STOCHASTIC MODEL OF THE RESIDUAL ACCELERATION ENVIRONMENT IN MICROGRAVITY

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ABSTRACT

We describe a theoretical investigation of the effects that stochastic residual accelerations (g-jitter) onboard spacecraft can have on experiments conducted in a microgravity environment. We first introduce a stochastic model of the residual acceleration field, and develop a numerical algorithm to solve the equations governing fluid flow that allow for a stochastic body force. We next summarize our studies of two generic situations: stochastic parametric resonance and the onset of convective flow induced by a fluctuating acceleration field.

STOCHASTIC MODEL OF G-JITTER

We have introduced a stochastic model to describe in quantitative detail the effect of the high frequency components of the residual accelerations onboard spacecraft (often called g-jitter) on fluid motion. Each Cartesian component of the residual acceleration field $\vec{g}(t)$ is modeled as a Gaussian narrow band noise characterized by three independent parameters: its intensity $< g^2 >$, a dominant frequency Ω, and a characteristic spectral width τ^{-1}. The autocorrelation function of g-jitter is defined by

$$< g(t)g(t') >=< g^2 > e^{-|t-t'|/\tau} \cos \Omega(t-t'). \tag{1}$$

The power spectrum for this autocorrelation function is,

$$P(\omega) = \frac{1}{2\pi} < g^2 > \tau \left(\frac{1}{1+\tau^2(\Omega+\omega)^2} + \frac{1}{1+\tau^2(\Omega-\omega)^2} \right). \tag{2}$$

For very small τ, $g(t)$ tends to white noise. For very large values of τ, each realization of $g(t)$ is a periodic function of angular frequency Ω. Up to statistical moments of second order, each realization of narrow band noise can be viewed as a temporal sequence of periodic functions of angular frequency Ω with amplitude and phase that remain constant only for a finite amount of time (τ on average). At random intervals new values of the amplitude and phase are drawn from prescribed distributions. This model is based on the following mechanism underlying the residual acceleration field: one particular natural frequency of vibration of the spacecraft structure (Ω) is excited by some mechanical disturbance inside the spacecraft, the excitation being of random amplitude and taking place at a sequence of unknown (and essentially random) instants of time. The power spectrum of such a process is the Lorentzian function given by Eq. (2), centered at $\omega = \Omega$, and of width $1/\tau$. This power spectrum has been shown to provide a reasonable fit to actual power spectra measured during the various microgravity missions, as illustrated in Fig. 1.

The values of the three parameters that define narrow band noise can be estimated from acceleration data measured onboard spacecraft. For example, from acceleration data from Spacelab 3 [1], we find $\sqrt{< g^2 >} \simeq 6 \times 10^{-4} g_E$, where g_E is the intensity of the Earth's gravitational field, $\Omega/2\pi \simeq 11 Hz$, and $\tau \simeq 0.16 s$. Therefore $\Omega\tau \simeq 11$, which is an intermediate value between the white noise and deterministic limits. Of course, different values will be obtained when data from different missions are analyzed, but the values given seem to be fairly typical [2].

Finally, and from a theoretical standpoint, narrow band noise provides a convenient way of interpolating between monochromatic noise (akin to the traditional studies involving a deterministic and periodic gravitational field), and white noise (in which no frequency component is preferred). In the limit $\Omega\tau \rightarrow 0$ with $D =< g^2 > \tau$ finite, narrow band noise reduces to white noise of intensity D; whereas, for $\Omega\tau \rightarrow \infty$ with $< g^2 >$ finite, monochromatic noise is recovered.

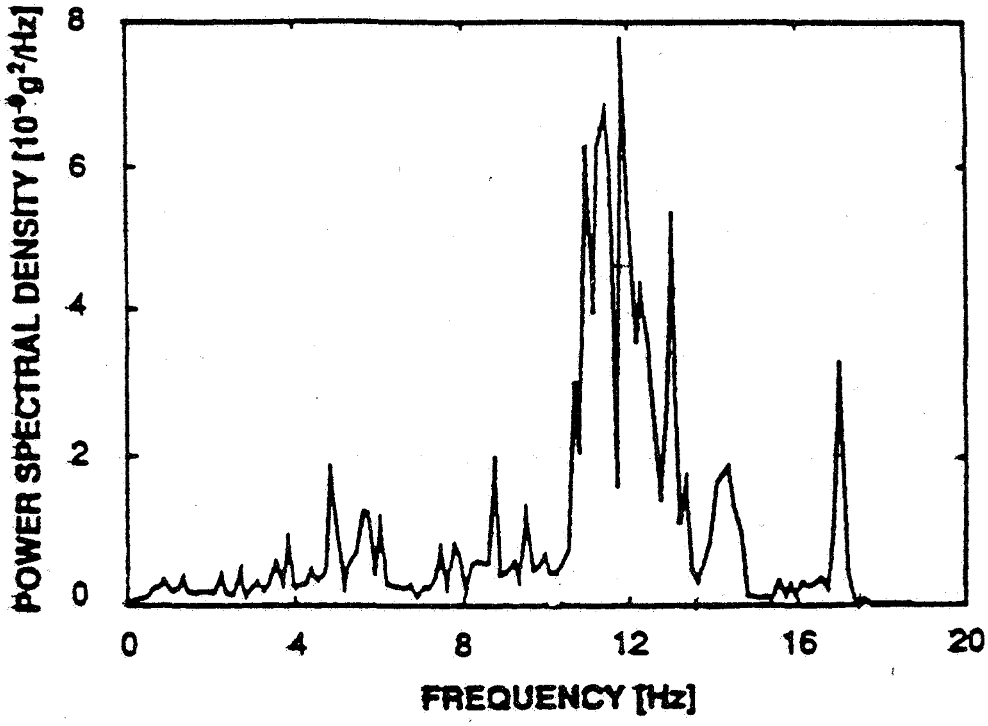

Figure 1: Power spectral density of a representative time window aboard Spacelab 3, [1]. The thick solid curve is the power spectrum given in Eq. (2) with the values of the parameters given in the text.

Numerical algorithm to generate narrow band noise

A realistic analysis of the effect of g-jitter on fluid flow will ultimately involve complex systems and geometries. It has therefore been necessary to develop a numerical algorithm that is able to simulate narrow band noise, and a method to integrate stochastic differential equations which contain this type of noise. We have first developed such a scheme to integrate the ordinary differential equation describing the motion of the parametric oscillator driven by narrow band noise [3]. The method developed is explicit and second order in time, and treats the stochastic contribution exactly.

Briefly, the equation to be solved is first expanded in powers of Δt, with Δt being the time step used in the numerical calculation. There appear terms of the form,

$$\Gamma_1(t,\Delta t) = \int_t^{t+\Delta t} dt' g(t'), \qquad \Gamma_2(t,\Delta t) = \int_t^{t+\Delta t} dt' \int_t^{t'} dt'' g(t''). \tag{3}$$

The number of multiple integrals of $g(t)$ equals the highest order in Δt retained in the discretization of the differential equation. The key point of the algorithm is that the integrals need not be done for each particular realization of the noise and iteration of the time stepping procedure. Instead, the variables $\Gamma_1(t,\Delta t)$ and $\Gamma_2(t,\Delta t)$ are themselves random variables with correlations that can be calculated exactly as a function of the parameters of the original noise and the time step Δt. Thus, during the numerical integration of the equation, a standard random number generator is used to sample directly the random variables Γ_1 and Γ_2 without ever explicitly specifying $g(t)$.

Finally, and in contrast with the case of a deterministic $g(t)$, the integration of the fluid equations has to be performed for an ensemble of realizations of $g(t)$, and the results averaged over the ensemble. Unfortunately, this adds considerably to the computational effort required.

STOCHASTIC PARAMETRIC RESONANCE

The response of an entire class of systems to a time dependent gravitational field can be described as parametric resonance. In simple cases, parametric resonance is modeled by the Mathieu equation for the most unstable mode of the system. The particular case that we have focussed on concerns the linear response of the free surface of an incompressible fluid subjected to a random effective gravity (parametric surface waves). In this case, each spatial Fourier component of the surface displacement satisfies the damped Mathieu equation [4]. Other examples of parametric resonance include Rayleigh-Bénard [5] and thermosolutal convection [6], both in an oscillatory gravitational field.

We have studied by analytical and numerical means the stability of the solutions of the parametric oscillator (Mathieu equation) when the driving force is a narrow band noise. Stability has been defined with respect to the second order statistical moments of the oscillator coordinate because of their relation to the energy of the oscillations. The neutral stability curve (Fig. 2) has been obtained analytically in the limit of low frequencies, and close to subharmonic resonance. An interpolation formula with no adjustable parameters has been derived that reduces to the two asymptotic forms in the two limits discussed. The main assumption underlying the interpolation formula is that, even in the stochastic case, the dominant response of a surface mode of natural frequency ω is subharmonic resonance, and its stability is determined by the intensity of the driving noise at 2ω. The approximation is seen to break down in the limit $\Omega\tau \gg 1$, but is found to be in agreement with the results of numerical computations up to $\Omega\tau \sim 30$. The values of $\Omega\tau$ estimated for g-jitter lie well within this range of validity.

The effects of varying the width of the noise spectrum on the stability of the parametric oscillator can be summarized as follows: In the region close to white noise ($\tau \simeq 0$), the noise intensity is determined by the parameter $D = \langle g^2 \rangle \tau$. The effect of increasing the correlation time τ while keeping D constant is in the direction of increasing stability. The same general conclusion is found to apply for $\Omega \neq 0$. We interpret this result to be a consequence of decreasing the intensity of the power spectrum at the corresponding subharmonic resonant frequency. In the opposite limit, $\Omega\tau \gg 1$, the strength of the external driving force is given by $\langle g^2 \rangle$, which is proportional to the area beneath the power spectrum. In this case, the broadening of the spectrum (decreasing τ) at constant $\langle g^2 \rangle$ has a stabilizing effect on the frequency at subharmonic resonance. This is interpreted as a lack of efficiency in exciting the resonance when the noise effectively spans a larger range of frequencies and cannot persist at resonance for very long times. The same trends would apply to a system which only has a discrete set of natural frequencies.

CONVECTION DRIVEN BY A FLUCTUATING GRAVITATIONAL FIELD

This research addresses the mechanical response of a fluid in which a smooth density gradient exists due, perhaps, to an imposed temperature of composition gradient. In this case, a fluctuating effective gravitational field may induce a number of convective instabilities due to its random value and orientation. The configuration studied is that of cavity flow in a "laterally" heated container [7]. The base state is a quiescent state in a two dimensional square geometry of side L, with an initial uniform temperature gradient along the x direction. We assume that the equation of state of the fluid is given by $\rho = \rho_0 [1 + \alpha(T - T_0)]$, where ρ is the mass density of the fluid at temperature T, ρ_0 is a reference density at some temperature T_0, and α is the thermal expansion coefficient. At $t = 0$, a time dependent gravitational field is turned on. The component of the gravitational field which is parallel to the initial density gradient is much too weak to trigger any fluid motion, although it will contribute to the flow once the fluid is set in motion. We have chosen to neglect this component in our initial study. On the other hand, components of the gravitational field perpendicular to the initial density gradient do not have to exceed a finite threshold in order to induce convection. In this case the quiescent state is not stationary. However, the facts that the actual values taken by the effective gravity are relatively small, and average to zero, raise the question of whether a significant convective flow can be generated, and how it may depend on the parameters of g-jitter.

Approximate analytic solutions to the flow field have been found in the limit in which the temperature field in the cavity is not appreciably distorted by the flow. In this limit of negligible heat transport, one focuses on the mechanical response of the fluid to the fluctuating acceleration field. We have been able to isolate a few important characteristics of cavity flow that result entirely from the stochastic nature of the acceleration field, and that would not have been obtained under a strictly periodic gravity modulation.

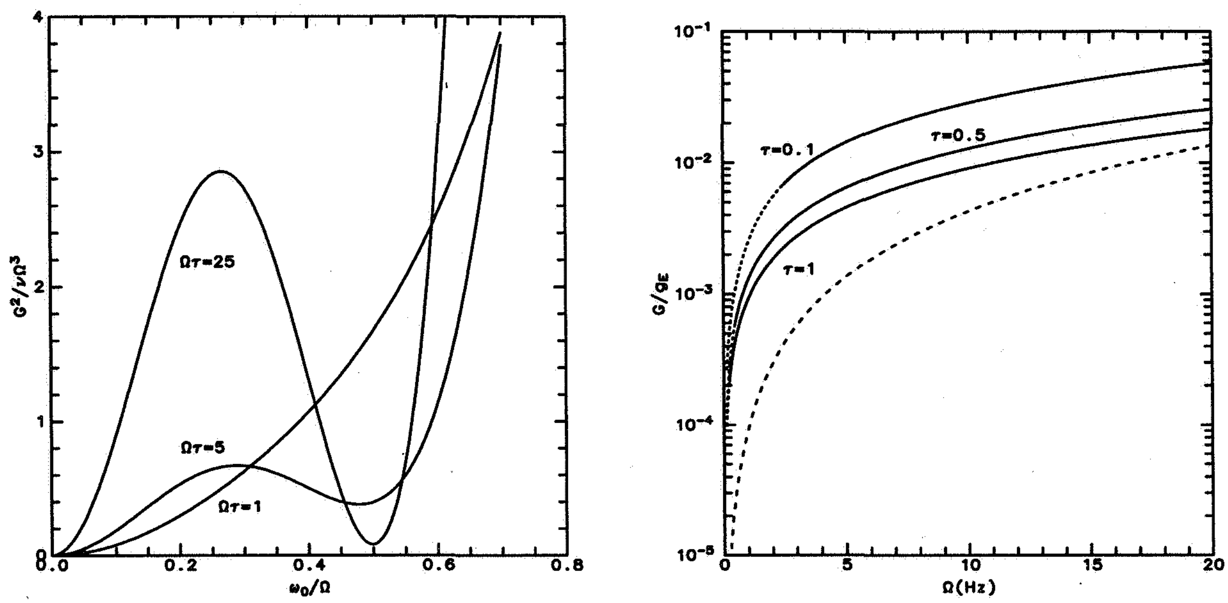

Figure 2: (Left) Stability boundaries for the second moments of a free fluid surface. The figure shows the dimensionless mean squared fluctuations in gravitational acceleration versus the dimensionless frequency of the surface modes. The intensity of the fluctuations in the gravitational field is given by $G^2 = \langle g(t)^2 \rangle$. Different curves show the stability boundaries for various values of the dimensionless correlation time $\Omega\tau$. (Right) Estimate of tolerable levels of g-jitter for instability of a planar water-air surface at room temperature. We show the normalized root mean squared g-jitter for instability as a function of the characteristic frequency of the driving noise (g_E is the intensity of the gravitational field on the Earth's surface). Three different correlation times are shown, as indicated in the figure. The dashed line is the stability curve for the Mathieu equation for the same driving frequency (deterministic case).

Although the imposed acceleration field, and hence the vorticity, average to zero, the vorticity itself can be described by a random walk in time. Specifically, at short times $t < \tau$, the mean square vorticity at the center of the cavity $\langle \xi^2 \rangle$ oscillates with angular frequency Ω, the characteristic frequency of the noise. The amplitude of the oscillatory component decays exponentially with a decay rate $1/\tau$. For $t \gg \tau$, $\langle \xi^2 \rangle$ increases linearly in time according to,

$$\langle \xi^2 \rangle \propto \langle g^2 \rangle \tau \left(\frac{\alpha \Delta T}{L} \right)^2 \frac{t}{1 + (\Omega\tau)^2}, \tag{4}$$

where ΔT is the imposed temperature difference across the cavity. This growth will not continue indefinitely, and $\langle \xi^2 \rangle$ is expected to saturate at a finite value due to viscous dissipation at the walls. The time required to reach saturation, and the maximum value of $\langle \xi^2 \rangle$ attained, are currently under investigation.

Another important consequence of our result is that there is fluid motion regardless of the value of the characteristic frequency Ω. It is often stated in the literature that the main effects of g-jitter on fluid motion are caused by those frequency components of the jitter that are close to inverse characteristic times of the process under study. The case of cavity flow provides a clear counterexample to that rule.

Numerical study of cavity flow

In order to study more realistic situations, a numerical algorithm to study cavity flow in two dimensions under the action of a fluctuating gravitational field has been developed [8]. We have solved the Navier-Stokes

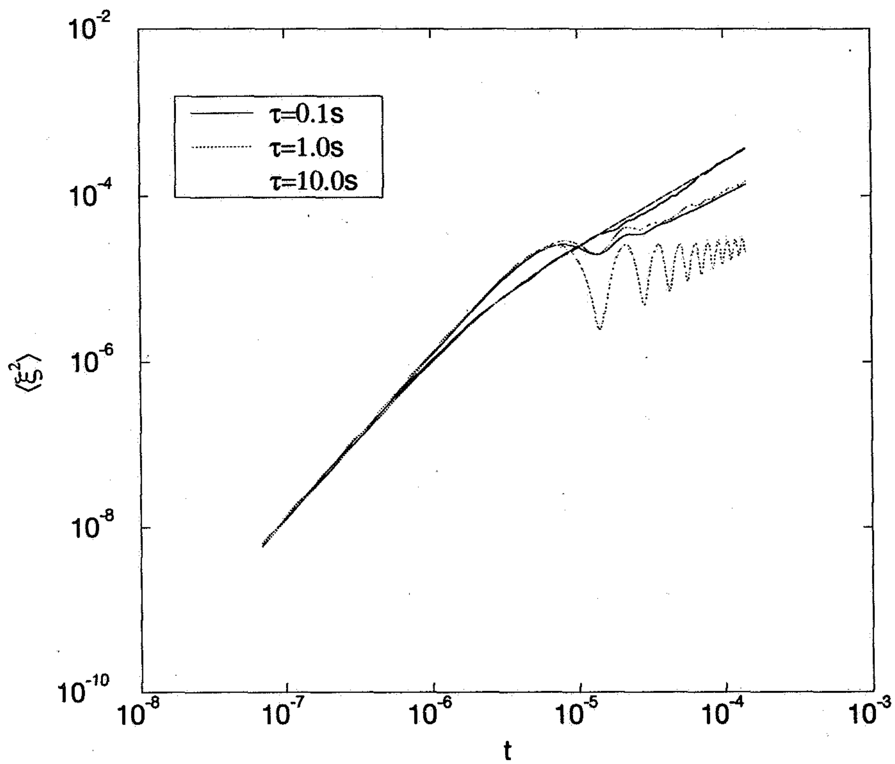

Figure 3: Second moment of the vorticity as a function of time for $\Omega/2\pi = 1Hz$ and (from top to bottom) $\tau = 0.1s, 1s$ and $10s$. The curves shown are the results of both analytic and numerical calculations. The numerical results are averages over 300 independent realizations of the random noise.

equation in the Boussinesq approximation by using a stream function-vorticity formulation. A Forward Time-Centered Space method is used which is first order accurate in time, and second order in space. The small Rayleigh numbers (appropriate for typical microgravity conditions) and the relatively large Prandtl numbers used in our calculations place us well within the range of stability of the technique when purely deterministic functions are involved. The Poisson equation that relates the stream function and the vorticity has been solved by a Successive Over Relaxation technique.

We have thus far explored values of the parameters within the range of validity of the analytic predictions described above. Due to the gentleness of the flow, our calculations are performed on a small, evenly spaced grid. The results for the mean square vorticity at the center of the cavity are shown in Fig. 3 (the numerical results are averages over three hundred independent realizations of the random function). The mean squared vorticity obtained numerically is in excellent agreement with the analytic calculations.

CONCLUSIONS

A stochastic model of the residual acceleration environment onboard spacecraft has been introduced. Not only does it provide a realistic description of the acceleration environment, but it also allows to smoothly interpolate between two well known limits: the white noise limit and the deterministic limit (a periodically modulated gravitational field). Two generic situations have been studied by both analytic and numerical means: a fluid system with a sharp density discontinuity (e.g., an interface separating two immiscible fluids or two coexisting phases of different density), and a system with a smooth density gradient due, perhaps, to an imposed temperature or composition gradient.

We have first concentrated on the response of an interface separating two regions of different density to a fluctuating acceleration field. Two main conclusions have emerged from our study. First, an external driving force with a broad frequency spectrum leads to parametric resonance in a wide frequency range. The resonant behavior is *not* given by the superposition of the resonances produced by each of the frequency components because of the nonlinear coupling between the external force and the oscillator coordinate. Second, the resonant behavior of the second moments (or the energy) is in general *weaker* than the resonance that would result from any of the frequency components alone.

In the case of a smooth density gradient, the component of the gravitational field which is parallel to the initial density gradient is much too weak to trigger any fluid motion in typical microgravity conditions (although it will contribute to the flow once the fluid is set in motion). The perpendicular component, however, does induce fluid flow with an amplitude that grows sublinearly in time: the flow field itself averages to zero but its mean squared value grows linearly with time (until viscous effects lead to saturation).

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References

[1] J.I.D. Alexander, Microgravity Sci. Technol. **3**, 52 (1990).

[2] G.L. Martin, C.R. Baugher and F.H. Henderson, "Summary report of mission acceleration measurements, STS-50", Acceleration Characterization and Analysis Project, NASA (1992).

[3] W. Zhang, J. Casademunt and J. Viñals, "Study of the parametric oscillator driven by narrow band noise to model the response of a fluid surface to time-dependent accelerations", Phys. Fluids A **5**, 3147 (1993).

[4] J. Casademunt, W. Zhang, J. Viñals and R.F. Sekerka, "Stability of a fluid surface in a microgravity environment", AIAA J. **31**, 2027 (1993).

[5] A.A. Wheeler, G.B. McFadden, B.T. Murray and S.R. Coriell, "Convective stability in the Rayleigh-Bénard and directional solidification problems: high-frequency gravity modulation", Phys. Fluids A **3**, 2847 (1991).

[6] B.V. Saunders, B.T. Murray, G.B. McFadden, S.R. Coriell and A.A. Wheeler, "The effect of gravity modulation on thermosolutal convection in an infinite layer of fluid", Phys. Fluids A **4**, 1176 (1992).

[7] J. Casademunt and J. Viñals, "Convection induced by a fluctuating acceleration field", in *Fluid Mechanics in Microgravity*, D.A. Siginer, R.L. Thomson and L.M. Trefethen, Eds., (ASME, New York, 1993), p. 133.

[8] J.R. Thomson, J. Casademunt and J. Viñals, "Cavity flow induced by a fluctuating acceleration field", Phys. Fluids, preprint.