

# PARTICLE EXPERIMENTS IN THERMAL AND VELOCITY GRADIENTS

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## ABSTRACT

The physical scales of velocity, length, time, thermal gradient magnitude and velocity gradient magnitude likely to be involved in gas-solid multiphase flight experiments are assessed for 1-100  $\mu\text{m}$  particles.

## INTRODUCTION

A complex interaction of forces normally governs the motion of solid particles which are conveyed pneumatically [1]. Forces due to viscous drag, gravity, inertia, velocity gradients, rotation, buoyancy, thermal gradients and several other causes may be present depending on the physical environment. The present research focuses on the effects of thermal and velocity gradients on the motion of solid particles in a laminar flow field. Thermal gradients induce an additional force on the particles, known as a thermophoretic force, which causes the particle trajectory to deviate from the fluid streamlines [2,3]. This force is dependent on the strength of the thermal gradient, the size of the particle and the thermophysical properties of both the particle and the fluid. Many numerical simulations rely on simple empirical expressions or an interpolation scheme based on Knudsen number developed by Talbot [2] to provide an expression for the thermophoretic force acting on a particle. Numerical investigations in laminar tube flows [4,5] and investigations of thermophoretic effects in stagnation point flows [6,7,8,9] show that additional terms must also be addressed in these non-isothermal systems to adequately characterize the motion of suspended particles. The adverse effects of thermophoresis on particle-based velocimetry instrumentation is also of concern [10].

Particles transiting a flow field containing a velocity gradient or shear will experience an additional force causing them to deflect from the surrounding fluid streamlines if they possess a non-zero velocity relative to the fluid. Saffman [12] demonstrated, for laminar flows, that the physical origins of this force lie in the variation of the pressure distribution acting on the surface of the particle. This force causes the particle to migrate to specific regions of shear. Segré and Silberberg [13] observed that small, neutrally buoyant spheres in Poiseuille flow migrate to a position 0.6 tube radii from the axis. This result has also been obtained analytically [12]. Experimental observations of liquid droplets in Poiseuille flow demonstrated that lift forces may direct the particles toward or away from the centerline depending on the particle and shear properties [15,16]. Lift forces acting on freely rotating particles in close proximity to walls have been investigated [17] and the effects of nonuniform particle concentrations in compressible shear flows and particle inertia on the accuracy of particle-based velocimetry instrumentation have also been studied [18].

## EXPERIMENTAL CONSIDERATIONS

A goal of this research is to identify the science requirements and experimental parameters necessary to design future flight experiment packages which can provide improved data on the effects of thermal and velocity gradients. The intent is to exploit the suppression of gravitational forces and convection effects in a reduced gravity environment to examine effects on gas-solid flows normally masked by gravitationally induced effects and to study flow regimes unattainable in a ground-based laboratory.

A uniformly accepted mathematical treatment for the motion of a solid particle through a compressible gas containing both velocity and thermal gradients does not exist in a simple form. For purposes of discussion, one can assume that any experiment package suitable for flight or drop tower tests will require that the experiment be contained in an enclosure. That is, the experiment will take place in an enclosed space or in a tube or duct. If the experiment involves a flowing gas or liquid then some entry length and characteristic time is required for the system to reach hydrodynamic and thermal equilibrium. A discussion of the characteristic lengths and times associated with a large enclosure is quite complex and will not be addressed here. However, a simple laminar flow model in a tube will provide a great deal of insight into the length and time scales likely to be encountered in a wide class of experimental configurations. This is the classic Poiseuille flow for which hydrodynamic and thermal entry lengths are well established. The hydrodynamic and thermal entry length equations are derived from conservation of momentum and energy equations by Kays and Crawford [19] in the following forms, neglecting axial conduction:

$$\text{Hydrodynamic entry length, } x \quad x = \frac{Re D}{20}$$

$$\text{Reynolds number} \quad Re = \frac{U D}{\nu}$$

$$\text{Thermal entry length, } x \quad x = \frac{Re Pr D}{20}$$

where  $x$  is the axial location of fully developed flow,  $U$  is the fluid velocity,  $D$  is the tube diameter,  $Re$  is the Reynolds number based on the tube diameter,  $\nu$  is the kinematic viscosity of the fluid, and  $Pr$  is the Prandtl number. The Prandtl number is the ratio of the momentum diffusivity to the thermal diffusivity. In fluids with  $Pr > 1$ , the hydrodynamic profile develops faster than the thermal profile. The thermal entry length solution above applies to cases where  $Re Pr > 100$ .

By selecting Reynolds numbers and tube diameters, the equations above may be used to estimate the hydrodynamic and thermal entry lengths and the characteristic velocity of the system. A worst-case time scale can also be derived by dividing the maximum of the two length scales above by the characteristic velocity scale. Table 1 presents the results of such calculations for some common materials whose Prandtl numbers span four orders of magnitude at 300K. The Reynolds numbers are fixed at 1000 and 500 to maintain a laminar flow. For comparison, a Reynolds number of 2300 would normally indicate a turbulent flow in a tube. Tube diameters are selected to be 2.54 cm and 1.27 cm. Selection of  $Re$  and  $D$  fixes the hydrodynamic entry length scale independent of the fluid under consideration. As expected from the linear relations above, reducing the Reynolds number for a given diameter reduces the fluid entry length, thermal entry length and characteristic velocity but maintains the same time scale. Reducing the diameter at a given Reynolds number reduces the fluid entry length, thermal entry length and time scale but increases the velocity scale. For enclosed laminar experiments involving gases, it is desirable to reduce the Reynolds number and increase the tube diameter. Among the possible costs of this solution are that the desired fluid dynamic regime may not be attained and/or gravitational terms may become significant in the transport of solid particles relative to the reduced hydrodynamic drag.

It is possible to assess the response of a solid particle to an abrupt change in the fluid velocity, in a general way, by consideration of the so-called particle relaxation time. That is, the time required for a particle to adjust its velocity to match a change in the fluid velocity to within a very small percentage. Neglecting gravity, an expression for the particle relaxation time is derived by Hinds [20] in the following form for particle motion in the Stokes region and  $Re$  (based on the particle diameter) less than one:

$$\text{Relaxation time, } \eta \quad \eta = \frac{\rho_p d^2 C_c}{18 \mu}$$

$$\text{Cunningham slip correction factor, } C_c \\ C_c = 1 + \frac{\lambda}{d} [2.514 + 0.800 \exp(-0.55 \frac{d}{\lambda})]$$

In these equations,  $d$  is the particle diameter,  $\rho_p$  the particle density,  $\mu$  the fluid dynamic viscosity,  $C_c$  the Cunningham slip correction factor, and  $\lambda$  is the mean free path of a fluid molecule at the temperature and pressure under consideration. In all that follows, atmospheric pressure will be assumed and the focus will be on the gas-solid flow case. A nondimensional parameter known as the Knudsen number may be defined as  $Kn = 2\lambda/d$  which relates the mean free path to the particle radius. By selecting specific particle and gas parameters, the equations above and the Reynolds condition for Stokes flow provide a means to calculate a particle relaxation time and determine the maximum velocity for which Stokes hydrodynamic drag applies. The results of such calculations are presented in Table 2 for two types of particles in a size range from 1-100  $\mu\text{m}$  in both hydrogen and air over a range of temperatures. The specific particles used in these calculations are alumina ( $\text{Al}_2\text{O}_3$ ) with a density of 3970  $\text{kg/m}^3$  and polystyrene latex (PSL) with a density of 1050  $\text{kg/m}^3$ . The diameters used are assumed to be equivalent spherical diameters and the intent is for the PSL particle to represent a low density particle and for the alumina to represent a medium density particle. Both materials are in common use as seed particles for particle-based velocimetry instruments. These simple calculations of the Knudsen number and the maximum velocity for which Stokes drag applies contain no particle properties other than the diameter. Hence, the results for alumina and PSL, or any other particle type, depends only on the thermodynamic properties of the fluid at the local temperature and pressure. The results indicate that maximum velocity for Stokes conditions on the particle is highly dependent on the local temperature and fluid properties. It may range from a few meters per second through supersonic velocities depending on the local conditions. These simple calculations also indicate that small particles may be expected to adapt to changes in the fluid velocity extremely rapidly. An incorrect conclusion may be drawn from such an analysis that the motion of sufficiently small

particles in laminar flow is governed solely by hydrodynamic drag for a given fluid and temperature. Indeed, hundreds of successful numerical models and particle-based experiments have been conducted based on this assumption. However, there are many forces besides drag which may be present in a given situation and which may impact the particle motion.

As described by Soo [21] and others, Tchen is generally credited with deriving the first widely accepted Lagrangian equation for the motion of a solid particle in an unsteady gas flow field. In what follows, the discussion of Tchen's equation provided by Nichols [22] will be paralleled. The basic equation, using index notation, is written in the form:

$$m_p \frac{dv_i}{dt} = -3 \pi \mu d (v_i - u_i) + m_f \frac{Du_i}{Dt} - \frac{1}{2} m_f \frac{d}{dt} (v_i - u_i) - \frac{3 d^2}{2} \sqrt{\pi \rho_f \mu} \int_0^t \frac{d}{d\tau} \frac{(v_i - u_i)}{\sqrt{t-\tau}} d\tau - g_i (m_p - m_f)$$

with

$$\frac{Du_i}{Dt} = \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \quad \frac{du_i}{dt} = \frac{\partial u_i}{\partial t} + v_j \frac{\partial u_i}{\partial x_j}$$

Here,  $v_i = v_i(t)$ ,  $u_i = u_i(t)$  and  $g_i$  are the component at time  $t$  in the  $x_i$  direction of the particle velocity, fluid velocity, and gravitational force, respectively.  $m_p$  is the mass of the particle,  $m_f$  is the mass of a volume of fluid equal to the particle volume, and  $\rho_f$  is the fluid density. The terms on the right of this expression represent forces acting on the particle associated with Stokes drag, the pressure distribution, the additional mass of fluid which must be accelerated along with the particle, a so-called Basset history integral over time  $\tau$  (related to the acceleration history of the particle) and gravity. Tchen's equation is normally solved numerically due to the complexities introduced by the Basset history term. More recent equations of motion have been developed, for example the equation of Maxey and Riley [23], which correct some of the shortcomings of the Tchen expression. However, the present discussion focuses on the general relationship of forces which are likely to be present in common gas-solid flows and the simpler equation is quite adequate.

An idealized, quasi-one-dimensional problem provides a convenient way to assess the relative contributions of the various forces acting on a solid particle in a gas flow field. The case of the gas flow undergoing a step change in velocity is considered. For this case the Tchen equation is solved either numerically using a 4th order Runge-Kutta technique, if the Basset term is included, or analytically if the Basset term is dropped. The analytic solution for the relative velocity between the particle and the gas is in the form of a decaying exponential which provides a more exact determination of the particle relaxation time than the expression above. For this case, the force terms in the Tchen equation are supplemented with expressions for a shear-induced lift force and a thermophoretic force arising from thermal gradients in a laminar flow. As noted by Nichols [22], the shear-induced lift force or Saffman force has the form:

$$\text{Shear-induced lift, } F_{L,i} \quad F_{L,i} = 1.6125 d^2 \sqrt{\pi \mu} \left| \frac{\partial u_i}{\partial x_j} \right|^{1/2} (v_i - u_i)$$

while Talbot [2] provided an expression for the force due to a thermal gradient in the form:

$$\text{Thermophoretic force, } F_{T,i} \quad F_{T,i} = \frac{\pi \mu v d}{2 |T|} \alpha(Kn) \frac{\partial T}{\partial x_i}$$

with

$$\alpha(Kn) = 12 C_s \frac{k_f / k_p + C_t Kn}{(1 + 3 C_m Kn)(1 + 2 k_f / k_p + 2 C_t Kn)}$$

representing a thermophoretic correction factor based on the Knudsen number. Thermal conductivities of the gas and particle are  $k_f$  and  $k_p$ , respectively, and  $C_s = 1.147$ ,  $C_t = 2.20$  and  $C_m = 1.146$  are empirically derived constants.

The equations for shear-induced lift and thermophoretic force combined with a solution to Tchen's equation for the one-dimensional step change in gas velocity provide a means to quantify the relative contributions of the force terms. By selecting the gas, temperature, magnitude of the step change in velocity, particle size, and particle type, the magnitudes of the velocity and thermal gradients required to produce a lift or thermophoretic force equal to the drag, gravitational, or Basset force acting on the particle may be determined. Table 3 presents the results of such a calculation for a  $1 \mu\text{m}$  particle experiencing a  $1 \text{ mm/s}$  relative velocity in both hydrogen and air over a range of temperatures. Table 4 presents similar calculations for the case in which the temperature is held fixed and the particle diameter is varied for the case of an alumina particle in hydrogen at 300K. Table 5 presents the case in which the gas, temperature, particle size and particle type are fixed and the relative velocity is varied from  $1 \text{ mm/s}$  to  $10 \text{ m/s}$ .

## CONCLUSIONS

The following observations may be made from these calculations. First, the thermodynamic properties of the gas under consideration are extremely important in determining the contributions of the various force terms. Second, for small relative velocities, the magnitude of the velocity gradient required to produce a lift force equal to either the hydrodynamic drag or gravitational force are quite large. However, at higher relative velocities, it is possible for relatively modest velocity gradients to result in lift forces which exceed the gravitational forces acting on a particle. Third, thermal gradients are much more likely to produce a significant force acting on the particle relative to drag and gravity. This is especially true for small particles in high temperature flows. The exact conditions for maximum thermally-induced forces are not linear in temperature and depend on the gas under consideration. Resolution of the physics involved in gas-solid flows could be expected to improve significantly from experiments in microgravity where access to experimental conditions involving low speed flows with small velocity and/or thermal gradients and unrestricted particle size are not precluded by ground-based convection and sedimentation effects.

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**Table 1:** Characteristic times, thermal entry lengths and velocities associated with combinations of Reynolds numbers and length scales for some common liquids and gases.

	Prandtl number 300K	Time scale (s)	Tube diameter 0.0254 m				Tube diameter 0.0127 m			
			Re 1000 Fluid scale 1.27 m		Re 500 Fluid scale 0.635 m		Re 1000 Fluid scale 0.635 m			
			Thermal entry length (m)	Velocity scale (m/s)	Thermal entry length (m)	Velocity scale (m/s)	Time scale (s)	Thermal entry length (m)	Velocity scale (m/s)	Velocity scale (m/s)
Mercury	2.48E-02	2.87E+02	3.15E-02	4.40E-03	1.57E-02	2.21E-03	7.17E+01	1.57E-02	8.86E-03	
Hydrogen	7.01E-01	2.91E-01	8.90E-01	4.37E+00	4.45E-01	2.19E+00	7.27E-02	4.45E-01	8.74E+00	
Air	7.07E-01	2.03E+00	8.98E-01	6.23E-01	4.49E-01	3.13E-01	5.07E-01	4.49E-01	1.25E+00	
Freon	3.50E+00	5.78E+02	4.44E+00	7.70E-03	2.22E+00	3.84E-03	1.45E+02	2.22E+00	1.54E-02	
Water	5.83E+00	2.20E+02	7.40E+00	3.37E-02	3.70E+00	1.68E-02	5.50E+01	3.70E+00	6.73E-02	
Ethylene Glycol	1.51E+02	3.45E+02	1.92E+02	5.55E-01	9.59E+01	2.78E-01	8.64E+01	9.59E+01	1.11E+00	

**Table 2:** Relaxation times, maximum velocities which maintain Stokes conditions and Knudsen number for 1, 10 and 100  $\mu\text{m}$  alumina and polystyrene latex particles.

Temp.	HYDROGEN				AIR			
	Alumina Relax. time	PSL Relax. time	Stokes Maximum Velocity	Kn	Alumina Relax. time	PSL Relax. time	Stokes Maximum Velocity	Kn
PARTICLE DIAMETER - 1 $\mu\text{m}$								
K	(s)	(s)	(m/s)		(s)	(s)	(m/s)	
100	5.84E-05	1.55E-05	1.76E+01	1.03E-01	3.18E-05	8.40E-06	2.32E+00	4.65E-02
300	3.65E-05	9.66E-06	1.10E+02	3.73E-01	1.51E-05	3.98E-06	1.53E+01	2.02E-01
500	3.35E-05	8.85E-06	2.59E+02	6.78E-01	1.22E-05	3.22E-06	3.70E+01	3.83E-01
1000	3.55E-05	9.38E-06	8.25E+02	1.53E+00	1.13E-05	2.98E-06	1.22E+02	8.56E-01
1500	3.96E-05	1.05E-05	1.62E+03	2.46E+00	1.18E-05	3.12E-06	2.45E+02	1.37E+00
2000	4.37E-05	1.16E-05	2.63E+03	3.44E+00	1.26E-05	3.32E-06	4.02E+02	1.97E+00
PARTICLE DIAMETER - 10 $\mu\text{m}$								
K	(s)	(s)	(m/s)		(s)	(s)	(m/s)	
100	5.24E-03	1.39E-03	1.76E+00	1.03E-02	3.02E-03	7.99E-04	2.32E-01	4.65E-03
300	2.59E-03	6.85E-04	1.10E+01	3.73E-02	1.23E-03	3.26E-04	1.54E+00	2.02E-02
500	1.91E-03	5.04E-04	2.59E+01	6.78E-02	8.57E-04	2.27E-04	3.70E+00	3.84E-02
1000	1.31E-03	3.47E-04	8.25E+01	1.53E-01	5.75E-04	1.52E-04	1.22E+01	8.56E-02
1500	1.10E-03	2.91E-04	1.62E+02	2.46E-01	4.67E-04	1.23E-04	2.45E+01	1.37E-01
2000	9.95E-04	2.63E-04	2.63E+02	3.44E-01	3.99E-04	1.06E-04	4.02E+01	1.97E-01
PARTICLE DIAMETER - 100 $\mu\text{m}$								
K	(s)	(s)	(m/s)		(s)	(s)	(m/s)	
100	5.18E-01	1.37E-01	1.76E-01	1.03E-03	3.00E-01	7.94E-02	2.32E-02	4.65E-04
300	2.49E-01	6.57E-02	1.10E+00	3.73E-03	1.20E-01	3.18E-02	1.54E-01	2.02E-03
500	1.77E-01	4.68E-02	2.59E+00	6.78E-03	8.22E-02	2.17E-02	3.70E-01	3.84E-03
1000	1.12E-01	2.97E-02	8.25E+00	1.53E-02	5.25E-02	1.39E-02	1.22E+00	8.56E-03
1500	8.65E-02	2.29E-02	1.62E+01	2.46E-02	4.05E-02	1.07E-02	2.45E+00	1.37E-02
2000	7.22E-02	1.91E-02	2.63E+01	3.44E-02	3.28E-02	8.67E-03	4.02E+00	1.97E-02

**Table 3: Magnitude of velocity and thermal gradients required to produce a lift or thermophoretic force equal in magnitude to the drag or gravitational force acting on a 1  $\mu\text{m}$  alumina particle experiencing a 1.0 mm/s relative velocity in hydrogen and air.**

Temp.	HYDROGEN				AIR			
	Velocity gradient		Thermal gradient		Velocity gradient		Thermal gradient	
	Drag (1/s)	Gravity (1/s)	Drag (K/m)	Gravity (K/m)	Drag (1/s)	Gravity (1/s)	Drag (K/m)	Gravity (K/m)
100	6.02E+08	1.24E+04	2.15E+04	1.09E+04	7.91E+07	2.49E+03	2.57E+05	7.55E+04
300	3.77E+09	1.49E+04	8.68E+03	2.11E+03	5.25E+08	2.76E+03	6.13E+04	7.21E+03
500	8.86E+09	1.62E+04	7.47E+03	1.28E+03	1.26E+09	2.91E+03	4.35E+04	3.48E+03
1000	2.82E+10	1.81E+04	7.57E+03	8.18E+02	4.17E+09	3.28E+03	3.56E+04	1.81E+03
1500	5.55E+10	1.94E+04	8.29E+03	6.82E+02	8.38E+09	3.52E+03	3.54E+04	1.38E+03
2000	9.00E+10	2.03E+04	9.06E+03	6.14E+02	1.37E+10	3.65E+03	3.75E+04	1.18E+03

**Table 4: Magnitude of velocity and thermal gradients required to produce a lift or thermophoretic force equal in magnitude to the drag or gravitational force acting on an alumina particle experiencing a 1.0 mm/s relative velocity in hydrogen at 300K.**

HYDROGEN				
Particle diameter (m)	Velocity gradient		Thermal gradient	
	Drag (1/s)	Gravity (1/s)	Drag (K/m)	Gravity (K/m)
5.0E-07	1.51E+10	7.44E+03	1.10E+04	6.67E+02
1.0E-06	3.77E+09	1.49E+04	8.68E+03	2.11E+03
1.0E-05	3.77E+07	1.49E+05	1.84E+04	4.45E+05
1.0E-04	3.77E+05	1.49E+06	1.09E+05	2.63E+08

**Table 5: Magnitude of velocity and thermal gradients required to produce a lift or thermophoretic force equal in magnitude to the drag or gravitational force acting on a 1  $\mu\text{m}$  alumina particle over a range of relative velocities in hydrogen at 300K.**

HYDROGEN				
Relative velocity (m/s)	Velocity gradient		Thermal gradient	
	Drag (1/s)	Gravity (1/s)	Drag (K/m)	Gravity (K/m)
1.0E-03	3.77E+09	1.49E+04	8.68E+03	2.11E+03
1.0E-02	3.77E+09	1.49E+03	8.68E+04	2.11E+03
1.0E-01	3.77E+09	1.49E+02	8.68E+05	2.11E+03
1.0E+00	3.77E+09	1.49E+01	8.68E+06	2.11E+03
1.0E+01	3.77E+09	1.49E+00	8.68E+07	2.11E+03