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PRODUCTION OF SQUEEZED STATES FOR MACROSCOPIC MECHANICAL OSCILLATOR

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Abstract

The possibility of squeezed states generation for macroscopic mechanical oscillator is discussed. It is shown that one can obtain mechanical oscillator in squeezed state via coupling it to electromagnetic oscillator (Fabry-Perot resonator) and pumping this Fabry-Perot resonator with a field in squeezed state. The degradation of squeezing due to mechanical and optical losses is also analysed.

Realization of quantum states such as squeezed, amplitude squeezed and others in real physical systems is of great importance for confirmation of predictions of quantum mechanics and its future development. There are a lot of papers concerning generation of squeezed states of electromagnetic fields [1, 2, 3]. However realization of such states in other systems, for example, mechanical is also of great importance. This problem arises in different high precision measurements, especially in gravitational wave experiment [4].

Let's consider a system of two coupled oscillators [5, 6]: electromagnetic (represented by Fabry-Perot resonator with a laser pump beam amplitude E_L and frequency ω_p near one of the resonant frequencies of resonator ω_0) and mechanical (represented by a moving mirror of Fabry-Perot resonator connected to a spring). Pump beam enters the resonator through fixed mirror with reflectance approaching 1 (this assumption is not critical for results and used only for simplicity, we also suppose that $|\, t_f \,| \rightarrow 0$ but $E_L \cdot |\, t_f \,| = \text{const.}$, where t_f is amplitude transmittance coefficient of fixed mirror). Usually one treats such a system in Hamiltonian formalism framework [7] when equations of motion are conservative and the system is in free evolution. However in our case it is more convenient to use Langevin approach with evolution of the system under the action of input fields: E_L (classical laser field) and E_{b0} (quantum field from field controller - device allowing to generate electromagnetic field in appropriate state). In our analysis E_{b0} is squeezed noise with two point of squeezing and it enters the resonator through moving mirror with nonzero amplitude transmittance coefficient t_m. Then linearised equations of motion for such system have the following form:

$$
\begin{aligned}
\ddot{\hat{x}} + \omega_\mu^2 \hat{x} &= -2\gamma \cdot [\hat{E}_1 \sin(\Delta t + \phi) + \hat{E}_2 \cos(\Delta t + \phi)] \\
\dot{\hat{E}}_1 + \delta_e \hat{E}_1 + 2\beta \hat{x} \cos(\Delta t + \phi) &= 2\delta_e \hat{E}_{b1} \\
\dot{\hat{E}}_2 + \delta_e \hat{E}_2 - 2\beta \hat{x} \sin(\Delta t + \phi) &= 2\delta_e \hat{E}_{b2}
\end{aligned}
\tag{1}
$$

where the following parameters are introduced

$$
\beta = \frac{\omega_0 E_0}{2l} \qquad \gamma = \frac{S E_0}{4\pi m t_m^2} \qquad \Delta = \omega_p - \omega_0 \tag{2}
$$

E_0 and ϕ are the amplitude and the phase of the field inside optical resonator due to the action of "laser force" E_L with mirrors fixed, S is the beam cross section, l is the length of optical resonator, δ_e is optical damping of resonator due to the leakage of inside field through moving mirror (with reflectivity less than 1), m and ω_μ are mass and frequency of mechanical oscillator and $\hat{x}$ is its coordinate operator. For simplicity we assume that one can omit the damping of mechanical system. We also assume that the fields could be represented in terms of their quadrature component operators with usual commutation relations [8, 9]:

$$\begin{aligned} \hat{E}_{ba} &= \hat{E}_{b1}\cos\omega_0 t + \hat{E}_{b2}\sin\omega_0 t \\ \hat{E}_{out} &= \hat{E}_1\cos\omega_0 t + \hat{E}_2\sin\omega_0 t \end{aligned} \tag{3}$$

Index "ba" fits a field entered the system from field controller and index "out" means that the field outputs from Fabry-Perot resonator through moving mirror.

Introducing quadrature components for mechanical oscillator through equation

$$\hat{x} = \hat{x}_c\cos(\Delta t + \phi) + \hat{x}_s\sin(\Delta t + \phi) \tag{4}$$

(frequency $\Delta \approx \omega_\mu$ is not exactly equal to the frequency ω_μ of single mechanical oscillator and matches the presence of coupling between two oscillators) one can easily obtain the following equations of motion (we suppose that the system is in steady state therefore $d\hat{x}_{c,s}/dt = 0$)

$$\begin{aligned} (\hat{x}_c\cos(\Delta t + \phi) + \hat{x}_s\sin(\Delta t + \phi))(\omega_\mu^2 - \Delta^2) &= -2\gamma \cdot [\hat{E}_1\sin(\Delta t + \phi) + \hat{E}_2\cos(\Delta t + \phi)] \\ \dot{\hat{E}}_1 + \delta_e\hat{E}_1 &= 2\delta_e\hat{E}_{b1} - \beta\cdot[\hat{x}_c + \hat{x}_c\cos(2\Delta t + 2\phi) + \hat{x}_s\sin(2\Delta t + 2\phi)] \\ \dot{\hat{E}}_2 + \delta_e\hat{E}_2 &= 2\delta_e\hat{E}_{b2} + \beta\cdot[\hat{x}_s - \hat{x}_s\cos(2\Delta t + 2\phi) + \hat{x}_c\sin(2\Delta t + 2\phi)] \end{aligned} \tag{5}$$

It is obvious from (5) that fluctuations of $\hat{x}_c$ and $\hat{x}_s$ depend on fluctuations of controller field quadrature components only near frequencies $\omega \approx 0$ and $\omega \approx 2 \mid \Delta \mid$ that means that controller field must consist of two modes with different frequencies ω_0 and $\omega_0 + 2\Delta$. Then one can introduce the following form for $\hat{E}_{ba}$:

$$\hat{E}_{ba} = \hat{E}_c\cos\omega_0 t + \hat{E}_s\sin\omega_0 t + \hat{E}_{2c}\cos((\omega_0 + 2\Delta)t + \chi) + \hat{E}_{2s}\sin((\omega_0 + 2\Delta)t + \chi) \tag{6}$$

where $\hat{E}_c$, $\hat{E}_s$, $\hat{E}_{2c}$, $\hat{E}_{2s}$ are quadrature component operators of two modes in narrow non overlapping bandwidths ($\delta\omega \ll \mid \Delta \mid$) near left and right sidebands which are detuned from the pump frequency ω_p by $\Delta : \omega_0 = \omega_p - \Delta$ and $\omega_0 + 2\Delta = \omega_p + \Delta$. Then introducing the coefficients

$$A = \omega_\mu^2 - \Delta^2 - \frac{4\beta\gamma\Delta}{\delta_e^2 + 4\Delta^2} \qquad B = \frac{8\beta\gamma\Delta^2}{\delta_e(\delta_e^2 + 4\Delta^2)} \tag{7}$$

one could obtain the following system of equations

$$\begin{aligned} A\hat{x}_c + B\hat{x}_s &= -4\gamma\hat{E}_s - 4\gamma\delta_e(\delta_e^2 + 4\Delta^2)^{-1/2}[\hat{E}_{2c}\cos(\chi - \zeta - 2\phi) + \hat{E}_{2s}\sin(\chi - \zeta - 2\phi)] \\ A\hat{x}_s - B\hat{x}_c &= -4\gamma\hat{E}_c + 4\gamma\delta_e(\delta_e^2 + 4\Delta^2)^{-1/2}[\hat{E}_{2c}\sin(\chi - \zeta - 2\phi) - \hat{E}_{2s}\cos(\chi - \zeta - 2\phi)] \end{aligned} \tag{8}$$

where phase $\zeta = \arctan(2\Delta/\delta_e)$ represents the delay of the pump field inside the cavity with respect to the laser field.

Let's assume that controller field E_{ba} is in squeezed state with $<\hat{E}_{ba}>=0$ and dispersions of quadrature components [8, 9]

$$<\Delta\hat{E}_s^2>=N_0/g_0 \qquad <\Delta\hat{E}_c^2>=N_0 g_0 \tag{9}$$

where $g_0>1$ is the squeezing coefficient of back action field for mode with frequency ω_0, N_0 is vacuum level of dispersion and we use the fact that $2\mid\Delta\mid\approx 2\omega_\mu \ll \omega_0$. The same expressions are valid for $\hat{E}_{2s}, \hat{E}_{2c}$ with obvious substitution $g_0\to g_2$ and correlations between $\hat{E}_s, \hat{E}_c$ and $\hat{E}_{2s}, \hat{E}_{2c}$ are zero. Factors g_0 and g_2 depends on the structure of back action field controller.

The mechanical oscillator would be in squeezed state only if the following special conditions are valid. The first condition concerns the detuning Δ of the system: it must satisfy the equation

$$A=0 \qquad \omega_\mu^2-\Delta^2-\frac{4\gamma\beta\Delta}{\delta_e^2+4\Delta^2}=0 \tag{10}$$

In this case contribution of "noisy component" $\hat{E}_c$ to $\hat{x}_s$ vanishes. For the contribution of another "noisy component" $\hat{E}_{2c}$ would be also unimportant one must choose the phase ϕ according to the following equation:

$$\chi-\zeta-2\phi=k\pi/2 \qquad k=0,\pm1,\pm2\ldots \tag{11}$$

that means that special phase correspondence must take place. In physical language this means that one must compensate the delay of electromagnetic field inside Fabry-Perot resonator with regard to the pump field. This can be done by appropriate phase correspondence between the pump beam and the field E_{ba} from the squeezed state controller. Then the system equations of motion occur ($k=1$)

$$\begin{aligned} B\hat{x}_s &= -4\gamma\hat{E}_s-4\gamma\delta_e(\delta_e^2+4\Delta^2)^{-1/2}\hat{E}_{2s} \\ B\hat{x}_c &= 4\gamma\hat{E}_c-4\gamma\delta_e(\delta_e^2+4\Delta^2)^{-1/2}\hat{E}_{2c} \end{aligned} \tag{12}$$

Therefore we obtain special quantum nondemolition coupling between optical and mechanical oscillator: one quadrature component of mechanical oscillator couples only with one quadrature component of optical field (on frequencies ω_0 and $\omega_0+2\Delta$) and the larger the squeezing of the field of two electromagnetic modes (with frequency ω_0 and $\omega_0+2\Delta$) the greater the squeeze factor of mechanical oscillator. It is worth mentioning that the squeezing g_2 of the mode with frequency $\omega_0+2\Delta$ could be smaller than the squeezing g_0 of the mode with frequency ω_0 of electromagnetic field provided $\delta_e<\Delta$ (because of the filtration of fluctuations by narrow bandwidth optical resonator) otherwise the squeezing of two modes must be equal.

Let's discuss the time of operating regime achievement and the influence of mechanical δ_μ and optical coherent (due to difraction and mirror absorbtion) δ_c losses. In practice optical damping δ_e due transmittance through moving mirror is much greater than mechanical damping δ_μ. Then the field inside resonator becomes squeezed through time δ_e^{-1}. After that through time $(2\beta\gamma/\omega_\mu)^{-1/2}$ (that is inverse value of coupling constant) the state of mechanical oscillator becomes also squeezed and the initial state is forgotten. It is obvious that mechanical losses must not be very large: $\delta_\mu<(2\beta\gamma/\omega_\mu)^{1/2}/g$ (g - required squeezing factor for mechanical oscillator) otherwise the rate of coherent pumping through losses would be larger than the rate of squeezing through action of controller field. Similarly δ_c must be smaller than $\delta_e/\max(g_0,g_2)$.

In conclusion let's discuss the structure of back action field controller. In accordance with the ideas of papers [10, 11, 12] it must contain two circulators for uncoupling the fields E_{out} and E_{ba}, the load (absorber black body with zero temperature) and two squeezers with pump frequencies $2\omega_0$ and $2(\omega_0+2\Delta)$ (for example, degenerate parametric amplifiers or four wave mixers). Then the field E_{out} comes through moving resonator mirror and two circulators to the load and dissipates in it. Zero fluctuations of the load enters through first circulator the squeezer with pump frequency $2\omega_0$, then gets through second circulator to the squeezer with pump frequency $2(\omega_0 + 2\Delta)$ and then enters the system through moving mirror.

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