N95- 13950

GENERAL PROPERTIES OF QUANTUM OPTICAL SYSTEMS IN A STRONG FIELD LIMIT

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Abstract

We investigate the dynamics of an arbitrary atomic system (n -level atoms or many n-level atoms) interacting with a resonant quantized mode of an em field. If the initial field state is a coherent state with a large photon number then the system dynamics possesses some general features, independently of the particular structure of the atomic system. Namely, trapping states, factorization of the wave function, collapses and revivals of the atomic energy oscillations are discussed.

1 Introduction

The Jaynes - Cummings Model is of principle importance in quantum optics. It consists of a single atom interacting with a single mode of quantized em field in a lossless cavity. The properties of JCM in the region of a strong coherent quantum field are:

1. Collapses and revivals of atomic inversion oscillations [1].
2. The existance of trapping states [2]. (These are initial atomic states which lead to the constant mean value of atomic energy in the course of the interaction with the field).
3. Wave Function (WF) factorization in the trapping states [3]. (This very unusual property of the JCM trapping states means that the field and atomic subsystems remain to a high accuracy in pure states, in spite of the presence of interaction).
4. WF factorization for an arbitrary initial atomic state at a half revival time. For this time moment the field state is a coherent superposition of macroscopically different states (so called Schrödinger cat) [4, 3].

Here, we address to ourselves the following question:
Which phenomena from this list will survive for an arbitrary atomic system interacting with a strong quantum field ?

2 What is an arbitrary atomic system ?

We start with some examples:

1. many two-level atoms in a cavity (the Dicke Model, [5]);
2. many three-level atoms of arbitrary level configurations in a cavity;
3. many n-level atoms of arbitrary degeneracy of the levels.

All these systems are described by Hamiltonians of the form:

$$\hat{H} = \omega\left(\hat{a}^{\dagger}\hat{a} + \hat{h}\right) + g\left(\hat{a}\hat{X}_{+} + \hat{a}^{\dagger}\hat{X}_{-}\right) \tag{1}$$

Here, $\hat{a}^{\dagger}, \hat{a}$ are photon operators; $\hat{h}$ is the bare Hamiltonian of the atomic system; $\hat{X}_{+}, \hat{X}_{-}$ are atomic transition operators. We suppose, that the following commutation relations are valid:

$$\left[\hat{h}, \hat{X}_{+}\right] = \hat{X}_{+} \qquad \left[\hat{h}, \hat{X}_{-}\right] = -\hat{X}_{-} \tag{2}$$

The model formulated in such a way implies the Rotating Wave Approximation and, therefore, the excitation number conservation:

$$[\hat{H}, \hat{N}] = 0, \qquad \hat{N} = \hat{a}^{\dagger}\hat{a} + \hat{h}. \tag{3}$$

It is usually the case in Quantum Optics. We do not impose any conditions on the commutator

$$[\hat{X}_{+}, \hat{X}_{-}]$$

which leads to a large freedom in the specification of the atomic subsystem.

We adopt also the exact resonance condition. It means that transition frequencies between neighboring levels are equal to the field frequency ω. This condition is imposed with a sake of simplicity (the arbitrary detunings can be also involved in our approach). Stress, that the cavity is supposed to be a perfect one. We do not discuss the dissipation processes here.

3 The classical field limit

For to approach the classical field limit one has to take a coherent field states (CS) with large photon numbers. Then one may substitute

$$\hat{a} \to \alpha, \quad \hat{a}^{\dagger} \to \overline{\alpha},$$

where $\alpha \equiv \sqrt{\overline{n}}e^{i\phi}$ is a CS parameter. Then the quantum interaction Hamiltonian becomes proportional to the operator

$$\hat{H}_{int} \to \hat{H}_{cl} = e^{i\phi}\hat{X}_{+} + e^{-i\phi}\hat{X}_{-}.$$

H_{cl} has the sense of the atomic Hamiltonian in a constant classical field. We will call the eigenvectors

$$\hat{H}_{cl} \mid \underline{p} >_{at} = \lambda_{p}^{0} \mid \underline{p} >_{at} \tag{4}$$

the *semiclassical eigenvectors.*

4 Factorization of the Wave Function

We now formulate our principle result [6]. *Let initial atomic state is semiclassical eigenstate and initial field state is CS with a large photon number:*

$$| in >=| \underline{p} >_{at} \otimes | \alpha > \tag{5}$$

Then the WF will be approximately factorized for the time up to $gt \sim \overline{n}$. The factorized WF has a form:

$$\begin{aligned} | \Psi(t) > &\cong | \Phi_p(t) > \otimes | A_p(t) >, \\ | \Phi_p(t) > &= \sum_n p_n e^{-igt\lambda_p^0\sqrt{n-C+1/2}} | n >_f, \\ | A_p(t) > &= e^{-ict} e^{-it\omega_p \hat{h}} | \underline{p} >_{at} \ . \end{aligned} \tag{6}$$

Here, p_n - initial CS amplitudes, C is the energy ground level of the bare atomic system, e^{-ict} is a phase factor which we do not write down explicitly here.

We do not give here the proof of eqs.(6) (see [6]). Instead, let us qualitatively describe the system behavior. Eqs.(6) stand that the field and the atomic subsystem remain approximately in pure states in the course of the evolution (not in the mixed ones!). However, the two subsystems essentially interact. The field evolution depends on the semiclassical atomic eigenfrequency λ_p^0. The field state rotates along the circle of radius $\sqrt{\overline{n}}$ in the phase space slowly loosing the shape of the initial CS (spreading in phase).

The atomic subsystem is rotated by the free atomic hamiltonian $\hat{h}$ with the angular velocity

$$\omega_p \equiv \frac{g\lambda_p^0}{2\sqrt{\overline{n} - C + 1/2}} \tag{7}$$

dependent on the initial photon number $\overline{n}$.

We have proved eqs.(6) by means of the perturbation theory with the operator-valued small parameter $1/(\hat{N} + 1/2)$. We neglected the terms of order $O(\overline{n}^{-1/2})$ the amplitudes. However, we kept higher accuracy in the frequencies (neglecting the terms of order $O(1/\overline{n})$). It means that our approximation holds for times up to $gt \sim \overline{n}$. It is a usual situation, that the system dynamics is more sensitive to the small corrections in frequencies, than in amplitudes.

5 Trapping states

It follows immediately from eqs. (6), that all the semiclassical eigenstates of an arbitrary atomic system are trapping states, i.e. the mean value of the atomic inversion does not evolve with time:

$$< \hat{h}(t) > = < \Psi(t) \mid \hat{h} \mid \Psi(t) > = < A_p(t) \mid \hat{h} \mid A_p(t) > =$$

$$_{at} < \underline{p} \mid \hat{h} \mid \underline{p} >_{at} = \ const.$$

This result holds with the accuracy $O(\overline{n}^{-1/2})$, due to our accuracy for the transition amplitudes.

6 Collapses and Revivals

This phenomenon, discovered initially for the JCM [1] appears for an arbitrary atomic system in a strong coherent field. Indeed, an arbitrary initial state of the atomic system can be expanded in the basis of the semiclassical eigenstates (4):

$$\mid \Psi >_{at} = \sum_{p} C_p \mid \underline{p} >_{at} .$$

Then, for the atomic inversion we get:

$$< \Psi(t) \mid \hat{h} \mid \Psi(t) > = \sum_{p,q} \overline{C}_q C_p < \Phi_q(t) \mid \Phi_p(t) >< A_q(t) \mid \hat{h} \mid A_p(t) > .$$

The atomic inversion is determined by the scalar products of the field states (6):

$$< \Phi_q(t) \mid \Phi_p(t) > = e^{-\overline{n}} \sum_{n} \frac{\mid \alpha \mid^{2n}}{n!} e^{-igt(\lambda_q^0 - \lambda_p^0)\sqrt{n-A/2+1/2}}.$$

This is a direct generalization of the well-known unharmonic series for the JCM, which also contains collapsing and reviving Rabi oscillations. The collapse, revival times and envelopes can be easily found.

The atomic matrix elements entering in eq.(6) are slowly varying functions oscillating with the frequency eq.(7). They may modulate the revival envelopes. However, for the Dicke model case these matrix elements to be equal to zero if $p \neq q \pm 1$ and the correspondent revivals disappear. If $p = q \pm 1$, the atomic matrix elements equals to 1 just for the time moments of revivals.

7 Schrödinger Cat.

For the JCM case there are only two factorized states. An arbitrary initial state leads to the superposition of them. The two JCM atomic states $\mid A_p(t) >$, $p = 0, 1$ are nothing but two different spin-1/2 coherent states and they can be transformed one to another by rotation. It just happens at a half revival time $t_R/2$ [3]:

$$\mid A_0(t_R/2) > = \mid A_1(t_R/2) > = \mid \psi_0 >_{at},$$

and then the system WF is factorized for an arbitrary initial atomic state:

$$\mid \Psi(t_R/2) > = \mid \psi_0 >_{at} (\mid \Phi_0(t_R/2) > + \mid \Phi_1(t_R/2) >)$$

Therefore, at this time moment the field WF is the quantum superposition of macroscopically different states $\mid \Phi_p(t) >$, $p = 0, 1$.

It is clear, that for larger atomic system (say for spin-1) not all different atomic semiclassical eigenstates can be transformed one into another by rotation, and the Schrödinger cat is absent for arbitrary initial atomic state (as it has been recently noticed in the work [7]

8 Conclusions

We have solved the problem of the interaction of an arbitrary atomic system with the strong quantized em field in a lossless cavity. The key point of our solution is the wave function factorization (6) for the specially chosen initial states (4).

Since these states form a complete basis, this gives possibilities for an exhaustive description of the system dynamics.

Being reduced to the JCM, our results reproduce the treatment of J. Gea-Banacloche [3].

For the case of the Dicke model, they correspond to the first two orders of the perturbation theory proposed in the work [8] and can be treated as a direct generalization of that scheme for the arbitrary atomic system. From the mathematical point of view, our treatment is connected with the concept of dynamical symmetry group for the quantum optical systems [9]. (Note, that this dynamical symmetry is approximate rather than exact one).

Stress, that our method allows to make explicit analytical calculations of any physical quantities for the systems under study.

9 Acknowledgments

S.Ch. and A.K. acknowledge the support from CONACYT.

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