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DECOHERENCE AND DISSIPATION FOR A QUANTUM SYSTEM COUPLED TO A LOCAL ENVIRONMENT

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Abstract

Decoherence and dissipation in quantum systems has been studied extensively in the context of Quantum Brownian Motion. Effective decoherence in coarse grained quantum systems has been a central issue in recent efforts by Zurek and by Hartle and Gell-Mann to address the Quantum Measurement Problem. Although these models can yield very general classical phenomenology, they are incapable of reproducing relevant characteristics expected of a local environment on a quantum system, such as the characteristic dependence of decoherence on environment spatial correlations. I discuss the characteristics of Quantum Brownian Motion in a local environment by examining aspects of first principle calculations and by the construction of phenomenological models. Effective quantum Langevin equations and master equations are presented in a variety of representations. Comparisons are made with standard results such as the Caldeira-Leggett master equation.

1 Introduction and Motivation

Decoherence via coarse graining has been studied in the context of quantum measurement theory by Zurek[1] and by Hartle and Gell-Mann[2] as a mechanism which leads to the emergence of classical properties. Recent efforts have focused on the decoherence effects of a heat bath, which has also been examined in detail in the study of quantum brownian motion. Decoherence is identified as the (effective) suppression of interference terms in the density operator ($\rho(x, x')$, $x \neq x'$). It has been pointed out that most of the models which have been considered are somewhat simplistic and cannot reproduce the phenomenological features expected of a system which interacts locally with a homogeneous and isotropic environment[3]. In this paper I describe the perceived shortcomings of existing models and illustrate the construction of a phenomenological quantum master equation which contains many features expected from local coupling to a homogeneous environment[4].

Although decoherence is the most interesting feature of the effects of a heat bath, dissipation (and other effects) also generally appear in the dynamics of the density operator:

$$\frac{\partial \rho(x, x'; t)}{\partial t} = \text{Hamiltonian terms} + \text{Dissipation terms} + \cdots - g(x, x')\rho(x, x'; t). \tag{1}$$

The decoherence term appears as a (spatially dependent) decay term in the evolution equation, and can be understood in terms of effective fluctuating forces, or potentials: [5, 6]

$$g(x, y) = (\frac{1}{\hbar^2})(c(x; x) + c(y; y) - 2c(x; y), \langle V(x, t)V(y, s)\rangle = c(x; y)\delta(t - s). \tag{2}$$

Typical models have a quadratic form, $g(x,y) \propto (x-y)^2$ for the decoherence term, corresponding to a fluctuating force which is independent of position. However, for a local bath one expects the correlation function to die off at some characteristic length scale (the correlation length of the environment), which has some important ramifications for decoherence. For a quadratic form of decoherence, the decay rate of the interference terms in the density matrix increases without bound, while for a local model the decay rate saturates at separations (between x and x') much larger than the correlation length of the environment, reflecting the independence of environment fluctuations at large separations. As it turns out, the quadratic form can be considered a short length scale approximation of a more detailed model.

To consider the decoherence effects of an environment, simultaneous treatment of dissipation is necessary since decoherence and dissipative effects both generally arise from the same source (the interaction with a heat bath). For simplicity, I consider only linear dissipation, that is

$$m\dot{\mathbf{x}} = \mathbf{p}, \quad \dot{\mathbf{p}} = -\frac{\eta}{m}\mathbf{p} + \mathbf{F}. \tag{3}$$

As an example of quantum dissipative evolution, Dekker[7] has constructed a phenomenological master equation which includes ohmic dissipation and quadratic decoherence:

$$\frac{\partial \rho}{\partial t} = \frac{1}{i\hbar}[H,\rho] - i\frac{\lambda}{\hbar}[x,\{p,\rho\}] + \frac{(D_{xp}+D_{px})}{\hbar^2}[x,[p,\rho]] - \frac{D_{xx}}{\hbar^2}[p,[p,\rho]] - \frac{D_{pp}}{\hbar^2}[x,[x,\rho]]. \tag{4}$$

The Caldeira-Leggett[8] master equation is obtained from a first-principle calculation for the effects of a simple thermal bath. With an appropriate choice of parameters for the Dekker model, the Calderia-Leggett master equation can be reproduced.

Many open system models can reduce to the same classical phenomenology, particularly in the Markov regime, and yet have significant differences for a quantum sytem in that same regime. To illustrate this "richness" of quantum dissipative models, consider a rather generic oscillator bath model (following Zwanzig[9]):

$$L = \frac{1}{2}m\dot{x}^2 - U(\mathbf{x}) + \sum_{\mu}\frac{m_\mu}{2}[\dot{q}_\mu^2 - \omega_\mu^2(q_\mu - a_\mu(\mathbf{x}))^2]. \tag{5}$$

The classical calculations (the results of which are presumably reproduced in at least some limit of the quantum model) are relatively straightforward. The classical fluctuation-dissipation relation between the fluctuating forces and the nonlinear dissipation kernal emerges naturally, and in the usual Markov limit becomes:

$$\langle f_i(\mathbf{x},t) f_j(\mathbf{y},s)\rangle = k_B T \eta_{ij}(\mathbf{x},\mathbf{y};t-s) = \overline{\eta}_{ij}(\mathbf{x},\mathbf{y}) 2k_B T\delta(t-s), \tag{6}$$

and a simple langevin equation can (at least in principle) be obtained:

$$\ddot{x}_i(t) = -\frac{\partial U(\mathbf{x}(t))}{\partial x_i} + f_i(\mathbf{x}(t),t) - \overline{\eta}_{ij}(\mathbf{x}(t))\dot{x}_j. \tag{7}$$

For a homogeneous environment, the dissipation constant would be independent of position.

Some observations about the Markov limit are in order. For the classical picture, the spatial correlations of the fluctuating forces are irrelevant. After all, the particle can only be in one place at one time. For a quantum system one must consider superpositions between the particle at different locations, i.e. superpositions between different trajectories for the particle. My point is that different models may produce the same classical phenomenology, but have some important differences for the quantum case, in particular for the effective decoherence due to the environment.

In order to help motivate some choices which will be required for the construction of the new model, consider a particle locally coupled to a scalar field. This particular model is a natural extension of one considered by Unruh and Zurek[10]. The action for this model is given by:

$$L = \int d^n r \left\{ \tfrac{1}{2}\left[\dot{\phi}^2 - c^2(\nabla_r \phi)^2\right] + \delta(\mathbf{r}-\mathbf{x})\left[\tfrac{m\dot{\mathbf{x}}^2}{2} - \varepsilon\phi(\mathbf{r},t) - V(\mathbf{x})\right]\right\}. \tag{8}$$

This model produces approximately ohmic dissipation in one dimension[8, 11]. In addition, one can extract from the influence functional the effective correlation function of the fluctuating forces[5, 11]:

$$< \mathbf{F}(\mathbf{x},\tau)\cdot\mathbf{F}(\mathbf{y},s) > 0 = \frac{\hbar\varepsilon^2}{2(2\pi)^d}\int d^d k k^2 \left\{\frac{\coth(\frac{\beta\hbar\omega}{2})}{\omega}\cos(\omega t)\cos(\mathbf{k}\cdot(\mathbf{x}-\mathbf{y}))\right\}. \tag{9}$$

This correlation function results from independent contributions from each mode of oscillation of the field. With some of the characteristics suggested by this local environment in mind, I now turn to the actual construction of the model.

2 The Phenomenological Model

The initial form of the evolution of the density operator is taken to be in the Lindblad[12] form (Schrödinger picture):

$$\frac{\partial \rho}{\partial t} = L[\rho] = \frac{1}{i\hbar}[H,\rho] + \frac{1}{2\hbar}\sum_\mu [V_\mu\rho, V_\mu^\dagger] + [V_\mu, \rho V_\mu^\dagger] = \frac{1}{i\hbar}[H,\rho] + \Delta L[\rho], \tag{10}$$

for which there is a corresponding form for the Heisenberg picture $L^*[O]$ which can readily be obtained from the cyclic properties of the trace. For a finite dimensional Hilbert space, this form is the most general for a completely positive dynamical semigroup. For infinite dimensional Hilbert spaces, it is a reasonable starting point. I will be focusing on the nonunitary part of the evolution, ΔL.

The construction of the model is essentially the determination of the operators V_μ, subject to the constraint that the dissipation is ohmic (expressed as an operator condition). This constraint produces the "correct" classical phenomenology, but does not completely determine the model. However, linear dissipation almost forces the V_μ to be at most linear in momentum, that is

$$V_\mu = A_\mu(\mathbf{x}) - \mathbf{B}_\mu(\mathbf{x})\cdot\mathbf{p}. \tag{11}$$

Homogeneity and isotropy also serve to constrain the model. Assuming some sort of mode by mode interaction with a field, a reasonable choice is given by:

$$\{V_\mu\} \;= \{\alpha(k)e^{i\mathbf{k}\cdot\mathbf{x}} - \beta(k)e^{i\mathbf{k}\cdot\mathbf{x}}\mathbf{k}\cdot\mathbf{p}\}. \tag{12}$$

The discrete index μ has been replaced by the continuous index $\mathbf{k}$. The model is then completely specified by the complex functions α and β.

The resulting nonunitary contribution to the Schrödinger equation is given by the expression:

$$\begin{aligned}\Delta L[\rho] &= -\int d^dk \tfrac{|\alpha(k)|^2}{\hbar}(\rho - e^{i\mathbf{k}\cdot\mathbf{x}}\rho e^{-i\mathbf{k}\cdot\mathbf{x}}) - \int d^dk \tfrac{|\beta(k)|^2}{\hbar}(\tfrac{1}{2}\{(\mathbf{k}\cdot\mathbf{p})^2,\rho\} - e^{i\mathbf{k}\cdot\mathbf{x}}\mathbf{k}\cdot\mathbf{p}\rho\mathbf{k}\cdot\mathbf{p}e^{-i\mathbf{k}\cdot\mathbf{x}}) \\ &- \int d^dk \tfrac{\mathrm{Re}[\alpha(k)^*\beta(k)]}{\hbar}(e^{i\mathbf{k}\cdot\mathbf{x}}\{\mathbf{k}\cdot\mathbf{p},\rho\}e^{-i\mathbf{k}\cdot\mathbf{x}}) - \int d^dk \tfrac{i\mathrm{Im}[\alpha(k)^*\beta(k)]}{\hbar}(e^{i\mathbf{k}\cdot\mathbf{x}}[\mathbf{k}\cdot\mathbf{p},\rho]e^{-i\mathbf{k}\cdot\mathbf{x}}). \quad (13)\end{aligned}$$

The position representation of the new model is given by:

$$\begin{aligned}\frac{\partial\rho(x,x';t)}{\partial t} &= \text{Hamiltonian terms} - \left(\int dk \frac{|\alpha(k)|^2}{\hbar}(1-\cos k(x-x'))\right)\rho(x,x';t) \\ &- \left(\frac{2}{\hbar}\int dk \mathrm{Re}[\alpha^*(k)\beta(k)]k\sin k(x-x')\right)\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)\rho(x,x';t) \\ &- \left(i\int dk \mathrm{Im}[\alpha^*(k)\beta(k)]k\sin k(x-x')\right)\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial x'}\right)\rho(x,x';t) \\ &+ \left(\int dk|\beta(k)|^2\hbar k^2\right)\left(\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial x'^2}\right)\rho(x,x';t) \\ &+ \left(\int dk|\beta(k)|^2\hbar k^2\cos k(x-x')\right)\left(\frac{\partial^2}{\partial x\partial x'}\right)\rho(x,x';t). \quad (14)\end{aligned}$$

The first nonhamiltonian term is responsible for decoherence. The corresponding noise spatial correlation is determined by $\alpha(k)$. The characteristic length should be on the order of the inverse of the "width" of $|\alpha(k)|^2$ in k space. The second nonhamiltonian term is responsible for the dissipation. Clearly the dissipation and other terms are more complicated in this new model. However, that would also be expected from a more elaborate first principle calculation.

By examining the Eherenfest relations of physical quantities using L^*, some interesting physical features of the new model emerge. By construction, the average position and momentum obey relations corresponding to ohmic dissipation:

$$\frac{d\langle P\rangle}{dt} = \frac{i}{\hbar}\langle[H,P]\rangle - \frac{\eta}{m}\langle P\rangle, \quad \frac{d\langle x\rangle}{dt} = \frac{i}{\hbar}\langle[H,x]\rangle = \frac{\langle P\rangle}{m}, \quad (15)$$

where

$$\frac{\eta}{m} = \int dk 2\mathrm{Re}(\alpha^*(k)\beta(k))k^2. \quad (16)$$

With only limited constraints on α and β (γ must be positive), the kinetic energy is seen to be thermalized:

$$\frac{d}{dt}\langle(\frac{P^2}{2m}-\frac{k_BT}{2})\rangle = \langle\frac{i}{\hbar}[H,(\frac{P^2}{2m}-\frac{k_BT}{2})]\rangle - \gamma\langle(\frac{P^2}{2m}-\frac{k_BT}{2})\rangle, \quad (17)$$

where

$$\gamma \equiv 2\frac{\eta}{m} - \int d^dk\frac{|\beta(k)|^2}{d}\hbar k^4, \; \frac{k_BT}{2} \equiv \frac{1}{\gamma}\int d^dk\frac{|\alpha(k)|^2}{2m}\hbar k^2. \quad (18)$$

The effective temperature is determined by α and β.

A low length scale approximation of the new model can be obtained by expanding the exponential terms in powers of $\mathbf{k}\cdot\mathbf{x}$:

$$\begin{aligned}\Delta L[\rho] \cong & -\int d^dk\frac{|\alpha(k)|^2}{2\hbar}[\mathbf{k}\cdot\mathbf{x},[\mathbf{k}\cdot\mathbf{x},\rho]]-\int d^dk\frac{|\beta(k)|^2}{2\hbar}[\mathbf{k}\cdot\mathbf{p},[\mathbf{k}\cdot\mathbf{p},\rho]] \\ & -\int d^dk\frac{i\mathrm{Re}(\alpha(k)^*\beta(k))}{\hbar}[\mathbf{k}\cdot\mathbf{x},\{\mathbf{k}\cdot\mathbf{p},\rho\}]+\int d^dk\frac{\mathrm{Im}(\alpha(k)^*\beta(k))}{\hbar}[\mathbf{k}\cdot\mathbf{x},[\mathbf{k}\cdot\mathbf{p},\rho]]. \quad (19)\end{aligned}$$

The lowest nonvanishing terms are second order, which exactly reproduces the Dekker master equation for 1 dimension. As a result, we can think of the Dekker or Caldeira-Leggett equations as a low length scale approximation for more general models.

On the other hand, the Caldeira-Leggett master equation,

$$\Delta L[\rho]=\frac{\eta}{i2m\hbar}[x,\{p,\rho\}]-\frac{\eta k_BT}{\hbar^2}[x,[x,\rho]], \quad (20)$$

can be considered a special case of the Dekker master equation, with the D_{xp} terms equal to zero (which Dekker has argued should be the case) and an additional low momentum approximation which ignores the D_{xx} term. With this type of special case in mind, we can construct a low momentum approximation for the new model which includes only the decoherence and dissipation terms:

$$\Delta L[\rho]= -\int d^dk\frac{|\alpha(k)|^2}{\hbar}(\rho-e^{i\mathbf{k}\cdot\mathbf{x}}\rho e^{-i\mathbf{k}\cdot\mathbf{x}}) -\int d^dk\frac{\mathrm{Re}(\alpha(k)^*\beta(k))}{\hbar}(\{\mathbf{k}\cdot\mathbf{p},e^{i\mathbf{k}\cdot\mathbf{x}}\rho e^{-i\mathbf{k}\cdot\mathbf{x}}\}). \quad (21)$$

This would seem to be a likely starting point for applications of this model. However, this approximation is not a positive form for the dynamics.

Finally, I would like to look at the Wigner representation of the new model, which has some interesting features. If we expand the terms of the evolution equation in powers of $\hbar$ (in the same manner as is typically done with the potential),

$$\begin{aligned}\dot{W}(q,p)= & -\frac{1}{m}\frac{\partial}{\partial q}(pW)+\frac{\partial}{\partial p}(V'(q)W)+\sum_{n=1}^{\infty}\frac{(\hbar)^{2n}(-1)^n2^{-2n}}{(2n)!}V^{(2n+1)}\frac{\partial^{2n+1}}{\partial p^{2n+1}}W \\ & +\lambda\frac{\partial}{\partial p}(pW)+\sum_{n=1}^{\infty}(\hbar)^{2n}\left(\int dk\frac{2\mathrm{Re}(\alpha^*\beta)k^{2n+1}}{(2n+1)!}\right)\frac{\partial^{2n+1}}{\partial p^{2n+1}}W \\ & +D_{pp}\frac{\partial^2}{\partial p^2}W+\sum_{n=1}^{\infty}(\hbar)^{2n+1}\left(\int dk\frac{|\alpha|^2k^{2n+2}}{(2n+2)!}\right)\frac{\partial^{2n+2}}{\partial p^{2n+2}}W \\ & +(D_{xp}+D_{px})\frac{\partial^2}{\partial p\partial q}W+\sum_{n=1}^{\infty}(\hbar)^{2n+1}\left(\int dk\frac{\mathrm{Im}(\alpha^*\beta)k^{2n+2}}{(2n+1)!}\right)\frac{\partial}{\partial q}\frac{\partial^{2n+1}}{\partial p^{2n+1}}W \\ & +D_{xx}\frac{\partial^2}{\partial q^2}W+\sum_{n=1}^{\infty}(\hbar)^{2n-1}\left(\int dk\frac{|\beta|^2k^{2n+2}}{(2n)!}\right)\frac{\partial^n}{\partial p^n}\left(\frac{\hbar^2}{4}\frac{\partial^2}{\partial q^2}+p^2\right)W, \quad (22)\end{aligned}$$

the lowest order terms correspond exactly to the Wigner representation of the Dekker equation. The Wigner representation of the Dekker equation is a standard classical type of diffusion equation. This illustrates the idea that the "classical" nature of the system emerges when coherent superpositions are not important in the dynamics. In this case, the relevant superpositions are between different locations separated by distances on the order of the environment correlation length.

The convolution theorem can also be used to write down the Wigner representation of the evolution:

$$\begin{aligned} W(q,p) &= (\text{Hamiltonian terms}) - \frac{2}{\hbar}\int dp'(p-p')W(q,p-p')\frac{p'\mathrm{Re}[\alpha^*\left(\frac{p'}{\hbar}\right)\beta\left(\frac{p'}{\hbar}\right)]}{\hbar^2} \\ &- \left(\int dk\frac{|\alpha|^2(k)}{\hbar}\right)W(q,p) + \int dp' W(q,p-p')\frac{|\alpha\left(\frac{p'}{\hbar}\right)|^2}{\hbar^2} \\ &- \frac{\partial}{\partial q}\int dp' W(q,p-p')\frac{p'\mathrm{Im}[\alpha^*\left(\frac{p'}{\hbar}\right)\beta\left(\frac{p'}{\hbar}\right)]}{\hbar^2} \\ &+ D_{xx}\left(\frac{1}{4}\frac{\partial^2}{\partial q^2} - \frac{p^2}{\hbar^2}\right)W(q,p) + \int dp'\frac{p^2|\beta\left(\frac{p'}{\hbar}\right)|^2}{\hbar^2}\left(\frac{1}{4}\frac{\partial^2}{\partial q^2} + \frac{(p-p')^2}{\hbar^2}\right)W(q,p-p'). \quad (23) \end{aligned}$$

One apparent effect in the new model is a spreading induced by these convolution terms.

In summary, a new phenomenological master equation for ohmic dissipation and decoherence has been constructed which has completely positive dynamics. The new model has the features expected from local coupling to a homogeneous environment: specifically, the evolution is istropic and translationally invariant. Spatial correlations of the environment appear explicitly in the models. The new model also includes previous results as low length scale approximations.

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