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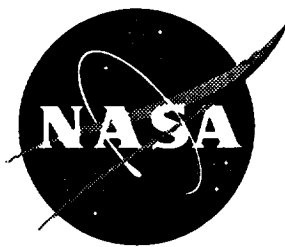
Accurate Interlaminar Stress Recovery From Finite Element Analysis

Alexander Tessler
Langley Research Center, Hampton, Virginia

H. Ronald Riggs
University of Hawaii at Monoa, Honolulu, Hawaii

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ACCURATE INTERLAMINAR STRESS RECOVERY
FROM FINITE ELEMENT ANALYSIS

A. Tessler

NASA Langley Research Center, Mail Stop 240, Hampton, VA 23681-0001

H. R. Riggs

University of Hawaii at Manoa, Honolulu, HI 96822

Introduction

The successful design of advanced composite aerospace structures requires a detail knowledge of strains and stresses that are induced by the applied loads. The inherent strength weakness between adjacent plies constitutes a weak link in a laminate which under the action of interlaminar stresses may result in the dominant interlaminar failure mode know as delamination. The delamination may then yield the ultimate laminate failure. Consequently, predicting accurate interlaminar strains and stresses is an essential facet of composites analysis. An comprehensive account of the currently available methods for the interlaminar stress recovery is presented by Byun and Kapania.[1]

The first-order shear deformation theory with the use of constitutive relations produces highly inaccurate interlaminar stresses that exhibit nonphysical discontinuity at the ply interfaces of composite laminates. This difficulty is overcome by integrating three-dimensional equilibrium equations of elasticity theory to recover the interlaminar stresses.[2] This alternative procedure, which is not entirely consistent with the assumptions of the first-order shear deformation theory, provides reasonably accurate predictions for the interlaminar stresses for closed form analytic solutions. When the procedure is applied to the results of finite element analysis, the accuracy of the interlaminar stress predictions tends to be rather poor. The main reason for this is that the integration of three-dimensional equilibrium equations involves expressions of strain gradients whose recovery from the finite element shape functions is known to be inferior.

This report discusses an accurate recovery procedure of interlaminar shear stresses from finite element solutions to laminated sandwich and composite plates. The procedure is general enough to be useful as a full-field post-processor of all finite element response quantities.

Approach

The preferred procedure for computing through-the-thickness distributions of interlaminar (transverse) stresses in laminated plates and shells is by means of integrating three-dimensional equilibrium equations of elasticity theory. The integrals contain partial derivatives of strain measures of plate/shell theory. These quantities are independent of the thickness coordinate and are commonly attributed to a reference plate/shell surface. For example, upon integration of the elasticity equilibrium equations, the interlaminar shear stress, τ_{xz} , corresponding to the first-order shear-deformation plate theory can be written as

$$\tau_{xz} = \mathbf{Q}_1 \cdot \mathbf{e}_{,x} + \mathbf{Q}_6 \cdot \mathbf{e}_{,y} \tag{1}$$

where a comma stands for partial differentiation; $\mathbf{e}$ is the vector of strain measures,

$$\mathbf{e} = \{\varepsilon_{xo}, \varepsilon_{yo}, \gamma_{xyo}, \kappa_{xo}, \kappa_{yo}, \kappa_{xyo}\} \tag{1a}$$

Q_j are the vectors whose components are functions of the thickness coordinate, z,

$$Q_j = \{q_{1j}, q_{2j}, q_{6j}, p_{1j}, p_{2j}, p_{6j}\} \qquad (1b)$$

where $j = 1,6$ and

$$q_{ij} = -\int_z C_{ij}^{(k)} dz, \quad p_{ij} = -\int_z C_{ij}^{(k)} z\, dz \qquad (1c)$$

with $C_{ij}^{(k)}$ denoting the elastic stiffness coefficients of the k-th ply.

The difficulties in obtaining reliable predictions of strain gradients from a finite element analysis are well established. For example, in first-order shear deformation elements based on Reissner-Mindlin type theories, the displacement (kinematic) fields are approximated with C^0-continuous shape functions ensuring continuity of displacements along element boundaries. The strains are represented by the first gradients of displacements; hence, they are only C^{-1} continuous, exhibiting non-physical discontinuities along element boundaries. Only at a limited number of discrete locations within the interior of a finite element reasonably accurate strain results can be calculated, these are referred to as *optimal* points. The first gradients of strain are obviously even less accurate, and hence reliable strain derivatives cannot be obtained directly from the element interpolation functions.

Tessler and co-workers[3-6] recently developed a smoothing variational formulation which combines discrete least-squares and penalty-constraint functionals in a single variational form. The strain (or stress) component to be smoothed and its Cartesian gradients enter in the variational principle as independent variables. For a sufficiently large value of the penalty number, the approach enables the resulting smooth strain field to be practically C^1-continuous throughout the domain of smoothing, exhibiting superconvergent properties of the smoothed quantity. The continuous strain gradients are also obtained

directly from the solution. The use of the recovered stress (strain) gradients has been demonstrated for assessing errors in finite element analysis by computing the residual in the strong form of the equilibrium equation. Also, the method has the versatility of being applicable to the analysis of rather general and complex structures built of distinct components and materials, such as found in aircraft design. For these types of structures, the smoothing is achieved with "patches", each patch covering the domain in which the smoothed quantity is physically continuous. The mathematical basis of the smoothing methodology is briefly summarized below.

Let $\hat{\varepsilon}_p = \hat{\varepsilon}(x_p)$ ($x_p = (x_p^1, x_p^2)$, $p=1,2,...,N$) represent a set of discrete *optimal* values for a specific strain component defined on the two-dimensional domain $\Omega = [x = (x^1, x^2) \subset \Re^2]$, with these strains resulting from a finite element analysis. We seek a smooth, continuous strain $\varepsilon(x)$ and its gradients $\varepsilon_{,i} \equiv \frac{\partial}{\partial x_i}\varepsilon$ (i=1,2) which can best represent the discrete data.

The problem can be cast as a minimization of a least-squares/penalty-constraint functional, such that

$$\delta\Phi = 0$$

where Φ is given by

$$\Phi = \frac{1}{N}\sum_{p=1}^{N}[\hat{\varepsilon}_p - \varepsilon(x_p)]^2$$
$$+\lambda\int_\Omega [(\varepsilon_{,1} - \theta_1)^2 + (\varepsilon_{,2} - \theta_2)^2]d\Omega \qquad (2)$$

where N is the total number of data points, x_p denotes the position vector of the data point, λ is a *penalty* number, and θ_1 and θ_2 are independent continuous functions also defined on Ω. The minimization of Φ is performed with respect to the coefficients in ε and θ_i which serve as the unknowns. The first term in Eq. (2) represents a discrete least-squares functional with the term

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$$\Delta_p = \hat{\varepsilon}_p - \varepsilon(\mathbf{x}_p) \tag{3}$$

signifying the *error* in $\hat{\varepsilon}_p$ as compared with the smooth solution $\varepsilon(\mathbf{x}_p)$. The second term in Eq. (2) is a penalty constraint functional which, for sufficiently large values of λ, yields the following constraint relations

$$\varepsilon_{,i} \to \theta_i \quad (i = 1,2) \quad \text{on} \quad \Omega \tag{4}$$

The proper interpretation of Eq. (4) is that for all practical purposes the θ_i $(i = 1,2)$ represent the gradients of ε. The variational principle in Eq. (2) is readily discretized with C^0-continuous element approximations, forming a mesh of "smoothing" elements. The smoothing finite element analysis then provides continuous and reliable solutions for the strain and its gradients. The latter quantities are then employed in Eq. (1) for the recovery of interlaminar shear stress distributions through the thickness.

In this paper, the accuracy and robustness of the two-dimensional smoothing discretization is examined for the problem of recovering accurate strain gradients, and subsequent integration of equilibrium equations to compute interlaminar shear stress distributions in laminated composites. The model problem is a simply-supported rectangular plate under a doubly sinusoidal transverse pressure load. The problem has an exact analytic solution which serves as a measure of goodness of the recovered interlaminar shear stresses.

The following charts describe the essential theoretical features of the approach and summarize the numerical results.

References

1. Byun, C., and Kapania, R. K., "Prediction of interlaminar stresses in laminated plates using global orthogonal interpolation polynomials," *AIAA Journal*, 30(11), (1992) 2740-2749.

2. Reddy, J. N., "A simple higher-order theory for laminated composite plates," *ASME Journal of Applied Mechanics*, 51, (1984) 745-752.

3. Tessler, A., Freese, C., Anastasi, R., Serabian, S., Oplinger, D. and Katz, A., "Least-squares penalty-constraint finite element method for generating strain fields from moire fringe patterns," *Proceedings of SPIE: Photomechanics and Speckle Metrology*, 814 (1987) 314-323.

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5. Tessler, A., Riggs, H. R., and Macy, S. C., "A variational method for finite element stress recovery and error estimation," *Computer Methods in Applied Mechanics and Engineering*, 111 (1994) 369-382.

6. Riggs, H. R., and Tessler, A., "Continuous versus wireframe variational smoothing methods for finite element stress recovery," *Proceedings of the Second International Conference on Computational Structures Technology*, Athens, Greece (1994)

Content

- **FSDT review**
- **Recovery Procedure**
  - Integration Scheme
  - Smoothing Analysis
- **Numerical results**
- **Summary**

Plate Theory (FSDT) Notation
5-Variable Approximation

$$u_x(x,y,z) = u + h\xi\,\theta_y$$
$$u_y(x,y,z) = v + h\xi\,\theta_x$$
$$u_z(x,y,z) = w$$

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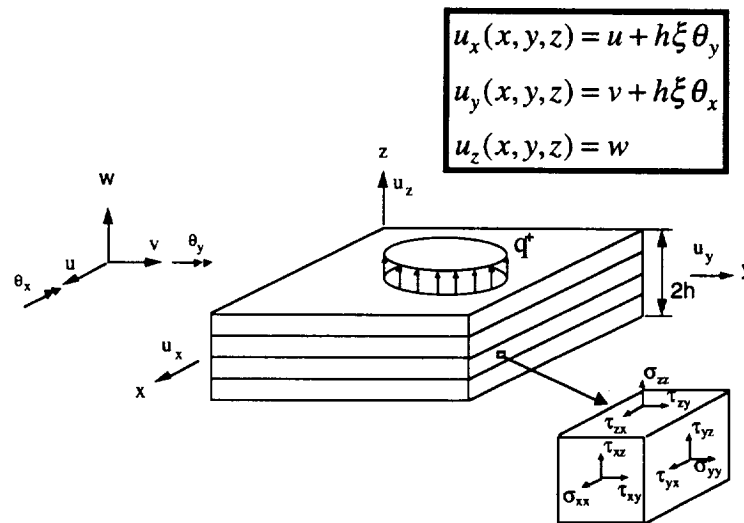
Eight Strain Measures:
First Partial Derivatives of Kinematic Variables

$$\left\{\begin{array}{c} \underline{\varepsilon}_o \\ \underline{\kappa}_o \\ \underline{\gamma}_o \end{array}\right\} = \left\{\begin{array}{c} u_{,x} \\ v_{,y} \\ u_{,y}+v_{,x} \\ \hline \theta_{y,x} \\ \theta_{x,y} \\ \theta_{x,x}+\theta_{y,y} \\ \hline w_{,x}+\theta_y \\ w_{,y}+\theta_x \end{array}\right\}$$

Inplane Strains

Bending Curvatures

Transverse Shear Strains
(uniform through thickness)

FEM: C^o-continuous interpolations u, v, w, θ_x, θ_y

Integration Scheme
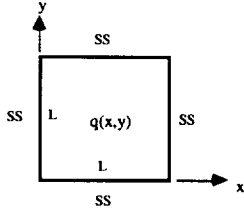
(1) $\tau_{xz} = \mathbf{Q}_1 \cdot \mathbf{e}_{,x} + \mathbf{Q}_6 \cdot \mathbf{e}_{,y}$ Shear Stress

(1a) $\mathbf{e} = \{\varepsilon_{xo}, \varepsilon_{yo}, \gamma_{xyo}, \kappa_{xo}, \kappa_{yo}, \kappa_{xyo}\}$ Strain Vector

(1b) $\mathbf{Q}_j = \{q_{1j}, q_{2j}, q_{6j}, p_{1j}, p_{2j}, p_{6j}\}$ Material-Thickness Coordinate Vector

(1c) $q_{ij} = -\int_z C_{ij}^{(k)} dz$, $p_{ij} = -\int_z C_{ij}^{(k)} z\, dz$ Q_j components

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Smoothing Analysis

$$(1) \quad \hat{\varepsilon}_p = \hat{\varepsilon}(\mathbf{x}_p)$$

| Sample optimal FEM discrete strain values at Gauss points of finite elements |

$$(2) \quad \Phi = \frac{1}{N} \sum_{p=1}^{N} [\hat{\varepsilon}_p - \varepsilon(\mathbf{x}_p)]^2$$
$$+ \lambda \int_{\Omega} [(\varepsilon_{,1} - \theta_1)^2 + (\varepsilon_{,2} - \theta_2)^2] d\Omega$$

| Apply Least-squares/ penalty-constraint finite element method to smooth FEM strains for entire domain |

$$(3) \quad \varepsilon_{,i} \rightarrow \theta_i \quad (i = 1, 2)$$

| Recover C^0-continuous, smoothed strain gradients throughout FEM domain |

Numerical Examples

SS Square Plate Under Doubly Symmetric Sinusoidal Normal Pressure

- **Loading: q sin(Πx/L) sin(Πy/L)**
- **Span/Thickness=30, 10, and 5**

Lamination
- **Gr/Ep:** $[0/90/0]_S$ **6-ply**
- **Gr/Ep:** $[0_3/90_3/0_3]_S$ **18-ply**
- **Sandwich:** $[(0/90)_S/h_c]_S$ **21-ply**

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FSDT vs. 3D Elasticity
Interlaminar Shear Stresses

Thin, 6 Ply Gr/Ep Laminate

$[0/90/0]_S, L/2h = 30$

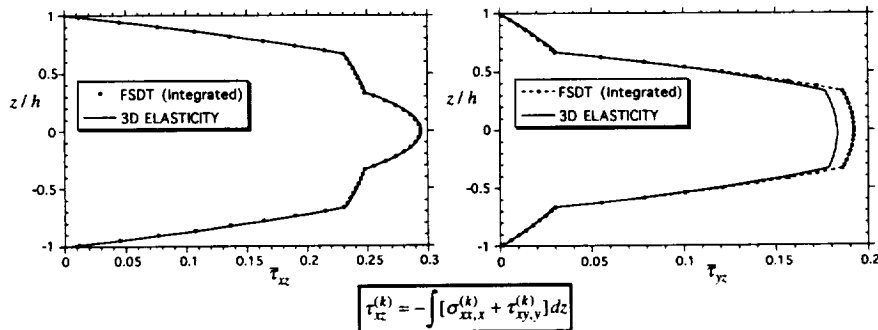$$\tau_{xz}^{(k)} = -\int [\sigma_{xx,x}^{(k)} + \tau_{xy,y}^{(k)}] dz$$

FSDT: First-order Shear Deformation Theory Solution
3D ELASTICITY: 3D Elasticity Theory Solution

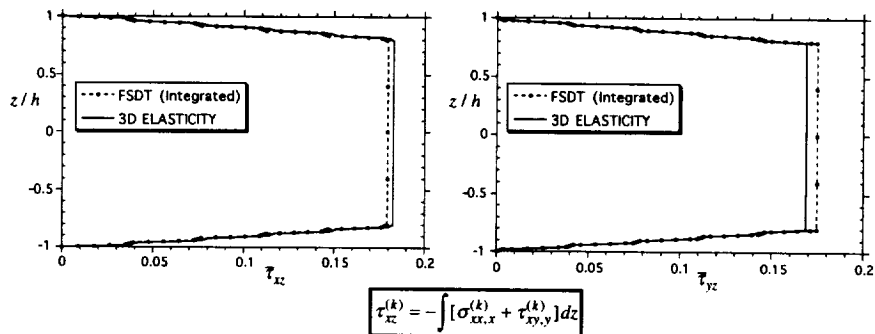# FSDT vs. 3D Elasticity
Interlaminar Shear Stresses

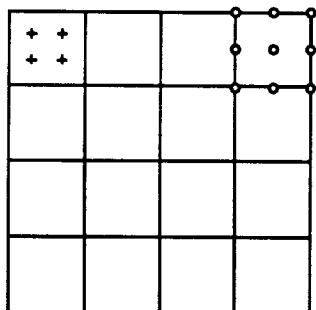Moderately Thick, 18 Ply Gr/Ep Laminate

$[0_3/90_3/0_3]_S, L/2h = 10$

$$\tau_{xz}^{(k)} = -\int [\sigma_{xx,x}^{(k)} + \tau_{xy,y}^{(k)}] dz$$

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FSDT vs. 3D Elasticity
Interlaminar Shear Stresses

Thick, 21 Ply Sandwich Laminate

$[(0/90)_5/h_c]_S$, $L/2h = 5$

$$\tau_{xz}^{(k)} = -\int [\sigma_{xx,x}^{(k)} + \tau_{xy,y}^{(k)}] dz$$

FEM and Smoothing Meshes

FINITE ELEMENT MESH
9-NODE ANS SHELL ELEMENTS

SMOOTHING ELEMENT MESH
3-NODE SMOOTHING ELEMENTS

o   u, v, w, θ_x, θ_y DOF

+   Optimal Curvatures @
    2x2 Gauss Points

● κ, θ_1, θ_2 DOF

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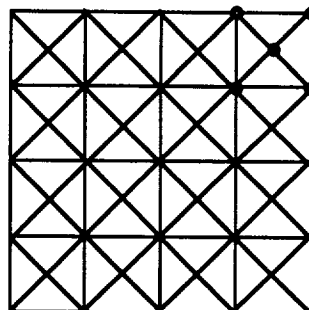
Finite Element Results

Thin, 6 Ply Gr/Ep Laminate

$[0 / 90 / 0]_S$, $L / 2h = 30$

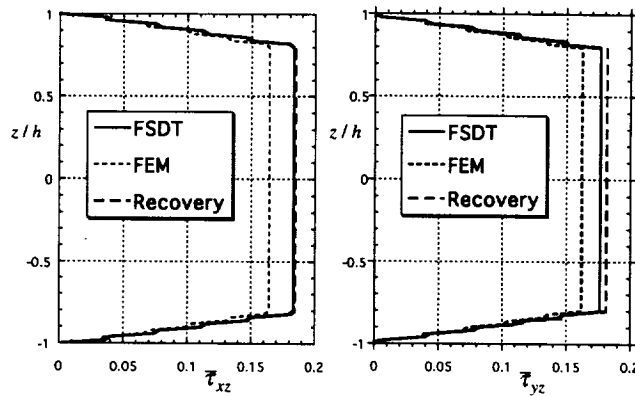| Response Quantity	% Error in Max. Value
Deflection	0.02
Rotation	0.03
Curvature	4.69
Interlaminar Shear τ_{xz}	8.80
Interlaminar Shear τ_{yz}	8.20

FEM vs. Recovered
Interlaminar Shear Stresses
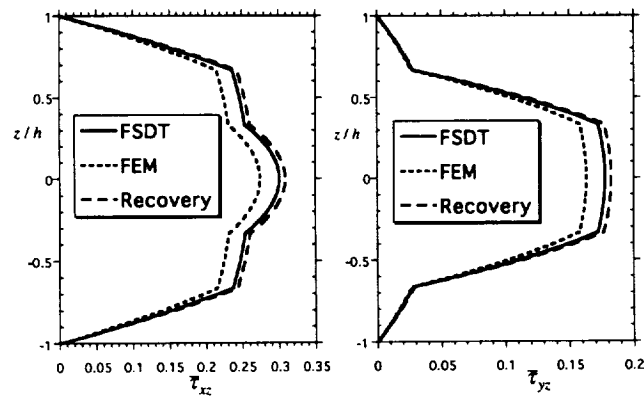
Thin, 6 Ply Gr/Ep Laminate

$[0 / 90 / 0]_S$, $L / 2h = 30$

FEM vs. Recovered Interlaminar Shear Stresses

Thick, 21 Ply Sandwich Laminate
$[(0/90)_5/h_c]_S$, $L/2h = 5$

Results Significance

- Direct evaluation of interlaminar shear stresses from FEM underestimates their value considerably (unconservative for design)
- FEM-based smoothing method recovers improved interlaminar shear stresses that are slightly overestimated (conservative for design)
- Recovered interlaminar shear stresses are improved by at least a factor of 3 [Error of 2.8%(Recovery) vs. 8.2%(FEM)]

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Summary

- Strain smoothing produces high-accuracy strains and their gradients that are C^0-continuous over full domain
- 'Smoothed' strain gradients yield superior recovery of interlaminar shear stresses over direct FEM recovery
- Effective for laminated composite and sandwich plates
- Viable full-field post-processor of all strains & stresses in general purpose FE software

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13. ABSTRACT (Maximum 200 words)

The accuracy and robustness of a two-dimensional smoothing methodology is examined for the problem of recovering accurate interlaminar shear stress distributions in laminated composite and sandwich plates. The smoothing methodology is based on a variational formulation which combines discrete least-squares and penalty-constraint functionals in a single variational form. The smoothing analysis utilizes optimal strains computed at discrete locations in a finite element analysis. These discrete strain data are smoothed with a smoothing element discretization, producing superior accuracy strains and their first gradients. The approach enables the resulting smooth strain field to be practically C^1-continuous throughout the domain of smoothing, exhibiting superconvergent properties of the smoothed quantity. The continuous strain gradients are also obtained directly from the solution. The recovered strain gradients are subsequently employed in the integration of equilibrium equations to obtain accurate interlaminar shear stresses. The model problem is a simply-supported rectangular plate under a doubly sinusoidal transverse pressure load. The problem has an exact analytic solution which serves as a measure of goodness of the recovered interlaminar shear stresses. The method has the versatility of being applicable to the analysis of rather general and complex structures built of distinct components and materials, such as found in aircraft design. For these types of structures, the smoothing is achieved with "patches", each patch covering the domain in which the smoothed quantity is physically continuous.

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