

1N-18  
25987  
P. 45

NASA Contractor Report 194987



# A Simple Analytical Aerodynamic Model of Langley Winged-Cone Aerospace Plane Concept

Bandu N. Pamadi  
*Vigyan, Inc.*  
*Hampton, Virginia*

Contract NAS1-19341  
October 1994

National Aeronautics and  
Space Administration  
Langley Research Center  
Hampton, Virginia 23681-0001

N95-12175

Unclass

G3/18 0025987

(NASA-CR-194987) A SIMPLE  
ANALYTICAL AERODYNAMIC MODEL OF  
LANGLEY WINGED-CONE AEROSPACE PLANE  
CONCEPT (Vigyan Research  
Associates) 45 p

# Contents

<b>1</b>	<b>Nomenclature</b>	<b>3</b>
<b>2</b>	<b>Introduction</b>	<b>5</b>
<b>3</b>	<b>Estimation of Lift Coefficient</b>	<b>6</b>
3.1	Subsonic and Supersonic Speeds . . . . .	6
3.1.1	Estimation of $C_{N,N}$ . . . . .	6
3.1.2	Estimation of $C_{N,e}$ . . . . .	8
3.1.3	Estimation of $K_B(W)$ and $K_W(B)$ . . . . .	10
3.1.4	Estimation of $C_{N,C}$ . . . . .	10
3.2	Hypersonic Speeds . . . . .	11
<b>4</b>	<b>Estimation of Pitching Moment Coefficient</b>	<b>12</b>
4.1	Subsonic and Supersonic Speeds . . . . .	12
4.2	Hypersonic Speeds . . . . .	15
<b>5</b>	<b>Estimation of Drag Coefficient</b>	<b>16</b>
5.1	Body Skin Friction Drag Coefficient . . . . .	16
5.2	Wing Skin Friction Drag Coefficient . . . . .	17
5.3	Vertical Tail Skin Friction Drag Coefficient . . . . .	17
5.4	Base Drag Coefficient . . . . .	18
5.5	Body Wave Drag Coefficient . . . . .	18
5.6	Wing Wave Drag Coefficient . . . . .	19
5.7	Aft Body Drag Coefficient . . . . .	20
5.8	Body Induced Drag Coefficient . . . . .	20
5.9	Wing Induced Drag Coefficient . . . . .	21
5.10	Canard Induced Drag Coefficient . . . . .	22
<b>6</b>	<b>Results and Discussion</b>	<b>22</b>
<b>7</b>	<b>Concluding Remarks</b>	<b>23</b>

### Abstract

A simple 3 DOF analytical aerodynamic model of the Langley Winged-Cone Aerospace Plane concept is presented in a form suitable for simulation, trajectory optimization, guidance and control studies. This analytical model is especially suitable for methods based on variational calculus. Analytical expressions are presented for lift, drag and pitching moment coefficients from subsonic to hypersonic Mach numbers and angles of attack up to  $\pm 20$  deg. This analytical model has break points at Mach numbers of 1.0, 1.4, 4.0 and 6.0. Across these Mach number break points, the lift, drag and pitching moment coefficients are made continuous but their derivatives are not. There are no break points in angle of attack. The effect of control surface deflection is not considered. The present analytical model compares well with the APAS calculations and wind tunnel test data for most angles of attack and Mach numbers.

# 1 Nomenclature

$A$	aspect ratio of the wing
$C_D$	drag coefficient = $\frac{Drag}{1/2\rho V^2 S_W}$
$C_{r,e}$	root chord of the exposed wing
$C_{Db}$	base drag coefficient
$C_{D,c}$	crossflow drag coefficient of the body
$C_{Df}$	skin friction drag coefficient
$C_{Di}$	induced drag coefficient
$C_L$	lift coefficient = $\frac{Lift}{1/2\rho V^2 S_W}$
$C_{L\alpha}$	= $\frac{dC_L}{d\alpha}$
$C_m$	pitching moment coefficient = $\frac{Pitching\ moment}{1/2\rho V^2 S_W L_{ref}}$
$C_N$	normal force coefficient = $\frac{Normal\ Force}{1/2\rho V^2 S_W}$
$C_{N\alpha}$	= $\frac{dC_N}{d\alpha}$
$C_{N,\alpha\alpha}$	second derivative of normal force coefficient with respect to $\alpha$
$D$	aerodynamic drag
$k_1, k_2$	apparent mass coefficients
$K_W(B)$	interference factor of wing on body
$K_B(W)$	interference factor of body on wing
$K_f$	sonic leading edge correction factor
$L$	aerodynamic lift
$L_{ref}$	reference length, equal to mean aerodynamic chord
$L_N$	length of the conical nose
$M$	Mach number
$r$	local radius of the body
$R$	maximum radius of the body
$S_B$	maximum cross sectional area of the body
$S_W$	reference wing area (planform area)
$S_{eff}$	flat plate area of the wing
$V_B$	volume of the body
$x$	axial distance measured from the leading edge of the body
$x'_{ac}$	distance of aerodynamic center from the apex of exposed wing
$\alpha$	angle of attack
$\beta$	$\sqrt{M^2 - 1}$
$\gamma$	ratio of specific heats
$\Lambda_{le}$	leading edge sweep angle of the wing
$\Lambda_{c/2}$	mid chord sweep angle of the wing

$\rho$  density of air

**Acronym**

APAS Aerodynamic Preliminary Analysis System

## 2 Introduction

The National Aero-Space Plane (NASP) program envisages the development of a manned, single-stage-to-orbit vehicle capable of horizontal take-off and landing using conventional runways. Towards this objective, NASA Langley has proposed a simple winged-cone configuration (Fig.1) as a baseline model for simulation, trajectory optimization, guidance and control research. Reference [1] presents a comprehensive tabular aerodynamic model using APAS [2]. However, for optimal control studies using variational calculus the tabular data is not very convenient. It is desirable to have an analytical aerodynamic model.

The purpose of this report is to present a simple, analytical model of Langley winged-cone aerospace plane concept suitable for 3 DOF simulation and trajectory optimization studies especially based on variational methods. Analytical expressions are presented for the estimation of lift, drag and pitching moment coefficients for subsonic, transonic, supersonic and hypersonic Mach numbers and angles of attack from 0 to  $\pm 20$  deg. The analytical model presented here is based on DATCOM [3] methods and is supplemented by the theoretical methods wherever they are available. However, a formal difficulty arises because different estimation/analytical methods have to be used at subsonic, supersonic and hypersonic speeds and this process introduces several break point Mach numbers. Across these break point Mach numbers, the aerodynamic coefficients become discontinuous. Using engineering approximations, the estimated aerodynamic coefficients have been made continuous across these break point Mach numbers. However, the derivatives of the aerodynamic coefficients with Mach number remain discontinuous across these break point Mach numbers. The analytical model presented in this report has four break points at  $M = 1.0, 1.4, 4.0$  and  $6.0$ . There are no break points in the angle of attack. The effects of sideslip, roll angle and control surface deflection are not considered in this study. The Langley winged-cone configuration has a forward mounted canard which is supposed to be operative only at subsonic speeds and retracted at speeds exceeding Mach one.

The present analytical model is shown to be in fair agreement with the available wind tunnel test data [6] and APAS calculations [1]. This analytical model may be used for 3 DOF simulations along with mass, inertia and propulsion models presented in [1].

### 3 Estimation of Lift Coefficient

In the following, we discuss the determination of normal force coefficient. Note that  $C_L = C_N \cos \alpha$ .

#### 3.1 Subsonic and Supersonic Speeds

The normal force coefficient of the vehicle is given by,

$$C_N = C_{N,WB} + C_{N,C} \quad (1)$$

where,  $C_{N,WB}$  and  $C_{N,C}$  are the normal force coefficients of the wing-body combination and canard respectively. The DATCOM [3] gives the following expression for the normal force coefficient of the wing body combination at subsonic and supersonic speeds:

$$C_{N,WB} = C_{N,N} + [K_W(B) + K_B(W)] C_{N,e} \frac{S_e}{S_W} \quad (2)$$

where,  $C_{N,N}$  is the normal force coefficient of the nose (body),  $K_W(B)$  and  $K_B(W)$  are the interference factors of wing on body and body on wing respectively,  $C_{N,e}$  is the normal force coefficient of the exposed wing,  $S_e$  is the exposed wing area and  $S_W$  is the wing plan form area. For the given wing-body configuration, the exposed wing is defined as the parts of the wing outboard of the largest body diameter at the wing body intersection [3] (Figure 2). From Table I, we have  $S_W = 3600 \text{ ft}^2$  and the exposed wing area  $S_e$  was estimated as equal to  $1176.8310 \text{ ft}^2$ .

In the following, we discuss the determination of  $C_{N,N}$ ,  $C_{N,e}$ ,  $K_W(B)$ ,  $K_B(W)$  and  $C_{N,C}$  at subsonic and supersonic speeds and match these parameters at the break point Mach numbers.

##### 3.1.1 Estimation of $C_{N,N}$

At subsonic speeds ( $M \leq 1.0$ ), from DATCOM [3]

$$C_{N,N} = 2(k_2 - k_1)\alpha \frac{S_B}{S_W} \quad (3)$$

where the apparent mass coefficient,  $k_2 - k_1$ , is equal to 0.92 for the subject configuration,  $S_B$  is the maximum cross sectional area of the body and  $\alpha$  is the angle of attack in radian. In this report, unless otherwise stated,  $\alpha$  is in radian. For the given configuration, we have  $S_B = 520.573 \text{ ft}^2$ . The

equation ( 3) is based on linear slender body theory and hence, we do not have nonlinear terms in the expression of  $C_{N,N}$  at subsonic speeds.

At low supersonic speeds ( $1.40 \leq M \leq 4.0$ ), the DATCOM [3] gives:

$$C_{N,N} = 2(k_2 - k_1)\alpha \frac{S_B}{S_W} + \frac{C_{d,c}S_p\alpha^2}{S_W} \quad (4)$$

where, the first term on the right hand side is the same linear slender body theory term of equation ( 3),  $C_{d,c}$  is the cross flow drag coefficient and  $S_p$  is the planform area of the body which was estimated as equal to 1893.1770ft<sup>2</sup> for the given configuration. It may be noted that the slender body result of equation ( 3) is supposed to be independent of Mach number. The nonlinear term on the right hand side of the equation ( 4) is due to the cross flow drag of the body. In general, the cross flow drag coefficient  $C_{d,c}$  depends on Reynolds number, Mach number and angle of attack. However, for simplicity we ignore the dependence of  $C_{d,c}$  on angle of attack and instead use a mean angle of attack of 10 deg to evaluate  $C_{d,c}$  at some selected Mach numbers in the range of 1.4 to 4.0. This data was then represented in the following analytical form:

$$C_{d,c} = c_1M^3 + c_2M^2 + c_3M + c_4 \quad (5)$$

where  $C_1 = 0.16018212e - 01$ ,  $C_2 = -0.21232216$ ,  $C_3 = 0.83332039$  and  $C_4 = 0.48085365$ .

At transonic ( $1.0 \leq M \leq 1.40$ ) and high supersonic Mach numbers ( $4.0 \leq M \leq 6.0$ ), we use interpolation to obtain  $C_{N,N}$ . The interpolated transonic normal force coefficient is matched with subsonic value at  $M = 1.0$  and low supersonic value at  $M = 1.4$ . Similarly, the interpolated high supersonic normal force coefficient is matched with the low supersonic data at  $M = 4.0$  and hypersonic data at  $M = 6.0$ . The interpolated expressions are:

for  $1.0 \leq M \leq 1.40$ ,

$$C_{N,N} = \frac{1.84\alpha S_B}{S_W} + 1.6766511(M - 1)\alpha^2 \quad (6)$$

for  $4.0 \leq M \leq 6.0$ ,

$$C_{N,N} = a_1\alpha^3 + a_2\alpha^2 + a_3\alpha + a_4 \quad (7)$$

where,

$$a_1 = 0.24197221M - 0.9678888 \quad (8)$$



$$a_2 = -0.32024002M + 2.0393596e - 01 \quad (9)$$

$$a_3 = -0.55238185e - 01M + 0.48702338 \quad (10)$$

$$a_4 = 1.5351068e - 04M - 6.1404271e - 04 \quad (11)$$

### 3.1.2 Estimation of $C_{N,e}$

For the exposed wing, at subsonic and supersonic speeds, DATCOM [3] gives the following relation:

$$C_{N,e} = \frac{C_{N\alpha,e} \sin 2\alpha}{2} + C_{N,aa} \sin \alpha | \sin \alpha | \quad (12)$$

where,  $C_{N\alpha,e}$  is the slope of normal force coefficient of the exposed wing with respect to  $\alpha$  and  $C_{N,aa}$  is the second derivative of the normal force coefficient with respect to  $\alpha$ . We assume  $C_{N\alpha,e} = C_{L\alpha,e}$ .

For subsonic ( $0 \leq M \leq 1.0$ ) Mach numbers,

$$C_{L\alpha,e} = \frac{2\pi A}{2 + \sqrt{\frac{A^2 \beta^2}{k^2} \left(1 + \frac{\tan^2 \Lambda_{c/2}}{\beta^2}\right)} + 4} \quad (13)$$

where,  $A$  is the wing aspect ratio equal to  $4 \tan \Lambda$  for a delta wing. Here,  $\Lambda = 90 - \Lambda_{le}$  and  $\beta = \sqrt{M^2 - 1}$ . Approximating the subject wing as a flat plate delta wing, we obtain  $k = 1$ . The parameter  $\Lambda_{c/2}$  is the sweep angle of the mid chord and is equal to  $\frac{1}{2 \tan \Lambda}$ . Assuming  $\Lambda$  to be small so that  $\tan \Lambda = \Lambda$ , which is justified for a highly swept delta wing like the one considered here, the equation (13) can be written as:

$$C_{L\alpha,e} = \frac{4\pi \Lambda}{1 + 2\sqrt{\Lambda^2(1 - M^2)} + 0.5} \quad (14)$$

From Table I, we have  $\Lambda_{le} = 76.0$  deg. so that approximately,  $\Lambda = 0.25$  radian. Using  $\pi = 3.1428$ , we obtain:

For  $0 \leq M \leq 1.0$ ,

$$C_{L\alpha,e} = \frac{3.1428}{1 + 0.5\sqrt{9 - M^2}} \quad (15)$$

At transonic speeds ( $1.0 \leq M \leq 1.40$ ), we use interpolation so that the interpolated coefficient matches with subsonic value of equation (15) at

$M = 1.0$  and supersonic value at  $M = 1.4$  given by equation (17). This interpolation formula valid for  $1.0 \leq M \leq 1.40$  is given in the following:

$$C_{N\alpha,e} = 0.35699257M + 0.94479781 \quad (16)$$

At low supersonic speeds ( $1.4 \leq M \leq 4.0$ ), using DATCOM [3] procedure, we obtain the linear term in normal force coefficient as follows:

$$C_{N\alpha,e} = K_f(1.56250 - 0.140620\sqrt{M^2 - 1}) \quad (17)$$

and for high supersonic speeds,  $4.0 \leq M \leq 6.0$ ,

$$C_{N\alpha,e} = \frac{4.0K_f}{\sqrt{M^2 - 1}} \quad (18)$$

In the above equations,  $K_f$  is the sonic leading edge correction factor. Using DATCOM [3] data and enforcing the matching of  $C_{N\alpha,e}$  at  $M = 4.0$  and  $6.0$  and curve fitting the numerical data, we obtain the following expressions for  $K_f$ :  
for  $1.40 \leq M \leq 4.0$ ,

$$K_f = -0.0086M^2 - 0.0385M + 1.08470 \quad (19)$$

for  $4.0 \leq M \leq 6.0$ ,

$$K_f = 0.25245975e - 01M^2 - 0.15027659M + 0.97881773 \quad (20)$$

Next we consider the evaluation of nonlinear term  $C_{N,aa}$ . Using DATCOM [3] procedure, for subsonic speeds ( $0 \leq M \leq 1.0$ ) we obtain,

$$C_{N,aa} = 4.7783 - 1.4504C_{N\alpha,e} \quad (21)$$

An approximation has to be introduced to evaluate  $C_{N,aa}$  at supersonic speeds because the given DATCOM [3] data do not cover the present configuration for all combinations of Mach numbers and angle of attack of interest. The parameter  $C_{N,aa}$  is almost constant with  $\alpha$  at low supersonic Mach numbers ( $1.0 \leq M \leq 2.0$ ) but starts varying with  $\alpha$  as Mach number increases above 2.0. At hypersonic speeds (as discussed later), this nonlinear term is proportional to  $\sin\alpha$ . Therefore, to retain this functional relationship, we will represent the available DATCOM [3] data as equations (22) and (23) with proper matching at  $M = 1.0, 4.0$  and  $6.0$ . (Note that the nonlinear term ( $C_{N,aa}$ ) doesn't have a breakpoint at  $M = 1.4$ ).

For  $1.0 \leq M \leq 4.0$ ,

$$C_{N,aa} = -0.8467277M + 3.736911 + 0.33333333(M - 1) \sin \alpha \quad (22)$$

For  $4.0 \leq M \leq 6.0$ ,

$$C_{N,aa} = -0.17500M + 1.05 + (1.90M - 6.6) \sin \alpha \quad (23)$$

### 3.1.3 Estimation of $K_B(W)$ and $K_W(B)$

From DATCOM [3], we find that for subsonic and transonic Mach numbers ( $0 \leq M \leq 1.4$ ),  $K_W(B) = 1.35$  and  $K_B(W) = 0.62$ . For supersonic speeds ( $1.4 \leq M \leq 6.0$ ), using DATCOM [3] data and matching of the normal force coefficient of wing-body combination, we obtain the following expressions:

$$K_W(B) = -0.83998215e - 01M + 1.46759750e00 \quad (24)$$

$$K_B(W) = a_1M^4 + a_2M^3 + a_3M^2 + a_4M + a_5 \quad (25)$$

where,  $a_1 = 0.13778753e - 02$ ,  $a_2 = -0.31571756e - 01$ ,  $a_3 = 0.26725216e00$ ,  $a_4 = -9.9452836e - 01$ , and  $a_5 = 1.5698651e00$ .

### 3.1.4 Estimation of $C_{N,C}$

The canard is proposed to be deployed only at subsonic speeds to augment the static longitudinal stability and is supposed to be withdrawn immediately for  $M \geq 1.0$  [1], i.e.,  $C_{N,C} = 0$  for  $M \geq 1.0$ . Using DATCOM [3] data, for subsonic speeds ( $0 \leq M \leq 1.0$ ),

$$C_{N,C} = 0.08\alpha \frac{S_C}{S_W} \quad (26)$$

where, angle of attack  $\alpha$  is in deg and  $S_C$  is the canard area. From Table I, we have  $S_C = 154.3 \text{ ft}^2$ . Further, to maintain the continuity of  $C_{N,C}$  at  $M = 1.0$ , we gradually reduce the contribution of the canard to zero at  $M = 1.4$  as follows, For  $1.0 \leq M \leq 1.4$ ,

$$C_{N,C} = (-0.20M + 0.28)\alpha \frac{S_C}{S_W} \quad (27)$$

### 3.2 Hypersonic Speeds

Here,  $C_{N,C} = 0$  so that  $C_N = C_{N,WB}$ . Further, at hypersonic speeds, the mutual interference between the wing and body can be ignored and the normal force coefficient of the wing-body configuration can be assumed as equal to the sum of the contributions from isolated wing and body components, i.e.,

$$C_{N,WB} = C_{N,B} + C_{N,W} \quad (28)$$

According to DATCOM [3], the normal force coefficient of the body based on base area is given by,

$$C_{N,B} = \frac{kl_B}{\pi R} \int_0^1 K_\theta(\alpha) \frac{r}{R} d\left(\frac{x}{l_B}\right) \quad (29)$$

where,  $k$  is a constant and equal to 2 for pointed bodies,  $l_B$  is the total body length,  $r$  is the local body radius,  $R$  is the maximum radius of the body,  $x$  is the local distance measured from the leading edge of the body and  $K_\theta(\alpha)$  is a function to be determined using DATCOM [3] data.

To evaluate  $K_\theta(\alpha)$ , the integral in equation (29) was divided into three parts, conical nose, cylindrical center body and aft boattail. Using DATCOM [3] data, the parameter  $K_\theta$  as a function of  $\alpha$  was evaluated for each part and least square curve fitted to obtain analytical expressions. Then these contributions were summed up to obtain the total  $K_\theta(\alpha)$  for the body. The result of equation (29) was multiplied by  $\frac{S_B}{S_W}$  to obtain the following expression for body normal force coefficient with respect to the reference wing area:

$$C_{N,B} = a_1\alpha^3 + a_2\alpha^2 + a_3\alpha + a_4 \quad (30)$$

where,  $a_1 = 4.8394443e - 01$ ,  $a_2 = 1.1791944e - 01$ ,  $a_3 = 1.5559427e - 01$ , and  $a_4 = 3.0702135e - 04$ .

Reference [4] gives the following expression for the normal force of the wing based on flat plate approximation and applicable for small angles of attack:

$$C_{L,W} = 2\alpha^2 \left[ \frac{2}{M\alpha} + \frac{(\gamma + 1)M\alpha}{6} \right] \quad (31)$$

Where  $\gamma$  is the ratio of specific heats and for air,  $\gamma = 1.4$ . We use equation (31) to approximate the normal force coefficient of the wing as follows:

$$C_{N,W} = \left[ \frac{4 \sin \alpha \cos \alpha}{M} + 0.8M \sin^3 \alpha \right] \frac{S_{eff}}{S_W} \quad (32)$$

Here,  $S_{eff}$  is the actual flat plate area of the wing and was estimated equal to 1357.6019 ft<sup>2</sup>. We have  $S_W = 3600$  ft<sup>2</sup>. Then,

$$C_{N,WB} = C_{N,B} + C_{N,W} \quad (33)$$

## 4 Estimation of Pitching Moment Coefficient

The estimation of pitching moment coefficient at subsonic and supersonic speeds is based on the linear aerodynamic theory [3] and the concept of aerodynamic center  $\left(\frac{x'_{ac}}{c_{re}}\right)$ . Once the aerodynamic center is determined, the pitching moment with respect to any desired moment reference point can be obtained. The moment reference center used in this report is shown in Fig 1. At hypersonic speeds, the method is more general and considers nonlinear variation of pitching moment coefficient with angle of attack.

### 4.1 Subsonic and Supersonic Speeds

In this section, we discuss the determination of  $C_m$  at subsonic ( $M \leq 1.0$ ) and low supersonic ( $1.4 \leq M \leq 4.0$ ) speeds using DATCOM [3]. The expressions for pitching moment coefficient at transonic ( $1.0 \leq M \leq 1.4$ ) and low supersonic ( $4.0 \leq M \leq 6.0$ ) speeds are obtained using interpolation. According to DATCOM [3], the location of the aerodynamic center of the wing body configuration is given by,

$$\left(\frac{x'_{ac}}{c_{re}}\right)_{WB} = \frac{\left(\frac{x'_{ac}}{c_{re}}\right)_N C_{L\alpha,N} + \left(\frac{x'_{ac}}{c_{re}}\right)_{W(B)} C_{L\alpha,W(B)} + \left(\frac{x'_{ac}}{c_{re}}\right)_{B(W)} C_{L\alpha,B(W)}}{C_{L\alpha,WB}} \quad (34)$$

where,

$$C_{L\alpha,WB} = C_{L\alpha,N} + C_{L\alpha,W(B)} + C_{L\alpha,B(W)} \quad (35)$$

Here,  $\left(\frac{x'_{ac}}{c_{re}}\right)$  terms are the chordwise distances measured in exposed wing root chords from the apex of the exposed wing to the aerodynamic center and positive aft,  $C_{L\alpha,N} = C_{N\alpha,N}$ ,  $C_{L\alpha,W(B)} = K_W(B)C_{L\alpha,e}$  and  $C_{L\alpha,B(W)} = K_W(B)C_{L\alpha,e}$ . The determination of each of the  $\left(\frac{x'_{ac}}{c_{re}}\right)$  is discussed in the following.

From subsonic to low supersonic speeds ( $0 \leq M \leq 4.0$ ), the aerodynamic center location of the fore body as a function of the root chord of the exposed wing and referred to the exposed wing apex as the origin is given by,

$$\left(\frac{x'_{ac}}{c_{re}}\right)_N = C_{m\alpha,B} \left[ \frac{-V_B^{1/3}}{c_{re} C_{L\alpha,B}} \right] \quad (36)$$

where,

$$C_{m\alpha,B} = \frac{2(k_2 - k_1)}{V_B} \int_0^{L_N} \frac{dS_x}{dx} (x_m - x) dx \quad (37)$$

Here,  $V_B$  is the volume of the forebody,  $C_{L\alpha,B} = C_{N\alpha,B} = C_{N\alpha,N}$ ,  $L_N$  is the length of the conical nose,  $x_m$  is the distance of the moment reference point from the leading edge and  $S_x$  is the local cross sectional area. As stated in DATCOM, we use  $x_m = L_N$ . Evaluating the integral of the equation (37), we obtain,

for  $0 \leq M \leq 4.0$ ,

$$\left(\frac{x'_{ac}}{c_{re}}\right)_N = -\frac{L_N}{3c_{re}} \quad (38)$$

For the given configuration,  $L_N = 147.10$  ft and the root chord of the exposed wing ( $c_{re}$ ) was estimated as equal to 68.70 ft. The negative sign indicates that the aerodynamic center of the forebody lies ahead of the apex of the exposed wing root chord.

Using DATCOM [3], the aerodynamic center of the exposed wing in the presence of body and as a fraction of the exposed wing root chord was estimated as,

for  $0 \leq M \leq 1.0$ ,

$$\left(\frac{x'_{ac}}{c_{re}}\right)_{W(B)} = -0.02915\beta + 0.65 \quad (39)$$

where  $\beta = \sqrt{M^2 - 1}$ .

For  $1.4 \leq M \leq 4.0$ ,

$$\left(\frac{x'_{ac}}{c_{re}}\right)_{W(B)} = 0.6650 \quad (40)$$

The calculation of the aerodynamic center of the wing-lift carry over on the body  $\left(\frac{x'_{ac}}{c_{re}}\right)_{W(B)}$  involves several intermediate steps. Following DATCOM [3]

we obtain,  
for  $0 \leq M \leq 1.0$ ,

$$\left(\frac{x'_{ac}}{c_{re}}\right)_{B(W)} = 0.50 - 0.023876\beta \quad (41)$$

and for  $1.4 \leq M \leq 4.0$ ,

$$\left(\frac{x'_{ac}}{c_{re}}\right)_{B(W)} = a_1 M^2 + a_2 M + a_3 \quad (42)$$

where,  $a_1 = -1.4300077e-01$ ,  $a_2 = 7.6328853e-01$  and  $a_3 = -9.9911864e-02$ .

The pitching moment coefficient of the wing-body combination with reference to the moment reference point (Fig. 1) is given by,

$$C_{mWB} = -\frac{C_{L\alpha, WB}\alpha(x'_{ac} - x_{ref})}{L_{ref}} \quad (43)$$

where,  $x_{ref}$  is the distance of the moment reference point from the leading edge of the body and  $L_{ref}$  is the reference length assumed equal to mean aerodynamic chord length. From Table I, we have,  $x_{ref} = 124.0$  ft and  $L_{ref} = 80.0$  ft.

The contribution of the canard to the pitching moment coefficient at subsonic speeds ( $0 \leq M \leq 1.0$ ) is given by,

$$C_{m,C} = C_{N,C} \left(\frac{x'_{ac}}{L_{ref}}\right)_C \quad (44)$$

For  $M \geq 1.4$ ,  $C_{m,C} = 0$ .

Then the total pitching moment coefficient of the vehicle at subsonic ( $0 \leq M \leq 1.0$ ) and low supersonic ( $1.4 \leq M \leq 4.0$ ) speeds is given by,

$$C_m = C_{m,WB} + C_{m,C} \quad (45)$$

At transonic ( $1.0 \leq M \leq 1.40$ ) and high supersonic ( $4.0 \leq M \leq 6.0$ ) speeds, we use interpolation. The transonic pitching moment coefficient matches with subsonic value at  $M = 1.0$  and supersonic value at  $M = 1.4$ . Similarly, the high supersonic ( $4.0 \leq M \leq 6.0$ ) pitching moment coefficient matches with low supersonic value at  $M = 4.0$  and hypersonic value at  $M = 6.0$ .

For  $1.0 \leq M \leq 1.40$ ,

$$C_m = (a_1 M + a_2) \alpha \quad (46)$$

where,  $a_1 = -7.12892780e - 01$  and  $a_2 = 7.391297636e - 01$ .

For  $4.0 \leq M \leq 6.0$ ,

$$C_m = a_1 \alpha^3 + a_2 \alpha^2 + a_3 \alpha + a_4 \quad (47)$$

where,

$$a_1 = -3.8724006205d - 01M + 1.54896024820d0 \quad (48)$$

$$a_2 = 2.2244718000d - 02M - 8.89788720000d - 02 \quad (49)$$

$$a_3 = -3.3111749650d - 02M + 1.54278592200d - 01 \quad (50)$$

$$a_4 = -1.5450500000d - 06M + 6.18020000000d - 06 \quad (51)$$

## 4.2 Hypersonic Speeds

As said earlier, at hypersonic speeds ( $M \geq 6.0$ ), the mutual interference between the wing and body can be ignored and the pitching moment coefficient of the wing-body configuration can be assumed as equal to the sum of the contributions from isolated wing and body components [4]. We divide the body into three parts, conical nose, cylindrical center body and aft boat tail. Then,

$$C_{m,B} = \frac{C_{N,N}(x_{ref} - x_{cp,N})}{L_{ref}} + \frac{C_{N,cyl}(x_{ref} - x_{cp,cyl})}{L_{ref}} + \frac{C_{N,aft}(x_{ref} - x_{cp,aft})}{L_{ref}} \quad (52)$$

where,  $C_{N,N}$ ,  $C_{N,cyl}$ , and  $C_{N,aft}$  are the normal force coefficients of the forebody, cylindrical center body and the aft boat tail respectively. These coefficients were evaluated using the equation (29). Further,  $x_{cp,N} = 2L_N/3$ ,  $x_{cp,cyl} = L_N + L_{cyl}/2$  and  $x_{cp,aft} = L_N + L_{cyl} + L_{aft}/2$ . For the given configuration,  $L_N = 147.10$  ft,  $L_{cyl} = 12.88$ ft and  $L_{aft} = 40.0$ ft. As before,  $x_{ref} = 124.0$  ft. Evaluating equation (52), we obtain the following expression:

$$C_{m,B} = a_1 \alpha^3 + a_2 \alpha^2 + a_3 \alpha + a_4 \quad (53)$$



where,  $a_1 = -3.9039436e - 02$ ,  $a_2 = 4.4489436e - 02$ ,  $a_3 = 5.7752634e - 02$  and  $a_4 = -3.0901224e - 06$ .

The pitching moment coefficient of the wing is given by,

$$C_{m,W} = \frac{C_{N,W}(x_{ref} - x_{cp,W})}{L_{ref}} \quad (54)$$

where,  $C_{N,W}$  is given by equation (32) and  $x_{cp,W}$  was estimated as equal to 156.50285. Then,

$$C_m = C_{m,B} + C_{m,W} \quad (55)$$

## 5 Estimation of Drag Coefficient

The total drag coefficient of the vehicle is given by,

$$C_D = C_{Df,B} + C_{Df,W} + C_{Df,V} + C_{Db} + C_{Dw,B} + C_{Dw,W} + C_{Di,B} + C_{Di,W} + C_{Di,C} \quad (56)$$

where,  $C_{Df,B}$  is the skin friction drag coefficient of the body,  $C_{Df,W}$  is the skin friction drag coefficient of the wing,  $C_{Df,V}$  is the skin friction drag coefficient of the vertical tail,  $C_{Db}$  is the base drag coefficient,  $C_{Dw,B}$  is the wave drag coefficient of the body,  $C_{Dw,W}$  is the wave drag coefficient of the wing,  $C_{Di,B}$  is the induced drag coefficient of the body,  $C_{Di,W}$  is the induced drag coefficient of the wing and  $C_{Di,C}$  is the induced drag coefficient of the canard. We ignore the skin friction of the canard. For drag estimation purposes, we ignore the mutual interference effects. In the following, we discuss the evaluation of each of these components of the drag coefficients at subsonic, supersonic and hypersonic Mach numbers and match these components of drag coefficients at the break point Mach numbers.

### 5.1 Body Skin Friction Drag Coefficient

For subsonic speeds ( $0 \leq M \leq 1.0$ ) DATCOM [3] gives,

$$C_{Df,B} = C_{f,B} \left[ 1 + \frac{60}{(l_B/d)^3} + 0.0025 \frac{l_B}{d} \right] \frac{S_s}{S_W} \quad (57)$$

where  $C_{f,B}$  is the skin friction coefficient of the body,  $\frac{l_B}{d}$  is the fineness ratio and  $S_s$  is the wetted area of the body. For the given configuration,  $l_B = 200.0\text{ft}$ ,  $d = 74\text{ft}$  and the wetted area ( $S_s$ ) was estimated as equal to

9426.7751 ft<sup>2</sup>.  $C_{fB}$  depends on body Reynolds number and Mach number. For simplicity, we use a mean Reynolds number (based on length) of  $0.8 \times 10^9$  and further assume that the flow is fully turbulent over the entire body surface. Using DATCOM [3], we obtain  $C_{fB} = 0.0015$ . With this, the equation ( 57) reduces to:

$$C_{Df,B} = 0.0044922 \quad (58)$$

In a similar fashion,  $C_{Df,B}$  was evaluated at supersonic and hypersonic speeds. It was observed  $C_{Df,B}$  at these speeds did not differ much from it's subsonic value. Therefore, for simplicity, we use equation ( 58) for supersonic and hypersonic Mach numbers.

## 5.2 Wing Skin Friction Drag Coefficient

Using DATCOM [3],

$$C_{Df,W} = C_{f,W} \left[ 1 + L \frac{t}{c} + 100 \left( \frac{t}{c} \right)^4 \right] R_{LS} \frac{S_{wet}}{S_W} \quad (59)$$

Where,  $C_{f,W}$  is the skin friction coefficient of the wing,  $\frac{t}{c}$  is the thickness ratio,  $S_{wet}$  is the wetted area and  $R_{LS}$  is a parameter to be obtained using DATCOM [3]. From Table I,  $\frac{t}{c} = 0.04$ . We assume  $\frac{S_{wet}}{S_W} = 1$ . From DATCOM [3], we get  $L = 1.2$  for the given wing for which maximum thickness is located at 50 percent chord. For a mean wing Reynolds number of  $0.4 \times 10^9$  based on root chord, we obtain  $C_{f,W} = 0.0022$ . Then, the equation ( 59) reduces to

$$C_{Df,W} = 0.0015 \quad (60)$$

As mentioned above, we also use equation ( 60) for supersonic and hypersonic Mach numbers.

## 5.3 Vertical Tail Skin Friction Drag Coefficient

$$C_{Df,V} = C_{f,V} \frac{S_V}{S_W} \quad (61)$$

Assuming  $C_{f,V} = 0.0022$ , we obtain

$$C_{Df,V} = 0.00039 \quad (62)$$

As in the case of the body and wing, we also use equation ( 62) for all supersonic and hypersonic Mach numbers.

#### 5.4 Base Drag Coefficient

From [5], for  $0 \leq M \leq 1.0$ ,

$$C_{Db} = \left[ 0.139 + 0.419(M - 0.161)^2 \right] \frac{S_{base}}{S_W} \quad (63)$$

For the given configuration,  $S_{base} = 134.150 \text{ ft}^2$ . For supersonic and hypersonic speeds ( $M \geq 1.4$ ),

$$C_{Db} = \left[ \frac{1}{M^2} - \frac{0.570}{M^4} \right] \left( \frac{S_{base}}{S_W} \right) \quad (64)$$

For transonic Mach numbers ( $1.0 \leq M \leq 1.4$ ), we use interpolation as follows,

$$C_{Db} = c_1 M + c_2 \quad (65)$$

where,  $c_1 = -1.80286235e - 01$  and  $c_2 = 6.14229134e - 01$ .

#### 5.5 Body Wave Drag Coefficient

For subsonic speeds,  $0 \leq M \leq 1.0$ , the body wave drag can be ignored. However, to avoid the discontinuity in body wave drag coefficient at  $M = 1.0$ , we assume,

$$C_{Dw,B} = C_{Dw,B1} M^6 \quad (66)$$

where,  $C_{Dw,B1}$  is value of body wave drag coefficient at  $M = 1.0$  given by the equation (67). At supersonic speeds, using DATCOM [3] the wave drag coefficient of the body was evaluated for  $1.0 \leq M \leq 4.0$  and the data was least square curve fitted to obtain the following expression:

$$C_{Dw,B} = \theta^2 \left( 8.0101 - 2.431M + 0.2443M^2 \right) \frac{S_B}{S_W} \quad (67)$$

From Table I,  $\theta = 5.0 \text{ deg}$  or  $\theta = 0.08726 \text{ rad}$ . As before,  $S_B = 520.573 \text{ ft}^2$  and  $S_W = 3600.0 \text{ ft}^2$ . For hypersonic speeds ( $M \geq 6.0$ ),

$$C_{Dw,B} = 2 \sin^2 \theta \left( \frac{S_B}{S_W} \right) \quad (68)$$

For  $4.0 \leq M \leq 6.0$ , we use interpolation as follows,

$$C_{Dw,B} = (c_1 M + c_2) \frac{S_B}{S_W} \quad (69)$$

where,  $c_1 = -7.61321184e - 04$  and  $c_2 = 1.97579419e - 02$  and

## 5.6 Wing Wave Drag Coefficient

For  $M \leq 0.60$ ,  $C_{Dw,W} = 0$ . Using DATCOM [3] and least square curve fitting the data, we obtain the wave drag coefficient of the wing for  $0.6 \leq M \leq \frac{1.05}{\sqrt{\cos \Lambda_{le}}}$ , or approximately,  $0.6 \leq M \leq 2.0$ ,

$$C_{Dw,W} = \frac{0.000585\Lambda(M - 0.6)}{1.05 - 0.6\sqrt{\Lambda}} \quad (70)$$

For the given configuration with  $\Lambda = 0.250$ , the equation ( 70) reduces to,

$$C_{Dw,W} = 0.0018745(M - 0.6) \quad (71)$$

For  $\frac{1.05}{\sqrt{\cos \Lambda_{le}}} \leq M \leq \frac{1}{\cos \Lambda_{le}}$  or approximately,  $2.0 \leq M \leq 4.0$ , we have,

$$C_{Dw,W} = k \cot \Lambda_{le} \left( \frac{t}{c} \right)^2 \quad (72)$$

For the given wing ( $\frac{t}{c} = 0.04$ ), the equation ( 72) reduces to,

$$C_{Dw,W} = 0.0016 \quad (73)$$

For  $\frac{1}{\cos \Lambda_{le}} \leq M \leq 6.0$ , or approximately,  $4.0 \leq M \leq 6.0$ ,

$$C_{Dw,W} = \frac{0.0064}{\sqrt{M^2 - 1}} \quad (74)$$

However, a formal difficulty arises in using equations ( 71) and ( 74) because the wing wave drag coefficient has new break points at  $M = 0.6$  and  $M = 2.0$  and the values of  $C_{Dw,W}$  at  $M = 4.0$  given by the equations ( 73) and ( 74) are not identical. To overcome this difficulty, we approximate  $C_{Dw,W}$  as follows so that we have the break points at  $M = 1.0, 1.4, 4.0$  and  $6.0$  as before:

for  $0 \leq M \leq 1.0$ ,

$$C_{Dw,W} = C_{Dw,W1}M^6 \quad (75)$$

where,  $C_{Dw,W1}$  is the value of wing wave drag coefficient at  $M = 1.0$  given by the equation ( 71).

for  $1.0 \leq M \leq 1.4$ ,

$$C_{Dw,W} = c_1M + c_2 \quad (76)$$

where,  $c_1 = 2.12550000e - 03$ , and  $c_2 = -1.375700e - 03$ .  
for  $1.4 \leq M \leq 4.0$ ,

$$C_{Dw,W} = 0.0016 \quad (77)$$

for  $4.0 \leq M \leq 6.0$ ,

$$C_{Dw,W} = c_1M + c_2 \quad (78)$$

where,  $c_1 = 4.8e - 4$ , and  $c_2 = 3.52e - 03$ .  
for  $M \geq 6.0$ ,

$$C_{Dw,W} = 0.0064 \quad (79)$$

### 5.7 Aft Body Drag Coefficient

For subsonic Mach numbers,  $0 \leq M \leq 1.0$ ;  $C_{D,aft} = 0$ . Using DATCOM [3] and curve fitting the data, we have for ( $1.0 \leq M \leq 6.0$ ),

$$C_{D,aft} = -0.0002M^3 + 0.003M^2 - 0.016M + 0.0342 \quad (80)$$

For  $M \geq 6.0$ ,  $C_{D,aft}$  is negligibly small and is ignored. However, to have a continuous variation of  $C_{D,aft}$  at break point Mach numbers of 1.0 and 6.0, we approximate the variation of  $C_{D,aft}$  at subsonic and hypersonic speeds as follows:

$$C_{D,aft} = C_{D,aft1}M^4; 0 \leq M \leq 1.0 \quad (81)$$

$$= C_{D,aft2}e^{-(M-6.0)}; M \geq 6.0 \quad (82)$$

where,  $C_{D,aft1}$  and  $C_{D,aft2}$  are the values of  $C_{D,aft}$  given by the equation (80) at  $M = 1$  and  $M = 6.0$  respectively.

### 5.8 Body Induced Drag Coefficient

The induced drag coefficient of the body at subsonic, supersonic and hypersonic Mach numbers is given by,

$$C_{Di,B} = C_{N,B}\alpha \quad (83)$$

Note that  $C_{N,B} = C_{N,N}$ .

## 5.9 Wing Induced Drag Coefficient

The estimation of the induced drag coefficient of the wing is based on linear aerodynamic theory. Note that  $C_{L,W} = C_{L\alpha,e}\alpha$  for subsonic and supersonic speeds and  $C_{L,W} = C_{N,W} \cos \alpha$  for hypersonic speeds. From DATCOM [3], the induced drag coefficient of the wing for  $0 \leq M \leq 0.8$  is given by,

$$C_{Di,W} = \frac{C_{L,W}^2}{\pi e_{av}} \quad (84)$$

where,  $e_{av} = 0.450$ . For  $0.8 \leq M \leq 1.4$ , using DATCOM [3],

$$C_{Di,W} = 0.4C_{L,W}^2 \quad (85)$$

For  $1.4 \leq M \leq 6.0$  from DATCOM [3],

$$C_{Di,W} = \left[ \frac{0.4010 + \sqrt{M^2 - 1}}{4.1887} \right] C_{L,W}^2 \quad (86)$$

For  $M \geq 6.0$ ,

$$C_{Di,W} = C_{L,W} \alpha \frac{S_{eff}}{S_W} \quad (87)$$

Once again a formal difficulty arises here because we have a new break point at  $M = 0.8$ . To avoid this, the data is approximated as follows,

$$C_{Di,W} = \left[ \frac{(1 - M^4)}{\pi e_{av}} + 0.4M^4 \right] C_{L,W}^2; 0 \leq M \leq 1.0 \quad (88)$$

$$= (0.35699257M + 0.94479781)C_{L,W}^2; 1.0 \leq M \leq 1.4 \quad (89)$$

$$= \left[ \frac{0.4010 + \sqrt{M^2 - 1}}{4.1887} \right] C_{L,W}^2; 1.4 \leq M \leq 4.0 \quad (90)$$

$$= aC_{Diw4} + bC_{Diw6}; 4.0 \leq M \leq 6.0 \quad (91)$$

$$= C_{L,W} \alpha \frac{S_{eff}}{S_W}; M \geq 6.0 \quad (92)$$

where  $a = -0.5M + 3.0$  and  $b = 0.5M - 2.0$ .  $C_{Diw4}$  and  $C_{Diw6}$  are the values of  $C_{Diw}$  at  $M = 4$  and  $M = 6$  obtained using equation (90) and (92) respectively.

### 5.10 Canard Induced Drag Coefficient

For  $0 \leq M \leq 1.0$ ,

$$C_{Di,C} = 0.909C_{N,C}\alpha \frac{S_C}{S_W} \quad (93)$$

For  $1.0 \leq M \leq 1.4$ ,

$$C_{Di,C} = (c_1M + c_2) \frac{S_C}{S_W} \quad (94)$$

where,  $c_1 = -1.35988656e - 02$  and  $c_2 = 1.90384119e - 02$ .

For  $M \geq 1.4$ ,

$$C_{Di,C} = 0 \quad (95)$$

## 6 Results and Discussion

The pictorial, three dimensional plots showing the variation of lift, drag and pitching moment coefficients with Mach number and angle of attack are shown in Figures 3 to 5. From these plots, it is observed that the coefficients vary smoothly with Mach number and angle of attack. Typical variations of base drag, wave drag and induced drag coefficients with Mach number at an angle of attack of 4.0 deg. are shown in Figures 6 to 8 to illustrate the continuity of these coefficients across the break point Mach numbers. The variation of total drag coefficient at  $\alpha = 4.0$  deg. is shown in Figure 9 along with APAS [1] and wind tunnel data [6]. It is observed that the present analytical model compares well with both data.

The variation of normal force coefficients of the body, wing and canard are shown in Figures 10 to 12 to illustrate the continuous variation of these coefficients across break point Mach numbers. The total lift coefficient at  $\alpha = 4.0$  compares well with APAS [1] and wind tunnel data [6] as shown in Figure 13. In comparison with APAS predictions [1], the lift coefficient predicted by the present analytical model is closer to the wind tunnel data [6] especially at high Mach numbers. The total pitching moment coefficient at  $\alpha = 4.0$  predicted by the present analytical model is shown in Figure 14 and is compared with APAS [1] and wind tunnel test data [6]. It is observed that significant differences exist between these data.

To illustrate the nature of comparison at higher angles of attack, the variation of total drag, lift and pitching moment coefficients at  $\alpha = 12.0$

deg. are shown in Figures 15 to 17 along with APAS [1] and wind tunnel data [6]. As before, the drag and lift coefficients compare fairly well and significant differences exist in pitching moment coefficient. In view of this, some degree of care and caution may be exercised in using the pitching moment coefficient data.

## 7 Concluding Remarks

We have presented a simple 3 DOF analytical aerodynamic model of the Langley Winged-Cone Aerospace Plane concept suitable for simulation, trajectory optimization, guidance and control studies especially for methods based on variational calculus. The present analytical aerodynamic model may be used along with the mass, inertia and propulsion models of [1].



## References

- [1] Shaughnessy, J.D., Pinckney, S.Z., McMinn, J.D., Cruz, C.I., and Kelley, Marie-Louse.; *Hypersonic Vehicle Simulation Model: Winged-Cone Configuration*, NASA TM 102610, 1990.
- [2] Sova, G., and Divan, P.; *Aerodynamic Preliminary Analysis System II, Part II, User's Manual*, NASA CR-182077,1990.
- [3] Hoak, D.E., and Finck, R.D.; *The USAF Stability and Control DATCOM*, Contract AF33(616)-6460, Project No. 8219, 1960.
- [4] Truitt, R. W.;*Hypersonic Aerodynamics*, The Ronald Press Company, N.Y. 1959.
- [5] Harloff, G. J., and Petrie, S. L.;*Preliminary Aero-thermodynamic Design Method for Hypersonic Vehicles*, AIAA Paper No. 87-2545.
- [6] Phillips, W.P., Brauckmann, G.J., Micoll, J.R., and Woods, W.C.; *Experimental Investigation of the Aerodynamic Characteristics for a Winged-Cone Concept*, AIAA Paper No. 87-2484.

Table I. Geometric Characteristics of The Configuration

**Wing**

Reference (planform) area, ft <sup>2</sup>	...	3600.0
Aspect ratio	...	1.0
Span, ft	...	60.0
Leading edge sweep angle, deg	...	76.0
Trailing edge sweep angle, deg	...	0.0
Mean aerodynamic chord, ft	...	80.0
Airfoil section	...	diamond
Airfoil thickness to chord ratio, ( $\frac{t}{c}$ )	...	0.04
Incidence angle	...	0
Dihedral	...	0

**Wing Flap**

Area each, ft <sup>2</sup>		92.30
Chord (constant), ft		7.22
Inboard section span location,ft	...	15.00
Outboard section span location,ft	...	27.78

**Vertical Tail**

Exposed Area each, ft <sup>2</sup>		645.70
Span, ft	...	32.48
Leading edge sweep angle, deg	...	70.0
Trailing edge sweep angle, deg	...	38.13
Airfoil section	...	diamond
Airfoil thickness to chord ratio, ( $\frac{t}{c}$ )	...	0.04

**Rudder**

Area, ft <sup>2</sup>	...	161.4
Span, ft	...	22.80
Chord to vertical tail chord ratio	...	0.25

Table I. Concluded

**Canard**

Exposed Area, ft <sup>2</sup>	...	154.30
Aspect ratio	...	5.48
Span, ft	...	33.6
Leading edge sweep angle, deg	...	16.0
Trailing edge sweep angle, deg	...	0.0
Airfoil section	...	NACA 65A006
Incidence angle	...	0
Dihedral	...	0

**Axisymmetric Fuselage**

Length		200.0
Cone half angle, deg		5.0
Cylinder radius (maximum), ft		12.87
Cylinder length, ft		12.88
Boattail half angle, deg		9.0
Boattail length, ft		40.0
Moment reference center, ft		124.0

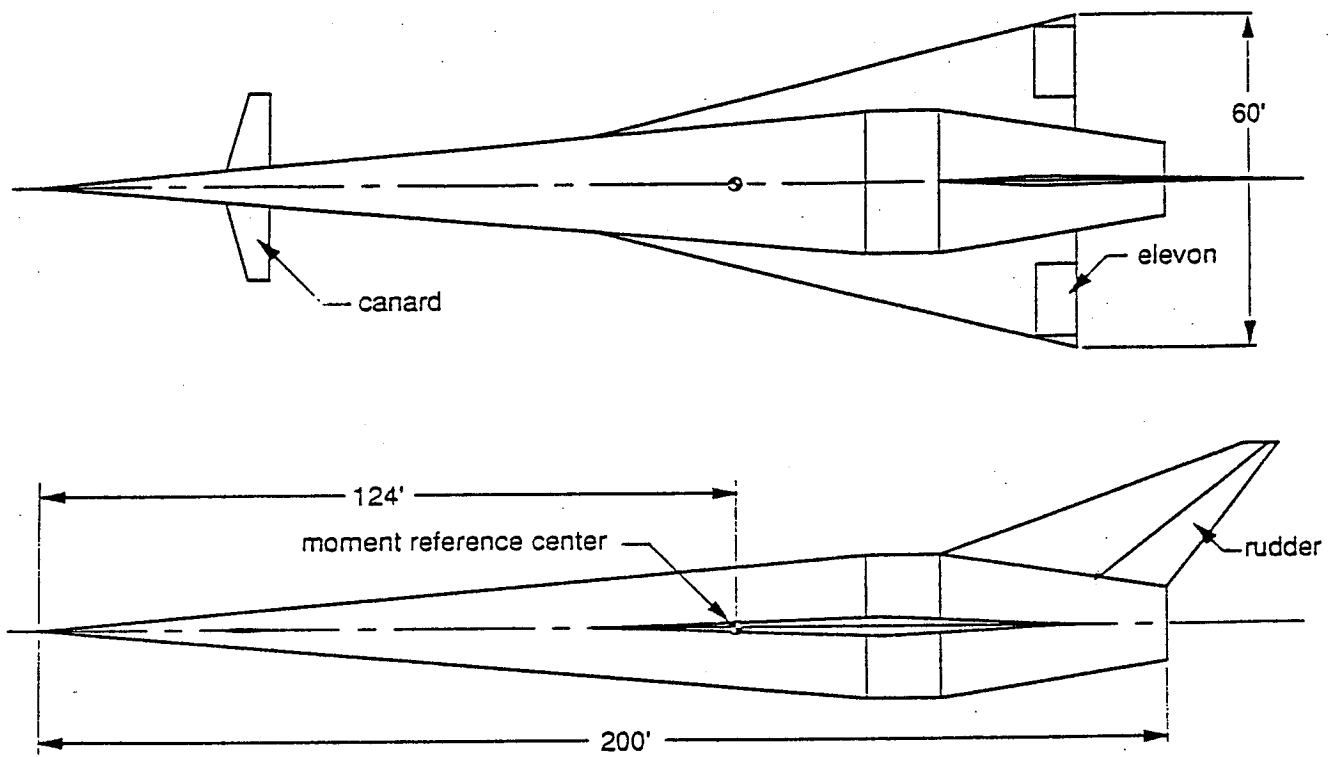


Figure 1.- Configuration geometry

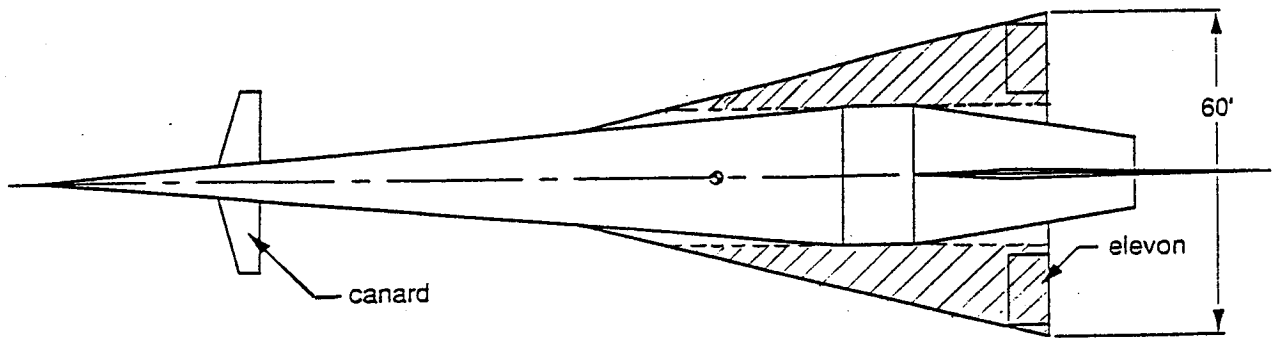


Figure 2. Schematic illustration of the concept of exposed wing area

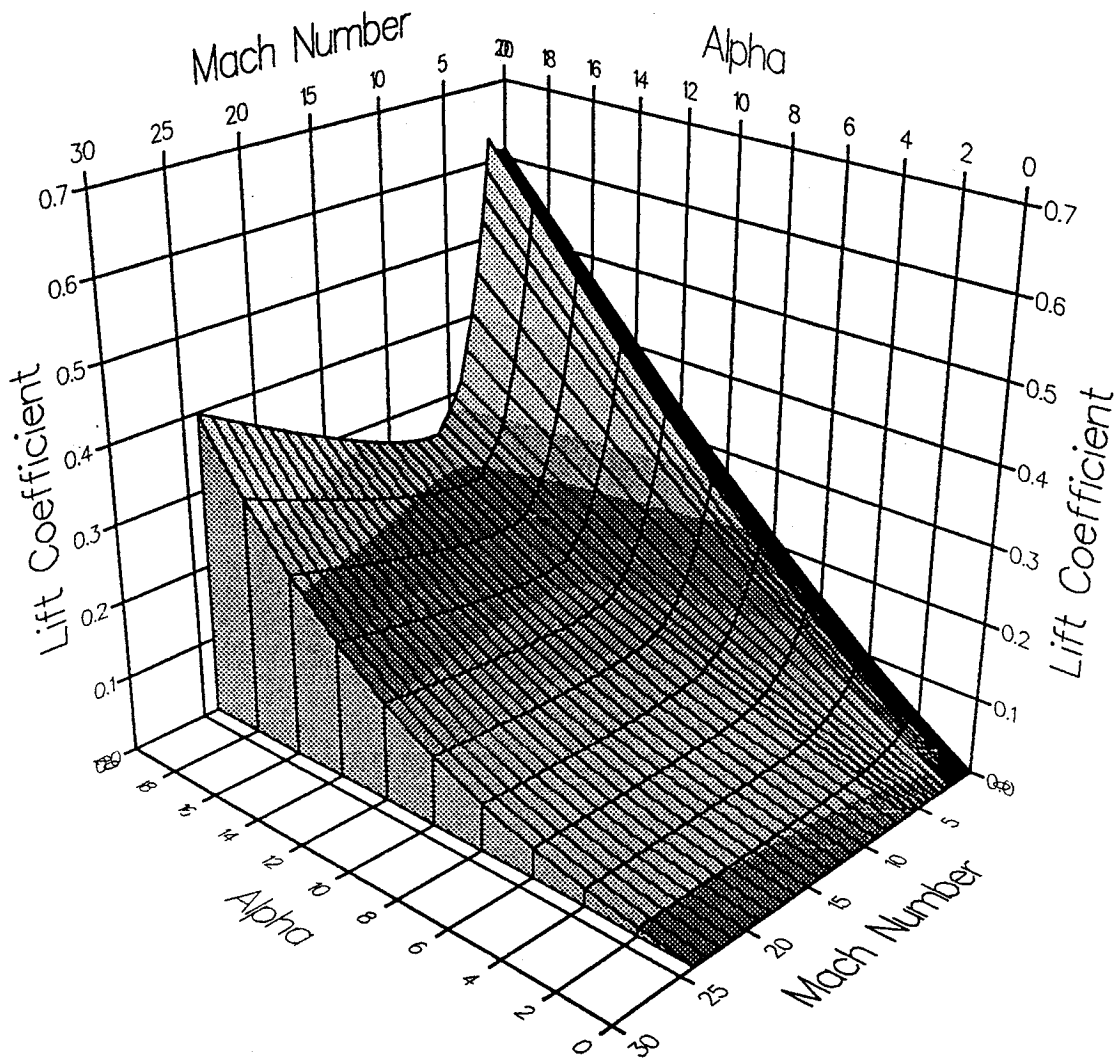


Figure 3. Variation of total lift coefficient with Mach Number and angle of attack

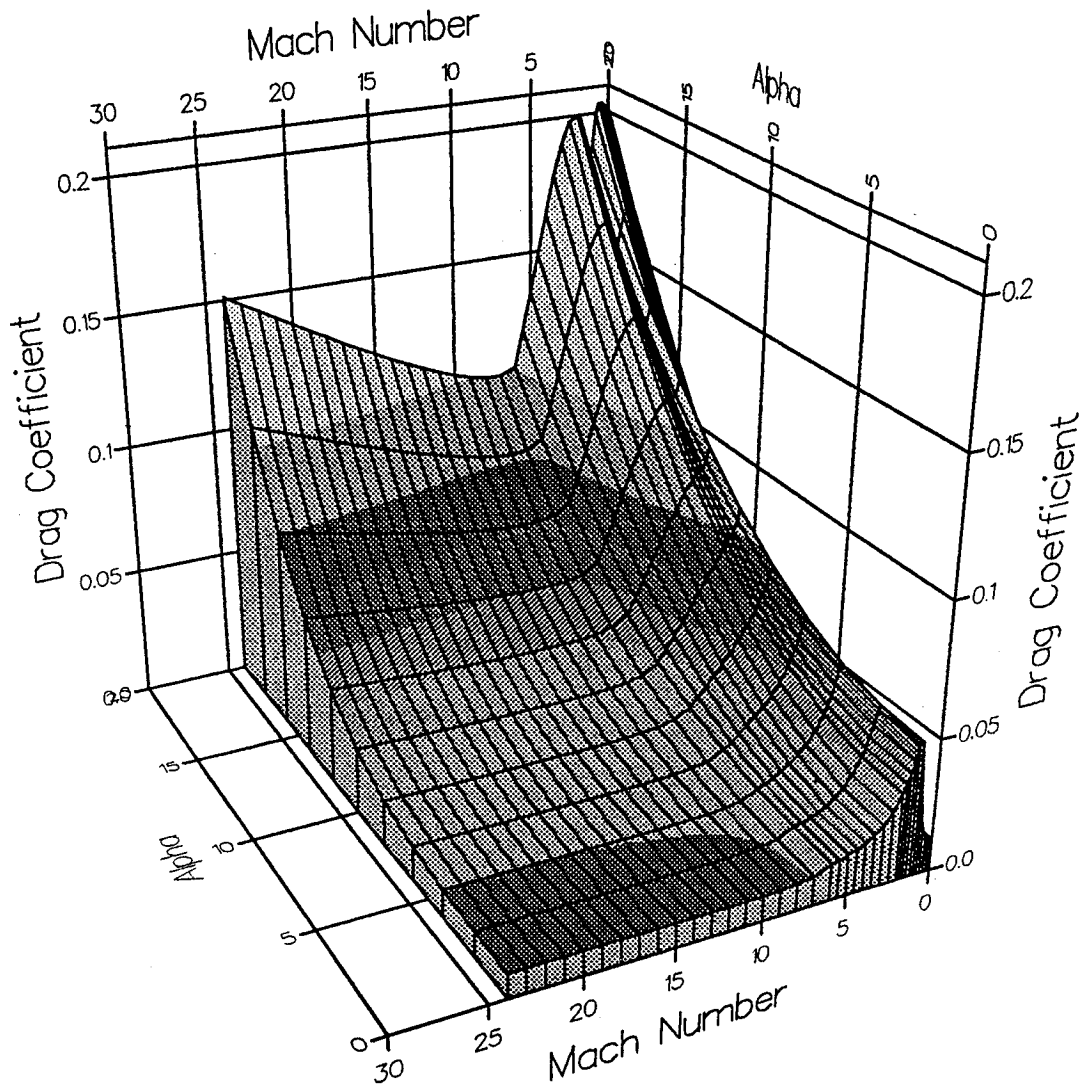


Figure 4. Variation of total drag coefficient with Mach Number and angle of attack

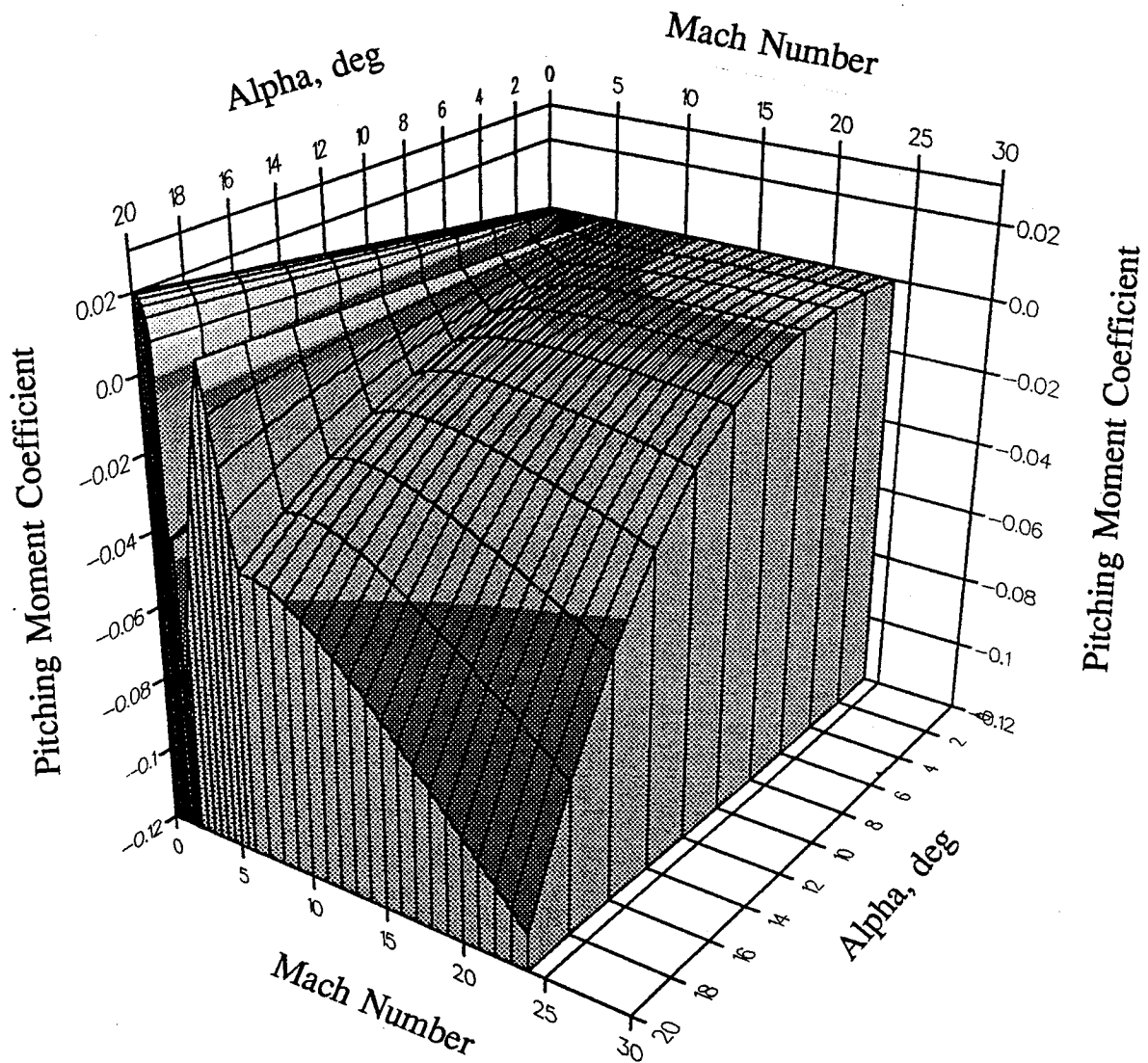


Figure 5. Variation of total pitching moment coefficient with Mach Number and angle of attack



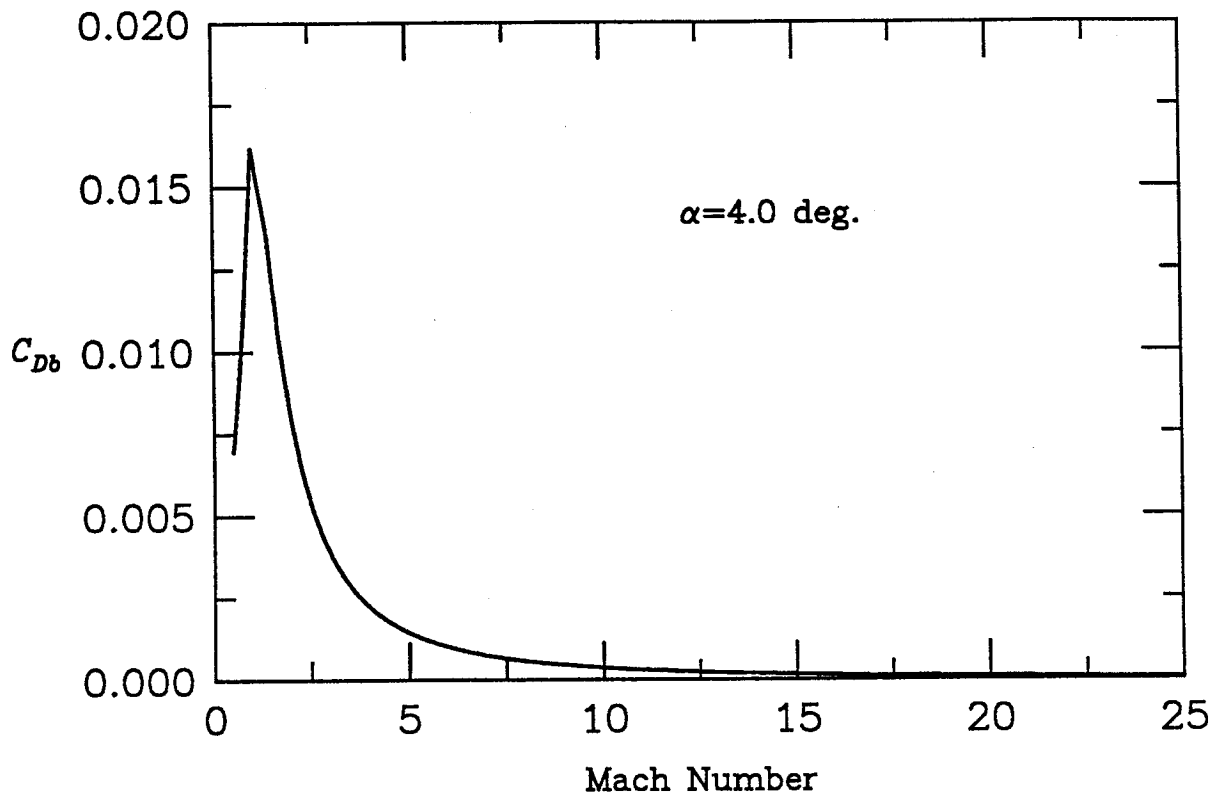


Figure 6. Variation of base drag coefficient with Mach number at  $\alpha=4.0$ deg.

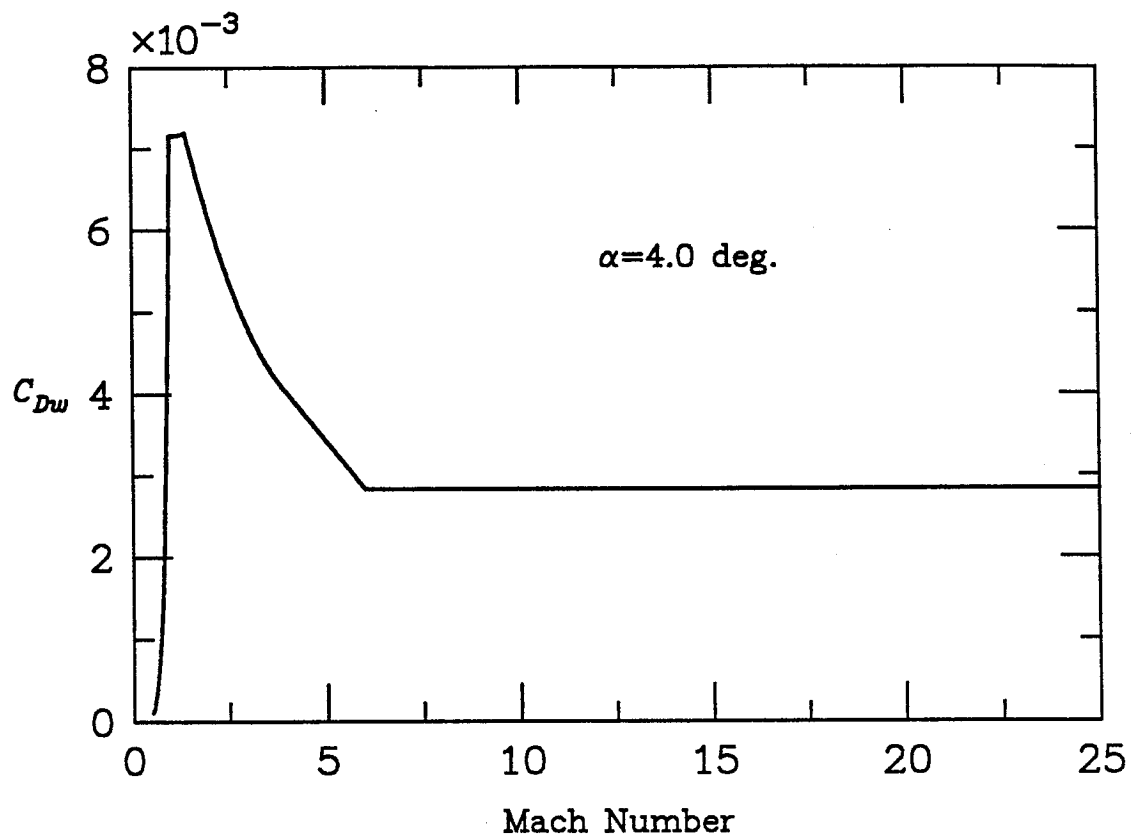


Figure 7. Variation of wave drag coefficient with Mach number at  $\alpha=4.0$  deg.

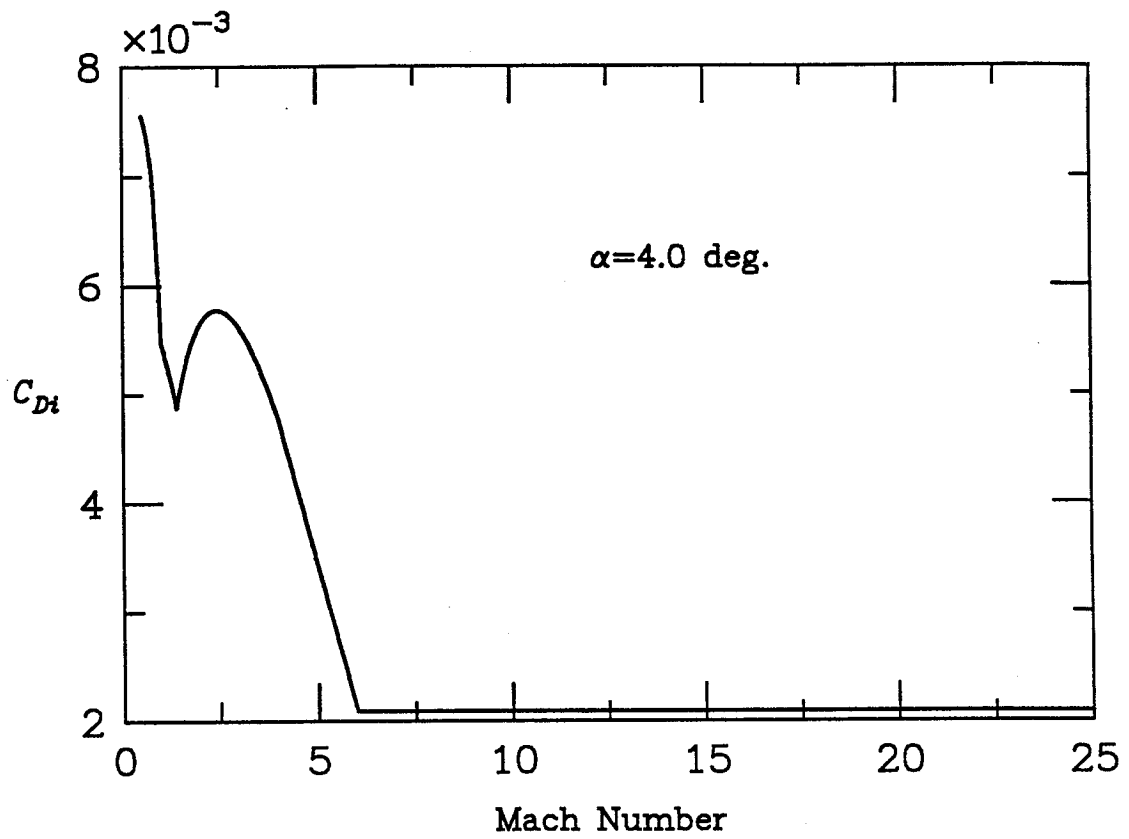


Figure 8. Variation of induced drag coefficient with Mach number at  $\alpha=4.0$  deg.

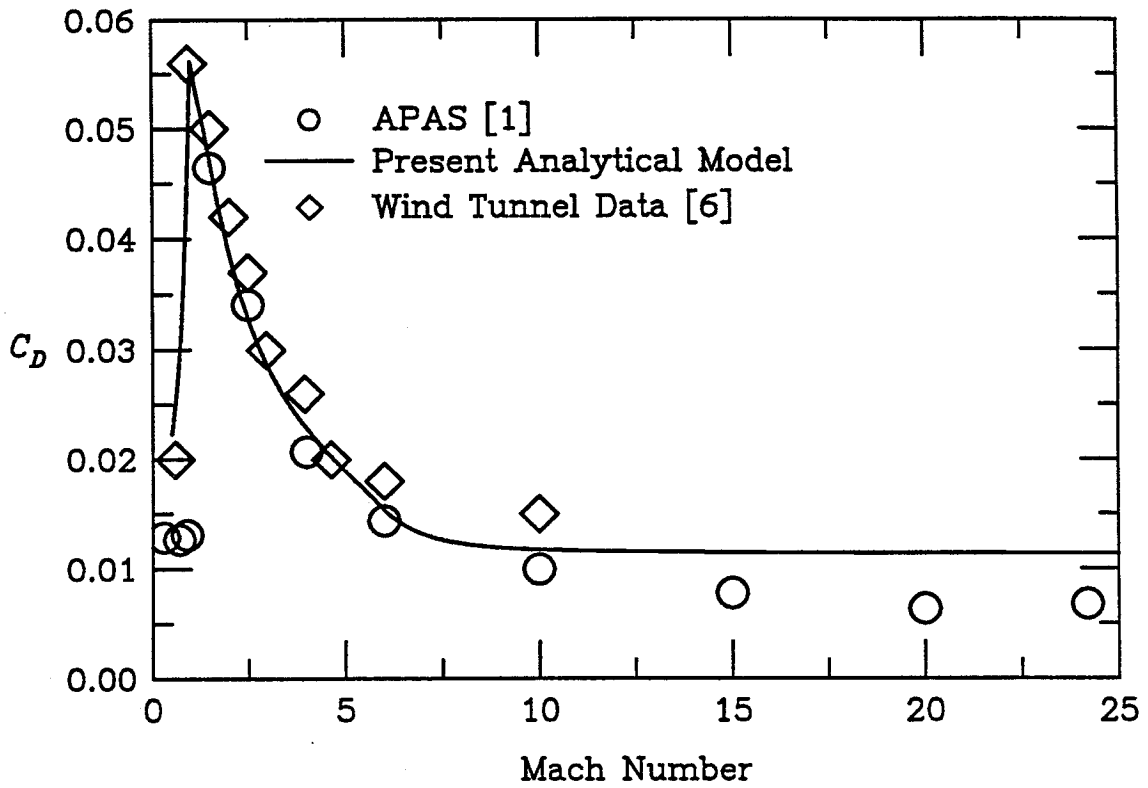


Figure 9. Variation of total drag coefficient with Mach number at  $\alpha=4.0$  deg.

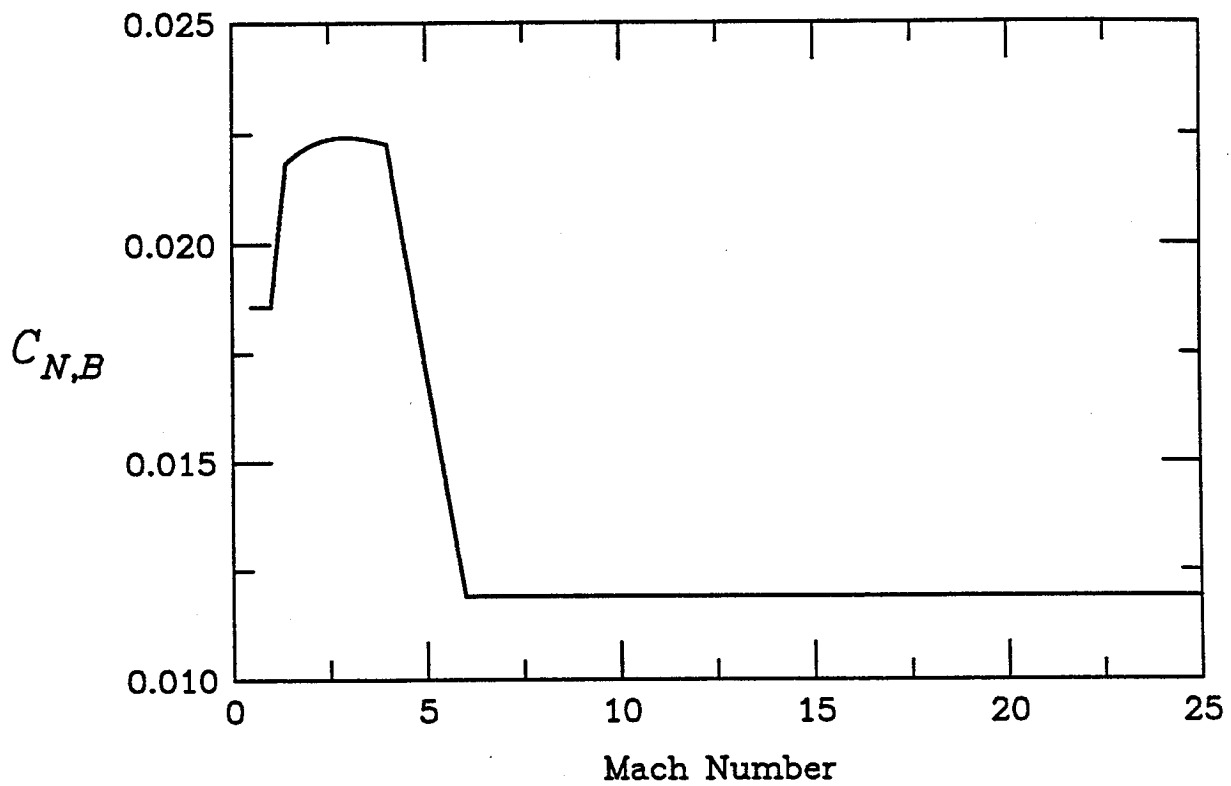


Figure 10. Variation of normal force coefficient of the body with Mach number at  $\alpha=4.0$  deg.

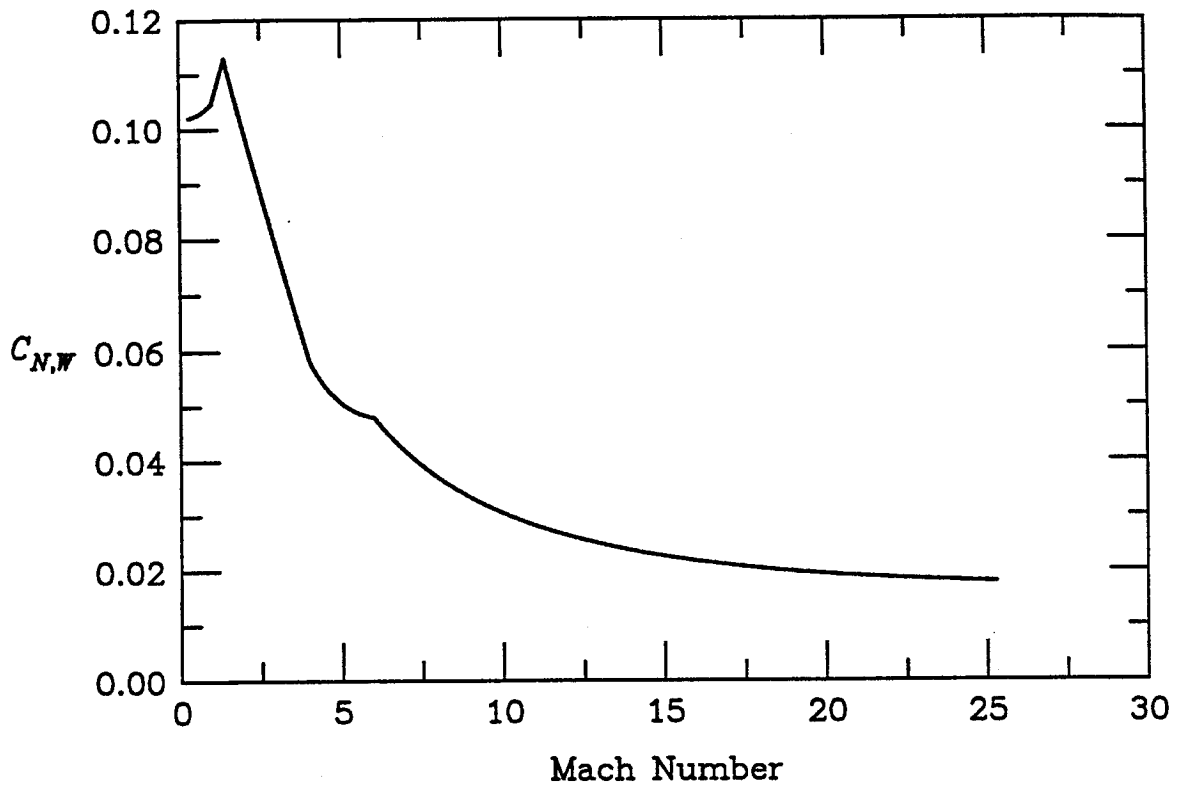


Figure 11. Variation of normal force coefficient of the wing with Mach number at  $\alpha=4.0$  deg.

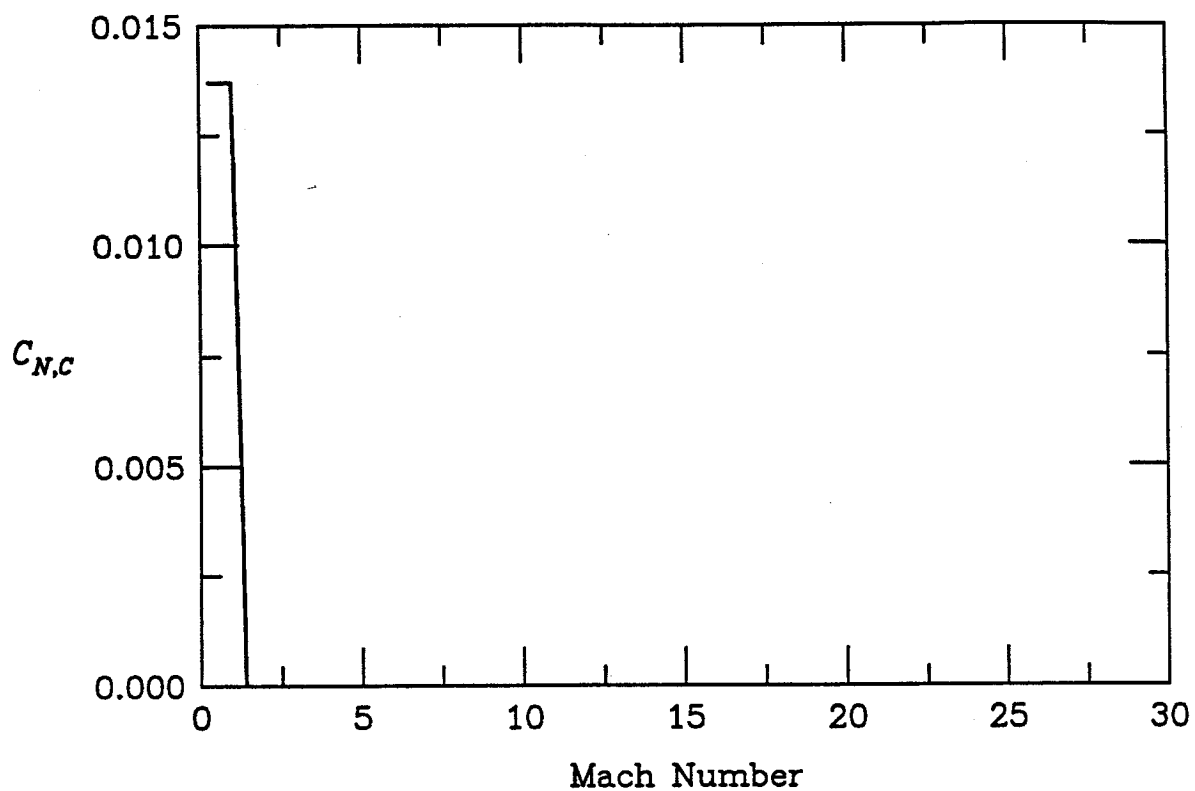


Figure 12. Variation of normal force coefficient of the canard with Mach number at  $\alpha=4.0$  deg.

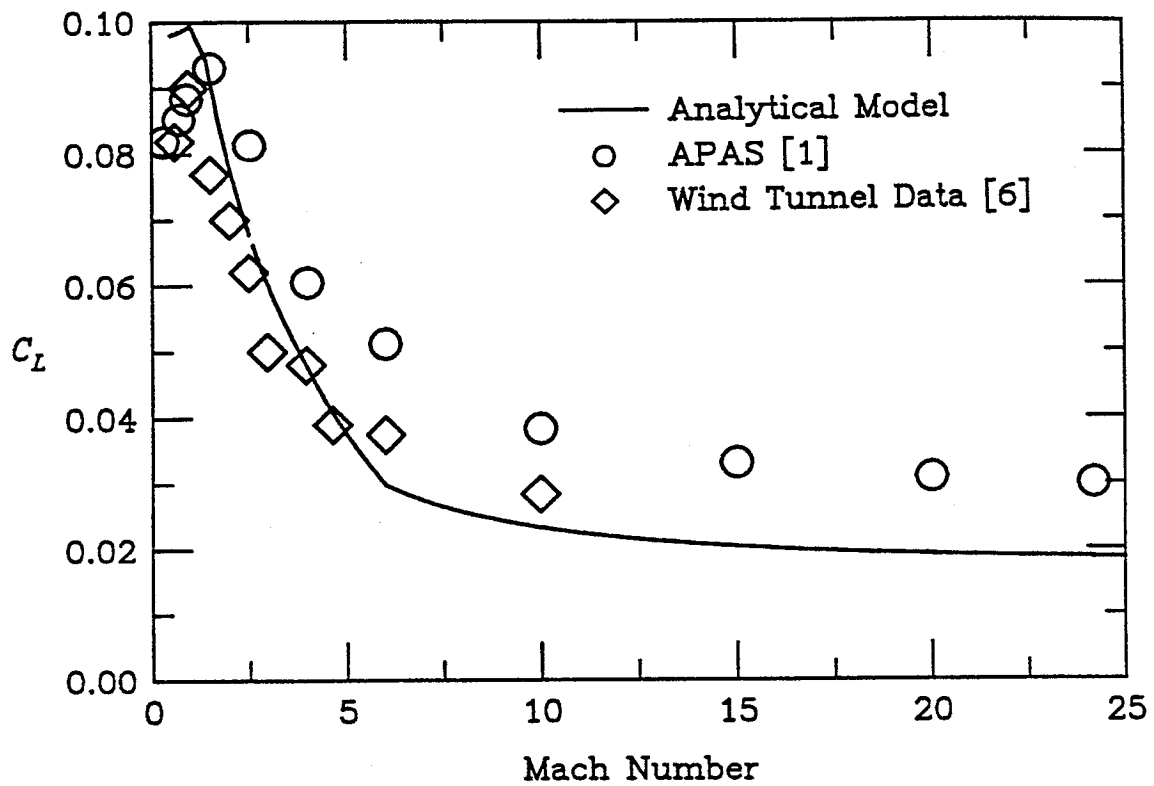


Figure 13. Variation of total lift coefficient with Mach number at  $\alpha=4.0$  deg.



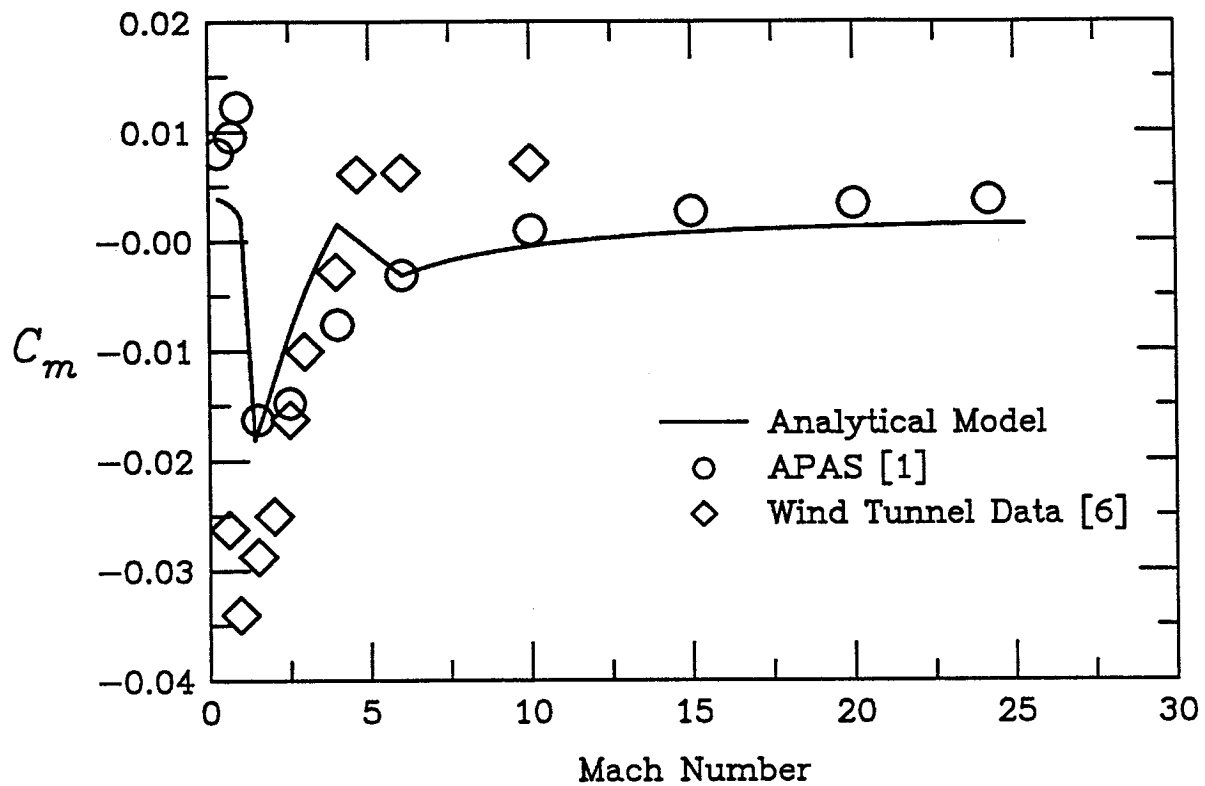


Figure 14. Variation of total pitching moment coefficient with Mach number at  $\alpha=4.0$  deg.

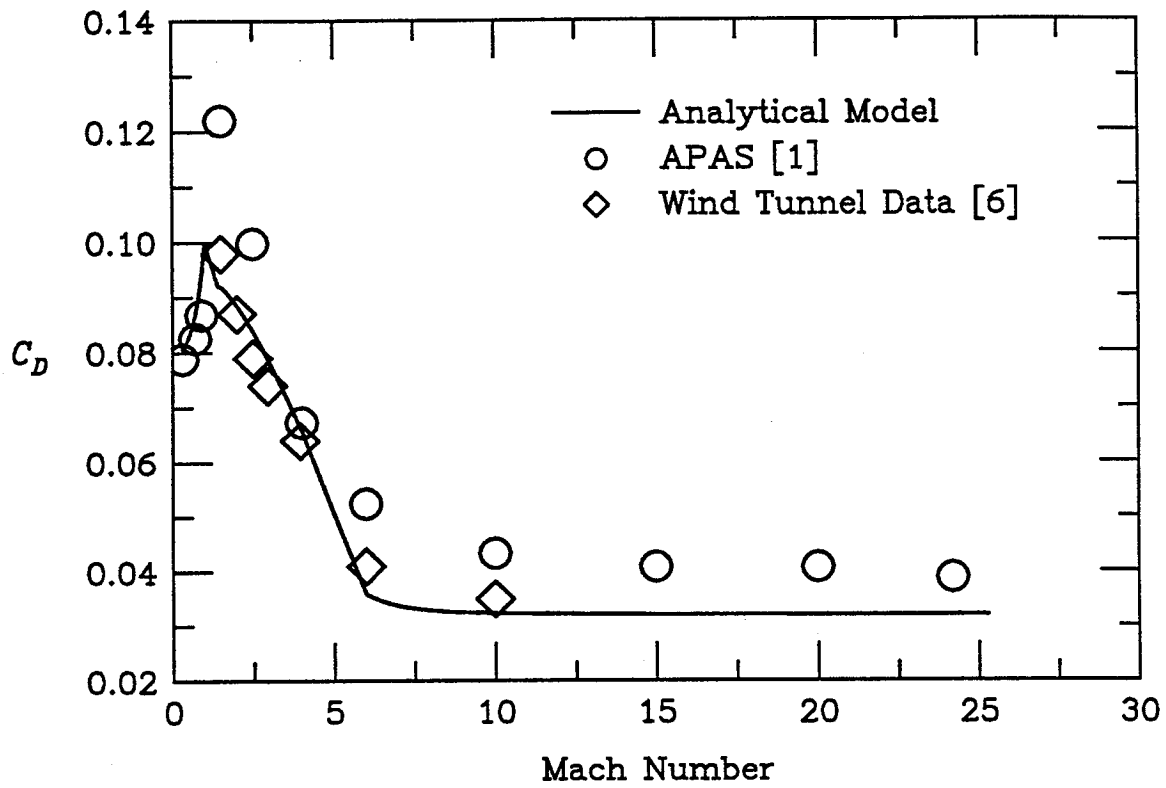


Figure 15. Variation of total drag coefficient with Mach number at  $\alpha=12.0$  deg.

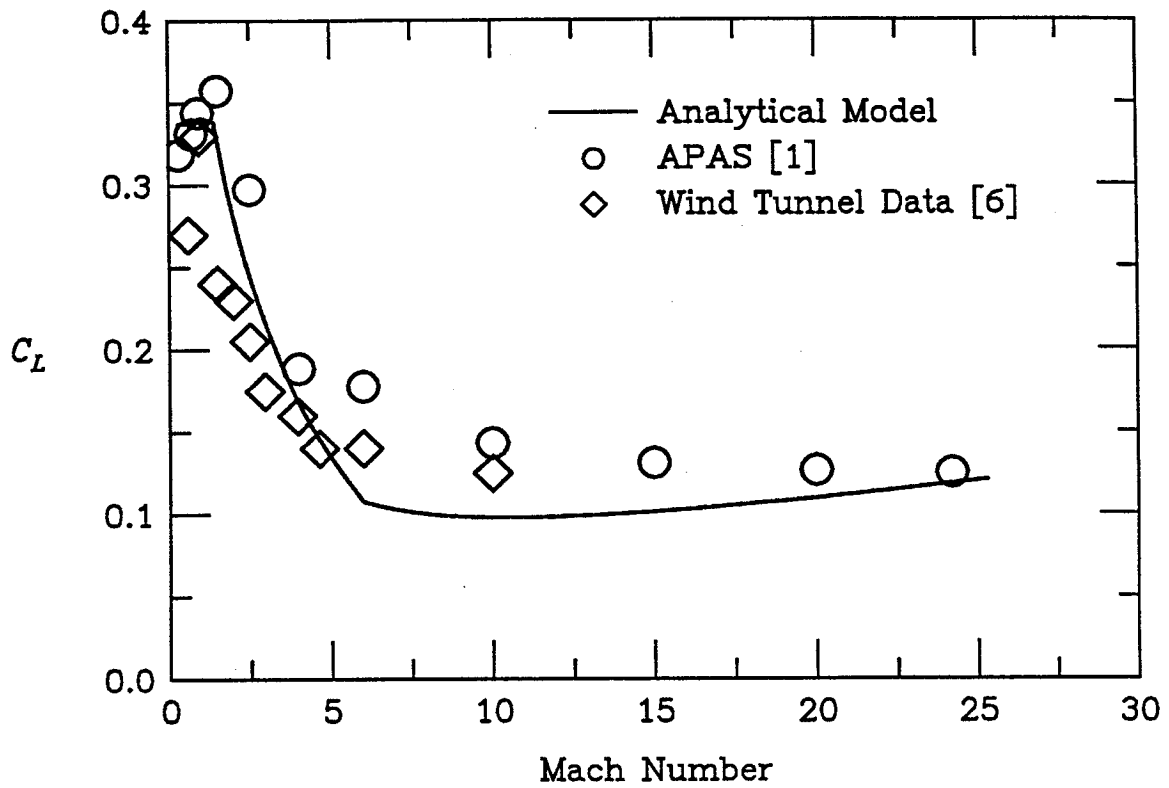


Figure 16. Variation of total lift coefficient with Mach number at  $\alpha=12.0$  deg.

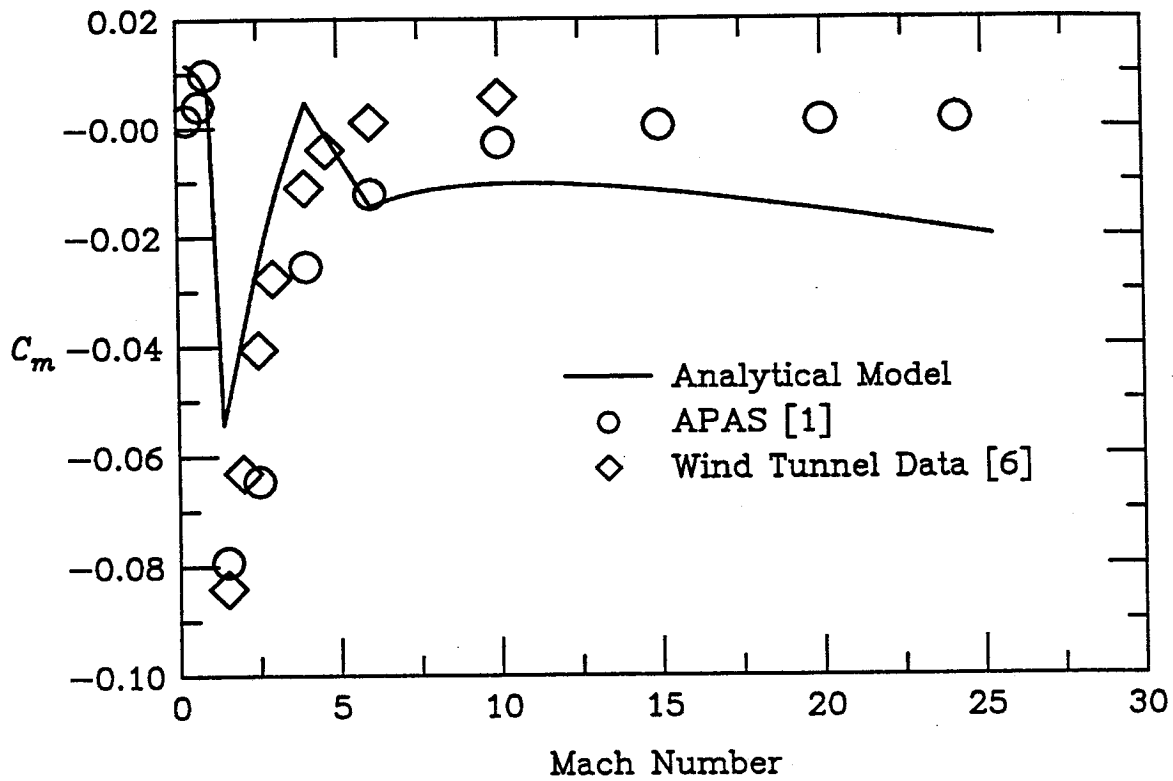


Figure 17. Variation of total pitching moment coefficient with Mach number at  $\alpha=12.0$  deg.

# REPORT DOCUMENTATION PAGE

*Form Approved*  
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE October 1994	3. REPORT TYPE AND DATES COVERED Contractor Report	
4. TITLE AND SUBTITLE A Simple Analytical Aerodynamic Model of Langley Winged-Cone Aerospace Plane Concept		5. FUNDING NUMBERS NAS1-19341 WU 232-01-04-05	
6. AUTHOR(S) Bandu N. Pamadi		8. PERFORMING ORGANIZATION REPORT NUMBER	
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) Vigyan, Inc. 30 Research Drive Hampton, VA 23666-1325			
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES) National Aeronautics and Space Administration Langley Research Center Hampton, Virginia 23681-0001		10. SPONSORING / MONITORING AGENCY REPORT NUMBER  NASA CR-194987	
11. SUPPLEMENTARY NOTES Langley Technical Monitor: William T. Suit			
12a. DISTRIBUTION / AVAILABILITY STATEMENT Unclassified - Unlimited  Subject Category 18		12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words)  A simple 3 DOF analytical aerodynamic model of the Langley Winged-Cone Aerospace Plane concept is presented in a form suitable for simulation, trajectory optimization, guidance and control studies. This analytical model is especially suitable for methods based on variational calculus. Analytical expressions are presented for lift, drag and pitching moment coefficients from subsonic to hypersonic Mach numbers and angles of attack up to +/- 20 deg. This analytical model has break points at Mach numbers of 1.0, 1.4, 4.0, and 6.0. Across these Mach number break points, the lift, drag and pitching moment coefficients are made continuous but their derivatives are not. There are no break points in angle of attack. The effect of control surface deflection is not considered. The present analytical model compares well with the APAS calculations and wind tunnel test data for most angles of attack and Mach numbers.			
14. SUBJECT TERMS Modeling, DATCOM, Hypersonic, Supersonic, Subsonic, Aerodynamic Coefficients, Analytical Model		15. NUMBER OF PAGES 44	
		16. PRICE CODE A03	
17. SECURITY CLASSIFICATION OF REPORT Unclassified	18. SECURITY CLASSIFICATION OF THIS PAGE Unclassified	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT