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CONTROL OF THE SCOLE CONFIGURATION USING DISTRIBUTED PARAMETER MODELS

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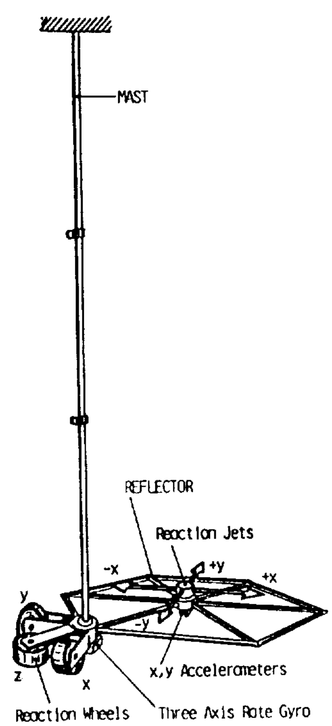


1. INTRODUCTION

(1) Assumptions

-- The reflector of SCOLE is a rigid body

-- The mast of SCOLE is an Euler beam

-- Actuators and sensors are collocated

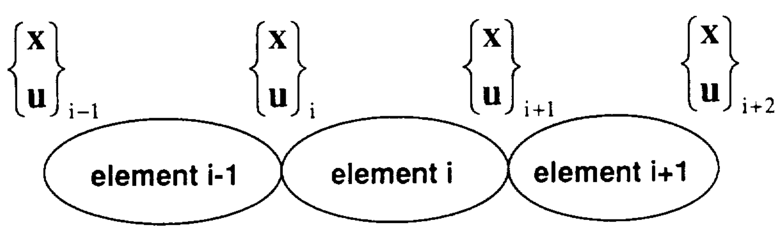
(2) Objectives

-- Vibration suppression

-- Controller designs using continuum models

2. DERIVATION OF THE CONTINUUM MODEL

• Holzer's Transfer Matrix Method



$$\begin{Bmatrix} \mathbf{x} \\ \mathbf{u} \end{Bmatrix}_{i+2} = \begin{bmatrix} \; \end{bmatrix}_{i+1} \begin{Bmatrix} \mathbf{x} \\ \mathbf{u} \end{Bmatrix}_{i+1}, \quad \begin{Bmatrix} \mathbf{x} \\ \mathbf{u} \end{Bmatrix}_{i+1} = \begin{bmatrix} \; \end{bmatrix}_{i} \begin{Bmatrix} \mathbf{x} \\ \mathbf{u} \end{Bmatrix}_{i}, \quad \begin{Bmatrix} \mathbf{x} \\ \mathbf{u} \end{Bmatrix}_{i} = \begin{bmatrix} \; \end{bmatrix}_{i-1} \begin{Bmatrix} \mathbf{x} \\ \mathbf{u} \end{Bmatrix}_{i-1}$$

$$\begin{Bmatrix} \mathbf{x} \\ \mathbf{u} \end{Bmatrix}_{i+2} = \begin{bmatrix} \quad \end{bmatrix}_{i+1} \begin{bmatrix} \quad \end{bmatrix}_{i} \begin{bmatrix} \quad \end{bmatrix}_{i-1} \begin{Bmatrix} \mathbf{x} \\ \mathbf{u} \end{Bmatrix}_{i-1}$$

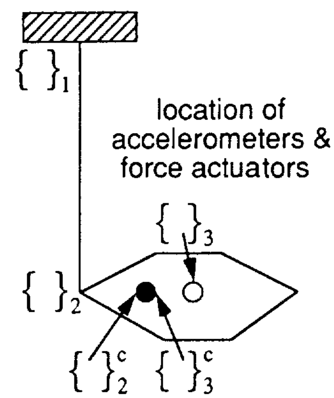
• Application to the SCOLE configuration

$$\begin{Bmatrix} \mathbf{x} \\ \mathbf{u} \end{Bmatrix}_3 = \begin{bmatrix} \text{transformation of} \\ \text{a force system} \end{bmatrix}_3^c \begin{Bmatrix} \mathbf{x} \\ \mathbf{u} \end{Bmatrix}_3^c$$

$$\begin{Bmatrix} \mathbf{x} \\ \mathbf{u} \end{Bmatrix}_3^c = \begin{bmatrix} \text{rigid body} \\ \text{dynamics} \end{bmatrix}_2^c \begin{Bmatrix} \mathbf{x} \\ \mathbf{u} \end{Bmatrix}_2^c$$

$$\begin{Bmatrix} \mathbf{x} \\ \mathbf{u} \end{Bmatrix}_2^c = \begin{bmatrix} \text{transformation of} \\ \text{a force system} \end{bmatrix}_2 \begin{Bmatrix} \mathbf{x} \\ \mathbf{u} \end{Bmatrix}_2$$

$$\begin{Bmatrix} \mathbf{x} \\ \mathbf{u} \end{Bmatrix}_2 = \begin{bmatrix} \text{beam} \\ \text{dynamics} \end{bmatrix}_1 \begin{Bmatrix} \mathbf{x} \\ \mathbf{u} \end{Bmatrix}_1$$



(1) Rigid Body--inertia and gravitational effects

$$m_i \ddot{\mathbf{w}}_i = \mathbf{f}_{i+1} - \mathbf{f}_i - \mathbf{G}_{1i}\boldsymbol{\theta}_i - \mathbf{f}_g$$

$$\mathbf{J}_i \ddot{\boldsymbol{\theta}}_i = \boldsymbol{\tau}_{i+1} - \boldsymbol{\tau}_i, \quad \mathbf{w}_{i+1} = \mathbf{w}_i, \quad \boldsymbol{\theta}_{i+1} = \boldsymbol{\theta}_i$$

where $\mathbf{G}_1 = \begin{bmatrix} 0 & -mg & 0 \\ mg & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\mathbf{f}_g = \begin{Bmatrix} 0 \\ 0 \\ m_i g \end{Bmatrix}$

$$\begin{Bmatrix} \mathbf{x} \\ \mathbf{u} \end{Bmatrix}_{i+1} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ s^2\overline{\mathbf{M}} + \mathbf{G} & \mathbf{I} \end{bmatrix}_i \begin{Bmatrix} \mathbf{x} \\ \mathbf{u} \end{Bmatrix}_i + \begin{Bmatrix} \mathbf{0} \\ \mathbf{u}_g \end{Bmatrix}$$

where $\mathbf{x} = \begin{Bmatrix} \mathbf{w} \\ \boldsymbol{\theta} \end{Bmatrix}$, $\mathbf{u} = \begin{Bmatrix} \mathbf{f} \\ \boldsymbol{\tau} \end{Bmatrix}$, $\overline{\mathbf{M}} = \begin{bmatrix} m\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{J} \end{bmatrix}$, $\mathbf{G} = \begin{bmatrix} \mathbf{0} & \mathbf{G}_1 \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$, $\mathbf{u}_g = \begin{Bmatrix} \mathbf{f}_g \\ \mathbf{0} \end{Bmatrix}$

(2) Rigid Body--transformation of a force system

$$\mathbf{w}_{i+1} = \mathbf{w}_i + \boldsymbol{\theta}_i \times \mathbf{r}_i = \mathbf{w}_i - \mathbf{r}_i \times \boldsymbol{\theta}_i = \mathbf{w}_i - \mathbf{R}_i\boldsymbol{\theta}_i$$

$$\boldsymbol{\theta}_{i+1} = \boldsymbol{\theta}_i, \qquad \mathbf{f}_{i+1} = \mathbf{f}_i$$

$$\boldsymbol{\tau}_{i+1} = \boldsymbol{\tau}_i - \mathbf{r}_i \times \mathbf{f}_i = \boldsymbol{\tau}_i - \mathbf{R}_i\mathbf{f}_i$$

$$\text{where } \mathbf{R}_i = \begin{bmatrix} 0 & -r_z & r_y \\ r_z & 0 & -r_x \\ -r_y & r_x & 0 \end{bmatrix}_i$$

$$\begin{Bmatrix} \mathbf{x} \\ \mathbf{u} \end{Bmatrix}_{i+1} = \begin{bmatrix} \mathbf{T}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{T}_2 \end{bmatrix}_i \begin{Bmatrix} \mathbf{x} \\ \mathbf{u} \end{Bmatrix}_i$$

$$\text{where } \mathbf{x} = \begin{Bmatrix} \mathbf{w} \\ \boldsymbol{\theta} \end{Bmatrix}, \ \mathbf{u} = \begin{Bmatrix} \mathbf{f} \\ \boldsymbol{\tau} \end{Bmatrix}, \ \mathbf{T}_1 = \begin{bmatrix} \mathbf{I} & -\mathbf{R} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}, \ \mathbf{T}_2 = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ -\mathbf{R} & \mathbf{I} \end{bmatrix}$$

(3) Beam-- elongation

$$E_i A_i w'' = \rho_i A_i s^2 w$$

$$\begin{Bmatrix} w \\ f \end{Bmatrix}_{i+1} = \begin{bmatrix} \cos\alpha^e L & -\frac{\sin\alpha^e L}{EA\alpha^e} \\ -EA\sin\alpha^e L & -\cos\alpha^e L \end{bmatrix}_i \begin{Bmatrix} w \\ f \end{Bmatrix}_i$$

$$= \begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix}_i \begin{Bmatrix} w \\ f \end{Bmatrix}_i$$

where, $\alpha_i^e = i\sqrt{\frac{\rho_i}{E_i}}s$

(4) Beam-- torsion

$$G_i J_i \theta'' = \rho_i J_i s^2 \theta$$

$$\begin{Bmatrix} \theta \\ \tau \end{Bmatrix}_{i+1} = \begin{bmatrix} \cos\alpha^t L & -\frac{\sin\alpha^t L}{GJ\alpha^t} \\ -GJ\sin\alpha^t L & -\cos\alpha^t L \end{bmatrix}_i \begin{Bmatrix} \theta \\ \tau \end{Bmatrix}_i$$

$$= \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}_i \begin{Bmatrix} \theta \\ \tau \end{Bmatrix}_i$$

where, $\alpha_i^t = i\sqrt{\frac{\rho_i}{G_i}}s$

(5) Beam-- bending

$$E_i I_i w'''' - \rho_i I_i s^2 + \rho_i A_i s^2 w = 0$$

$$\begin{Bmatrix} w \\ \theta \\ f \\ \tau \end{Bmatrix}_{i+1} = \frac{1}{2} \begin{bmatrix} cn+ch & \frac{sn}{\beta_1}+\frac{sh}{\beta_2} & -\frac{sn}{h_1}+\frac{sh}{h_2} & -\frac{cn}{h_3}+\frac{ch}{h_4} \\ -\beta_1 sn+\beta_2 sh & cn+ch & -\frac{\beta_1}{h_1}cn+\frac{\beta_2}{h_2}ch & \frac{\beta_1}{h_3}sn+\frac{\beta_2}{h_4}sh \\ -(h_1 sn+h_2 sh) & \frac{h_1}{\beta_1}cn-\frac{h_2}{\beta_2}ch & -(cn+ch) & \frac{h_1}{h_3}sn-\frac{h_2}{h_4}sh \\ h_3 cn-h_4 ch & \frac{h_3}{\beta_1}sn-\frac{h_4}{\beta_2}sh & -\frac{h_3}{h_1}sn-\frac{h_4}{h_2}sh & -(cn+ch) \end{bmatrix}_i \begin{Bmatrix} w \\ \theta \\ f \\ \tau \end{Bmatrix}_i$$

where $\beta_1{}^2 = \frac{-\rho I s^2 + s\sqrt{(\rho I s)^2 - 4\rho AEI}}{2EI}$, $\beta_2{}^2 = \frac{\rho I s^2 + s\sqrt{(\rho I s)^2 - 4\rho AEI}}{2EI}$

$sn=\sin\beta_1 L$, $cn=\cos\beta_1 L$, $sh=\sinh\beta_2 L$, $ch=\cosh\beta_2 L$, $k_1=EI$, $k_2=\rho I$

$h_1=k_1\beta_1^3+k_2\beta_1 s^2$, $h_2=k_1\beta_2^3-k_2\beta_2 s^2$, $h_3=k_1\beta_1^2$, $h_4=k_1\beta_2^2$

(6) Beam-- overall transfer matrix

$$
\begin{Bmatrix} w_x \\ w_y \\ w_z \\ \theta_x \\ \theta_y \\ \theta_z \\ f_x \\ f_y \\ f_z \\ \tau_x \\ \tau_y \\ \tau_z \end{Bmatrix}_{i+1}
=
\begin{bmatrix}
b_{11} & 0 & 0 & 0 & -b_{12} & 0 & b_{13} & 0 & 0 & 0 & b_{14} & 0 \\
0 & b_{11} & 0 & b_{12} & 0 & 0 & 0 & b_{13} & 0 & -b_{14} & 0 & 0 \\
0 & 0 & e_{11} & 0 & 0 & 0 & 0 & 0 & e_{12} & 0 & 0 & 0 \\
0 & b_{21} & 0 & b_{22} & 0 & 0 & 0 & b_{23} & 0 & -b_{24} & 0 & 0 \\
-b_{21} & 0 & 0 & 0 & b_{22} & 0 & -b_{23} & 0 & 0 & 0 & -b_{24} & 0 \\
0 & 0 & 0 & 0 & 0 & t_{11} & 0 & 0 & 0 & 0 & 0 & t_{12} \\
b_{31} & 0 & 0 & 0 & -b_{32} & 0 & b_{33} & 0 & 0 & 0 & b_{34} & 0 \\
0 & b_{31} & 0 & b_{32} & 0 & 0 & 0 & b_{33} & 0 & -b_{34} & 0 & 0 \\
0 & 0 & e_{21} & 0 & 0 & 0 & 0 & 0 & e_{22} & 0 & 0 & 0 \\
0 & -b_{41} & 0 & -b_{42} & 0 & 0 & 0 & -b_{43} & 0 & b_{44} & 0 & 0 \\
b_{41} & 0 & 0 & 0 & -b_{42} & 0 & b_{43} & 0 & 0 & 0 & b_{44} & 0 \\
0 & 0 & 0 & 0 & 0 & t_{21} & 0 & 0 & 0 & 0 & 0 & t_{22}
\end{bmatrix}_i
\begin{Bmatrix} w_x \\ w_y \\ w_z \\ \theta_x \\ \theta_y \\ \theta_z \\ f_x \\ f_y \\ f_z \\ \tau_x \\ \tau_y \\ \tau_z \end{Bmatrix}_i
$$

2. DERIVATION OF THE CONTINUUM MODEL

$$
\begin{Bmatrix} \mathbf{x} \\ \mathbf{u} \end{Bmatrix}_3 = \begin{bmatrix} \mathbf{T_1} & \mathbf{0} \\ \mathbf{0} & \mathbf{T_2} \end{bmatrix}_3^c \left(\begin{bmatrix} \mathbf{I} & \mathbf{0} \\ s^2\overline{\mathbf{M}}+\mathbf{G} & \mathbf{I} \end{bmatrix}_2^c \begin{bmatrix} \mathbf{T_1} & \mathbf{0} \\ \mathbf{0} & \mathbf{T_2} \end{bmatrix}_2 \begin{bmatrix} \mathbf{F_1} & \mathbf{F_2} \\ \mathbf{F_3} & \mathbf{F_4} \end{bmatrix}_1 \begin{Bmatrix} \mathbf{x} \\ \mathbf{u} \end{Bmatrix}_1 + \begin{Bmatrix} \mathbf{0} \\ \mathbf{u_g} \end{Bmatrix} \right)
$$

$\Rightarrow \mathbf{M}s^2\mathbf{x} + \mathbf{K}(s)\mathbf{x} = \mathbf{u} - \bar{\mathbf{u}}_g$ for boundary condition $\mathbf{x}_1 = \mathbf{0}$

where $\mathbf{M} = [\mathbf{T_2}]_3^c[\overline{\mathbf{M}}]_2^c[\mathbf{T_1}]_3^{c^{-1}}$, $\mathbf{K} = [\mathbf{T_2}]_3^c(\mathbf{G} + [\mathbf{T_2}]_2[\mathbf{F_4}]_1[\mathbf{F_2}]_1^{-1}[\mathbf{T_1}]_2^{-1})[\mathbf{T_1}]_3^{c^{-1}}$

$\mathbf{x} = \mathbf{x}_3$, $\mathbf{u} = \mathbf{u}_3$, $\bar{\mathbf{u}}_g = [\mathbf{T_2}]_3^c\mathbf{u_g}$

- Feedforward control

  $\mathbf{u} = \mathbf{K}_0\mathbf{x}_d + \bar{\mathbf{u}}_g$ where $\mathbf{K}_0 = \mathbf{K}|_{s=0}$ and $\mathbf{x}_d$ is the desired output

- Continuum Model

$\Rightarrow \mathbf{M}s^2\mathbf{x} + \mathbf{K}(s)\mathbf{x} = \mathbf{u}$, where $\mathbf{u}$ is the feedback control

3. SYSTEM PARAMETERS

(1) Rigid Body

element	$\bar{x}$	$\bar{y}$	$\bar{z}$ (in)	W(lb)	J_{xx}	J_{yy}	J_{zz} ($slug-in^2$)
rate gyro	0	0	-129.3	1.69	0	0	0
accelerometers	10	20.8	-129.3	0.17	0	0	0
	12	18.8	-129.3	0.17	0	0	0
thrusters	12	20.8	-124.3	1	0	0	0
reaction wheels	0	-6	-125.8	4.28	0	0	0
	6	0	-125.8	4.28	0	0	0
	-4.5	-4.5	-125.8	4.28	0	0	0
reflector	12	20.8	-125.8	4.76	24.8	24.6	49.67
solenoid	4	7	-123.8	5.5	0	0	0
manifold	0	0	-124.5	1.68	0	0	0

(2) Beam

E=30 Mpsi, $I_{xx}=I_{yy}=6.66\times10^{-3}$ in^4, $I_{zz}=2I_{xx}$

G=15 Mpsi, L=125.5 in, m=4.48 lb, A=0.108 in^2

- Identified Modal Frequencies

	measured(Hz)	identified(Hz)
1st out-of-plane bending	.4545	.4609
1st in-plane bending	.4764	.4707
1st torsion	1.98	1.9240
2nd in-plane bending	3.13	3.1455
2nd out-of plane bending	4.63	4.6839

4. CONTROL DESIGN USING CONTINUUM MODELS

(1) Pole-Assignment Technique

- Continuum Model $\mathbf{M}s^2\mathbf{x} + \mathbf{K}(s)\mathbf{x} = \mathbf{u}$
- Approximations $\mathbf{K}(s) \approx \mathbf{K}_0$ or $\mathbf{K}(s) \approx \mathbf{K}_0 + \mathbf{K}_2 s^2$

  where $\mathbf{K}_0 = \mathbf{K}|_{s=0}$ and $\mathbf{K}_2 = \frac{1}{2}\mathbf{K}''|_{s=0}$

- Pole - Assignment

  LTI system $\mathbf{M}_t\ddot{\mathbf{x}} + \mathbf{K}_0\mathbf{x} = \mathbf{u}$ where $\mathbf{M}_t = \mathbf{M}$ or $\mathbf{M}_t = \mathbf{M} + \mathbf{K}_2$

  Desired damping matrix ζ

  Rate feedback control $\boxed{\mathbf{u} = -2\mathbf{M}_t\mathbf{V}\zeta\Omega\mathbf{V}^{-1}\dot{\mathbf{x}}}$

  where $\mathbf{V}$ satisfies $\mathbf{M}_t^{-1}\mathbf{A}\mathbf{V} = \mathbf{V}\Omega^2$, Ω^2 is diagonal

(1) Pole-Assignment Technique

- Advantages

  -- Easy to implement (use constant controller gains)

- Disadvantages

  -- Stability is not guaranteed (due to approximation of **K**(s) at the beginning of the design process)

(2) Linear Dynamic Controller

- Continuum Model $(\mathbf{M}s^2 + \mathbf{D}s + \mathbf{K})\mathbf{x} = \mathbf{B}\mathbf{u}$

$$\mathbf{y} = (\mathbf{H}_a s^2 + \mathbf{H}_v s + \mathbf{H}_d)\mathbf{x}$$

- Controller Dynamics $(\mathbf{M}_c s^2 + \mathbf{D}_c s + \mathbf{K}_c)\mathbf{x}_c = \mathbf{B}_c \mathbf{u}_c$

$$\mathbf{y}_c = (\mathbf{H}_{ac} s^2 + \mathbf{H}_{vc} s + \mathbf{H}_{dc})\mathbf{x}_c$$

$$\mathbf{u} = \mathbf{y}_c,\ \mathbf{u}_c = -\mathbf{y}$$

- Controller Transfer Function $\mathbf{\Psi}(s)$

$$\mathbf{u} = \mathbf{y}_c = -(\mathbf{H}_{ac} s^2 + \mathbf{H}_{vc} s + \mathbf{H}_{dc})(\mathbf{M}_c s^2 + \mathbf{D}_c s + \mathbf{K}_c)^{-1}\mathbf{B}_c \mathbf{y}$$

$$\mathbf{\Psi}(s) = -(\mathbf{H}_{ac} s^2 + \mathbf{H}_{vc} s + \mathbf{H}_{dc})(\mathbf{M}_c s^2 + \mathbf{D}_c s + \mathbf{K}_c)^{-1}\mathbf{B}_c$$

(2) Linear Dynamic Controller

- Overall System Dynamics

$$\begin{bmatrix} \mathbf{M}s^2 + \mathbf{D}s + \mathbf{K} & \mathbf{B}(\mathbf{H}_{ac} s^2 + \mathbf{H}_{vc} s + \mathbf{H}_{dc}) \\ -\mathbf{B}_c(\mathbf{H}_a s^2 + \mathbf{H}_v s + \mathbf{H}_d) & \mathbf{M}_c s^2 + \mathbf{D}_c s + \mathbf{K}_c \end{bmatrix} \begin{Bmatrix} \mathbf{x} \\ \mathbf{x}_c \end{Bmatrix} = 0$$

- Overall Characteristic Equation $\det[\ ] = 0$
- Design Parameters $\mathbf{M}_c, \mathbf{D}_c, \mathbf{K}_c, \mathbf{B}_c, \mathbf{H}_{ac}, \mathbf{H}_{vc}$, and $\mathbf{H}_{dc}$ by using knowledge of system parameters to achieve better performances

(2) Linear Dynamic Controller

• Advantages

-- Explicit transfer function for continuum models
-- Possible for guaranteed stability
-- More design flexibilities

• Disadvantages

-- Need approximations for **K**(s) to realize controllers
-- Hard to implement (need Runge-Kutta algorithm to solve for controller dynamics)

(3) LQG CONTROLLER

• Continuum Model $(\mathbf{M}s^2 + \mathbf{K})\mathbf{x} = \mathbf{B}(\mathbf{u} + \mathbf{n}_a)$

$$\mathbf{y} = \mathbf{B}^T\mathbf{x} + \mathbf{n}_r$$

where $\mathbf{n}_a \to N(0, d_a\mathbf{I})$ and $\mathbf{n}_r \to N(0, d_r\mathbf{I})$

• Performance Index $\lim_{T\to\infty} \frac{1}{T}\left\{\int_0^T (\|\mathbf{B}^T\dot{\mathbf{x}}\|^2 + \lambda\|\mathbf{u}\|^2)dt\right\}$

• Optimal Controller Transfer Function

$$\mathbf{\Psi}(s) = -h\mathbf{B}^T s(\mathbf{M}s^2 + \gamma\mathbf{B}\mathbf{B}^T s + \mathbf{K})^{-1}\mathbf{B}$$

where $h = \frac{\sqrt{d_a/d_r}}{\sqrt{\lambda}}$, $\gamma = \sqrt{d_a/d_r} + \frac{1}{\sqrt{\lambda}}$

5. NUMERICAL EXAMPLE

(1) Pole-Assignment Technique

Open loop	ζ_d (%)	$K(s) \approx K_0$		$K(s) \approx K_0 + K_2 s^2$	
f(Hz)		f(Hz)	ζ(%)	f(Hz)	ζ(%)
.4609	10	.4586	9.790	.4586	10.000
.4707	10	.4684	9.790	.4683	10.000
1.9240	10	1.9145	9.960	1.9144	10.000
3.1455	10	3.1328	9.200	3.1310	9.840
4.6839	10	4.6741	7.800	4.6706	9.110
10.9675		10.9652	.370	10.9648	.390
12.0448		12.0325	1.340	12.0279	1.570
28.6392		28.6391	.023	28.6378	.025
29.0288		29.0278	.090	29.0275	.110
55.6990		55.6990	.006	55.6990	.007
55.8956		55.8957	.016	55.8957	.018
91.8160		91.8419	.036	91.8439	.037

(2) Linear Dynamic Controller

Design Parameters

$M_c = M + K_2$

$K_c = K_0$

D_c 10% damping for the first five modes

$H_{ac} = H_{dc} = H_a = H_d = 0$

$B_c = B = I$

$H_{vc} = H_v = I$

f(Hz)	ζ(%)
.3757	4.88
.5677	4.91
1.7300	4.95
1.9144	10.00
2.1721	4.93
2.9420	4.92
4.6928	2.79
10.9420	.19
12.0455	.38
28.6431	.013
29.0289	.020
55.6989	.006

6. CONCLUSION

(1) A continuum model for the SCOLE configuration has been derived using transfer matrices.

(2) Controller designs for distributed parameter systems have been analyzed.

(3) Pole-assignment controller design is easy to implement but stability is not guaranteed.

(4) Explicit transfer function of dynamic controllers has been obtained and no model reduction is required before the controller is realized.

(5) One specific LQG controller for continuum models had been derived, but other optimal controllers for more general performances need to be studied.