# Interactive Orbital Proximity Operations Planning System Instruction and Training Guide 

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## Symbols and Terms

## Symbols:

c en route closing rate
d viewing range, being the distance between the viewpoint and the intersection of the viewing axis with the orbital plane (focus of interest)
t
time
$t_{0}$ starting time of the maneuver
$t_{f} \quad$ final time of the maneuver, predetermined and not to be exceeded
$\mathrm{s}, \mathrm{r}, \mathrm{w} \quad$ right-hand, space-station-based coordinate system; the s -axis points in the direction of the orbital velocity vector (V-bar), the r-axis points upward away from the center of Earth (R-bar) and the w-axis points out of the orbital plane toward the viewer
$\mathrm{X}_{\mathrm{e}} \mathrm{ye}_{\mathrm{e}} \mathrm{z}_{e} \quad$ right-hand viewing system; the $\mathrm{x}_{\mathrm{e}}$-axis is the viewing axis, which also coincides with the center of the monitor; the $y_{e}$-axis points to the right; the $z_{\mathrm{e}}$-axis points downward.
$\Psi, \Theta \quad$ "tethered" rotation angles, for viewing the orbital plane obliquely in s-direction and rdirection, respectively

Nomenclature:

AWP
Active way point
Arrival way point

Burn arrow

Constraint envelope

Departure way point

En route closing rate

## Active way point

Terminal or transfer way point, currently under control of the operator.
Way point at which rendezvous takes place; operator can adjust the time of arrival (TOA) at the arrival way point, but its position is constrained to be located on the target trajectory.
Red arrow with tail, visualizing direction and magnitude of a maneuvering burn on the display. The length of the arrow is proportional to the magnitude of the burn. A scale factor of $500-\mathrm{m}$ length per meter per second velocity change, has been chosen.
Boundaries of a volume through which the chaser is not allowed to pass. The envelope is displayed in the main window as a dotted wireframe image, which changes from dim to bright when the constraint volume is violated.

Way point, located at the initial chaser trajectory, at which the departure maneuvering burn is given, which inserts the chaser on a transfer trajectory toward the rendezvous with the target. The operator can adjust the TOA at the departure way point, but its position is constrained to be located on the initial chaser trajectory.

Component of relative approach velocity vector between chaser and target, along their mutual line of sight (slant range).

| En route passage clearance | Minimal distance between chaser and target, when passing each other en route. |
| :---: | :---: |
| En route passage closing time | Time in which the chaser would reach the target if the closing rate remained constant. |
| Final coasting | Advancing the rendezvous to a time $t=t_{\text {arr }}$ before the final time of the mission $\mathrm{t}=\mathrm{t}_{\mathrm{f}}$, so that in the interval $\mathrm{t}_{\mathrm{arr}}<\mathrm{t}<\mathrm{t}_{\mathrm{f}}$ the chaser coasts along on the target trajectory. |
| IOP | In-orbital plane |
| Initial chaser trajectory | The relative motion trajectory of the chaser before the departure maneuvering burn. |
| Initial coasting | Delaying the departure to a time $t=t_{\text {dep }}$ after the start of the mission at $\mathrm{t}=\mathrm{t}_{0}$, so that in the interval $\mathrm{t}_{0}<\mathrm{t}<\mathrm{t}_{\text {dep }}$ the chaser coasts along on the initial chaser trajectory. |
| In-orbital plane | Refers to a position or maneuvering burn in the ( $\mathrm{s}, \mathrm{r}$ ) plane. |
| IOP position pointer | IOP component of the optimization vector, visualized as an arrow, attached to the active transfer way point, which indicates the IOP direction in which to move the way point in order to reduce fuel use. |
| IOP position pointer trace | Locus of points obtained by following the direction of the IOP position pointer in a sequence of small correcting steps, while keeping the TOA and the OOP position of the active transfer way point fixed. |
| LEO | Low-Earth orbit |
| Maneuvering burn | An impulsive force of short duration, which varies the parameters of the orbit. The maneuvering burns are materialized by activating the orbital maneuvering system of the spacecraft. This system applies a calibrated, constant force in a given direction, for a precisely measured interval of time. The magnitude of the maneuvering burn is measured in meters-per-second velocity change. |
| OOP | Out-of-orbital plane |
| OOP position pointer | OOP component of the optimization vector, visualized as an arrow, attached to the active way-point time bar in the OOP window, which indicates the OOP direction in which to move the transfer way point in order to reduce the fuel cost. |
| Optimization pointer | An arrow that indicates the direction in space and time in which to move the transfer way point in order to reduce fuel cost. |
| Optimization vector | Indicates the direction in three-dimensional space in which to move the transfer way point in order to reduce fuel use; the vector is visualized by decomposing it into an IOP component (the IOP position pointer) and an OOP component (the OOP position pointer). |


| Orbital plane | Plane through the center of Earth, on which the circular space station <br> orbit is located. In the display, the orbital plane is represented by a <br> rectangular grid. |
| :--- | :--- |
| Out-of-orbital plane | Refers to a position or maneuvering burn in w-direction, perpendicular <br> to the (s,r) plane. |
| Primer vector | Solution to the adjoint system, used in Lawden's classic trajectory <br> optimization technique; for a fuel optimum, the primer vector has to <br> satisfy four necessary conditions. |
| Primer-vector diagram | Graph of the primer-vector distribution, shown in the primer-vector <br> window. |
| Primer-vector | Magnitude p of the primer-vector solution as a function of time; <br> the solution is obtained by using Lawden's second necessary |
| condition. |  |


| Timing pointer | An arrow, attached to the active way-point time bar in the primer- <br> vector window; it indicates the direction in which to change the TOA <br> of the way point in order to reduce fuel use. |
| :--- | :--- |
| TOA-adjusted IOP | Optimal position of a transfer way point, obtained by alternate <br> optimal position <br> adjustments in ( $s, r$ r) and TOA space, while the OOP position remains <br> fixed. This position is indicated by a star with an octagonally shaped <br> reticle at its center. |
| Transfer trajectory | Chaser's relative motion trajectory resulting from the departure <br> maneuvering burn, which sends the chaser on way toward a <br> rendezvous with the target. |
| Transfer way point | A way point on the transfer trajectory, other than the departure or <br> arrival way point. |
| Way point | Relative three-dimensional position (in space-station coordinates) at <br> which the orbital maneuvering system is activated. |
| Way-point pedestal | A pole placed at the center of a way point, perpendicular to the orbital <br> plane. The end-point of this pole is marked by a small open circle, <br> which indicates the true three-dimensional position of the way point. |

# Interactive Orbital Proximity Operations Planning System 

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## SUMMARY

This guide instructs users in the operation of the Proximity Operations Planning System. This system uses an interactive graphical method for planning fuel-efficient rendezvous trajectories in the multi-spacecraft environment of the space station and allows the operator to compose a multi-burn transfer trajectory between arbitrary initial chaser and target trajectories. The available task time (window) of the mission is predetermined, and the maneuver is subject to various operational constraints, such as departure, arrival, spatial, plume-impingement, and en route passage constraints. The maneuvers are described in terms of the relative motion experienced in a space-station centered coordinate system. Both in-orbital-plane and out-of-orbital-plane maneuvering are considered. A number of visual optimization aids are used for assisting the operator in reaching fuel-efficient solutions. These optimization aids are based on the primer-vector theory and include (1) a primervector diagram, which assists the operator in determining the optimal number of way points, and in adjusting the departure or arrival timing; and (2) optimization pointers, which indicate the direction in space and time in which to move the intermediate way points for reducing fuel use to a minimum. The visual feedback of trajectory shapes, operational constraints, and optimization functions provided by user-transparent and continuously active background computations, allows the operator to make fast, iterative design changes which rapidly converge to fuel-efficient solutions. The planning tool is an example of operator-assisted optimization of nonlinear cost functions.

### 1.0 INTRODUCTION

### 1.1 Purpose of This Guide

This guide instructs users in the operation of the Proximity Operations Planning System. It includes (1) a brief background in orbital dynamics and orbital maneuvering, (2) a detailed description of the system, its purpose, and its control modes, (3) a description of the graphical representation of the various mission constraints and optimization functions, and (4) a number of exercises that demonstrate each of the system functions.

### 1.2 Background

The purpose of the interactive graphical system is to enable the operator to design a fuel-efficient, time-limited rendezvous trajectory between spacecraft in the proximity of the space station, subject to a variety of constraints, the constraints ensure, among other things, that there will be sufficient clearance between spacecraft when passing each other en route, as well as sufficient clearance from large space structures. The basic idea underlying the system is to present the maneuver, the constraints, the optimization functions, and the total fuel cost in an easily interpretable pictorial format.

The future proximate orbital environment of spacecraft in low-Earth orbit (LEO) may include a variety of spacecraft co-orbiting in close vicinity. Most of these spacecraft will be "parked" in stable locations with respect to each other, that is, they will be on the same circular orbit. However, some
missions will require unforeseen repositioning or transfers among them as in the case of the retrieval of an accidentally released object. In this case, complex maneuvers are anticipated that will involve several spacecraft that might be in relative motion with respect to each other.

This multivehicle environment poses new requirements for control and display of the relative positions of the vehicles. Conventional scenarios involve proximity operations between two vehicles only. In these two-spacecraft missions, the maneuver may be optimized and computed in advance of the time of the actual mission. However, since the variety of possible scenarios in a multivehicle environment is large, a future spacecraft environment could require astronauts to execute maneuvers that may not have been precomputed. This demand will require an on-site planning tool that allows fast, interactive, informal creation of fuel-efficient maneuvers that meet all constraints set by the flight rules.

### 1.3 Difficulties in Orbital Maneuvering

The difficulties encountered in planning and executing orbital maneuvers have three principal causes (refs. 1-6). The first one is the counterintuitive and nonlinear character of orbital motions as experienced in a relative-motion reference frame. The orbital motions are expressed and tend to be perceived in a coordinate frame attached to a large proximate vehicle such as the space station and, thus, represent relative rather than absolute motions. This is demonstrated in the following example.

Figure 1 shows the relative reference frame as seen from the side, with two spacecraft: a target craft, flying on the same circular orbit at a fixed distance ahead of a chaser craft. The s-axis or "V-bar" points in the direction of flight, and the r-axis or "R-bar" points away from the center of Earth. From experience in inertially fixed environments, it would be intuitively assumed that an impulsive force (relatively large force of short duration), applied on the chaser in the forward direction, would move the chaser toward the target. The effect of this impulse on the relative motion pattern of the chaser is shown in Figure 1a. The numbers indicate the time in minutes. After several minutes, orbital mechanic forces will dominate the motion pattern and move the chaser spacecraft "upward," that is, to a higher orbit. These forces will result in a backward relative motion, since by Kepler's third law objects in a higher orbit have a slower orbital rate. Thus, a forward thrust ultimately has the opposite effect from that intended. Figure 1 b shows that impulses in the forward and downward directions will eventually bring the chaser to the target.

The second cause of difficulties in planning and executing orbital maneuvers is the safety constraints to which multi-vehicle orbital missions are subject. These include clearance between spacecraft passing each other en route and clearance from existing structures, allowable closing rates between spacecraft, angles of departure and arrival, and maneuvering burn restrictions caused by plume impingement or payload characteristics. The design of a fuel-efficient trajectory which satisfies these constraints is a nontrivial task.

For example, since the relative orbital motions are cyclic, proper timing of the transfer maneuver by choosing suitable departure and arrival times might lead to considerable fuel savings.

Visualization of the relative trajectories, control forces, and optimization functions in an easily interpretable graphical format, might improve the feel for orbital motions and control forces and will
provide direct feedback of the operator's control actions. Furthermore, visualization of the constraints in a pictorial format will enable interactive, graphical trajectory planning in which the design may be iteratively modified until all constraints are satisfied.


Figure 1. Counterintuitive character of orbital motion: (a) An impulsive force applied on the chaser in forward direction, has the opposite effect from that intended; (b) An impulsive force in forward and downward directions will bring the chaser to rendezvous with the target.

### 1.4 Orbital Motion: Background

Spacecraft orbiting Earth move in an orbital plane that passes through the center of gravity of Earth. The orientation of this plane with respect to Earth is almost entirely determined by the launch parameters and is specified by two angles, the longitude $\Omega$ of the ascending node, AN, (the intersection of the north-bound orbital path with the equatorial plane) and the inclination of the plane i. (fig. 2a). In the orbital plane, the shape of the orbit can be either circular, elliptic, parabolic, or hyperbolic, depending on the total energy. In our case all orbits are either circular or near-circular elliptic. Figure 2 b shows the shape of the orbit in the orbital plane. The ellipse is located with one of its foci $F$ at the Earth center of mass. The orientation of the ellipse is determined by the angle of the line of the apsides $\omega$, which is the angle between the major axis of the ellipse and the line o-AN. The ellipse is characterized by the semimajor axis a and the eccentricity e. For a circular orbit the eccentricity is zero. For Earth orbit, the point closest to Earth is called the perigee PE and the point farthest from Earth is the apogee AP. Finally, the position of the spacecraft in orbit is fully determined by the time of passing perigee, or equivalently, by the angle of the true anomaly $v$, which is the angle traveled by the spacecraft since perigee passing. Thus, for a given planet the spacecraft position is entirely determined by six parameters. Orbital maneuvering takes place by altering the parameters of the orbit by applying an impulsive force in a given direction. Forces, applied in the orbital plane, will alter the eccentricity or the height of the orbit as well as the position of the apogee, whereas forces applied perpendicular to the orbital plane will alter the plane's AN longitude or inclination.


Figure 2a. Definition of the orbit: orientation of the orbital plane.


Figure 2b. Definition of the orbit: shape of the orbit in the orbital plane.
In our case all orbital maneuvering takes place about the space station. The station is stabilized in circular orbit at an altitude of 480 km above Earth's surface and at an orbital velocity of about $\mathrm{V}=7,623 \mathrm{~m} / \mathrm{sec}$.

As mentioned before, impulsive forces can alter the absolute elliptical shape of the orbit, as well as the orientation of the orbital plane. However, in our display we do not visualize this shape. Instead, we visualize the relative motion patterns which develop between two spacecraft, co-orbiting close to each other, for example, within several kilometers of each other. Since the space station has a stabilized, fixed circular orbit, we consider and visualize the relative motion of all co-orbiting spacecraft with respect to the space-station-centered coordinate system. This is a right-hand system with its origin located at the station's center of mass, the s-axis pointing in the direction of the orbital velocity vector, and the r-axis pointing "upward" away from the center of Earth. The s-o-r plane constitures the station's orbital plane, and the $w$-axis is perpendicular to this plane, pointing toward the viewer.

When two spacecraft share the same circular orbit, they fly along on this orbit while the distance between them remains stationary. However, when the orbit of one of the spacecraft is altered from a circular orbit to a slightly elliptical one, a relative in-orbital-plane motion will develop between the spacecraft. This motion will continue until the elliptical orbit of the spacecraft is changed back to the original circular one.

### 1.5 Orbital Maneuvering

Impulsive forces vary the parameters of the orbit; they occur through activation of the orbital maneuvering system of the spacecraft, which applies a calibrated constant force, referred to as "maneuvering burn," in a given direction for a precisely measured interval of time, such as several tens of seconds. The magnitude of the impulsive force, also referred to as a "velocity impulse," is thus determined by the duration of the burn, which is also proportional to the fuel used. Since the time interval of the orbital-maneuvering bum is relatively short in comparison to the duration of the mission (several tens of seconds vs. tens of minutes), the burn can be considered as an instantaneous velocity change. If the burn is applied in the orbital plane, the orientation of the orbital plane of the spacecraft in inertial space, is not altered. This is not the case for a burn perpendicular to the orbital plane. In this latter case, the orbital plane of one of the spacecraft will rotate slightly, which will be experienced by the other spacecraft as a periodic relative motion in the out-of-orbital-plane direction. Thus, space flight is mainly unpowered, except for the instances in which the orbital maneuvering system is activated. The relative positions at which this is done are referred to as the way points of the maneuver.

Figure 3 describes a typical three-burn maneuver for bringing a chaser to a rendezvous with a target craft. The mission is to take place between the starting time $t=t_{0}$ and the final time $t=t_{f}$. The initial relative motion trajectories of chaser and target are shown in the space station coordinate system $(s, r)$ where the $s$-axis is again in the direction of flight and the $r$-axis away from the center of Earth. Following the departure burn $\Delta \mathrm{v}_{1}$ at time $\mathrm{t}=\mathrm{t}_{1}$, the chaser leaves its initial trajectory to start the transfer toward the target. Following the intermediate burn $\Delta v_{2}$ at time $t=t_{2}$, the chaser continues along the modified transfer trajectory, and reaches the target craft at time $t=t$, where the final burn (retro-burn) $\Delta v_{3}$ is applied, to bring the velocity between chaser and target to zero.


Figure 3. Typical in-orbital-plane three-burn maneuver to bring the chaser to rendezvous with the target; the maneuver is subject to spatial constraints.

In this case the rendezvous maneuver is composed of four sections: a section of initial coasting along the chaser trajectory ( $\mathrm{t}_{0}<\mathrm{t}<\mathrm{t}_{1}$ ), two transfer sections, and a section of final coasting along the target trajectory ( $\mathrm{t}_{3}<\mathrm{t}<\mathrm{t}_{\mathrm{f}}$ ). In this example, the second transfer section violates a spatial constraint. This requires a revision of the transfer trajectory by changing the departure, arrival, or intermediate burn timing, by relocating the intermediate way point, or by adding additional way points.

The total fuel used in the maneuver is equivalent to the total duration of activating the orbital maneuvering system. For a spacecraft with a given mass, the total amount of orbital fuel used is thus proportional to the sum of the velocity changes caused by all the maneuvering burns, including the final burn at rendezvous. Therefore, it is customary to indicate the fuel used in terms of meters-persecond total velocity change.

## 1.6 "Inverse" Method for Solving Orbital Motion

Interactive trajectory design demands that the operator be given free and unconstrained control over the positioning and timing of way points. However, the input variables to the "forward" equations of orbital motion, describing out-of-the-window maneuvering, are the direction and magnitude of maneuvering burns, rather than the position and timing of way points. The inverse solution, which is performed in user transparent, continuously active background computations, computes the maneuvering burns necessary to bring the spacecraft to a position in space and time specified by the operator. In the case of a multibum maneuver like the example in section 1.5 , the situation is even more complex. Modifying the position or timing of the intermediate way point demands changes in maneuvering burns of the neighboring way points to keep their time and position fixed. The preservation of way-point times and positions is a key design feature of the planning process, which enables solutions to complex orbital maneuvering problems involving interacting constraints.

### 1.7 Concept of the "Active Way Point"

Although a trajectory may be composed of several way points, only one way point at a time, the "active" way point, is controlled by the operator. Although the position and timing of the active way point can be varied, the position and timing of all other way points remain unchanged. However, variations in the active way point will cause changes in the shape of the trajectory sections and waypoint maneuvering burns just preceding and just following the active way point. The usertransparent background computations visualize these changes almost instantaneously and provide the operator with on-line feedback on design actions.

### 2.0 DESCRIPTION OF THE SYSTEM

### 2.1 Display Layout

The display includes five windows, (fig. 4) as implemented on Silicon Graphics IRIS and Indigo computers using GL. The main window shows a bird's eye view of the orbital plane and the vicinity of the space station. Two smaller windows are located at the bottom. The left bottom window displays optimization information in the form of the primer-vector diagram, and also allows adjustment of the time-of-arrival (TOA) of way points. The right bottom window shows the out-of-
orbital-plane (OOP) situation and allows adjustment of the OOP position of way points. An information window in the upper left comer lists various optimization settings, the elapsed time since the start of the planning procedure, and the orbital-plane cursor position. A fuel use indicator is at far right; it indicates the fuel required for a particular maneuver in meters-per-second of total velocity change.

The Main Window- The birds-eye perpendicular view of the orbital plane shows the space station at the center, where the dotted wire-frame box represents the spatial constraint through which other spacecraft are not allowed to pass. All orbital maneuvers are expressed in terms of the relative motion with respect to the space-station coordinate system, ( $\mathrm{s}, \mathrm{r}, \mathrm{w}$ ), with its origin at the space-station center-of-mass. The s-axis, pointing in the direction of the orbital velocity vector, is the horizontal line through the center of the space station. The right-pointing arrows located on this line indicate the flight direction. The main window is also a section of the circular orbit of the space station referred to as the V-bar. Spacecraft to the right of the space station "fly ahead" of it and have a positive s-coordinate. The r-axis, pointing upward, away from the center of Earth, is the vertical line through the space station and is referred to as the space station $R$-bar; the $w$-axis is perpendicular to the orbital plane, and points toward the viewer.

Orbital grid spacing is 100 m , both in the V-bar and R-bar directions. Thus, at the present view an area of about $2,000 \times 1,600 \mathrm{~m}$ or $20 \times 16$ blocks, is visible within the viewport. To the right of the space station the space shuttle is visible, which is the target craft with which the rendezvous is to take place. The mission is to take place between the starting time of the planned maneuver, $t=t_{0}$, and the final time, $t=t_{f}$. At $t=t_{0}$, the shuttle is located $1,000 \mathrm{~m}$ in front of the station and 400 m above the V -bar, and at the final time it is $1,000 \mathrm{~m}$ behind the station and 600 m above the V -bar. Although the relative motion trajectories are three-dimensional, that is, they constitute motion in the orbital plane as well as out of it, only the projections of the trajectories on the orbital plane are shown. These projections are presented by the colored, curved traces. The way points are indicated by a small, hollow circle. At the center of this circle a "pedestal" is shown, perpendicular to the orbital plane, of which the end-point indicates the actual three-dimensional way-point position. Although in the present plan view this pedestal is hardly noticeable, it will be clearly visible when the orbital plane is viewed obliquely.

The blue trace is the trajectory the shuttle will follow, where the markers with numbers indicate the time-of-arrival in minutes after $t=t_{0}$. The initial chaser trajectory is indicated by the purple trace; the transfer trajectory, which eventually brings the chaser to rendezvous with the target, is indicated by the green trace. In this example, a departure bum is initiated 6 min after $t=t_{0}$ which inserts the chaser on its transfer trajectory toward the target. The departure way point is indicated by DEP WP. An intermediate burn is given at the transfer way point WP1, 35 min after $t=t_{0}$. The hollow blue circles on the target trace indicate the position of the target at the TOA of the corresponding chaser way points. The rendezvous takes place 1 hr after $\mathrm{t}=\mathrm{t}_{0}$ at the arrival way point, indicated by ARR WP. Below each way point, the TOA is displayed in hours (h), min ('), and sec ("). The departure burn is indicated by the red arrow, where the length is made proportional to the magnitude of the burn. A fixed scale factor of $500-\mathrm{m}$ length per meter-per-second burn has been chosen. At the arrival way point a retro-burn is to be initiated, in order to bring the relative velocity between the two spacecraft to zero. However, rather than showing the burn, the relative velocity at which the chaser approaches the target, is shown. The relative approach velocity vector is indicated by the orange
arrow (without tail). Again, a scale factor of $500-\mathrm{m}$ length per meter-per-second relative approach velocity, has been chosen.


Figure 4. Display layout.

The Fuel-Use Indicator- The fuel-use indicator on the right-hand side of the display (fig. 4) shows two thermometer-like scales and a numerical read-out at the bottom. The green tape in the right-hand scale indicates the total fuel used, computed as the sum of all impulses, both in-orbital plane (IOP) and out-of-orbital plane (OOP), including the retro-burn at rendezvous. The uppermost portion of the right-hand tape (dark green section) is shown in expanded form on the left-hand scale. Since the left scale is eight times more sensitive, it can be used effectively in fine-tuning a planned maneuver.

The Primer-Vector Window- The horizontal axis in the primer-vector window represents the time from the start of the maneuver, expressed in fractions of an orbital period. Here, only one orbital period is shown. The dotted vertical lines represent the TOA at the departure, at the transfer, and at the arrival way point. As mentioned before, the primer-vector window serves in adjusting the TOA of way points. In addition, the primer-vector curve includes information that might assist the operator in obtaining more fuel-efficient trajectory designs. Its functionality is discussed in section 6.

The Out-of-Orbital-Plane Window- The out-of-orbital-plane window shows the OOP situation in the so-called time-axis format. This format is primarily intended to assist in the OOP trajectory design by unambiguously displaying spatial constraints. The basic idea of the time-axis format is shown in figure 5 . From the perspective view in figure 5a alone, it cannot be clearly determined whether the spatial constraint is violated or how the trajectory should be planned to avoid it. The downward view along the R-bar (fig. 5b) is even less clear, because of the curved trajectory. In the time-axis format (fig. 5 c ) the OOP deviation is plotted as a function of the time traveled along the trajectory. The spatial constraints are visualized as follows. At each point on the time axis, at the corresponding location on the trajectory, a line is placed perpendicular to the orbital plane. If this line intersects a spatial constraint envelope, the intersection points of this line with this envelope are computed and sections of the line that fall within the spatial constraint are identified. The area traced by these line sections is plotted on the time-axis format of figure 5 c as the yellow polygon.


Figure 5a. Principle of the "time-axis" format; perspective view of the orbital plane.


Figure 5b. Principle of the "time-axis" format; downward view along the $R$-bar (toward the center of Earth).


Figure 5c. Principle of the "time-axis" format; time-axis format.

The horizontal axis in the OOP window again represents the time in fractions of an orbit. The way points are indicated by small circles, the OOP departure burn by the red arrow (with tail), and the OOP relative approach velocity vector at rendezvous by the orange vector (without tail). The bright yellow area indicates the violated section of the constraint, whereas the dark yellow area indicates the section that is not violated.

## Exercise w. Determining hion bitalspane cursor fosition

To determine the in-orbital-plane cursor position, proceed as follows.

1. Move the mouse and note that the left-pointing slanted little arrow (cursor) moves accordingly in the orbital plane.
2. Move the cursor to the center of the shuttle (starting location of the target trace). Verify that the cursor position reading in the information window is $(1000,400)$, that is, $1,000 \mathrm{~m}$ in front of the space station, and 400 m above the V -bar.
3. Move the cursor to position $(0,0)$. Verify that the cursor is at the center of the dotted box.
4. Move the cursor to the center of the dotted polyhedron (to the right of the space station) and verify that the center is at $(300,0)$.

### 2.2 Control Devices

The operator interacts with the planning system by means of a mouse, on which only the right button is active. The mouse is used both in the trajectory design process and in viewpoint operations, which allow the operator to explore the spatial situation about the space station. The various operational modes can be accessed through pop-up menu choices. In general, pop-up menus are brought up by double clicking the right mouse button (clicking twice in rapid succession). The right mouse button is hereafter referred as "the mouse button."

## 

To call up and use the pop-up menus, proceed as follows.

1. With the cursor in the main window (not the fuel window) and away from the transfer trajectory, double click the mouse button, but do not hold the button down. The menu shown in figure 6 a appears.
2. Note that the shape of the cursor changes from a horizontal right-pointing arrow, when outside the menu, to a small, black right-slanted arrow, when inside the menu. Place the cursor on the View option and note that this selection will be highlighted. This option also shows a triangular arrow, to the right of which a submenu appears; the submenu offers additional choices, as shown in figure 6 b .
3. Place the cursor on the Out-of-Plane option. A submenu with two additional options appears, each of which again shows a triangular arrow, to indicate that additional choices are possible. Place the cursor on the Range option. A sub-submenu with various choices appears (fig. 6c).
4. Move the cursor over each one of the range options. Note that each one of the options can be highlighted, except for the Auto option, which is "grayed-out." This indicates that this option is presently active and cannot be chosen. To verify this, place the cursor on the Auto option and click the mouse button once. The original Program Options menu reappears, indicating that no selection has been registered.
5. Move the cursor to the Main/Select option and click the mouse button. The menu disappears, indicating that the selection has been successful.


Figure 6. Example of a pop-up menu with a submenu structure: (a) Program Options menu; (b) View options submenu; (c) Out-of-Plane options submenu with Range options sub-submenu.

### 2.3 Operational Modes

The system allows two basic operations: (1) viewpoint operations, in which the operator is able to explore the spatial situation about the space station and thus choose a viewpoint position and viewing direction that focus and range-in on the momentary area of interest; and (2) trajectory design operations, in which way points are created, selected, changed, and deleted, in order to obtain a multi-burn trajectory that complies with the given set of mission constraints. These operations are discussed in detail in sections 3 and 4, respectively. They are governed by four basic modes: the main mode, the select mode, the change mode, and the viewpoint operation mode (fig. 7).

The main mode is the starting mode of the program, from which other modes can be accessed. The cursor, which is inactive in this mode, is a small slanted arrow; it can be moved around the orbital plane freely. As in Exercise 1, the $(s, r)$ coordinates of the cursor position in the orbital plane are displayed in the numerical window.

As mentioned in section 1.7, in the trajectory design process only the active way point can be changed by the operator. The select mode allows existing way points to be selected for activation, or new way points to be created, with the mouse. The procedure by which this is done is outlined in section 4.

The actual trajectory design takes place in the change mode. In this mode the IOP position, the OOP position, or the TOA of a transfer way point can be changed. Furthermore, the change mode allows the TOA at the departure or at the arrival way point to be adjusted.

The viewpoint operation mode allows the operator to change his viewpoint and viewing direction, by means of a translate, rotate, or ranging operation. Furthermore, it allows the operator to return to a preset default viewing situation.

In addition to these four basic modes, the operator can delete way points, change the scaling of the fuel dial or set a fuel reference marker, change the parameters of the graphical optimization aids, and change the dimensions and scaling of the out-of-plane situation display. The various choices are made through pop-up menus. The seven different menu operations are shown in the fourth row of the diagram in figure 7.

### 2.4 Sizing and Scaling the OOP Window

Apart from the "normal" size at which the OOP window is presently displayed, this window can be expanded to a larger size, or can be turned off. The OOP window scaling can be chosen from a set of fixed values, or set to an auto-scaling mode in which the scale is automatically adjusted to the smallest possible value for which the OOP trajectory profile is still within the window boundaries.

Figure 7 shows that the OOP window and scale-setting operations (fourth row, second from right) are accessible through menus, called up in the Main Mode (Main Menu) or in the transfer way-point Change Mode (Transfer WP Menu).

## 

To set the size and scale of the OOP Window proceed as follows.

1. With the cursor in the main window and away from the transfer trajectory, double click and release the mouse button. The Program Options menu appears. Move the cursor successively over the Out-of-Plane option, the Window submenu option, and the Expanded sub-submenu option, and click the mouse button once. Note that the OOP window has changed into its expanded size. Repeat this step, but this time select the Off sub-submenu
option, and note that the OOP window has disappeared. Repeat this step once more, but select the Normal sub-submenu option to change the OOP window back to its normal size.
2. Double click to call up the Program Options menu again and select the Out-of-Plane option and Range submenu option. Note that the Auto option is grayed out, indicating that the system is presently set to the auto-scale mode, by which the scale has been set to $\pm 100$. Select the " 200 " sub-submenu option, click the mouse button once, and verify that the OOP window scale has changed to the $\pm 200$ range. Repeat this step, but this time select the Auto sub-submenu option to return to the auto scaling mode.

### 2.5 Scaling and Marking the Fuel Use Indicator

The fuel-use indicator scaling can be chosen either from a set of fixed values, or set to an autoscaling mode, in which the scale is automatically adjusted to the smallest possible value for which the tape is still within the indicator's boundaries.

In fine-tuning a planned maneuver, it is sometimes useful to memorize the fuel use of a certain solution so that it can be compared with the fuel use of alternative solutions. The fuel use is memorized by setting a reference marker which remains unchanged until it is either reset or turned off.

Figure 7 shows that the fuel use indicator operations (at the far left of the fourth row) are accessible from the Main Mode or from the transfer way point Change Mode, through a menu which is called up while the cursor is in the fuel window.

## 

To set the scaling and reference marker of the fuel-use indicator, proceed as follows.

1. Move the cursor into the fuel window and note that the cursor has changed from the small slanted arrow into a horizontal right-pointing arrow. Note also the appearance of the label Double Click for Fuel Menu. Double click the mouse button to call up the Fuel Dial menu. Select the Range option and note that the Auto option has been grayed out from its submenu, indicating that the system is presently set to the auto-scale mode. Select the " 0.8 " submenu option, click the mouse button once and note that the scale has changed accordingly and that the cursor has changed back into the small slanted arrow. Repeat this step but this time select the Auto submenu option to return to the auto-scaling mode.
2. Move the cursor into the fuel window and double click to call up the Fuel Dial menu once more. Select the Ref Bar option and Set from the submenu and click the mouse button once. Note that purple reference bars have appeared on top of the two indicator tapes. These bars will stay in position until they are reset by the above procedure. Repeat this step, but this time select the Off submenu option. Verify that the reference bars have disappeared.


Figure 7. Operational system modes routing chart.

### 2.6 Exiting the Program

Figure 7 shows that exiting the program takes place through a selection from the Main menu, called up in the Main Mode (at the far right of the fifth row). To prevent accidentally exiting the program, this menu selection requires confirmation. The program exit is used when the trajectory design has been completed. The design is memorized for later retrieval and processing. In case the program-exit option has been chosen and confirmed, the mission constraints will be checked for possible violations. If one or more constraints are still violated, an additional confirmation is required to exit the program.

## 

To exit the program, proceed as follows.

1. With the cursor in the main window and away from the transfer trajectory, double click to call up the Program Options menu. Select Program Exit and then select Yes really! from the CONFIRM menu. The program does not exit, since several mission constraints are still violated. A second confirmation menu entitled CONSTRAINTS VIOLATED!! has appeared. The constraints that are violated are listed in the flashing information window. Select No, do NOT exit. from the confirmation menu to return to the Main Mode.

### 3.0 VIEWPOINT OPERATIONS

The viewpoint mode allows the spatial situation about the space station to be explored. The geometry of the viewing situation is shown in figure 8. The viewing system ( $\mathrm{x}_{\mathrm{e}}, \mathrm{y}_{e}, \mathrm{z}_{\mathrm{e}}$ ) has its origin at the viewpoint $A$, the $x_{e}$-axis coincides with the viewing direction (which is the center of the monitor screen), the $\mathrm{ye}_{\mathrm{e}}$-axis points to the right, and the $\mathrm{z}_{\mathrm{e}}$-axis points downward. Point B indicates the intersection of the viewing axis with the orbital plane. This point is the focus of interest and is located at the center of the screen. Although the viewpoint position A and the angular orientation of the viewing system ( $\mathrm{x}_{\mathrm{e}}, \mathrm{y}_{e}, \mathrm{z}_{e}$ ) are defined by three displacements and three angles (which can all be controlled independently), it is useful to constrain the motion to the following three types: tethered rotation, translation, and ranging.

In the tethered-rotation motion, the viewing axis $x_{e}$ pivots (like a tether ball) about point $B$, which is kept fixed on the orbital plane, while the distance $d$ between points $A$ and $B$, is kept constant. The lines $s^{\prime}$ and $w^{\prime}$, passing through point $B$, are parallel to the $s$ - and $w$-axes, respectively. The $x_{e}$ and $y_{e}$ axes of the viewing system are located at all times in the plane $P_{1}$ which passes through the line $s^{\prime}$. The plane $P_{2}$ is perpendicular to the line $s^{\prime}$ at point $B$. The intersection of $P_{1}$ and $P_{2}$ is the line $w^{\prime \prime}$. The tethered rotation is controlled by the angles $\Psi$ and $\Theta$, where $\Psi$ is the angle between $\mathrm{x}_{\mathrm{e}}$ and $\mathrm{w}^{\prime \prime}$, and $\Theta$ is the angle between $w^{\prime \prime}$ and $w^{\prime}$. Thus, the angles $\Psi$ and $\Theta$ control the obliquity of viewing along the orbital plane in the $s$ - and r-directions, respectively. This tethered rotation is very useful for the following reasons: First, while the area of interest remains in the center of the display, it allows the operator to explore other areas of interest by bringing them into the viewport by making appropriate changes in the angles $\Psi$ and $\Theta$. Second, the line s' will appear on the screen at all times


Figure 8. Geometry of the viewing situation.
as a horizontal line through the center of the display, and will represent a line parallel to the V-bar. Thus, while the viewing direction might change, the direction of the V -bar is at all times recognizable as the horizontal line passing through the center of the display.

The translational motion relates to the position of the tether attachment point $B$ (focus of interest) in the orbital plane. Here the ( $\mathrm{s}, \mathrm{r}$ ) coordinates of point B are varied, while the angles $\Psi$ and $\Theta$ remain unchanged. This type of translational motion enables the operator to move areas of interest to the center of the display.

In the ranging operation, all parameters are kept constant except for the range d . Thus, the viewpoint is moved along the viewing axis toward areas of interest, so that they can be studied in detail.

The viewing system reset will return the viewing situation to a default setting which is defined a priori.

## Exercise 5 : yevpolnt ©perations.

To demonstrate the viewpoint operations, proceed as follows.

1. With the cursor in the main window and away from the transfer trajectory, double click the mouse button. The Program Options menu appears. Move the cursor over the View option,
select Rotate from the submenu, and click the mouse button once. The program is now in the tethered rotation mode. This mode is identified by the Press to Rotate icon.
2. Refer to the diagram in figure 7 and confirm that starting from the Main Mode, we have moved to the Program Options Main Menu by double clicking (DC) and moved to the Viewing Menu and to the Rotate option through menu selections (M). Trace the route with a highlighter, and number each section sequentially.
3. Move the mouse (without pressing the mouse button) and note the rotations of the icon Press to Rotate. Other than notifying the operator of being in the Rotate mode, the mouse motions do not have any effect.
4. With the mouse button pressed, move the mouse first left and right and after that back and forth, to experience the effects of changing the angles $\Psi$ and $\Theta$, respectively. Note that, regardless of the rotation, the grid line parallel to the V-bar (with the orbital velocity vector arrows) and through the center of the display, remains at all times horizontal. The $\Psi$ and $\Theta$ angles, in degrees, are displayed in the information window.
5. Release the mouse button and double click to call up the View Options Viewing Menu. Select Reset and click the mouse button once. Notice that the system has returned to its default viewing situation, and that the View Options menu is still visible. Select Translate and click the mouse button once. The program is now in the Translate mode. Refer to the diagram in figure 7 to verify that we have moved from Rotate to the Viewing Menu by double clicking, to Reset by menu selection, back to the Viewing Menu by default and after that to Translate through a menu selection. Trace the route and number the sections sequentially (Note: Sections that are passed more than once should be marked and numbered accordingly.)
6. The Translate Mode is identified by the icon Press-to-move-to-cursor-position, and by the appearance of a cross-hair cursor. Note that the IOP cursor position is again displayed in the information window. A translation can be performed in two ways: (1) by placing the cursor on the desired IOP position and clicking the mouse button, hereinafter referred to as the cursor mode; (2) by moving the mouse while keeping the mouse button pressed, referred to as the "drag" mode.
7. Without pressing the mouse button, move the cursor to the nose of the shuttle (position 1000,420 ) and click the mouse button once. Note that the tether attachment point B "slues" to the new location. Note also that this sluing motion resembles the response of a welldesigned position control system, that is, fast at the start and slower toward the end. Although the transfer to the new location could have been done instantaneously, such a jump would disorient the operator. The sluing function is designed to give the operator the feel of dealing with "natural" physical objects which have mass, rather than with massless computerized synthetic images.

Remark: If the viewing menu is called up during a sluing translation, the tether attachment point $B$ will return to its originating position. Therefore, the menu should be called up after the sluing motion has been fully completed.
8. Repeat step 7 for different locations in the orbital plane and finish by moving again to the nose of the shuttle. Press and hold down the mouse button. Note that a cross-hair cursor has appeared in the center of the display. The system is now in the drag mode. Move the mouse and verify that the position in the orbital plane varies accordingly. The IOP position of the cross-hair is again displayed in the information window. Note also that the sensitivity of motion is much less than in the cursor mode. The drag mode is very useful in fine-tuning procedures. While holding down the mouse button, place the cross-hair at $(1000,400)$ and release the mouse button.
9. Double click again to call up the View Options Viewing Menu. Select Range and click the mouse button once. The program is now in the Ranging mode. Trace the route and number the sections of this mode change in the diagram in figure 7. The Ranging Mode is identified by the icon Press to Range. Move the mouse back and forth (without pressing the mouse button) and note the expansion and contraction of the icon. Other than notifying the operator of being in the Ranging mode, the mouse motions do not have any effect.
10. With the mouse button pressed, move the mouse back and forth and note the change in the viewing range d, and the corresponding range displayed in the information window. Set the range at 75 . Release the mouse button and double click again to call up the View Options menu. Select Rotate and click the mouse button once. As in step 4, use the rotate mode to view the shuttle from different viewing angles.
11. Double click, select Reset from the View Options menu and click the mouse button once to return to the default viewing situation. Select Translate and use the cursor mode to translate to the center of the space station.
12. Use the viewing modes to observe parts of the space station in detail. (There is no need to mark the necessary mode changes in the diagram of fig. 7.)
13. Finish by selecting Reset to have the system return to its default viewing situation, and then select Return, to return to the originating mode, that is, the Main Mode.

### 4.0 TRAJECTORY DESIGN

Trajectory design involves creating and editing the minimal number of way points, necessary to define a fuel-efficient trajectory that satisfies all mission constraints. The editing of intermediate transfer way points involves determining their position, both in the in-orbital plane and in the out-oforbital plane, as well as their time-of-arrival. In contrast, the editing of terminal way points is limited to adjusting their TOAs only, since the positions of the departure and arrival way points are constrained to be located on the initial chaser and target trajectories, respectively. The departure and arrival timing, in turn, is constrained to be within the time-span available for the mission. This time span is predetermined by the specific mission requirements and cannot be changed by the operator. Delaying the departure time and advancing the arrival time are often referred to as initial and final coasting, respectively.

### 4.1 Selecting, Creating, and Activating Way Points

Existing way points are selected, or new way points are created, in the select mode. This mode is activated by pressing and holding down the mouse button, (action P in the diagram of fig. 7). The cursor in this mode is a large slanted arrow. Selection takes place by pointing with the arrow toward the desired way point. The selected way point changes from a hollow circle to a filled one. Both terminal and transfer way points can be selected. When the arrow is pointed toward a section of the transfer trajectory on which no way points exist, a new way point can be created. The location on the trajectory at which the new way point will appear is indicated by a hollow circle. Activation of the selected way point takes place by releasing the mouse button (action RS in fig. 7). Following a successful activation, the program will switch to the change mode. If the mouse button is released while no way point is selected, the program will return to the main mode (action R in fig. 7).

## 

To select, create, and activate way points, proceed as follows.

1. Move the cursor to IOP position $(400,-200)$ and press and hold down the mouse button. Verify that the cursor has changed to a large slanted arrow (the select cursor). While keeping the cursor on about the same IOP position, release the mouse button, and verify that the cursor has changed back to the regular small slanted arrow. Refer to the diagram of figure 7 and confirm that we have moved from the Main Mode to the Select Mode (action P), and back to the Main Mode (action S). Trace and number the route of these mode changes in the diagram of figure 7.
2. Press and hold down the mouse button, move the select cursor to the intermediate way point WP1, and point the slanted arrow at the way-point circle. Verify that the selected way point changes from a hollow circle to a filled one. (Note: Do not release the mouse button yet. In case you accidentally released the mouse button while the way point was selected, double click and select Main/Select from the menu and repeat step 2.) Move the select cursor away from the way point, and note that the way point has changed back into a hollow circle. Repeat this step several times, and note that the selected way point is indicated in the primer-vector and OOP windows by the highlighted time bars. Note also that the little hollow circle on the target trace, which indicates the position at which the target will be at the TOA of the selected way-point, has changed into a filled circle.
3. While still holding down the mouse button, move the select cursor along the transfer trajectory from the departure way point DEP WP to the arrival way point ARR WP and back. Note that a hollow "shadow" circle follows the select cursor along the path. The hollow circle is the position at which the new way point is to be created. A second hollow circle will move along the target trajectory. The latter one is the position where the target will be at the TOA of the new way point. Note also the dim timing bars, indicating the TOA of the new way point in the primer-vector and OOP windows. The IOP cursor position (not the position of the selected way point!) is displayed in the information window. The TOA of the selected way point is displayed next to the time bar in the primer-vector window. Note also, that as the select cursor passes an existing way point on the path, the hollow circle changes into a
filled one, indicating the selection of an existing way point, rather than the creation of a new one. The selection of an existing way point is indicated in the primer-vector and the OOP windows by the highlighted timing bars.
4. Select WP1, verify that the way point has been selected by checking that it appears as a filled circle, and release the mouse button. The selected way point has now been activated and the program has changed to the Change Mode, (action RS in fig. 7). Trace and number the mode change in figure 7. The cursor has changed into a cross-hair with open center (the "change" cursor), and the active way point is surrounded by a blinking circle. The active way point is also easily recognizable in the primer-vector and OOP window by the blinking time bar and circle, respectively. The corresponding location of the target on the target trace at the active way-point TOA has been marked by a solid circle.
5. Double click the mouse button to call up the Transfer WP menu. Select the Main/Select option and click the mouse button to return to the Main Mode Trace and number the mode change in figure 7.
6. Select a way-point position at the $20-\mathrm{min}$ marker on the trajectory section between DEP WP and WP1, verify that the position at which the new way point is to be created has been selected by checking that this point appears as a hollow circle, and release the mouse button. A new way point has now been created and activated, and the program has again changed to the Change Mode. Note that the newly created way point has now been labeled WP1 and the previously existing way point has been labeled WP2.
7. Repeat step 5 to return to the Main/Select mode (no need to mark the mode changes in figure 7).

### 4.2 Deleting Way Points

Intermediate-transfer way points that the operator finds are no longer useful, can be deleted. Terminal way points, however, cannot be deleted. In order to delete a way point, it has to be activated first. The deletion is accomplished by means of a pop-up menu selection.

## Frercise B. Dellig a May mom:

To delete a way point, proceed as follows.

1. Select and activate WP1, the way point created in Exercise 7. Double click the mouse button to call up the Transfer WP menu. Select the Delete WP option and click the mouse button. A confirmation menu appears. Select the Yes really! option. Verify that the way point has disappeared and that the previously existing way point has been relabeled WP1. Note that the program has returned to the Main Mode, recognizable by the small slanted arrow. Trace and number the mode changes in figure 7.
2. Select and activate the terminal way point ARR WP. Double click the mouse button to call up the Arrival WP menu. Note that the Delete WP option has been grayed out. Select this
option, click the mouse button, and note that the menu reappears, indicating that no legal choice was made. Select the Main/Select option to return to the Main Mode.

### 4.3 Terminal Way-Point Time-of-Arrival Changes

As mentioned previously, the editing of the departure and arrival way point is limited to adjusting their TOA. If the available time-span of the mission would be fully utilized, the chaser would depart at the starting time $t=t_{0}$ and the rendezvous would take place at the final time $t=t_{f}$. However, under certain conditions, a delayed departure or advanced arrival might result either in a lower fuel cost or in the avoidance of a spatial constraint.

## 

To change the TOA of the departure or arrival way point, proceed as follows.

1. As outlined in step 4 of Exercise 7, activate the departure way point DEP WP and do not hold down the mouse button. Note the appearance of the label Set TOA in Primer Window, just above the way point. Note also the appearance of the cursor in the primer window, consisting of a pair of inward-pointing horizontal arrows. The system is now in the terminal way-point TOA change mode (right-hand side of the third row in fig. 7). Trace and number the mode change in figure 7.
2. Move the mouse left and right and note that the arrow-pair cursor in the primer vector window is moving accordingly. Note that the motions of the cursor are limited to the yellow area, which is the time span from the start of the mission at $t=t_{0}$ to the TOA of the transfer way point, WP1 (from $0^{\prime} 00^{\prime \prime}$ to $35^{\prime} 00^{\prime \prime}$ after $t=t_{0}$, or equivalently, $0-0.372$ orbital periods). Note also that as you move the cursor in the primer-vector window, a single-arrow cursor will slide along the initial chaser trace in the main window. The TOA of the cursor is displayed in the information window.
3. A TOA change can be performed in two ways: (1) by placing the primer-vector window cursor at the desired TOA, or by placing the single-arrow cursor at the desired departure location on the initial chaser trace, and clicking the mouse button (the cursor mode); (2) by moving the mouse while keeping the mouse button pressed (the drag mode).
4. Without pressing the mouse button, move the primer-vector window arrow-pair cursor to the 0.25 orbital period time bar (TOA setting of $23^{\prime} 33^{\prime \prime}$ in the information window) and click the mouse button once. The TOA of the departure way point will "jump" to the new setting. Again without pressing the mouse button, place the single-arrow cursor at the $30-\mathrm{min}$ mark on the initial chaser chase, and click the button once. The TOA of the departure way point will jump to 30 min .
5. Repeat step 4 for different positions of either the primer-vector window cursor or the singlearrow cursor on the initial chaser trace, and finish by setting the latter at the $10-\mathrm{min}$ marker. Press and hold down the mouse button. Note that the single-arrow cursor has disappeared,
that the arrows of the cursor in the primer-vector window are now pointing outward, and that in addition to the display in the information window, the TOA is also displayed to the right of this cursor. The system is now in the drag mode.
6. While holding down the mouse button, change the TOA from $0^{\prime} 00^{\prime \prime}$ to $30^{\prime} 00^{\prime \prime}$ and back, several times. Note the change in the shape of the first transfer trajectory section, as well as the changes in the direction and magnitude of the departure and transfer way-point maneuvering burns. Finish by setting the TOA to $6^{\prime} 00^{\prime \prime}$ and release the mouse button. Double click to call up the Departure WP menu. Select the Main/Select option to return to the Main Mode. Trace and number the mode change in figure 7.
7. Repeat steps 1 and 2 for the arrival way point ARR WP. Note that the allowable TOA setting range is again indicated by the yellow area in the primer-vector window and that it is the time span from the TOA of the transfer way point, WP1 to the final time $t=t_{f}$, (from $35^{\prime} 00^{\prime \prime}$ to $1 \mathrm{~h} 5^{\prime} 55^{\prime \prime}$ after $\mathrm{t}=\mathrm{t}_{0}$, or equivalently, 0.372 - 0.7 orbital periods).
8. Use the cursor mode and the drag mode to set the arrival TOA to different values. Finish by setting the arrival way-point TOA to $1 \mathrm{~h} 0^{\prime} 00^{\prime \prime}$ and double click to call up the Arrival WP menu. Select the Main/Select option to return to the Main Mode Trace and number the mode change in figure 7.

### 4.4 Transfer Way-Point Changes

Once a transfer way point has been activated, the system allows changing its IOP position, its OOP position, and its TOA with simple cursor operations. If the change cursor (see step 4, Exercise 1) is located in the main window, the IOP position can be changed. If this cursor is moved to either the OOP window or the primer-vector window, the OOP position or the TOA of the way point can be changed.

## 

To change the IOP position of transfer way points, proceed as follows.

1. As outlined in step 4 of Exercise 7, activate transfer way point WP1. Verify that the program is in the Change Mode by noting that the active way point is surrounded by a blinking circle and that the cursor has changed into an open centered cross-hair.
2. As with the TOA terminal way-point changes, an IOP position change can also be performed in two ways: (1) by placing the change cursor at the desired IOP position, and clicking the mouse button (the cursor mode); or (2) by moving the mouse while keeping the mouse button pressed (the drag mode). Without pressing the mouse button, use the IOP cursor position readout in the information window to move the change cursor to position $(200,200)$, and click the mouse button once. The way point WP1 will jump to the new position. Repeat this for position $(200,600)$.

Remark: If the cursor was accidentally moved into either the primer-vector window or the OOP window, its shape will change into an arrow pair. In this case, move the cursor upward until the cross-hair cursor reappears in the main window.
3. Press and hold down the mouse button. Note that the cross-hair cursor has disappeared. While holding down the mouse button, slide the way point WP1 along the current R-bar (vertical line) between position $(200,600)$ and $(200,-200)$ and back. Note the changes in shape of the trajectory sections preceding and following the active way point, and the changes in maneuvering burns and terminal approach velocity. Terminate by placing WP1 at position ( $200,-200$ ) and releasing the mouse button.
4. Use the cursor mode, to move the way point WP1 to position ( $-200,0$ ) and the drag mode to slide the way point along the V-bar, between position $(-200,0)$ and $(200,0)$ and back. Terminate by sliding the way point down to position ( $200,-200$ ) and releasing the mouse button. Double click and select Main/Select from the Transfer WP menu, to return to the Main Mode.

## 

To change the OOP positions of transfer way points, proceed as follows.

1. Select and activate the way point WP1, and without pressing the mouse button move the cross-hair cursor into the OOP window. Regardless of where the window was entered, the cross-hair cursor will change in to a pair of inward-pointing vertical arrows, centered at the active way point. Note also the appearance of the label Set Position in OOP Window. Move the mouse back and forth and note that the arrow-pair cursor is moving accordingly. Note also that the horizontal motions of the mouse are disabled. The system is now in the OOP position change mode (left-hand side of the third row in fig. 7). Trace and number the mode change in figure 7.
2. An OOP position change can also be performed either in the cursor mode or in the drag mode. Without pressing the mouse button, use the OOP cursor position readout in the information window to move the cursor to position $w=-50$ and click the mouse button once. The way point will jump to the new position. Repeat this for position $w=50$.
3. Press and hold down the mouse button. Note that the arrows of the cursor are now pointing outward, and that the OOP position is displayed to the right of them. The system is now in the drag mode. While holding down the mouse button, move the way point between $w=-50$ and $w=50$ and back. Note the changes in the shape of the OOP trajectory and the changes in magnitude of the OOP maneuvering burns. Finish by moving the way point to $w=45$ and release the mouse button.
4. Return to the IOP position-change mode by moving the arrow-pair cursor upward outside the OOP window until it has changed back into the cross-hair cursor. Trace and number the mode change in figure 7.
5. Without pressing the mouse button, move the cross-hair cursor back into the OOP window until the arrow-pair cursor reappears. Double click to call up the Transfer WP menu and select the Fix/Change option to return to the IOP position change mode. Note that in this alternative way of returning to the IOP mode, the cross-hair has jumped directly to the way point WP1. Double click and select Main/Select from the Transfer WP menu, to return to the Main Mode. Trace and number the mode change in figure 7.

## Exercise 12 Changing to A of Iranster Way Points

To change the TOA of transfer way points, proceed as follows.

1. Select and activate the way point WP1 and without pressing the mouse button, move the cross-hair cursor into the primer-vector window. Regardless of where the window was entered, the cross-hair cursor will change in to the pair of inward-pointing horizontal arrows as seen in Exercise 9, centered at the active way-point time bar. Move the mouse left and right and note that the arrow-pair cursor is moving accordingly. The TOA of the cursor is shown in the information window. Note again the label Set TOA in Primer Window just above the active way point in the main window. The system is now in the transfer way point TOA change mode (third row, second from right in fig. 7). Trace and number the mode change in figure 7.
2. Note again that the motions of the arrow-pair cursor are limited to the yellow area, which is the time span between the TOA of the departure way point and the arrival way point, (from $6^{\prime} 00^{\prime \prime}$ to $1 \mathrm{~h} 0^{\prime} 0^{\prime \prime}$ after $\mathrm{t}=\mathrm{t}_{0}$, or equivalently, $0.372-0.637$ orbital periods). The TOA change can again be made either in the cursor mode or the drag mode. Without pressing the mouse button, move the arrow-pair cursor to the 0.25 orbital period time bar and click the mouse button once. The TOA of the departure way point will jump to the new setting.
3. Press and hold down the mouse button. Note that the arrows of the cursor are now pointing outward, and that in addition to the display in the information window, the TOA is displayed to the right of the arrows. While holding down the mouse button, change the TOA in the range from $15^{\prime} 00^{\prime \prime}$ to $50^{\prime} 00^{\prime \prime}$ and back, several times. Note the changes in shape of the trajectory sections preceding and following the active way point and the changes in maneuvering burns and terminal approach velocity. Finish by setting the TOA to $35^{\prime} 00^{\prime \prime}$ and releasing the mouse button.
4. Return to the IOP position change mode by moving the arrow-pair cursor upward outside the OOP window, until it has changed back into the cross-hair cursor. Trace and number the mode change in figure 7.
5. Without pressing the mouse button, move the cross-hair cursor back into the primer-vector window until the arrow-pair cursor reappears. Double click to call up the Transfer WP menu and select the Fix/Change option to return to the IOP position change mode. Note that in this alternative way of returning to the IOP mode, the cross-hair has jumped directly to the way point WP1. Double click and select Main/Select from the Transfer WP menu to return to the Main Mode. Trace and number the mode change in figure 7.

## 

Some situations might require accurate positioning of way points. The following exercises demonstrates the use of a ranging operation to allow more accurate control over the position of way points. Proceed as follows.

1. Select and activate the way point WP1 and double click to call up the Transfer WP menu. Select the View and Translate option. Verify that the system is in the translate mode of the Viewpoint Operations by identifying the icon Press-to-move-to-cursor-position. Use the cursor mode to translate to the way point WP1, and verify that this way point is still at position $(200,-200)$ as it was placed there in step 4 of Exercise 10.
2. Double click to bring up the View Options menu. Select the Range option and verify that the range is still set at 1650 . Set the range at 300 . Double click again to bring up the View Options menu and select Return. Verify that the system has returned to the originating mode, that is, the Change Mode.
3. Use the cursor mode to move the way point to position ( $200,-100$ ) and the drag mode to place it back at position $(200,-200)$. Note that at the present range the way point can be positioned quite accurately. Double click and select Main/Select from the Transfer WP menu to return to the Main Mode.

## 

As mentioned in section 2.1, the traces shown in the main window are the projections of threedimensional trajectories on the orbital plane. However, the actual spatial position of the trajectory way points is indicated by the end-point of a "pedestal," centered at the way point and perpendicular to the orbital plane. In this exercise we will observe these pedestals by viewing the orbital plane obliquely.

1. Select and activate the way point WP1 and double click to call up the Transfer WP menu. Select the View and the Rotate options. Set the angles $\Psi$ and $\Theta$ to $(10,-50)$. Note that the pedestal at the way point WP1 is now clearly visible. The smaller circle at the end indicates the true spatial position of the way point.
2. Double click to bring up the View Options menu. Select Return to return to the Change Mode. Verify that the system is in the change mode by noting the cross-hair cursor and the blinking active way-point circle.
3. Move the cross-hair cursor into the OOP window. Use the drag mode to move the way point between $w=-50$ and $w=50$ and back. Note the changes in the pedestal end-point position. Finish by placing the way point at $\mathrm{w}=45$.
4. Double click and select Main/Select from the Transfer WP menu to return to the Main Mode. Press and while holding down the mouse button, move the select cursor along the transfer trajectory from the $30-\mathrm{min}$ marker to the $50-\mathrm{min}$ marker and back. Note that a hollow circle with pedestal follows the select cursor along the path. The end-point of the pedestal traces the actual spatial trajectory of the chaser. Note also that a similar pedestal moves along the target trajectory. The end-point of this pedestal traces the spatial trajectory of the target.
5. With the cursor in the main window, double click to call up the Program Options menu. Select View and Reset to have the system return to its default viewing situation. From the View Options menu, which is still visible, select Return to return to the originating mode, that is, the Main Mode.

## 4.5 "Undoing" Way-Point Changes

The planning system offers the option to "undo" way-point changes, that is, to return to the situation before the way-point change took effect. This option is particularly useful in trajectory design aimed at reaching fuel-efficient solutions in which, starting from a given situation, various alternative solutions are to be considered. The undo option can be employed on changes that took effect either in the terminal or the transfer way points.

Generally, the undo operation returns the system to the situation that existed at the moment the way point was activated. However, in a lengthy design process, it might be desirable to "memorize" a particular situation to which an undo operation would return. In principle, a situation can be memorized by subsequently returning to the Main Mode and reactivating the way point. However, the program offers a shortcut, which enables it to memorize the desired situation without the need for leaving the Change Mode.

## 

To memorize and undo way-point changes, proceed as follows.

1. Activate the way point WP1 and verify that this way point is located at position (200,-200). Use the cursor mode to place the way point at position ( 800,0 ). Move the cross-hair cursor into the OOP window and verify that the OOP way point position is $w=45$. Use the cursor mode to place the way point at $w=-50$. Without pressing the mouse button move the cursor upward and back into the main window and then into the primer-vector window and verify that the TOA is $35^{\prime} 00^{\prime \prime}$. Use the cursor mode to set the TOA to $20^{\prime} 00^{\prime \prime}$.
2. Double click to call up the Transfer WP menu, select the Undo Change option and then the Yes, really! option to confirm the undo operation. Verify that the way point has returned to IOP way-point position ( $200,-200$ ), OOP position $w=45$ and TOA $35^{\prime} 00^{\prime \prime}$, which is the situation that existed at the time the way point was activated (check the OOP position and TOA by moving the cursor into the OOP window and primer-vector window, respectively). Verify also that the system is still in the Change Mode.
3. Repeat step 1 and double click to call up the Transfer WP menu. Select the Fix/Change option to memorize the changes. Verify that the system is still in the IOP position change mode. Use the cursor mode to place the way point at position ( 0,500 ). Double click again and select the Undo Change option from the Transfer WP menu. Select Yes, really! to confirm the undo operation and verify that the way point has returned to position $(800,0)$.
4. Use the cursor mode to place the way point at position (200,-200). Move the cursor into the OOP window and use the cursor mode to place the way point at $w=45$. Move the cursor upward and back into the main window and then into the primer-vector window, and use the cursor mode to set the TOA to $35^{\prime} 00^{\prime \prime}$. Double click and select the Main/Select option from the Transfer WP menu to return to the Main Mode.
5. Activate the departure way point DEP WP and verify that the TOA is $6^{\prime} 00^{\prime \prime}$. Use the cursor mode to set the TOA to $20^{\prime} 00^{\prime \prime}$. Double click and select the Undo Change option from the Departure WP menu. Select Yes, really! to confirm the undo operation and verify that the TOA has returned to $6^{\prime} 00^{\prime \prime}$. Note that the undo operation has no effect on the other way points, since no changes have taken place in those way points since the departure way point was last activated. Double click and select Main/Select from the Departure WP menu in order to return to the Main Mode.

### 5.0 MISSION CONSTRAINTS

In the multi-spacecraft environment, stringent safety rules will be imposed regarding clearance from existing structures, clearance between spacecraft en route, restrictions on angles and velocities of departure and arrival, and restrictions to prevent plume impingement on nearby spacecraft. In the proximity operations planning system, the constraints are visualized in a graphical format, which clearly informs the operator of possible violations and assists in taking the correct actions to resolve them.

### 5.1 Spatial Constraints

Spatial constraints constitute volumes through which the chaser is not allowed to pass. Thus, spatial envelopes can be defined that are visualized on the display by the dotted rectangular outline around the space station and the polyhedron outline located to the right of it. The operator must be able to make a clear judgment as to whether the planned trajectory clears the spatial constraint. In case the trajectory does not clear the constraint, the operator must be able to decide whether to avoid the constraint by means of an in-plane maneuver or an out-of-plane maneuver. However, it might not always be possible to make these judgments on the basis of the main window view. Therefore, as outlined in section 2.1, the spatial constraint is also presented unambiguously in the "time-axis" format in the OOP window by the yellow polygon.

If the chaser trajectory violates a spatial constraint envelope, the dotted outline in the main window turns from dim to bright. Since the polyhedron volume is violated, it appears as a bright outline, whereas the space station volume, which is not violated, appears as a dim one. In addition, violated sections of the trajectory are highlighted. In the OOP window, the violated section of the constraint appears as the bright yellow areas, whereas the non-violated sections appear as dim areas. The violated section of the OOP trajectory profile is highlighted as well.

## Eisercisemb. Resolving Spatal constramts

This exercise demonstrates how the main window view, in combination with the situation shown in the OOP, can be used effectively for solving spatial constraints.

1. Activate way point WP1 and use the drag mode to move the way point slowly along the horizontal grid line between position $(200,-200)$ and $(-400,-200)$. Note that at position $(100,-200)$ the trajectory no longer passes through the polyhedron volume, and has changed into a dim outline. The yellow polygon in the OOP window has disappeared.
2. Note that at position $(-100,-200)$ the trajectory passes through the space station volume, that the violating trajectory sections are highlighted, and that the constraint appears in the OOP window as the yellow rectangle. By sliding the way point along the same horizontal line ( $r=-200$ ), find the locations at which the space station volume is cleared either from the front section or from the aft one. Verify that these locations are $(30,-200)$ and $(-284,-200)$, respectively.
3. Use the cursor mode to place the way point at position (200,-200) and release the mouse button. Move the cursor into the OOP window. Use again the drag mode to move the way point slowly between $w=45$ and $w=100$. Note that for $w>60$, dim areas appear; they indicate that for this part of the trajectory the spatial constraint is not violated. Note that in the main window the violated highlighted section of the trajectory has become smaller. Verify that for $w=100$ the spatial constraint is no longer violated. The polygon has turned entirely dim, and the polyhedron outline appears dim as well. Finish by placing the way point at $w=100$ and releasing the mouse button.

Remark: As the way point is moved beyond $w=100$, the scale of the OOP window changes automatically to a larger one. The constraint and the trajectory trace will suddenly become smaller accordingly.
4. Double click to call up the Transfer WP menu. Select the Main/Select option to return to the Main Mode. Double click again to call up the Program Options menu. Select the View and Translate option. Translate to position ( 300,0 ), which is the center of the polyhedron. Double click to bring up the View Options menu and select the Range option. Set the range at 800 . Double click to bring up the View Options menu again. Select the Rotate option and set the angles $\Psi$ and $\Theta$ to $(-35,-46)$. The polyhedron is now viewed obliquely. Double click to bring up the View Options menu again and select Return to return to the main mode. Press and while holding down the mouse button move the select cursor along the transfer trajectory from WP1 to the $50-\mathrm{min}$ marker. Note that at the position for which the TOA is
$40^{\prime} 20^{\prime \prime}$ (use the TOA readout in the primer vector window) the end-point of the pedestal just clears the spatial constraint. Move the select cursor away from the chaser trajectory and release the mouse button.
5. Double click to bring up the Program Options menu again. Select View and Reset to have the system return to its default viewing situation. Select Return from the View Options menu to return to the Main Mode. Activate the way point WP1 and without pressing the mouse button, move the cursor into the OOP window. Use the cursor mode to place the way point at $w=45$ and double click to bring up the Transfer WP menu. Select Main/Select to return to the Main Mode.

### 5.2 Departure and Arrival Constraints

Restrictions on angles of departure and arrival may originate from structural constraints at the departure gate or from the orientation of the docking gate or grapple device at the target craft. In addition, departure velocities and terminal approach velocities at the target might be limited as well. The terminal velocity might be limited by the characteristics of the grapple mechanism or the docking procedure.

The IOP departure constraints are visualized in figure 9 by the bracketed arc. Since the departure burn vector indicates the direction and velocity at which the chaser craft separates from its initial trajectory, the departure constraints are satisfied when the burn vector is within the solid bracketed arc. If the burn vector is within the brackets, but exceeds the arc, the departure velocity is violated (fig. 9a). If the burn vector is outside the brackets, the departure direction is incorrect (fig. 9b). It should be noted that although the symbols in figure 9 relate to the general case of a chaser, departing from the gate of a vehicle moving with respect to the space station, it is valid for departure from a vehicle at a stationary location as well.

The IOP arrival constraints are visualized by a similar bracketed arc. Thus the arrival constraints are satisfied when the relative approach velocity vector at rendezvous is within the bracketed arc.

The OOP departure and arrival constraints are shown in figure 10. The OOP component of the departure burn vector and of the relative approach velocity have to be within the constraint brackets.

If a departure or arrival constraint is being violated, it changes from a dim arc or bracket into a bright one. On the other hand, in order to keep the display free of unnecessary symbols, the arcs are turned off when the burn vector is within less than $75 \%$ of the allowable magnitude and angular range (shaded areas in figs. 9-11).

## 

In this example the departure burn has the correct direction, but its magnitude exceeds the allowable departure velocity. At the rendezvous, the direction of the relative approach velocity vector is incorrect. Departure or arrival constraint violations can frequently be resolved by adding an additional way point just after departure or before arrival. At departure, the new way
point can be placed at a position such that only a small departure burn will be needed to leave the departure gate. A larger burn will be given at the new way point, at a safe distance away from the departure gate, which sends the chaser on its way to the target. At the new way point, just before rendezvous, a relatively large retro-burn is given safely away from the target, so that the chaser will slowly drift toward the target satisfying the specified arrival constraints.

1. With the cursor in the main window and away from the transfer trajectory, double click to call up the Program Options menu. Select View and Translate. Use the cursor mode to translate to the departure way point DEP WP (position $-461,67$ ) and double click to call up the View Options menu. Select Range and set the range to 950 . Double click to bring up the View Options menu again and select Return to return to the Main Mode.
2. Create a new way point at the $20-\mathrm{min}$ marker and use the drag mode to move the way point slowly along the horizontal grid line between positions $(-300,-100)$ and $(-700,-100)$. Note that at position $(-521,-100)$ the departure constraint has turned into a dim arc, and at position $(-590,100)$ again into a bright arc, indicating that the departure direction has been violated. Move the way point slowly along the vertical grid line between positions ( $-600,-100$ ) and $(-600,0)$. Note that the constraint turns into a dim arc again at position $(-600,-84)$ and disappears at position $(-600,-31)$. Finish by placing the way point at position $(-525,-100)$. Double click to call up the Transfer WP menu, and select Main/Select to return to the Main Mode.
3. With the cursor in the main window and away from the transfer trajectory, double click to call up the Program Options menu. Select View and Translate and translate to position $(-300,300)$. Double click to call up the View Options menu and select Return to return to the Main Mode.
4. Create a new way point at the $50-\mathrm{min}$ marker and use the drag mode to move the way point slowly along the horizontal grid line between positions $(-100,400)$ and $(-300,400)$. Note that the arrival constraint turns into a dim arc at position ( $-110,400$ ), disappears at position $(-122,400)$, reappears as a dim arc at position $(-175,400)$, and turns into a bright arc again at position $(-203,400)$. Finish by placing the way point at position $(-175,400)$.
5. Double click to call up the Transfer WP menu and select View and Reset to have the system return to its default viewing situation. Select Return from the View Options menu to return to the originating mode, which is the Change Mode. Double click to call up the Transfer WP menu and select Main/Select to return to the Main Mode.

## 

To resolve OOP departure and arrival constraints, proceed as follows.

1. Activate the way point WP1 and without pressing the mouse button, move the cursor into the OOP window. Use the drag mode to move the way point between $w=90$ and $w=-90$. Note that for $w>76$ and $w<-68$ the constraint is violated and appears as a bright set of brackets.

For $-50<w<58$ the constraint is not drawn, and for all other values it is drawn as a dim set of brackets. Finish by setting the way point at $w=3$, and release the mouse button.
2. Double click to call up the Transfer WP menu and select Main/Select to return to the Main Mode. Activate the way point WP3 and without pressing the mouse button, move the cursor into the OOP window. Use the drag mode to verify that the arrival constraint is violated for $w>102$ and $w<-10$, and that in the range $4<w<88$, the constraint is not visible $4<w<88$. Verify also that for all other values the constraint is drawn as a dim set of brackets. Finish by setting the way point at $\mathrm{w}=45$ and releasing the mouse button. Double click to call up the Transfer WP menu, and select Main/Select to return to the Main Mode.



Figure 9. In-orbital-plane departure constraints.


Figure 10. Out-of-orbital-plane departure and arrival constraints.


Figure 11a. In-orbital-plane arrival constraints; approach velocity violated.


Figure 11b. In-orbital-plane arrival constraints; approach direction violated.

## 

This exercise demonstrates that the solution of the various violations of mission constraints can be separated from one another. This is an essential characteristic of the system, since it allows a complex trajectory design problem to be separated in to a sequence of relatively simple independent design steps. In this example the departure and arrival constraints are satisfied, but the spatial constraints are still violated.

1. Activate the way point WP2 and use the drag mode to move the way point slowly between position $(200,-200)$ and $(-100,-200)$. Note that since only the maneuvering burns at the neighboring way points, WP1 and WP3 are affected, changes in the position of the way point WP2 do not affect the departure and arrival situation.
2. Place the way point WP2 at position $(65,-200)$ and verify that departure, arrival and spatial constraints are satisfied. Double click to call up the Transfer WP menu and select Main/Select to return to the Main Mode.

### 5.3 Plume-Impingement Constraints

Way-point maneuvering burns are subject to plume-impingement constraints. Hot exhaust gases from the orbital maneuvering system might damage the reflecting surfaces of sensitive optical equipment, such as telescopes or infrared sensors. Maneuvering burns aligned toward such equipment are restricted in direction and magnitude; limits for the allowable direction and magnitude of the burns are a function of the distance to the equipment and of plume characteristics. IOP constraints are visualized by the bracketed impingement-constraint arc shown in figure 12a. The arc
is centered on the mutual line of sight between the two spacecraft. Plume-impingement constraints are satisfied when the burn vector does not cross the arc. When a plume-impingement constraint is being violated, the arc changes from dim to bright, and when the burn vector is within less than $75 \%$ of its allowable range, the arc is turned off. When the chaser is very close to the target, the constraint arc becomes very small, see figure 12 b . In this case, the brackets are extended and ticks are added to make the constraint clearly visible.


Figure 12a. Plume-impingement constraints; normal range.


Figure 12b. Plume-impingement constraints; close range.

## Exercise 20: Besoling Pume mpligemen: Consizants

To resolve plume-impingement constraints, proceed as follows.

1. With the cursor in the main window, double click to call up the Program Options menu. Select View and Translate. Translate to position $(100,100)$ and double click to call up the View Options menu. Select Range and set the range to 675. Double click to bring up the View Options menu again and select Return to return to the Main Mode.
2. Create a new way point approximately at the position at which the transfer trajectory intersects the $s=100$ R-bar. The new way point (WP3) will be approximately at position $(100,113)$ and its TOA will be $42^{\prime} 20^{\prime \prime}$ (refer to the TOA readout in the primer-vector window). Note the solid circle on the target trace, which indicates the position of the target at the new TOA of the way point.

Remark: If the TOA of way point WP3 is not $42^{\prime \prime} 20^{\prime \prime}$, release the mouse button and move the cursor into the primer-vector window to adjust the TOA to the correct value. Double click and select the Fix/Change option from the Transfer WP menu to return to the IOP position change mode.
3. Use the drag mode to slide the way point slowly upwards along the $s=100 \mathrm{R}$-bar. Note that at position ( 100,140 ) a dim plume-impingement arc appears, which turns into a bright arc at position $(100,149)$. Continue to slide the way point upward and note that at position $(100,240)$ the arc has changed its appearance for close-to-target situations, shown in figure 12b. Release the mouse button and use the cursor mode to move the way point WP3 to position $(300,148)$. Note that although the arc itself is very small, it is clearly marked by the extended brackets.
4. Use the cursor mode again to place the way point at position ( 100,175 ). Move the cursor into the primer-vector window to adjust the way point's TOA. Again, use the drag mode to slowly vary the TOA between $42^{\prime} 20^{\prime \prime}$ and $44^{\prime} 00^{\prime \prime}$. Note that as the TOA advances, the solid circle on the target trace advances accordingly. Note that at a TOA of $43^{\prime} 10^{\prime \prime}$ the arc turns dim, and at a TOA of $43^{\prime} 30^{\prime \prime}$ the arc disappears. Finish by setting the TOA at 42'30" and releasing the mouse button. Move the arrow-pair cursor upward outside the primer-vector window and move the cross-hair cursor into the OOP window. Set the OOP position of the way point WP3 to $w=50$, double click, and select the Main/Select option from the Transfer WP menu to return to the Main Mode.

### 5.4 En Route Passage Constraints

For complex trajectories, the chaser trajectory might cross the target trajectory one or more times before the final rendezvous. Flight safety requires that chaser and target craft maintain a minimal separation at the trajectory crossings because of uncertainty in tracking or in actual thrust generation. Furthermore, the en route closing rate between the two vehicles is subject to an approach velocity limit. In conventional docking procedures, these limits are proportional to the range. A previously
used rule of thumb was to limit the closing rate to $0.1 \%$ of the range. This conventional rule is quite conservative and originates from visual procedures, in which large safety margins are taken into account to correct for human or system errors. Although the future traffic environment will be far more complex, and will therefore demand larger safety margins, more advanced and reliable measurement and control systems may relax these demands.

In this display system, the en route closing rate c is defined as the component of the relative approach velocity vector between chaser and target, $\underline{V}_{a}$, along their mutual line of sight (slant range) (fig. 13). This relative approach velocity vector is given by $V_{a}=V_{c}-V_{t}$, where $\underline{V}_{c}$ and $V_{t}$ are the relative velocities of chaser and target, respectively. Since the allowed closing rate is proportional to the range between the spacecraft, it is useful to express the constraint in terms of the minimal allowed "closing time." The closing time is defined as the time in which the chaser would reach the target if the closing rate remained constant. The $0.1 \%$ rule corresponds to a closing time of 1000 sec , or $16^{\prime} 40$ ".

The en route passage constraint (either passage clearance or passage closing time) is visualized in figure 13. The bold lines of the chaser and target trajectory indicate the corresponding chaser and target positions for which the constraint is violated. The centers of these sections are indicated by the bright dots. The circle, centered on the target dot, indicates the minimal allowed range between the two dots. In case the passage clearance constraint is violated, the circle radius equals the minimal allowed clearance, and in case the closing time constraint is violated, the radius equals the closing rate multiplied by the minimal allowed closing time. Thus, the constraint is violated when the chaser dot intrudes the circle. When a passage constraint is being violated, it changes from a dim circle into bright one, and when the range between the dots exceeds $133 \%$ of its allowable range, the circle is turned off.

## Exercise 24: Resolving En Route Passage Constrathts

To resolve en route passage constraints, proceed as follows.

1. Activate the way point WP3 and double click to call up the Transfer WP menu. Select View and Range and set the range to 1000 . Double click to call up the View Options menu and select Return to return to Change Mode.
2. Use the cursor mode to move the way point WP3 to position $(0,500)$. Use the drag mode to move the way point slowly along the horizontal grid line, between positions $(0,500)$ and $(400,500)$. Note that at position $(73,500)$ the dim en route passage constraint circle appears, centered on the target dot. Note also that the chaser dot is still well outside the circle, indicating that the chaser passes safely in front of the target. Note also that at position $(111,500)$ the chaser dot intrudes the circle and the constraint turns into a bright circle. The sections of the chaser and target traces that are violated are highlighted. These sections appear as double lines, since the three-dimensional trajectory, as well as its projection on the orbital plane, is drawn. Note that at position $(315,500)$ the constraint turns again into a dim circle, indicating that the chaser passes safely behind the target, and at position $(363,500)$ the circle
disappears. Finish by placing the way point at position $(200,500)$ and releasing the mouse button.
3. Double click to call up the Transfer WP menu. Select View and Translate and translate to the target dot at position $(259,285)$. Double click to call op the View Options menu, select Range, and set the range to 325 . Double click again to call up the View Options menu, select Rotate and set the angles $\Psi$ and $\Theta$ to $(-30,-50)$. The main window now shows a closeup oblique view of the en route passage situation. Double click to call up the View Options menu and select Return to return to the Change Mode. Double click to call up the Transfer WP menu again and select Main/Select to return to the Main Mode.
4. Press and hold down the mouse button while moving the select cursor along the section of the transfer trajectory enclosed within the constraint circle. Verify that the end-points of the pedestals of chaser and target pass each other at an unacceptably close range. Move the select cursor away from the chaser trajectory and release the mouse button.
5. Activate the way point WP3 and without pressing the mouse button, move the cursor into the OOP window. Use the drag mode to move the way point to $\mathrm{w}=160$. Note that the constraint has again turned into a dim circle. Double click to call up the Transfer WP menu and select Main/Select to return to the Main Mode.
6. Repeat step 4 and verify that the chaser passes well above the target. Activate the way point WP3 and double click to call up the Transfer WP menu. Select View and Reset to return to the default viewing situation. Select Return from the View Options menu to return to the Change Mode. Double click to call up the Transfer WP menu again. Select Delete WP and Yes, really! to delete the way point WP3.

### 6.0 OPTIMIZATION AIDS

As is true of the relative orbital motions, the optimization functions involved in orbital maneuvering are also frequently counterintuitive. In contrast to the Earth-surface environment in which the energy needed to move objects from one location to another is largely attributed to friction and atmospheric drag, the energy involved in reaching a certain position in space is not a direct function of the speed and distance traveled. Since orbital maneuvers are basically undamped and cyclic, fuel-efficient solutions require the proper placement, timing, and phasing of the maneuvering burns, rather than minimization of the speed and distance traveled. The operation planning system includes a number of optimization aids that can assist the operator in reaching fuel-efficient solutions.

### 6.1 Fuel Optimization and the Primer Vector

The optimization aids are based on the classic primer-vector orbital trajectory optimization technique developed by Lawden (ref. 4) and extended to non-optimal trajectories by Lion and Handelsman (ref. 5). Lawden's primer vector is the solution to the adjoint system. For impulse-type maneuvers, Lawden shows that the primer-vector time-solution has to satisfy four necessary conditions for the maneuver to be fuel-optimal: (1) the primer vector and its first time-derivative must be continuous everywhere along the trajectory; (2) at the maneuvering burns, the primer vector
aligns with the direction of the thrust, and has unity magnitude; (3) the primer-vector magnitude must be less than unity at all other points on the trajectory; and (4) at intermediate burns (not the initial or the final one) the magnitude of the primer-vector time-derivative (primer-vector rate) must be zero. Condition 2 is used to compute the primer-vector time-history along the path, whereas the interactive trajectory design is aimed at satisfying the remaining conditions.


Figure 13. En route passage constraints.

Lion and Handelsman showed that the primer-vector time-history can indicate whether the fuel consumption is optimal, whether additional intermediate burns are useful, or whether to introduce initial or final coasting. By extending the definition of the primer vector to non-optimal trajectories, they indicated how to modify the position and timing of intermediate way points for bringing the fuel used to a minimum. Their extension forms the basis for the optimization aids used in the system.

### 6.2 Description of Optimization Aids

The optimization aids include a diagram of the primer-vector distribution, which is the magnitude $p$ of the primer-vector time-history, and a number of optimization pointers, which indicate the direction in space and time in which to move the transfer way points.

Following Lawden's third condition, the primer-vector diagram provides a clear indication of whether fuel-efficiency might benefit from inserting additional way points, or from initial or final coasting (i.e., delaying the departure time or advancing the arrival time). This is demonstrated in figure 14, which shows typical primer-vector magnitude shapes. The time is given in units of the orbital period T. Trajectories for which the primer-vector magnitude p is entirely below unity are
optimal and will not benefit from additional way points (fig. 14a). On the other hand, when this magnitude is partially or completely above unity, condition 3 is violated, and additional way points (fig. 14b), initial coasting (fig. 14c), or final coasting (fig. 14d), might yield a lower fuel cost. Any changes made by the operator in the position or timing of way points will be reflected immediately in changes in the primer-vector distribution.



Figure 14. Typical primer-vector magnitude shapes: (a) Fuel-optimal trajectory; (b) Non-optimal trajectory; (c) Trajectory that requires initial coasting; (d) Trajectory that requires final coasting.

Conditions 1 and 4 of Lawden's primer-vector time-solution determine how the way points should be moved in order to reduce the fuel cost. Figure 15 shows the primer-vector diagram for a three-burn maneuver in which the intermediate way point is non-optimally timed or placed. At this way point, the primer-vector rate is non-zero (a non-zero slope, violating condition 4) and also discontinuous (violating condition 1). The difference in the Hamiltonian function just before and after the burn, indicates how to change the way-point timing. This difference is used to device the timing pointer, attached to the intermediate way point time bar at $t=t_{1}$ in the primer-vector diagram (fig. 15).

Likewise, the vector difference in primer-vector rate just before and after applying the burn, indicates the direction in three-dimensional space in which to move the way point in order to reduce the fuel cost, whereas the average primer-vector rate magnitude just before and after the burn indicates the degree of sub-optimality.

Thus, a three-dimensional optimization vector is defined, for which the direction aligns with the primer-vector rate difference, and for which the length is proportional to the average primer-vector rate magnitude. This vector is decomposed into an IOP component in the ( $\mathrm{s}, \mathrm{r}$ ) plane, and an OOP component in the w-direction. The IOP component is visualized by a position pointer, attached to the transfer way point, which indicates the IOP direction in which to move the way point (see fig. 16a). Likewise, the OOP component is visualized in the time-axis format by the OOP position pointer shown in figure 16 b .


Figure 15. Primer-vector magnitude diagram for three-burn maneuver with non-optimally placed transfer way point. The timing pointer indicates how to change the TOA of the way point.

### 6.3 Primer-Vector Diagram

As mentioned earlier, the shape of the primer vector assists the operator in evaluating the optimality of a maneuver. Changes in the trajectory design are reflected immediately in a change in the primervector distribution, so that the effect of these changes on the optimality can be evaluated almost instantaneously. In contrast to the optimization pointers, the primer-vector diagram does not give a clear direction in which to move way points in order to reach optimality. Instead, it allows a broader insight in to the optimization functions.

## Ekerchse 22 . Ining Pimpergecto: Bugram

In this exercise the primer-vector distribution is examined to determine whether a maneuver might benefit from additional way points. Furthermore, it is shown that optimality is characterized by a smooth distribution (no discontinuities in the slope) which is entirely below unity. Since this exercise is intended to demonstrate the shape of the optimal primer-vector distribution, the optimal solution is presented directly, without elaborating how it is derived. However, in subsequent exercises, various optimization tools will be employed to derive this optimal solution.

1. Select and activate the way point WP3, double click to call up the Transfer WP menu, select Delete WP, and select Yes, really! to delete the way point. Repeat this step for the remaining transfer way points WP2 and WP1. Verify that the remaining trajectory is a two-burn trajectory.
2. Select and activate the arrival way point ARR WP and set its TOA to $1 \mathrm{~h} 5^{\prime} 00^{\prime \prime}$. Double click to bring up the Arrival WP menu and select Main/Select to return to the Main Mode. Select and activate the departure way point DEP WP and set its TOA to $0^{\prime} 00^{\prime \prime}$. Verify that the primer-vector distribution is entirely above unity. This indicates that, since Lawden's third condition is violated, the solution is non-optimal and that an additional way point might yield a lower fuel cost.
3. Use the drag mode to slowly vary the TOA between $0^{\prime} 00^{\prime \prime}$ and $15^{\prime} 00^{\prime \prime}$. Note the changes in the primer-vector distribution and the change in fuel cost. Verify that the fuel cost decreases from an initial value of $0.844 \mathrm{~m} / \mathrm{sec}$ at TOA $0^{\prime} 00^{\prime \prime}$ to $0.645 \mathrm{~m} / \mathrm{sec}$ at TOA $8^{\prime} 00^{\prime \prime}$. At TOA $12^{\prime} 50^{\prime \prime}$ the primer-vector distribution is entirely below unity, indicating that for the chosen departure and arrival times, the fuel cost cannot be reduced by introducing additional way points. Finish by setting the TOA of the departure way point to $0^{\prime} 00^{\prime \prime}$, double click to bring up the Departure WP menu, and select Main/Select to return to the Main Mode.
4. Create and activate a new way point on the transfer trajectory at the 20 -min marker (use the TOA readout in the primer-vector window for accuracy). Use the cursor mode to move the way point to position $(100,-100)$. Note that the shape of the primer-vector distribution has changed and that its first section is below unity. Use the drag mode to slide the way point downward along the $s=100$ R-bar. Verify that at position $(100,-400)$ the distribution is entirely below unity, and that it is discontinuous in the slope at the active way-point bar (it shows a sharp edge). Although Lawden's third condition is satisfied, the discontinuity indicates that Lawden's first and fourth conditions are still violated, and that the design is still not optimal. This is confirmed by noting that the fuel cost has increased to $1.314 \mathrm{~m} / \mathrm{sec}$. Finish by placing the way point at position ( $100,-100$ ).
5. Double click to call up the Transfer WP menu, select View and Translate, and use the cursor mode to translate to position $(-100,0)$. Double click again to call up the View Options menu, select Range, and set the range to 750 . Double click to call up the View Options menu and select Return to return to the Change Mode.
6. Use the cursor mode to move the way point WP1 to position $(-478,-70)$ and release the mouse button. Move the cursor into the primer-vector window and set the TOA of the way point to $17^{\prime} 00^{\prime \prime}$. Move the cursor back into the main window and then into the OOP window. Set the OOP position of the way point at $w=-6.4$. Note that as you change the OOP position slightly about this value, the primer-vector distribution changes accordingly. Fine tune the OOP position such that the primer-vector distribution is entirely below unity. Verify that the distribution is now smooth at the active way-point bar, that is, that the primer-vector and its first time-derivative are continuous. Note that the fuel cost for the three-burn maneuver is $0.562 \mathrm{~m} / \mathrm{sec}$ which is an improvement of $33 \%$ over the two-burn maneuver.
7. Double click to call up the Transfer WP menu, select Delete WP and after that Yes, really! to delete the way point.

### 6.4 Optimization Pointers

The optimization pointers assist the operator in moving transfer way points in a direction that will result in a lower fuel cost. In contrast to the primer vector diagram, which assists in evaluating the optimality of the entire maneuver, the optimization pointers relate to the active transfer way point only. The terminal way points do not have optimization pointers. The pointers make it possible to find the best possible position and TOA of the active way point, while the parameters of all other way points remain fixed. However, the search for a global optimum requires variations in the parameters of all the way points involved. Since only one way point can be active at a time, the
search for a global optimum involves an iterative sequence, in which each way point is changed individually, until the primer-vector distribution indicates that the necessary conditions for a global optimum are met.


Figure 16. Position pointers: (a) IOP component; (b) OOP component of the optimization vector.

The search for a fuel minimum can be compared to the problem of finding the lowest spot in a mountain landscape, which is done by continuously descending in a direction in which the local down slope is the steepest. The optimization pointer visualizes the direction of the steepest descend.

The length of the pointer is proportional to the steepest local slope. At the minimum, the slope is zero and the pointer collapses into a point. On the other hand, at a location far away from the minimum, the slope, and consequently the length of the pointer, can be large. Therefore, locations close to the optimum might require a larger gain between the slope and the length of the vector than locations that are far away from the optimum. The program allows the gain to be adjusted between 1 and 100. The gain adjustments can be made through selections of the transfer way-point menu, called up from the Change Mode.

## Exercise 23 U Sthy fosmon Pomters

In this exercise the use of the position pointers in finding the three-dimensional optimal waypoint position is demonstrated. It will be shown that alternate adjustments in the IOP and the OOP position, will eventually bring the way point to its optimal position. In this exercise, the way-point TOA will be set directly to its optimal value. Adjustments of the way-point TOA will be discussed in the next exercise.

1. Create and activate a way point on the transfer trajectory at the 20 -min marker. Move the cursor into the primer-vector window and set the TOA of the way point to $17^{\prime} 00^{\prime \prime}$. Double click to call up the Transfer WP menu. Select the Optimization option, select the Vector option from the submenu, and select the " 1.0 " option from the Gain sub-submenu. Verify this gain setting in the information window. Note that both the IOP position pointer in the main window and the OOP position pointer in the OOP window have appeared.
2. Use the drag mode to move the way point WP1 slowly in the direction of the pointer and note that the fuel cost reduces accordingly. Note that as you continue following the direction of the pointer, its length becomes increasingly smaller. Continue until the pointer is no longer visible.
3. Double click to bring up the Transfer WP menu, select View and Translate, and use the cursor mode to translate to position $(-500,-100)$. Double click to call up the View Options menu, select Range, and set the range to 250 . Double click to call up the View Options menu once more, and select Return to return to the Change Mode.
4. Double click to call up the Transfer WP menu, select Optimization, select Vector from the submenu, and " 10 " from the Gain sub-submenu. Verify the new gain setting in the information window. Use the drag mode to again move the way point in the direction of the pointer until the pointer is no longer visible. Verify that the way-point position is $(-557,-30)$ and that the fuel cost is $0.726 \mathrm{~m} / \mathrm{sec}$.
5. Note that the OOP position pointer indicates that the OOP position still needs adjustments. Note also that the primer-vector distribution indicates that the Lawden conditions are not yet met (the curve is partly above unity and discontinuous in the rate). Repeat the procedure in step 4 to again set the pointer gain to 1 .
6. Move the cursor in the OOP window. Use the drag mode to move the OOP way-point position slowly in the direction of the pointer and note that the fuel cost reduces accordingly.

Continue following the direction of the pointer, until the pointer is no longer visible. Verify that the OOP position is $w=-6.5$ and that the fuel cost is $0.572 \mathrm{~m} / \mathrm{sec}$.
7. Note that the IOP pointer indicates that an additional adjustment of the IOP position is required. Double click and choose Fix/Change from the Transfer WP menu to return to the IOP position change mode. Adjust the way-point position in the direction of the pointer until the pointer is no longer visible. Repeat step 4 to again set the pointer gain to " 10 " and make the final adjustments. Verify that the way point is now at position $(-478,-70)$ and that the fuel cost is $0.562 \mathrm{~m} / \mathrm{sec}$. This is the optimal setting, shown earlier in step 5 of Exercise 22. Finish by repeating step 4 to again set the pointer gain to 1 . Double click and select Main/Select from the Transfer WP menu, to return to the Main Mode.

## Exercise 24. Using Tining Pointer

In this exercise the use of the timing pointer in finding the optimal way-point TOA, is demonstrated. It is shown that convergence to sub-optimal "local" minima is possible. Suboptimal solutions, however, are easily identified by observing the primer-vector distribution. The global minimum is found by searching the complete TOA range.

1. Select and activate the way point WP1. Double click to call up the Transfer WP menu, select Optimization, select Hamiltonian from the submenu and select " 0.2 " from the Gain subsubmenu. Verify this gain setting in the information window.
2. Move the cursor into the primer-vector window and use the cursor mode to set the TOA of the way point to $5^{\prime} 00^{\prime \prime}$ (use the TOA readout in the information window). Note that the timing pointer is now clearly visible and that it points to the right to indicate that the TOA has to be increased. Use the drag mode to adjust the TOA of the way point slowly in the direction of the pointer and note that the fuel cost and length of the pointer reduce accordingly. Verify that at $17^{\prime} 00^{\prime \prime}$ the pointer has vanished.
3. Continue to hold down the mouse button and continue to slowly increase the TOA. Note that at TOA $17^{\prime} 30^{\prime \prime}$ the pointer points to the left to indicate that the TOA has to be reduced. Note, however, that at TOA 17'55" the pointer suddenly changes polarity and points to the right. Continue to increase the TOA and verify that at TOA $20^{\prime} 34^{\prime \prime}$ the pointer has again vanished. The fuel cost is now at a "local" minimum of $0.603 \mathrm{~m} / \mathrm{sec}$. The local minimum is easily identified from the non-optimal primer-vector distribution and also from the non-zero position pointers. Continue to increase the TOA and verify that another local minimum exists at TOA $50^{\prime} 20^{\prime \prime}$, with a fuel cost of $1.752 \mathrm{~m} / \mathrm{sec}$. Finish by setting the TOA to the optimal value at $17^{\prime} 00^{\prime \prime}$ and fine-tune the setting until the timing pointer has vanished. Double click and select Main/Select from the Transfer WP menu, to return to the Main Mode.

### 6.5 Automatic TOA Adjustments

From Exercises 23 and 24 it is clear, that finding the optimal way-point setting involves alternate adjustments in the ( $\mathrm{s}, \mathrm{r}$ )-space, in the w -space, and in the TOA-space. Although individual adjustments in the direction of the pointer in each of these spaces is guaranteed to reduce the fuel cost, adjustments in one space will require re-adjustments in another space. The repetitive character of this process calls for partial automatization of the design steps.

In this display, an automatic TOA adjustment mode is introduced. If this mode is activated, the system continuously adjusts the TOA in the direction of the timing pointer until it has reached its optimal setting. Thus, the operator can make adjustments in ( $\mathrm{s}, \mathrm{r}$ )-space or w -space, while the TOA will be adjusted automatically by the system. However, the system does not guarantee whether it has locked on a local or a global minimum. This largely depends on the initial setting. The automated TOA adjustment mode should therefore be used in situations in which the initial setting guarantees convergence to a global minimum.

## Exercise 25. Using the Automated TOA Adusiment Mode

This exercise demonstrates that in the automated TOA mode, the TOA is continuously adjusted by the system when changes are made in the ( $\mathrm{s}, \mathrm{r}$ )- or w -space. It will also be shown that the system can lock either on local minima or on the global minimum, dependent on the initial situation.

1. Select and activate the way point WP1. Double click to call up the Transfer WP menu. Select View and Range and set the range to 500 . Double click to call up the View Options menu and select Return to return to the Change Mode.
2. Use the drag mode to slide the way point WP1 slowly along the $s=-500 \mathrm{R}$-bar between positions $(-500,100)$ and $(-500,-100)$. The timing pointer in the primer-vector diagram indicates that adjustments of the TOA are necessary.
3. Double click to call up the Transfer WP menu, select Optimizations, select TOA from the submenu, and select Auto from the Mode sub-submenu. The system is now in the automated TOA adjustment mode. Verify this setting by checking the information window.
4. Repeat step 2 and verify that the system adjusts the TOA from about $10^{\prime} 00^{\prime \prime}$ at position $(-500,100)$ to $17^{\prime} 00$ " at position $(-500,-100)$. (The TOA is listed under the way point WP1.) Finish by placing the way point at the optimum at $(-478,-70)$.

Remark: The TOA adjustment is an iterative process which takes place by background computations. At each system update, the TOA is adjusted in the direction of the minimum. The amount of adjustment in each step is carefully controlled by the system to maintain numerical stability. On the average, between 3 and 6 iterations are needed to reach the minimum. After making an adjustment in the $(s, r)$-space or the $w$-space, it is advisable to wait until the TOA adjustment has fully converged to its minimum.
5. Move the cursor in the primer-vector window, and (without pressing the mouse button and while using the TOA readout in the information window) move the cursor to TOA $4^{\prime} 00^{\prime \prime}$. Press and hold down the mouse button, and note that the automated adjustment is temporarily disabled and that the TOA remains fixed at the initial setting of $4^{\prime} 00^{\prime \prime}$. Release the mouse button and verify that the TOA converges to the optimal value of $17^{\prime} 00^{\prime \prime}$ (the TOA is listed under the way point WP1). Without moving the mouse, click the mouse button to have the system converge once more from TOA $4^{\prime} 00^{\prime \prime}$ to $17^{\prime} 00^{\prime \prime}$.
6. Repeat step 5 for an initial TOA setting of $35^{\prime} 00^{\prime \prime}$, and verify that the TOA now converges to the local minimum at $20^{\prime} 34^{\prime \prime}$. Also repeat step 5 for the initial setting of $1 \mathrm{~h} 00^{\prime} 00^{\prime \prime}$ and verify that the TOA converges to the second local minimum at 50'20". This exercise clearly shows that the convergence depends on the initial setting.
7. Double click to call up the Transfer WP menu, select View and Translate, use the cursor mode to translate to position ( $-400,-100$ ), and select Return from the View Options menu to return to the Change Mode. Double click again and select Optimization, TOA and Manual to return to the manual TOA adjustment mode. Verify this setting in the information window. Double click once more, select Delete WP, and Yes, really! to delete the way point and to return to the Main Mode.

### 6.6 IOP Position Pointer Trace

In Exercise 23 it was shown that the fuel cost can be reduced by moving the way point in the direction of the position pointer. Although the length of the pointer indicates the slope of the cost function, and thus the extent of the necessary change, the pointer does not indicate the amount by which to move the way point in the given direction. If this amount is too large, the pointer might "bounce around" and sharply change direction at each correction, which will make following the general direction toward the minimum quite difficult. This bouncing around is noted in particular in the vicinity of the optimum, where the direction of the pointer becomes undefined.

A solution for this problem has been found by displaying, in addition to the position pointer, a position pointer trace. This trace is computed at each background computation by adjusting the ( $\mathrm{s}, \mathrm{r}$ ) values in the direction of the pointer, in a sequence of correcting steps. However, the TOA and OOP position remain fixed at the active way-point values. The amount of correction in each step is carefully controlled by the system, so that numerical stability is maintained. The number of correcting steps can be set between 10 and 100. The larger the number of steps, the longer the trace, but the slower the system update rate. The position pointer trace is shown in figure 17. At the origin, the trace is tangential to the position pointer (fig. 17a). The end-point is marked by a diamondshaped reticle. By moving the way point to the end-point of the trace, the IOP position is adjusted towards the optimum. The trace thus provides both the direction as well as the necessary amount of correction. It is clear that toward the optimum, the necessary extent of the correction will become increasingly smaller. Accurate positioning of the way point near the optimum might require rangingin on the diamond-shaped reticle (fig. 17b). The way point is optimally adjusted in the ( $\mathrm{s}, \mathrm{r}$ )-space by centering the way-point dot at the reticle (fig. 17c).


Figure 17a. The IOP position pointer trace: the end-point of the trace is marked by the diamondshaped reticle; way point far away from the optimum.


Figure 17b. The IOP position pointer trace: way point near the optimum (close-up).


Figure 17c. The IOP position pointer trace: IOP position of the way point adjusted optimally (close-up).

## Exercise 26 Using lop Position Pointer Trice

To use the IOP position pointer trace, proceed as follows.

1. Create and activate a new way point on the transfer trajectory at the 10 -min marker (while in the Select Mode, use the TOA readout in the primer-vector window for accuracy). Double click to call up the Transfer WP menu, and select Optimization, Trace and " 50 ," to set the number of correcting steps in the computation of the trace to 50 . Verify this setting in the information window.
2. Use the drag mode to slide the way point WP1 along the horizontal line between positions $(-200,-100)$ and $(-500,-100)$. Verify that the trace remains tangential to the position pointer and that its end-point remains more or less in the same position.
3. Use the cursor mode to center the green dot of WP1 at the diamond-shaped end-point of the trace. Verify that the position pointer has vanished and that the fuel cost is $0.6 \mathrm{~m} / \mathrm{sec}$. Note the timing pointer in the primer-vector window, indicating that the TOA still needs adjustment. Double click, select Optimization, TOA, and Auto to activate the automated TOA adjustment mode. Verify that the TOA has been automatically adjusted to $11^{\prime} 36^{\prime \prime}$, and that the end-point of the trace has moved slightly, indicating that an additional IOP position adjustment is required.
4. Use the cursor mode repeatedly to re-center the green dot of WP1 at the end-point of the trace.

Note that the correction has to be carried out several times, since the automatic TOA adjustment after each correction in ( $\mathrm{s}, \mathrm{r}$ )-space requires additional re-adjustments until both the ( $\mathrm{s}, \mathrm{r}$ ) values and the TOA value have converged to their optimum values. Verify that the fuel cost is now $0.582 \mathrm{~m} / \mathrm{sec}$ and that the TOA has been adjusted to $12^{\prime} 47^{\prime \prime}$.

Remark: Instead of using the cursor mode repeatedly, it might be more convenient to use the drag mode in a continuous manner. The cursor mode was chosen here to demonstrate the interaction between the adjustments in the different optimization spaces.
5. Note that the OOP position pointer in the OOP window indicates that the OOP position still needs adjustment. Move the cursor into the OOP window. Take care to center the arrow-pair cursor at the way point before pressing the mouse button. Press the mouse button and use the dra g mode to slowly adjust the OOP position in the direction of the pointer until the pointer has vanished. Verify that the OOP position is now $w=-14.2$ and that the fuel cost is $0.571 \mathrm{~m} / \mathrm{sec}$. Note that the $(\mathrm{s}, \mathrm{r})$ position needs an additional adjustment.
6. Release the mouse button and move the cursor back into the main window. Repeat step 4 to re-adjust the ( $\mathrm{s}, \mathrm{r}$ ) values. This time use the drag mode for the adjustments. Verify that the fuel cost is now $0.565 \mathrm{~m} / \mathrm{sec}$.
7. Repeat steps 4 and 5 in sequence to bring the values to the global minimum. Verify that the system is at the global minimum by checking the primer-vector distribution and by comparing the setting with the one in Exercise 22, step 5.

Remark: Too rapid adjustment of the OOP position, or failure to center the arrow-pair cursor at the way point before pressing the mouse button, might cause the automatic TOA adjustment to lock on to a local minimum. In this case, set the OOP position to $w=-15$, carry out step 5 of Exercise 25, and repeat step 7 of this exercise.
8. Double click to call up the Transfer WP menu, and select Optimization TOA and Manual, to return to the manual TOA adjustment mode. Double click once more, select Delete and Yes, really! to delete the way point and to return to the Main Mode.

### 6.7 TOA-Adjusted IOP Optimal Position

In Exercise 26 it was shown that the position pointer trace indicates both the direction and the amount of the necessary ( $(\mathbf{r}, \mathrm{r}$ ) correction. However, finding the fuel optimum still requires alternate adjustments in the ( $\mathrm{s}, \mathrm{r}$ )-space, the TOA-space and the w -space, since adjustments in one space require re-adjustments in the other spaces until all values have converged to their optimum values. The TOA-adjusted IOP optimal position provides a shortcut in the adjustment procedure that eliminates the need for alternating between the ( $\mathrm{s}, \mathrm{r}$ )-space and the TOA-space.

The optimal position is computed by alternate adjustments in ( $\mathbf{s , r}$ )- and TOA-space, in a manner similar to that of step 4 in exercise 26. The initial values used in the computations are the position and TOA of the active way point. The OOP position remains unchanged. The number of iterations can be set between 10 and 100 . Since this process requires between 0.5 and 2 sec of computation time, it is not executed at each background computation, as with the position pointers or the position pointer trace. Instead, the computations are started only after the mouse has remained inactive for 3 sec .

The optimal position is indicated by a star with an octagonally shaped reticle at its center (fig. 18). Figure 18a shows the situation for which neither the way-point position, nor the way-point TOA is adjusted to its optimal value. In this case the way point, the end-point of the trace, and the optimum will be separated from one another. In figure 18 b only the ( $\mathrm{s}, \mathrm{r}$ ) values are adjusted, by placing the way point on the end-point of the trace. Since the TOA is not yet adjusted, the end-point is still separated from the optimum. The same is the case in figure 18c, in which the way point is placed at the optimum, but the TOA for this position is incorrect. In figure 18d both the position and the TOA are adjusted correctly, so that way point, end-point, and optimum coincide.


Figure 18. TOA adjusted IOP optimal position for various situations: (a) Way point before optimization; (b) Way point centered on end-point.


Figure 18. TOA-adjusted IOP optimal position for various situations: (c) Way point centered at optimum; (d) Way point, end-point, and optimum coincide.

## Exercise 27. Using toA-Adjusted iof Opilinal Position

To use the TOA-adjusted IOP optional positions, proceed as follows.

1. Create and activate a new transfer way point at $10^{\prime} 00^{\prime \prime}$. Double click to call up the Transfer WP menu, and select Optimization, Optimum, and " 50 ", to set the number of iterations in the computation of the optimum to 50 . Verify this setting in the information window. Note that the star-shaped optimum has appeared slightly below the diamond-shaped end-point of the trace.
2. Use the cursor mode to place the way point at position ( $-300,-100$ ). Do not move the mouse for several seconds. Note that after 3 sec of inactivity, the star-shaped optimum disappears for about 0.5 sec . This indicates that the optimum is recomputed. Use the cursor mode to move the way pointback and forth several times between locations $(-300,-100)$ and $(-400,-100)$.

Between each move, wait until the optimum has disappeared and again reappeared. Finish by placing the way point at the end-point of the trace.
3. Double click to call up the Transfer WP menu, select Optimization, TOA, and Auto to activate the automated TOA adjustment mode. Use the cursor mode to move the way point to the center of the optimum. Verify that the green way-point dot, the end-point, and the optimum coincide, which indicates that the ( $\mathrm{s}, \mathrm{r}$ ) and TOA values have been adjusted.
4. The OOP position pointer in the OOP window indicates that the OOP position still needs adjustment. Move the cursor into the OOP window and set the OOP position at $w=-6.5$. Release the mouse button and move the cursor back into the main window and then into the primer-vector window. Without pressing the mouse button, move the cursor to $5^{\prime} 00$ " (use the TOA readout in the information window), and press and release the mouse button. This procedure will guarantee that the system will converge to the global minimum.
5. Without pressing the mouse button, move the cursor back into the main window and use the cursor mode to readjust the way-point to re-center the way point at the optimum. Check the primer-vector distribution to verify that the system is at the global minimum and compare the setting with the one in Exercise 22, step 5.
6. Double click to call up the Transfer WP menu, and select Optimization, TOA, and Manual, to return to the manual TOA adjustment mode. Double click once more, and select Delete and Yes, really! to delete the way point and to return to the Main Mode. Double click to call up the Program Options menu, select View and Reset to return to the default viewing situation, and select Return from the View Options menu to return to the Main Mode.

### 6.8 Convergence Problems

Occasionally, the selected number of correcting steps in the computation of the position pointer trace, or the number of iterations in the computation of the optimum, is not sufficient for convergence of the solution within preset limits. In this case the trace, or the optimum, is drawn with dotted lines as a wire-frame only. Although the optimization aids still point in a direction in which fuel use will be lower, the lack of convergence indicates that the adjustments are sub-optimal. In some cases the problem can be solved by choosing a larger number of correcting steps or iterations, or by changing the initial way-point position or timing.

Frequently, the position pointer trace does not appear immediately after a way point has been created. This is because the way-point bum at the new way point has a magnitude of zero. Thus, the direction of the burn is undetermined and the computation of the primer-vector distribution is incorrect. A simple solution to this problem is to slightly displace the way point so that the magnitude of the maneuvering burn is no longer zero.

### 6.9 Optimization in the Presence of Constraints

In some cases a global fuel-optimal trajectory might violate one or more mission constraints. Although the optimization aids do not indicate a global minimum in the presence of constraints, they can be used to readily find fuel-efficient sub-optimal solutions. The following exercises demonstrate this concept.

Exercise 28. Using opimization Alds in tre Presence on: consizam:s:1.

1. Create and activate a new way point on the transfer trajectory at the 60 -min marker (use the TOA readout in the primer-vector window for accuracy). Double click to call up the Transfer WP menu, select View and Translate, and translate to the new way point WP1. Double click to call up the View Options menu, select Range, and set the range to 650. Double click to call up the View Options menu again and select Return to return to the Change Mode.
2. Note that both the position pointer trace and the optimum are drawn as dotted wire-frames, indicating that the number of steps in the computation of the optimum is not sufficient for
convergence of the solution within preset limits. The end-point of the trace is located on the target trajectory. Use the drag mode to position the way-point dot at the end-point and note that fuel use has been reduced to $0.774 \mathrm{~m} / \mathrm{sec}$.
3. Use the drag mode to move the way point along the horizontal grid line at $\mathrm{r}=500$ from position $(-600,500)$ to $(-700,500)$. Find the position on this line for which the relative approach velocity vector is just within the constraint bracket. Verify that this position is $(-642,500)$ and that the fuel use has increased to $1.077 \mathrm{~m} / \mathrm{sec}$. Note that the en route closing rate constraint is still violated, since the chaser dot intrudes the constraint circle.
4. Double click to call up the Transfer WP menu, select View and Reset to return to the default viewing situation, and select Retum from the View Options menu to return to the Change Mode. Double click to call up the Transfer WP once more, and select Main/Select to return to the Main Mode.
5. Note that the first section of the primer-vector diagram is entirely above unity, which indicates that an additional way point might improve the fuel efficiency. An additional way point is required in any case, to resolve the violated constraints. Create and activate this new way point on the transfer trajectory at the $30-\mathrm{min}$ marker. Use the drag mode first to move the way point in the direction of the position pointer and then to center the way-point dot at the end-point of the trace. Verify that for this position the en route passage constraints and the spatial constraints are resolved.
6. Double click to call up the Transfer WP menu, select View and Translate, and translate to the position of the departure way point DEP WP. Double click to bring up the View Options menu, select Range, and set the range to 500 . Double click once more and select Return to return to the Change Mode. Use the drag mode to accurately center the way-point dot at the end-point. Verify that the fuel use is $0.718 \mathrm{~m} / \mathrm{sec}$ and that the departure constraint is still violated. Note also that the TOA-adjusted IOP position is to the left, indicating that a TOA adjustment might reduce the fuel used and increase the fuel efficiency.
7. Double click to call up the Transfer WP menu, select Optimization, TOA and Auto to activate the automated TOA-adjustment mode. Use the cursor mode to center the way-point dot at the optimum. Verify that fuel use has been reduced to $0.675 \mathrm{~m} / \mathrm{sec}$ and that the departure constraint is no longer violated.
8. The OOP position pointer in the OOP window indicates that the OOP position still needs adjustment. Move the cursor in the OOP window, and use the drag mode to adjust the OOP position in the direction of the pointer until the pointer has vanished. Release the mouse button and move the cursor back into the main window. Re-center the way-point dot at the optimum. Alternate between the ( $\mathrm{s}, \mathrm{r}$ ) adjustment and the w adjustment until the OOP pointer is no longer visible and the way-point dot is centered at the optimum.

Remark: After each ( $s, r$ ) adjustment it is recommended that the operator wait for afew seconds until the optimum has disappeared and again reappeared. This indicates that the optimum has been re-computed. After each re-computation, the optimum might move slightly.
9. Double click to call up the Transfer WP menu, select View and Reset to return to the default viewing situation, and select Return from the View Options menu to return to the Change Mode. Verify that the fuel cost is $0.656 \mathrm{~m} / \mathrm{sec}$ and that all mission constraint violations are resolved.
10. Double click to call up the Transfer WP menu, and select Optimization, TOA, and Manual, to return to the manual TOA adjustment mode. Double click once more, and select Delete and Yes, really! to delete the way point WP1 and to return to the Main Mode. Activate the remaining transfer way point, double click again, and select Delete and Yes, really! to also delete this way point and to return to the Main Mode. Verify that the remaining trajectory is again a two-burn trajectory.

Remark: In this particular situation, the optimum position and timing of way point WP1, subject to the earlier choice of way point WP2, results in a trajectory for which all the violated mission constraints are resolved. This is not necessarily the case in all situations. Frequently the way point can not be positioned at the optimum, since it results in one or more mission constraints being violated. In that case a position on the position pointer trace has to be found for which the constraints are not violated. This situation is demonstrated in the next exercise. The above solution is sub-optimal, since the rather arbitrary choice of way point WP2 is not guaranteed to lead to a global constrained optimum.

Exercise 29: Using epilmization Alds in the fresence of constraints:2

To use optimization aids in the presence of constraints, proceed as follows

1. Create and activate a new way point on the transfer trajectory at the $50-\mathrm{min}$ marker (use the TOA readout in the primer-vector window for accuracy). Double click to call up the Transfer WP menu, select View and Translate and translate to position (-400,400). Double click to call up the View Options menu, select Range, and set the range to 875. Double click to call up the View Options menu again and select Return to return to the Change Mode.
2. Note that this time the end-point of the trace is slightly below the target trajectory. Use the drag mode to move the way-point dot to the end-point of the trace and verify that the arrival constraint and en route closing rate constraint are still violated. Use the cursor mode to move the way point to position $(-245,320)$ and verify that both constraints are now resolved. Verify that the fuel use is $0.872 \mathrm{~m} / \mathrm{sec}$. Use the cursor mode repeatedly to move the way point WP1 along the position pointer trace toward the end-point, in small steps (about half the width of the change cursor). Note that at each step the fuel used reduces. Continue until either the arrival constraint or the en route closing rate constraint is violated. Back up slightly to resolve the constraint. Verify that the fuel use is now $0.849 \mathrm{~m} / \mathrm{sec}$ and that the position of the way point WP1 is $(-228,320)$. Note that because of inaccuracies in following the trace, the setting you arrived at might vary slightly from the above one.
3. Double click to call up the Transfer WP menu, select View and Translate and use the drag mode to move to position ( $-300,0$ ). Double click to call up the View Options menu, and select Return to return to the Change Mode. Double click to call up the Transfer WP menu once more and select Main/Select to return to the Main Mode.
4. Create and activate a new way point on the transfer trajectory at the 30 -min marker, (use the TOA readout in the primer-vector window). Use the procedures in steps 7 and 8 of Exercise 28 to bring the way point WP1 to its optimal position and timing (subject to the given position and timing of way point WP2). Verify that the fuel cost is now $0.734 \mathrm{~m} / \mathrm{sec}$, the IOP position of the way point WP1 is $(-419,-69)$, the OOP position is $w=-11.5$, its TOA is $15^{\prime} 32^{\prime \prime}$, and the spatial constraint is still violated.
5. Double click to call up the Transfer WP menu, select Optimization, TOA, and Manual to return to the manual TOA-adjustment mode. Move the cursor into the primer-vector window and use the drag mode to increase the TOA slowly until the spatial constraint is no longer violated (the constraint envelope turns dim). Release the mouse button and move the cursor back into the main window and then into the OOP window. Use the drag mode to adjust the OOP position in the direction of the OOP pointer until the pointer has vanished. Note that the fuel cost that satisfies the mission constraints is $0.804 \mathrm{~m} / \mathrm{sec}$.
6. Without pressing the mouse button, move the cursor back into the main window and then into the primer-vector window. Use the cursor mode to adjust the TOA to $19^{\prime} 00^{\prime \prime}$ and, without pressing the mouse button, move the cursor back into the main window.
7. Use the cursor mode to move the way point to position $(-560,60)$. Verify that both the departure and the spatial constraints have been resolved, but that the plume-impingement constraint resulting from the maneuvering burn at the way point WP2 has been violated. Note that the fuel cost is rather high, that is $0.987 \mathrm{~m} / \mathrm{sec}$. Use the cursor mode repeatedly to move the way point WP1 along the position pointer trace toward the end-point in small steps (about half the width of the change cursor). Note again that at each step the fuel used reduces. Continue until either the departure constraint or the spatial constraint is violated. Back up slightly to resolve the constraint. Verify that the way-point position is $(-543,-48)$.
8. Move the cursor into the OOP window and use the drag mode to adjust the OOP position in the direction of the OOP pointer until the pointer has vanished. Note that the fuel cost that satisfies the mission constraints is now $0.796 \mathrm{~m} / \mathrm{sec}$.
9. Double click to call up the Transfer WP menu, select View and Reset to return to the default viewing situation, and select Return from the View Options menu to return to the Change Mode. Verify that all mission constraints have been resolved. Double click and select Main/Select from the Transfer WP menu to return to the Main Mode. Double click once more to call up the Program Options menu, and select Program Exit and Yes, really! to terminate the design.

Remark: The initial choice of the TOA of the way point WPI has dominated the subsequent trajectory design. The two arbitrary choices have evolved into two entirely different solutions. Unfortunately, the system does not yet provide clues as to how to choose the TOA
of this way point. Nor does the system provide a strategy for the sequence in which to resolve mission constraints. The interactive nature of the system does, however, allow the operator to quickly explore the various possibilities and to single out feasible and fuelefficient solutions.

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