6P

STABILIZATION OF THERMOCAPILLARY CONVECTION BY MEANS OF NONPLANAR FLOW OSCILLATIONS

R. E. Kelly
Arthur Or
Mechanical, Aerospace and Nuclear Engineering Department
University of California
Los Angeles, CA 90024-1597

ABSTRACT

Nonplanar flow oscillations have been shown to be effective in stabilizing buoyancy-induced Rayleigh-Benard convection. The present study was initiated to see if thermocapillary convection of the Marangoni type might also be stabilized by the same means. When surface deflection can be ignored, significant stabilization occurs. However, when the operating parameters are such that surface deflection is nonnegligible, destabilization can occur, in contrast to Rayleigh-Bénard convection. Mechanisms for both stabilization and destabilization are discussed.

INTRODUCTION

Thermal convection of the classical Rayleigh-Bénard type is predicted to begin in the form of convection rolls (or vortices) for the case of a Boussinesq fluid. Due to horizontal isotropy, no preferred direction occurs for the rolls and, unless a well-defined initial disturbance is imposed, the pattern of convection in a large aspect ratio container is somewhat jumbled. If a horizontal, unidirectional shear flow exists in the fluid layer that is heated, convection begins at the same value of critical Rayleigh number (Ra_c) as without shear but a well-defined pattern of rolls with axes in the flow direction (called "longitudinal rolls") is predicted to be the preferred pattern of convection and has been often observed in experiments (for a review, see ref. 1). This result means that the shear has a stabilizing effect upon all disturbances that have a nonzero wavenumber in the direction of the shear. For longitudinal rolls, the value of this wavenumber component is zero, and the stabilizing effect does not occur.

Now consider the situation when the shear flow has two horizontal components of velocity that vary in the vertical direction so that the direction of shear varies continuously with height. It is then impossible for a disturbance to orient itself so as to escape the effect of shear, and so we should expect disturbances associated with buoyancy to be stabilized, i. e. Ra_c will then increase with the Reynolds number, Re. This result should hold even when the shear is periodic in time, meaning that the effect can in principle be observed in a laboratory experiment by suitably oscillating one or both surfaces in their own planes, thereby eliminating the need for the kind of flow apparatus required for a steady flow with a net mass through-flow. Kelly and Hu (ref. 2) have made a detailed analysis of the case of oscillatory shear for small values of Re and concluded that stabilization does indeed occur, that the degree of stabilization increases with the Prandtl number (Pr), and that maximum stabilization occurs when the nondimensional forcing frequency is of order unity. They also predicted that the pattern of convection consists of rolls with axes in the direction of the dominant velocity component. In further work, Hu and Kelly (ref. 3) report results for finite values of Re which indicate that quite significant stabilization might occur, e. g. , Ra_c is increased by a factor of sixteen for $Pr = 10$ as Re increases up to a value of 100, which is probably small enough to avoid any hydrodynamic shear instability (although the critical value of Re for such an instability is difficult to predict).

It is felt that this result might be of value to certain applications in materials processing when it is desirable to avoid thermal convection in order to achieve a more uniform product. Whether or not it is of practical value depends on whether the effect can be realized in more practical configurations, such as an oscillating disk.

Due to these encouraging results, we decided to see if similar stabilization is predicted for thermocapillary convection of the Marangoni type that occurs via an instability. Marangoni convection is generally regarded as being of importance in space applications when Rayleigh-Bénard convection is no longer dominant.

THE CASE OF MARANGONI CONVECTION

Marangoni convection (or thermocapillary convection of the Bénard type) occurs in a fluid layer with at least one free surface along which surface tension can act so as to drive convection if the surface tension varies in magnitude along the surface due to its dependence upon a spatially varying temperature associated with a thermal disturbance. The general problem of a deformable surface involves several nondimensional parameters, such as the Marangoni number (Ma), the Prandtl number (Pr), the Bond number (Bo), the Biot number (Bi), and the Crispation number (Cr) which tends to characterize the deformability of the surface. If gravity is nonnegligible, then the Rayleigh number must also be considered. Due to this complexity, a clear picture of the overall problem can be difficult to achieve, although understanding of the problem associated with a nondeformable surface has been available since the pioneering work of Pearson (ref. 4). Nonetheless, by considering various limits it is now understood that two distinct modes with different physical characteristics can participate in the instability, although each naturally involves the thermocapillary effect (e. g. , see ref. 5). For the Pearson mode, surface deformation is not essential, and the resulting convection is described by a wavelength of the same order as the layer depth. For the other mode which is associated with Scriven and Sternling (ref. 6) and Smith (ref. 7), surface deformation is essential, and the characteristic wavelength can be much greater than the layer depth. Hence, it will be referred to as the long wavelength mode in this paper, with the caveat that a clear distinction between the two modes is possible only under certain operating conditions.

We have considered the effects of a nonplanar flow oscillation upon the critical Marangoni number (Ma_c) by two approaches (ref. 8). In the first approach, the Reynolds number is assumed to be small, and an expansion is made in terms of Re, i. e.,

$$Ma_c = Ma_o + ReM_1 + Re^2M_2 + \ldots$$

At each order of Re, the corresponding value of M_j is determined (e. g., $M_1 = 0, M_2 \neq 0$) by means of a solvability condition. For small Re, the change in Ma_c from the corresponding value without shear is small, but this approach helps us to obtain some feeling for how the result depends qualitatively on the various physical parameters. For finite values of Re, a numerical solution to the linear stability equations is obtained by means of a Fourier-Chebyshev expansion with time-dependent amplitudes. In this way, a set of coupled ordinary differential equations with time-periodic coefficients is obtained which can be analyzed by the use of Floquet theory (see ref. 8).

Some results for the case of a nondeformable surface as obtained from the small Re expansion are shown in Fig. 1. The factor M_2 is plotted versus the nondimensional frequency β. It is clear that stabilization occurs ($M_2 > 0$) with the degree of stabilization increasing with Pr. Furthermore, an optimal value of β exists that gives maximum stabilization. In Fig. 2, the effect of Biot number is shown, and it is clear that M_2 decreases as Bi increases. When the surface is allowed to be deformable, the same analysis indicates that long wavelength disturbances can be destabilized by the oscillation, as shown in Fig. 3. Stabilization occurs only for the highest wavenumber that is associated with the Pearson mode. Calculations are currently being done for the disturbance energy budget in order to gain insight into the destabilizing mechanism. It might be associated with the action of the perturbation shear stress at the deformed surface, which is the mechanism associated with the instability of steady film flow down an inclined plane (ref. 9). This conjecture is made on the basis of other results (not shown) that indicate that M_2 for this case is rather insensitive to changes in Bi and Pr. As a result, the effect of nonplanar oscillations upon the onset of Marangoni convection depends upon which mode tends to be more unstable, which in turn depends upon the operating conditions (ref. 5). A more detailed presentation of the small Re results is forthcoming (ref. 10).

For cases when surface deformation can be ignored, Fig. 4 indicates that substantial stabilization is possible. Even for a relatively low value of $Re = 20$, an almost two-fold increase in Ma_c is possible for this high Pr case at B_i=0.5. The same numerical code is currently being used to determine the maximum value of Re at which stabilization still occurs for the nondeformable case, as well as predicting the amount of destabilization possible at finite values of Re for the deformable case.

Some preliminary results for the deformable case when Re=25 are shown in Fig. 5, where Ma_c is shown as a function of wavenumber for two different values of β; the neutral curve for $Re = 0$ is also provided as a reference and indicates that long waves are most unstable for these operating conditions. The effect of the oscillations is shown to be strongly destabilizing, even causing instability in the normally stable regime $Ma < 0$. It should be noted that Yih (ref. 11) has shown already that instability can occur due to shear oscillations for the isothermal case ($Ma = 0$).

CONCLUSIONS

It is evident that nonplanar flow oscillations can have a significant effect upon the onset of Marangoni convection. In contrast to Rayleigh-Bénard convection, the effect can be stabilizing or destabilizing, depending upon the operating conditions. The present analysis has been done for a system that is infinite in both horizontal directions. Side boundaries must, of course, generally be considered for actual applications; these can affect the basic flow as well as stabilize the long wavelength disturbances. For the oscillating disk configuration, the effect of spatial inhomogeneity in the flow could also have a strong effect on the long wavelength disturbances.

Finally, it should be remarked that the oscillations should profoundly affect the subcritical instability characteristic of Marangoni convection, in a manner similar to that discussed in (ref. 12) for convection in a slightly non-Boussinesq fluid.

REFERENCES

1. Kelly, R. E., Adv. Appl. Mech. **30** (1994), 35-112.
2. Kelly, R. E. and Hu, H. -C., J. Fluid Mech. **249** (1993), 373-390.
3. Hu, H. C., and Kelly, R. E., Heat Transfer 1994 (Proc. 10^{th} Int. Heat Transfer Conf.) to appear.
4. Pearson, J. R. A., J. Fluid Mech. **4** (1958), 489-500.
5. Goussis, D. A. and Kelly, R. E., Int. J. Heat Mass Transfer **33** (1990), 2237-2245.
6. Scriven, L. E. and Sternling, C. V., J. Fluid Mech. **19** (1964), 321-340.
7. Smith, K. A., J. Fluid Mech. **24** (1966), 401-414.
8. Or, A. C. and Kelly, R. E., AIAA Paper 94-0242 (1994).
9. Kelly, R. E., Goussis, D. A., Lin, S.P., and Hsu, F. K., Phys, Fluids A **1** (1989), 819-828.
10. Or, A.C. and Kelly, R.E., to be published in Int. J. Heat Mass Transfer (1994).
11. Yih, C. -S., J. Fluid Mech. **31** (1968), 737-751.
12. Hall, P. and Kelly, R.E., (1994), submitted for publication.

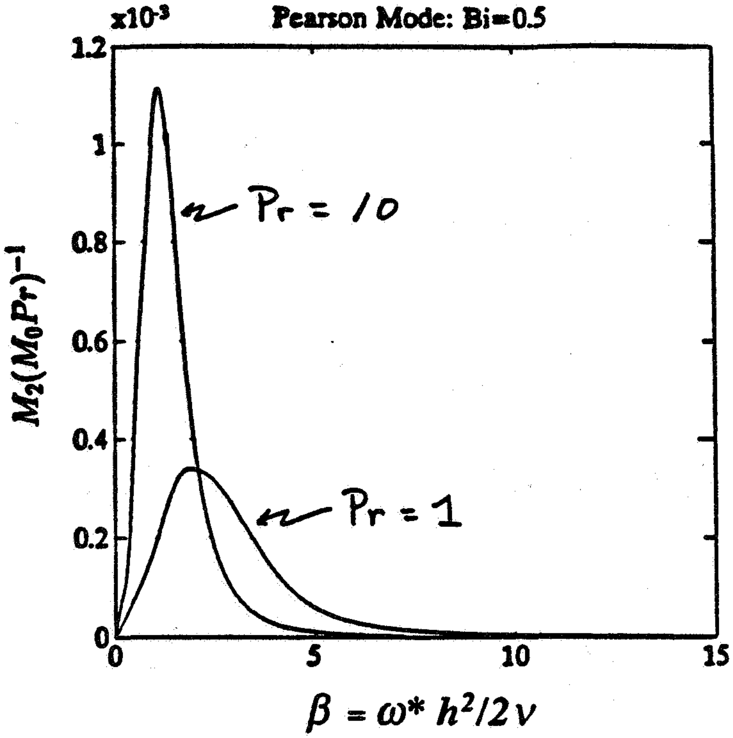

Fig. 1. The change (M_2) in the critical Marangoni number as a function of nondimensional frequency (β) at small values of *Re*. Nondeformable surface.

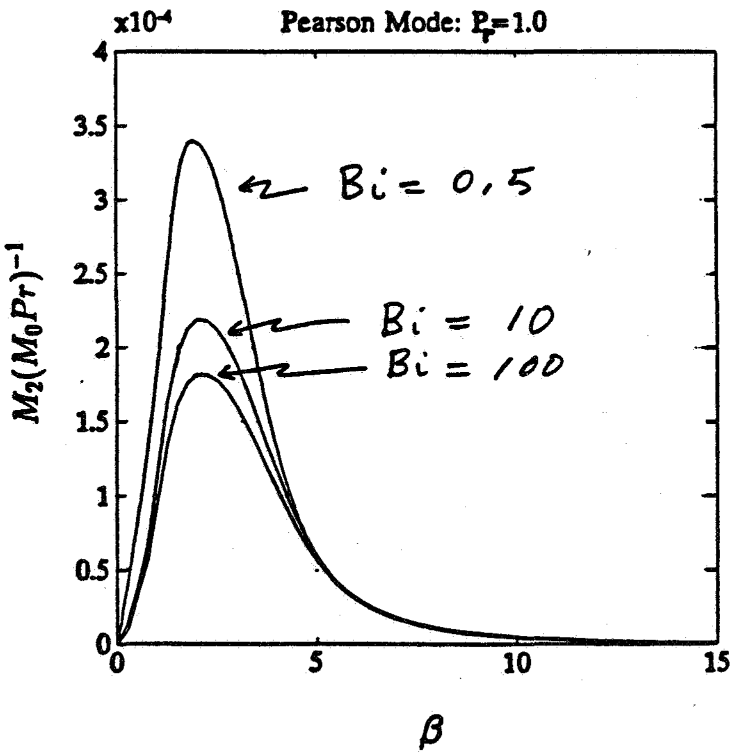

Fig. 2. The change (M_2) in the critical Marangoni number as a function of nondimensional frequency (β) at small values of *Re* for different values of Biot number. Nondeformable surface.

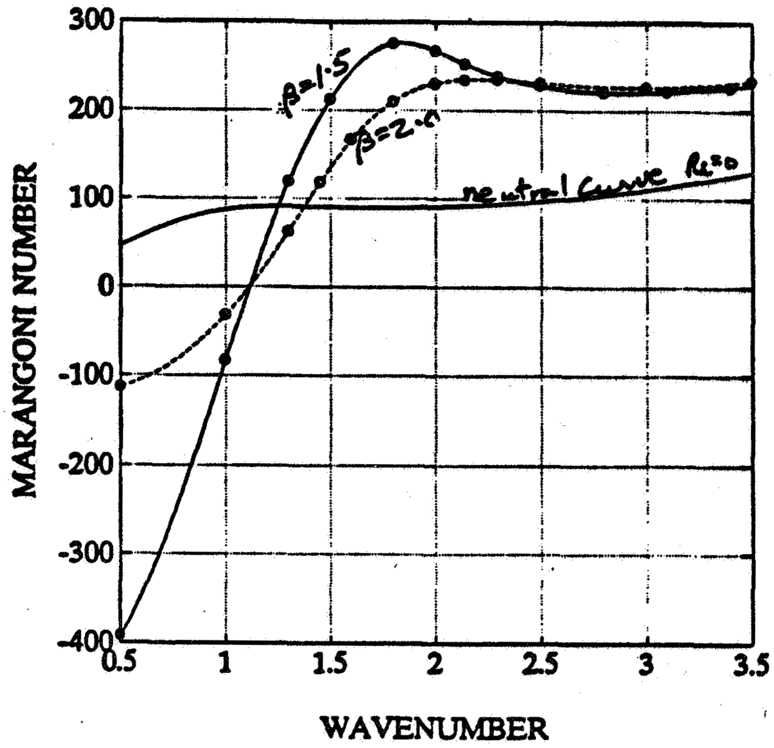

Fig. 5. The critical Marangoni number as a function of wavenumber for the case of a deformable surface with $Re = 25, Pr = 7, B_i = 0.5, Bo = 0.1$, and $\chi = 1.0$.

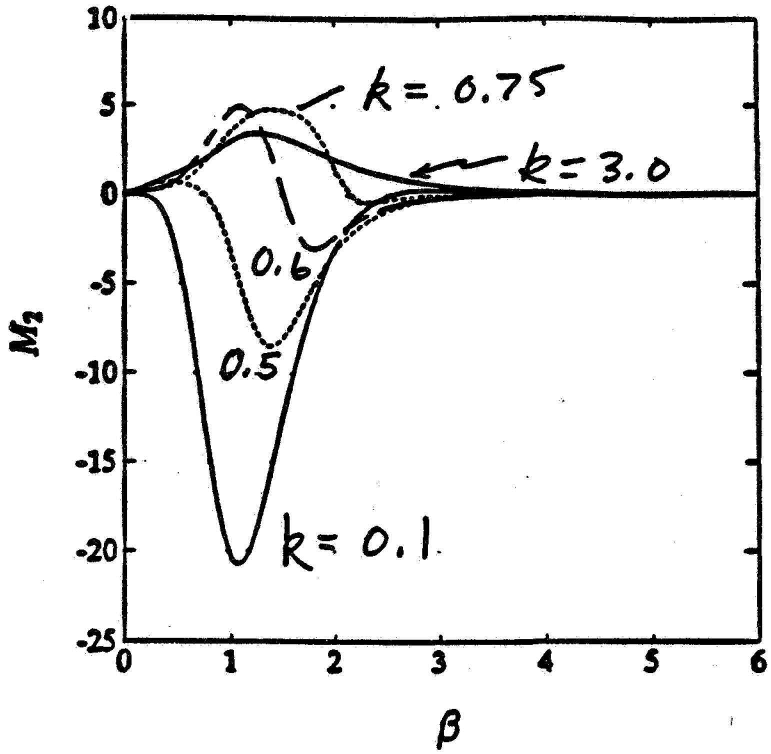

Fig. 3. The change (M_2) in the critical Marangoni number for a deformable surface as a function of frequency (β) for different wavenumbers at small Re. All other parameters are fixed.

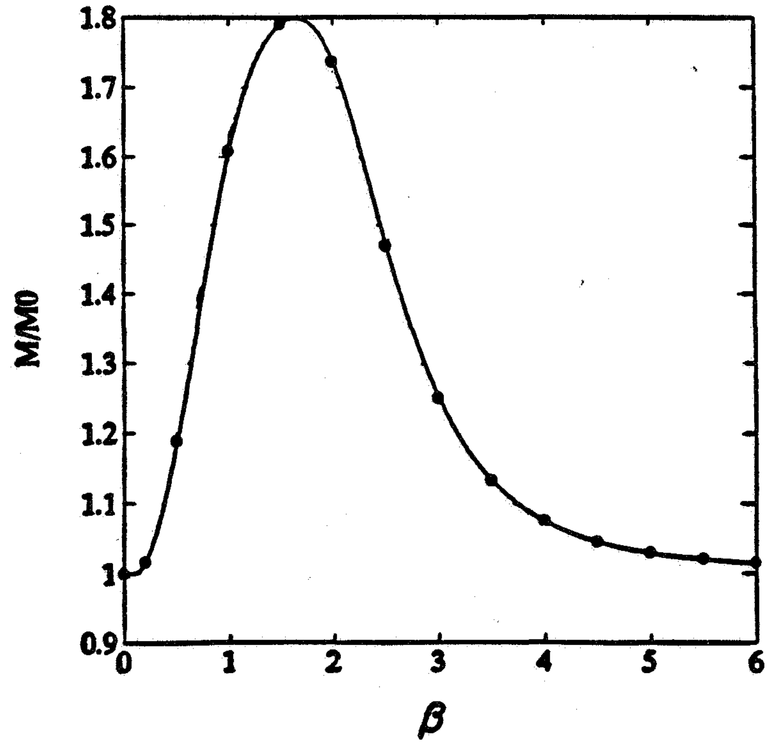

Fig. 4. The critical Marangoni number as a function of nondimensional frequency β for $Re = 20$, $Pr = 7$ and $B_i = 0.5$, with a nondeformable surface for the critical wavenumber at 2.14.