquad (19)$$

is the performance measure to be minimized with respect to the parameter desired to be found.

PARAMETER IDENTIFICATION TECHNIQUES

Linear least-square estimation (LSE) is used to identify the ARMA model parameters. This method is selected because of its computation efficiency and implementation simplicity. The identification process is carried out for each reaction torque wheel and for each mode. The test data in the identification process of the SCOLE problem is processed in a two-step operation.

For the first step, the AR coefficients a_1 and a_2 of the ARMA are identified using the free-decay portion of the collected data. The spectral content of the free-decay portion of the data is examined using the Discrete Fourier Transform and the Hamming window. The free-decay data are filtered to suppress noise and signals due to any modes not wanted in the model. The filtered data is processed using the standard least-square estimation to identify the AR coefficients, a_1 and a_2, of the modelled mode. The identification of the a_1 and a_2 coefficients for each mode generally depends on the data base used in the estimation. As more significant data are added, the estimates should converge to a value and the variance of the estimates will improve to a limit, which depends on the measurement noise and the model. After convergence occurs, the mean and

variance of the estimates should remain constant. Therefore, the variation of the estimates is examined as data is added to the data base and the mean of the estimates is taken over the last several data base additions. To ascertain confidence in the estimates, the variance of the estimates is also taken into consideration. Also, the damping ratios and frequency of each mode can be computed from the a_1 and a_2 coefficients. In the second step of the parameter identification procedure for the ARMA model, the control effectiveness coefficient b_{TW} of the torque wheel used is determined. Once the values of the a_1 and a_2 coefficients for the mode of interest are determined, a similar linear least-square scheme is employed on the excitation portion of the test data to obtain the control effectiveness of the torque wheel actuator with respect to the mode of interest. Again, the mean and variance of the estimate of b_{TW} is taken into consideration.

EXPERIMENTAL PROCEDURES

Experimental work and testing can be conducted on SCOLE either at the NASA Langley Research Center in Hampton, VA, where SCOLE is located, or at any remote site that has a computer terminal and modem communications capability. The work presented in this paper was conducted both at NASA and from the UNC Charlotte College of Engineering.

A manual structural excitation test is performed wherein data is collected and analyzed to verify physical modal directions and frequencies as predicted by computer simulations of SCOLE. Bias readings of all inertial sensors are always taken before each run to establish a reference frame. For mode 1 testing, the structure is hand held by the reflector and pulled in the center of the +x and +y-axis directions, as shown in Figure 3, approximately six inches or until the displacement angle about the z-axis reaches five degrees. When the reflector is released, free decay data is recorded and collected. The same process occurs for mode 2 testing, with the reflector being released from the center of the -x and +y-axis directions as shown in Figure 4. Data is collected and verified against predicted natural frequencies of modes 1 and 2.

The vibrational dynamic model we seek for SCOLE incorporating the actual natural frequencies, damping ratios, and control effectiveness coefficients are obtained in this work by using the mast-end mounted reaction wheels to excite the structure. Structural excitation tests are individually conducted wherein the structure, initially at rest, is sinusoidally forced by a single reaction wheel for 30 seconds at the predicted mode of interest. The data recording is continued for 60 seconds to obtain free-decay

data. Tests are carried out for each of the x,y, and z-axis torque wheels and for both modes 1 and 2 and are summarized in Table 1.

DISCUSSIONS

Experimental data and results of the system identification process are summarized and illustrated in Table 2 and Figures 5 through 10. Tests and analysis using each of the x, y, and z-axis torque wheels at both the first and second modes, .4401 and .4764 Hz respectively, are carried out. The identified parameters a_1, a_2, b_{TW}, and the computed values of £ and f for all of the tests are tabulated in Table 2.

Figure 5 shows the input excitation signal of the reaction torque wheel. Before each test, the structure was steadied and bias readings of the sensors were taken and accounted for. A sinusoidal forcing signal of amplitude 20 at the desired mode test frequency was applied to the structure for 30 seconds. For mode 1 frequency of .4401 Hz, the x-axis reaction torque wheel excited the structure the most. This was in agreement with the predicted first bending mode shape shown to occur closer to the pitch or y-axis direction. The y-axis torque wheel had the greatest effect on exciting the structure for mode 2 tests, which agreed with the predicted second bending mode shape occurring in the direction of the roll or x-axis. The z-axis torque wheel had the least effect on exciting the structure, as it tended to excite the structure in the yaw or about the z-axis direction. Since the first two vibrational modes occur dominantly in the x-y plane, the z-axis torque wheel had little effect on exciting the structure at the first two modal frequencies.

Figures 6 and 7 illustrate the frequency spectral analysis on the free decay portion of the data using the DFT and Hamming window. The structure would non-periodically vibrate in both translational and rotational directions after about 30 seconds of free decay. Therefore, only the first 30 seconds were considered due to other modes becoming dominant in the decay. The length of the data record used by the DFT was chosen to be equal to an integer number of periods of the sequence. The presence of leakage or significant non-zero frequency components occurred when the data record was improperly truncated. Therefore, an integer number of periods represented by the value of N was chosen. The frequency magnitudes were also proportional to the number of periods included in the record length. The greater the number of periods , the larger the magnitude. To further reduce the effects of the discontinuities introduced by truncating the sequence, Hamming windows were

used. The Hamming windows represent a noticeable improvement in suppressing the magnitude of the side lobes and the unwanted non-zero frequencies while broadening the main lobe frequency. The use of Hamming windows was extremely valuable in detecting and identifying the first and second modal frequencies, which were very close together.

Figures 8 and 9 of each test provide a description of the parameter identification process for the free decay coefficients a_1 and a_2. From these figures, the identified parameters are seen to converge as data is added to the data base for estimation. The identification process was terminated when convergence was achieved based on the deviation of each of the last three iterations from their arithmetic average with a convergence bandwidth of 5 percent. The mean values of a_1 and a_2 for each test are shown in Table 2. The natural frequency and its damping factor were computed based upon the coefficients a_1 and a_2, and are also shown in Table 2. The accuracy of the modal frequency was affected by the characteristics of the LSE identification technique. The frequency calculated from the LSE varied slightly from the frequency given by the DFT as shown on the figures. The characteristics and accuracies of the LSE algorithm and the DFT accounted for a part of the difference.

Figure 10 of each test shows the control effectiveness parameter of the reaction torque wheel, b_{TW}. Control effectiveness parameters were particularly difficult to accurately determine because of weak actuators and were small when compared to their associated frequency and damping parameters. Also, due to the difference between the forced and resonant frequencies of the system, the value of b_{TW} found may be highly inaccurate. The estimate converged as more data was added. It is important to use the mean of b_{TW} taken over a converged portion of the data and not a single point. The mean and variance of b_{TW} are shown on the figures. The presence of a disturbance was induced when the reaction torque wheel was initially started. It is believed that this was the result of start up friction of the reaction torque wheel assembly. Therefore, the first 5 seconds of data were not included in the data base for estimation.

SUMMARY

In this work, an empirical model of the vibrational dynamics of the first two flexible modes of SCOLE was found using the linear least square identification procedure. The experimental apparatus and procedures followed and the system model assumed were also discussed in this study.

Testing was done by exciting the structure from a quiescent state with torque wheels and recording the time history data of rate gyro sensors and accelerometers. The torque wheels were then shut down and free-decay data recorded. The DFT and Hamming window were used to analyze the free decay portion of the data. The coefficients of an autoregressive model to the data were determined using linear least square analysis. Next, the control effectiveness of the torque wheels was found using the excitation portion of the test data, again utilizing linear least squares. Experimental data and graphs were also presented to provide a description of the digital signal process algorithms and techniques used in determining the first two flexible modes of SCOLE. The experimental results presented in this paper have the potential to be extremely useful in modelling vibrational dynamics of large flexible spacecraft structures.

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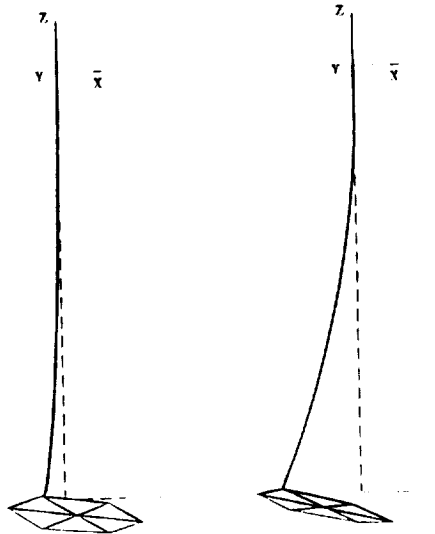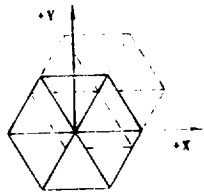
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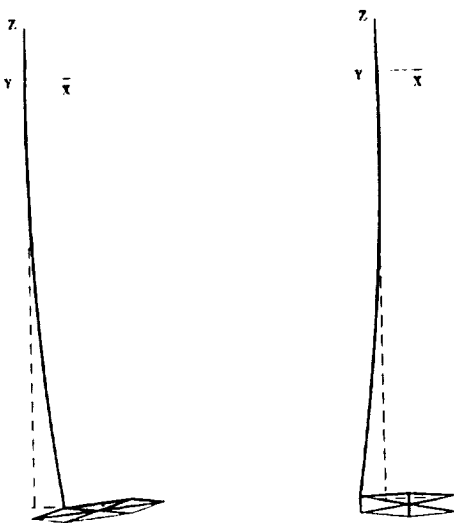
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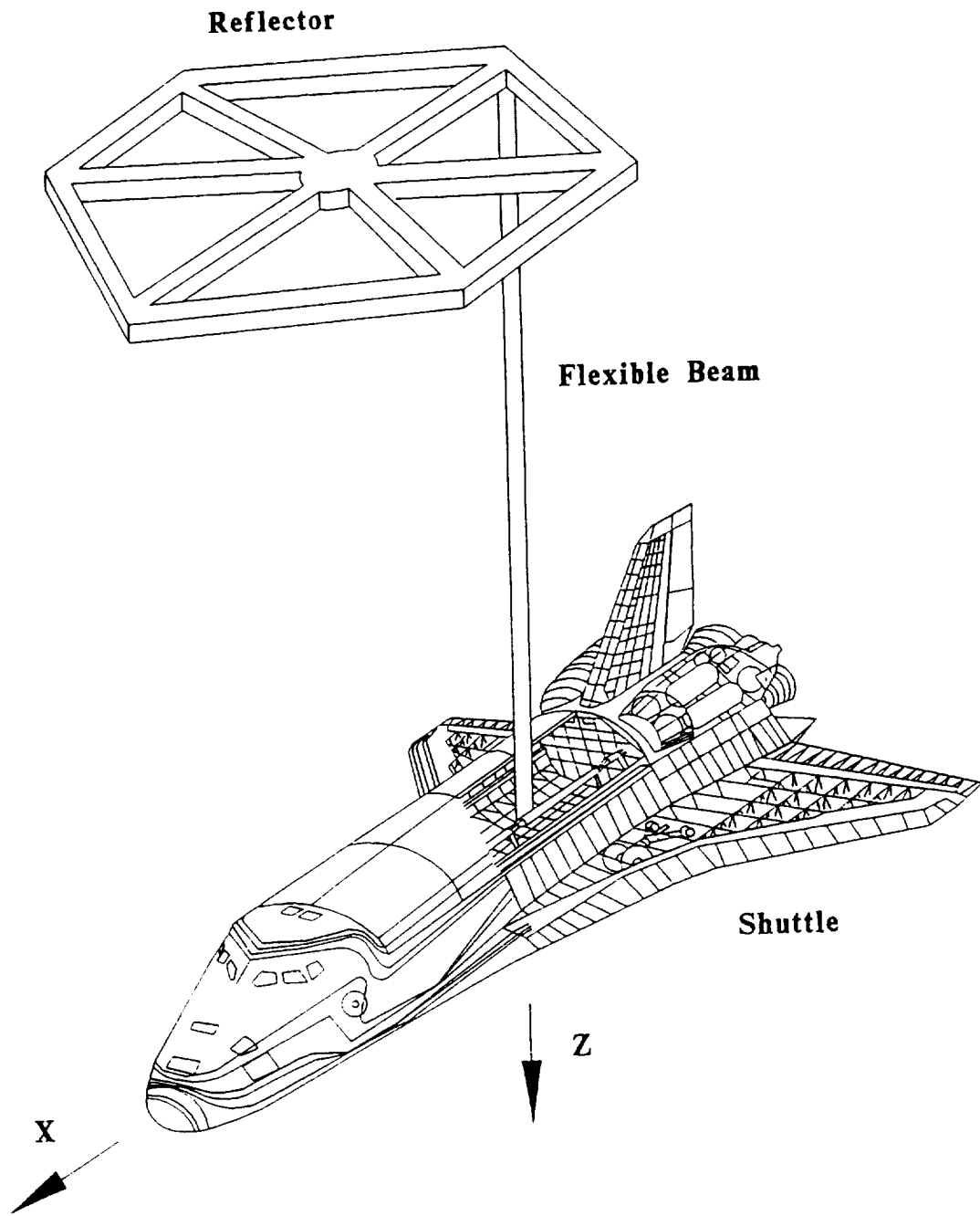
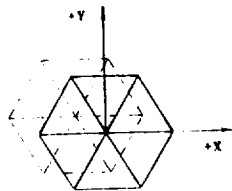The Space Shuttle Model
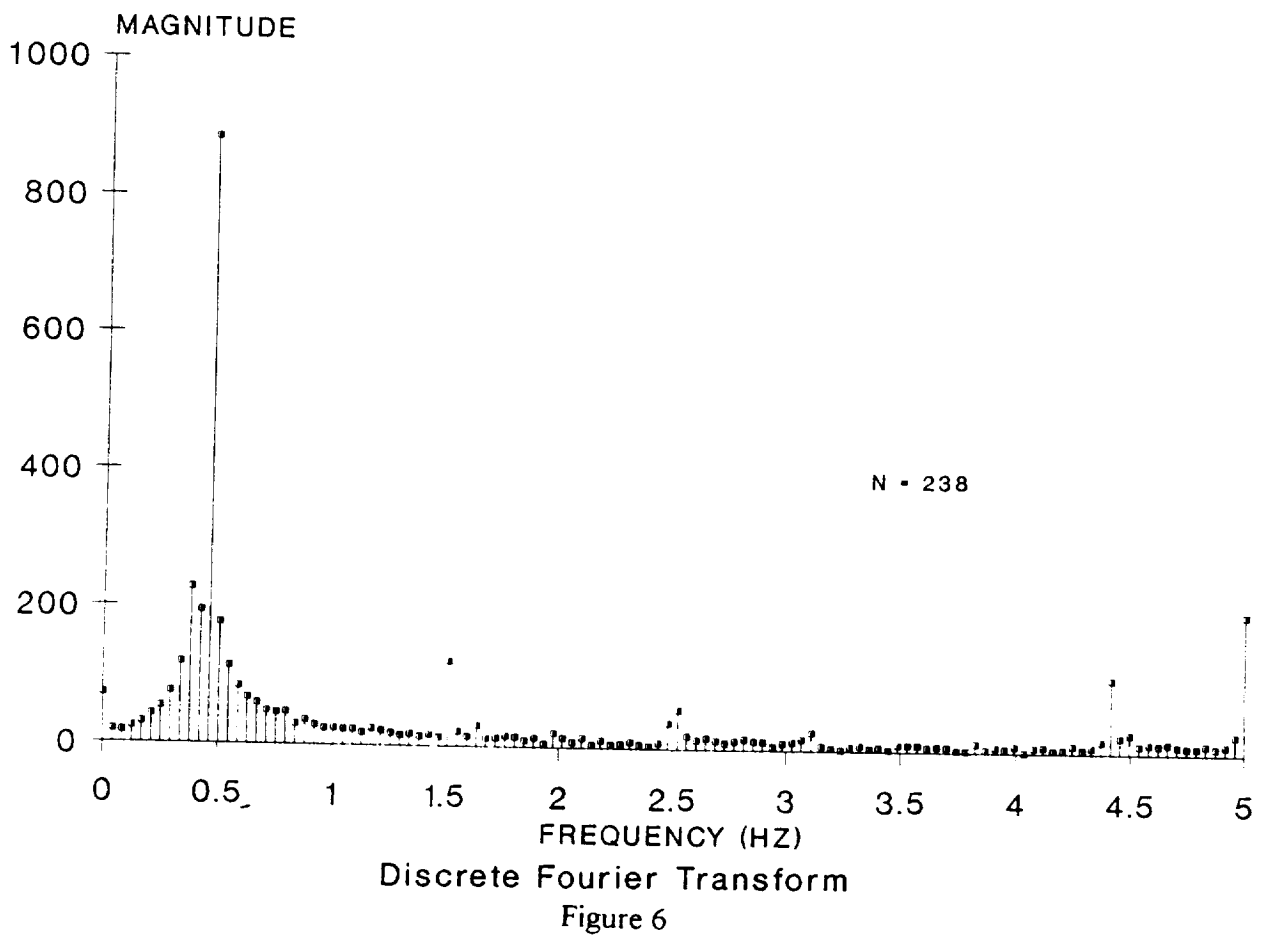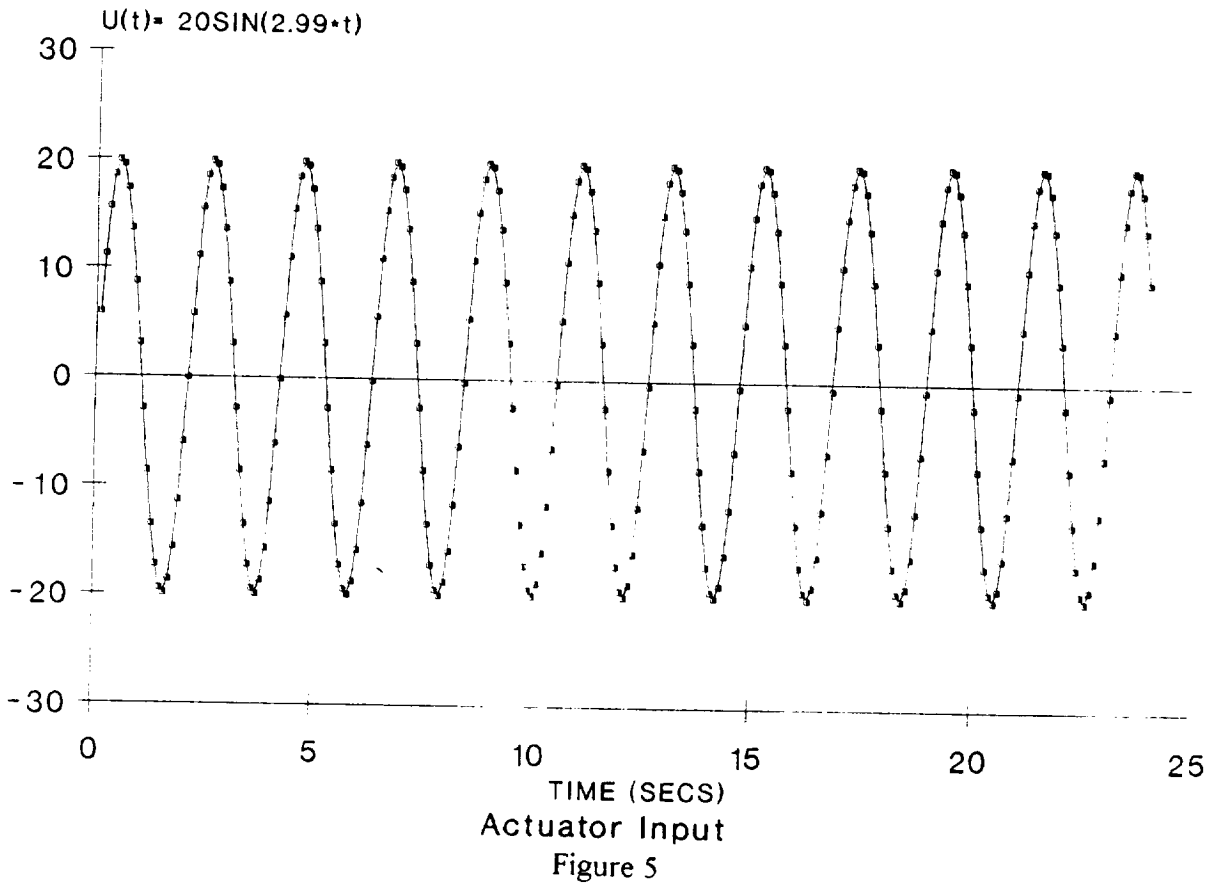
Figure 1

The SCOLE Experimental Apparatus

Figure 2

Original photograph unavailable at time of publication

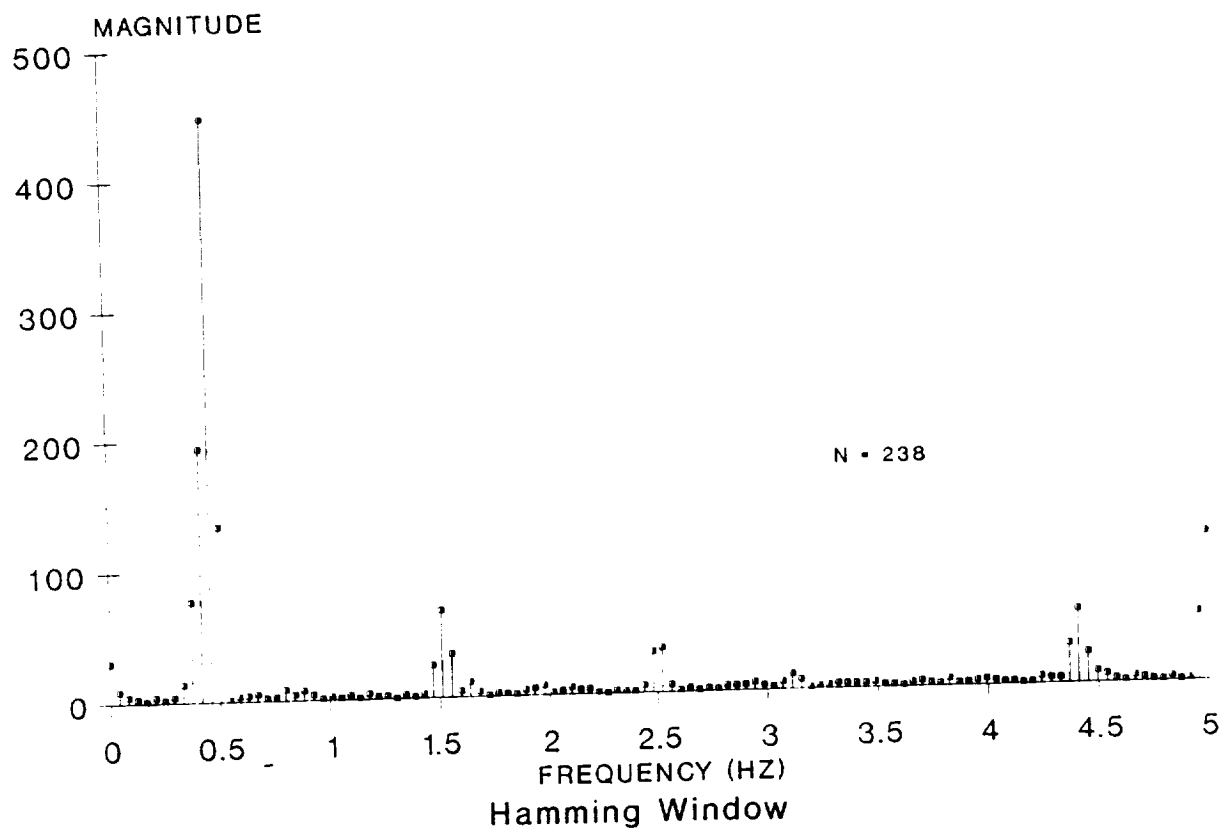**Mode 1 Testing**
Figure 3

Mode 2 Testing
Figure 4

U(t)= 20SIN(2.99*t)

TIME (SECS)
Actuator Input
Figure 5

MAGNITUDE

N = 238

FREQUENCY (HZ)
Discrete Fourier Transform
Figure 6

MAGNITUDE

N = 238

FREQUENCY (HZ)

Hamming Window

Figure 7

MEAN A1 = 1.7073690
VARIANCE A1 = .0001020

TIME (SECS)

Arma Coefficient A_1

Figure 8

MEAN A2 = -.7947280
VARIANCE A2 = .0000996

Arma Coefficient A_2

Figure 9

MEAN B = 1.5270996
VARIANCE B = .1837367
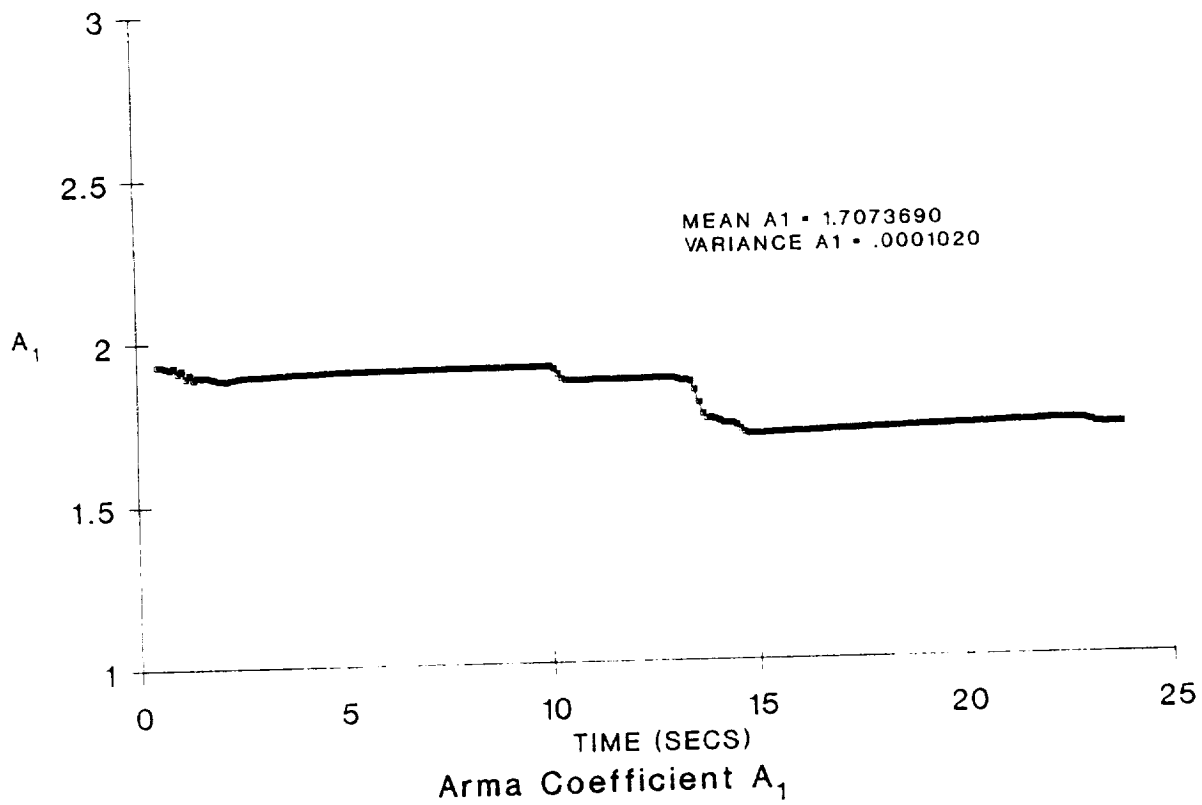
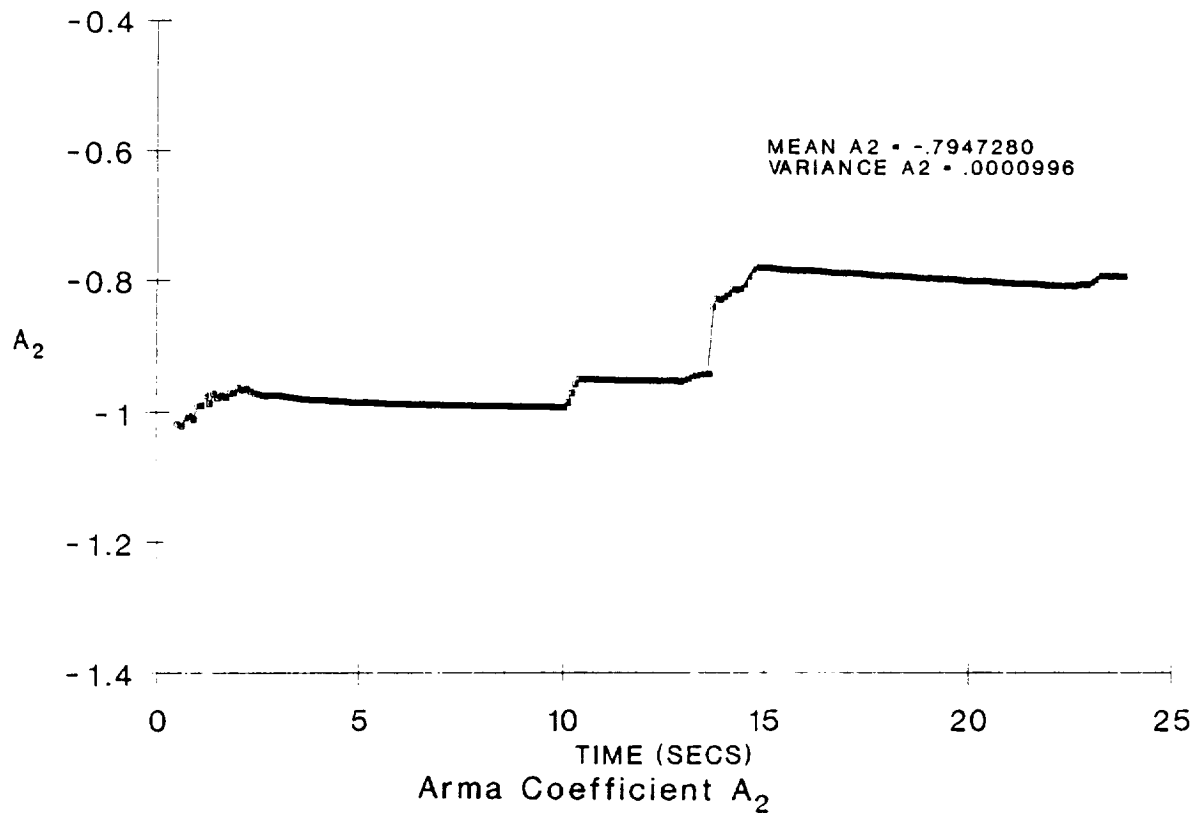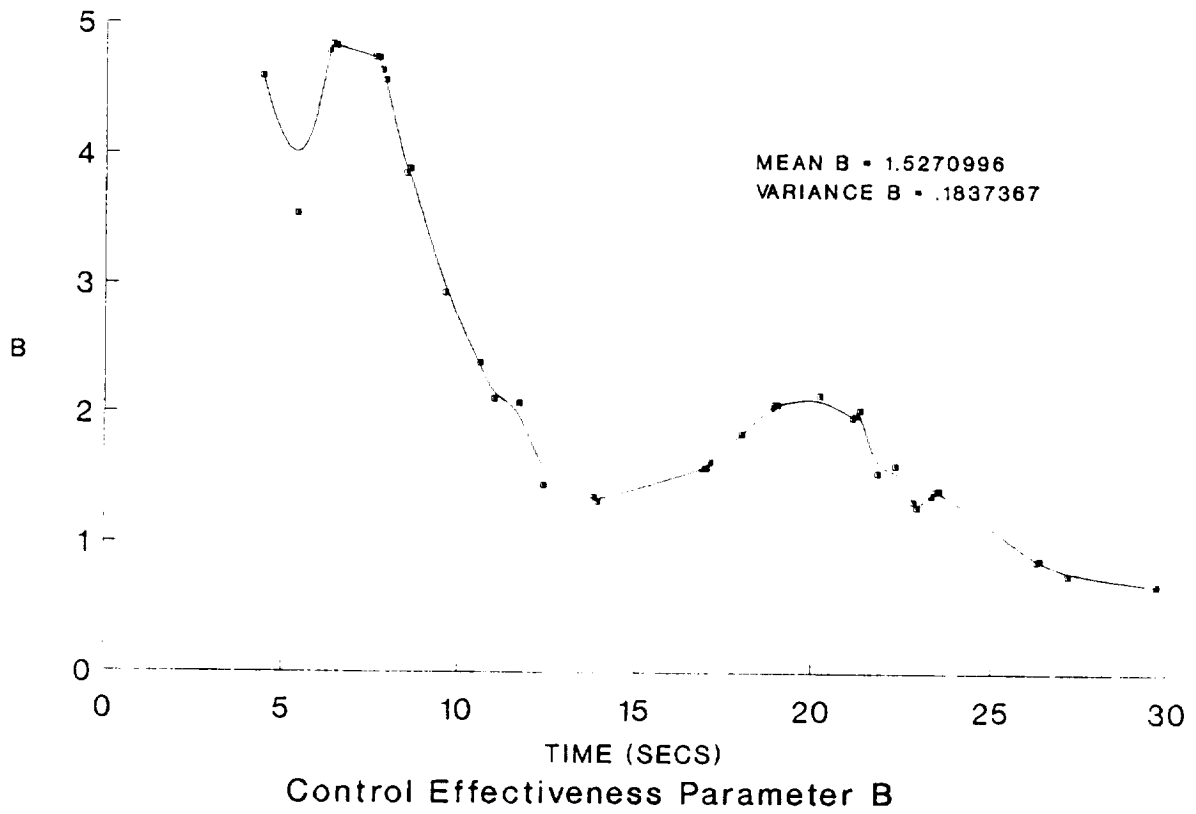Control Effectiveness Parameter B

Figure 10

Table 1

Test Parameters

TEST #	REACTION TORQUE WHEEL	MODE	EXCITATION TIME (SECS)	FREQUENCY (Hz)	AMPLITUDE	FREE DECAY TIME (SECS)	SAMPLE INTERVAL (SECS)
1	X-AXIS	1	30	0.4401	20	60	0.1
2	Y-AXIS	1	30	0.4401	20	60	0.1
3	Z-AXIS	1	30	0.4401	20	60	0.1
4	X-AXIS	2	30	0.4764	20	60	0.1
5	Y-AXIS	2	30	0.4764	20	60	0.1
6	Z-AXIS	2	30	0.4764	20	60	0.1

Table 2

Experimental Identification Results

TEST #	REACTION TORQUE WHEEL	MODE	a_1	a_2	GRAPH FREQ (Hz)	LSE FREQ (Hz)	ζ	b_{TW}
1	X-AXIS	1	1.5235475	-0.6120654	0.44715	0.5351800	0.7299576	-5.5898004
2	Y-AXIS	1	1.6931082	-0.7789838	0.44715	0.4978276	0.3992480	0.0080636
3	Z-AXIS	1	1.5868783	-0.6751711	0.44715	0.5223736	0.5983680	-0.8804742
4	X-AXIS	2	1.8732456	-0.6641920	0.46218	0.5164785	0.4766312	0.6327561
5	Y-AXIS	2	1.7073690	-0.7947280	0.46218	0.4997192	0.3658724	1.5270996
6	Z-AXIS	2	1.6953652	-0.7836996	0.46610	0.5042274	0.3846554	-1.4330764