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Magnetometer-Only Attitude and Rate Determination for a Gyro-Less Spacecraft

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ABSTRACT

Attitude determination algorithms that require only the Earth's magnetic field will be useful for contingency conditions. One way to determine attitude is to use the time derivative of the magnetic field as the second vector in the attitude determination process. When no gyros are available, however, attitude determination becomes difficult because the rates must be propagated via integration of Euler's equation, which in turn requires knowledge of the initial rates. The spacecraft state to be determined must then include not only the attitude but also the rates.

This paper describes a magnetometer-only attitude determination scheme with no a priori knowledge of the spacecraft state, which uses a deterministic algorithm to initialize an extended Kalman filter. The deterministic algorithm uses Euler's equation to relate the time derivatives of the magnetic field in the reference and body frames and solves the resultant transcendental equations for the coarse attitude and rates. An important feature of the filter is that its state vector also includes corrections to the propagated rates, thus enabling it to generate highly accurate solutions.

The method was tested using in-flight data from the Solar, Anomalous, and Magnetospheric Particles Explorer (SAMPEX), a Small Explorer spacecraft. SAMPEX data during several eclipse periods were used to simulate conditions that may exist during the failure of the on-board digital Sun sensor. The combined algorithm has been found effective,

yielding accuracies of 1.5 deg in attitude (within even nominal mission requirements) and 0.01 degree per second (deg/sec) in the rates.

INTRODUCTION

The coarseness of the attitude information derived from the Earth's magnetic field, \vec{B} , limits the usefulness of magnetometers in accurate attitude determination systems. On the other hand, magnetic field measurements offer several advantages: (1) the sensors are inexpensive, (2) measurements can be made any time regardless of the spacecraft's orientation in space, and (3) \vec{B} usually changes direction rapidly enough to make computation of its time derivative possible and these changes during the orbit are sufficiently large to enable determination of all three Euler angles using only a three-axis magnetometer (TAM).

The first and second advantages make a TAM attractive for Small Explorer missions that have modest attitude requirements. The third advantage prompts a closer look at contingency attitude algorithms that use only TAM measurements and are the subject of this paper. In fact, the third advantage allows the spacecraft rates to be computed, in principle, by examining time derivatives of \vec{B} .

Therefore, we address here the following nontrivial problem: Can we reliably estimate both attitude *and* rates of the spacecraft using only TAM measurements and *no* a priori information? If so, we can provide for sensor contingencies of a gyro-less

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spacecraft such as SAMPEX, as well as of a gyro-based spacecraft when the gyros are not functional.

Note that the second situation is not hypothetical. For example, the Earth Radiation Budget Satellite (ERBS) experienced a control anomaly (Kronenwetter and Phenneger, 1988, and Kronenwetter et al., 1988) during a hydrazine thruster-controlled yaw inversion maneuver that resulted in the spacecraft tumbling with rates of over 2 deg/sec. As a result, both Sun and Earth sensor readings became unreliable, and the gyro output was saturated. Similarly, control of the Relay Mirror Experiment (RME) satellite was lost after the failure of the Earth sensors (Natanson, 1992). In both cases, a TAM became the only functional attitude instrument.

We present here a combined scheme invoking two different algorithms—deterministic attitude determination from magnetometer-only data (DADMOD) and the Real-Time Sequential Filter (RTSF)—both of which have been tested successfully for SAMPEX in giving the positive answer to the above question. The DADMOD (Natanson et al., 1990; Natanson et al., 1991; and Natanson, 1992) is an algorithm that relates the time derivatives of \vec{B} in inertial and spacecraft body coordinates to determine the attitude and the body rates. DADMOD has been successfully tested for ERBS under normal conditions as well as for RME after the aforementioned horizon sensor failure (Natanson, 1992).

The RTSF (Challa, 1993, and Challa et al., 1994) is a novel extended Kalman filter that estimates, in addition to the attitude, errors in rates propagated via Euler's equations. The RTSF is sensitive to rate errors as small as 0.0003 deg/sec (Natanson et al., 1993), and this feature makes it a very robust and accurate real-time algorithm. In particular, it has been shown (Challa, 1993, Challa et al., 1994) that the RTSF converges successfully in TAM-only situations using *inertial initial conditions*; i.e., the spacecraft is assumed at rest in the geocentric inertial coordinates (GCI) with its axes coinciding with the GCI axes. Note that the RTSF does not explicitly compute the time derivatives of \vec{B} , which

are the main source of errors in the deterministic scheme.

The combined method suggested here uses the deterministic solution for initializing the RTSF to guarantee and speed up its convergence. In this scheme, the initial conditions for the RTSF are determined by the DADMOD using a 100-second batch of magnetometer measurements. The method is applied here to flight data for SAMPEX during eclipse periods. During these periods, the magnetic torquer is turned off, so that the spacecraft attitude is controlled only by the momentum wheel (Forden et al., 1990, and Frakes et al., 1992); this situation is similar to the aforementioned contingency conditions for RME. Remarkably, the accuracy of our attitude estimates is less than 2 degrees, which is within the SAMPEX requirements under normal conditions (Keating et al., 1990).

MAGNETOMETER-ONLY DETERMINISTIC ATTITUDE/RATE DETERMINATION

The deterministic scheme starts by constructing the second vector measurement from the first time derivatives of \vec{B} resolved in the reference and body frames. This gives the usual transformation equations

$$A\vec{B}^R = \vec{B}^A, \quad (1a)$$

and

$$A\dot{\vec{B}}^R = \dot{\vec{B}}^A + \vec{\omega}^A \times \vec{B}^A \quad (1b)$$

where A is the attitude matrix, $\vec{\omega}$ is the angular velocity vector, and superscripts R and A imply that that the corresponding vectors are resolved in the reference and body frames, respectively. If the initial value of $\vec{\omega}^A$ is known, then $\vec{\omega}^A$ can be obtained by integrating Euler's equation, and the TRIAD algorithm (Wertz, 1984) can be used to compute the attitude matrix A from the vector pairs

$$(\vec{B}^R, \vec{B}^A)$$

and

$$(\dot{\vec{B}}^R, \dot{\vec{B}}^A + \vec{\omega}^A \times \vec{B}^A)$$

as has been done by Natanson et al. (1993). The nontrivial nature of the problem considered here arises from the unknown initial conditions for Euler's equation.

As shown by Natanson et al. (1990), the problem can be cast in the form of transcendental equations as follows. Taking into account that the vector lengths must be the same, regardless of the frame in which it is resolved, the projection $\vec{\omega}_\perp$ of $\vec{\omega}^A$ onto the plane perpendicular to \vec{B}^A can be expressed as a function of an unknown angle Φ between the vectors $A[\vec{B}^R \times \dot{\vec{B}}^R]$ and $[\vec{B}^A \times \dot{\vec{B}}^A]$. The problem thus reduces to two unknown variables: the angle Φ and the projection ω_1 of $\vec{\omega}$ in the direction of \vec{B} , with the attitude matrix A dependent only on the angle Φ . To find Φ and ω_1 , Equations (1a) and (1b) must be supplemented by Euler's equations, which can be written in the following schematic form:

$$\dot{\vec{\omega}}^A = \vec{\Omega}_0(\Phi) + \vec{\Omega}_1(\Phi)\omega_1 + \vec{\Omega}_2\omega_1^2 \quad (2)$$

where the vectors $\vec{\Omega}_0(\Phi)$, $\vec{\Omega}_1(\Phi)$, and $\vec{\Omega}_2$ are given by Equations (25a) through (25c) of Natanson (1992).* The kinematic equation relating the second derivatives $\ddot{\vec{B}}^A$ and $\ddot{\vec{B}}^R$ is then formally represented as

$$\vec{\Lambda}_0(\Phi) + \vec{\Lambda}_1(\Phi)\omega_1 + \vec{\Lambda}_2\omega_1^2 = \vec{0} \quad (3)$$

where the vectors $\vec{\Lambda}_0(\Phi)$, $\vec{\Lambda}_1(\Phi)$, and $\vec{\Lambda}_2$ are defined by Equations (23a) through (23c) of Natanson (1992).

Two nontrivial equations (transcendental in Φ) are obtained by projecting the vector equation (3) on two directions perpendicular to \vec{B} . One of the resultant equations is then analytically solved with respect to ω_1 at different values of Φ , and one of

* Note that the cited equations erroneously used

$$I^{-1}[\vec{Y} \times \vec{Z}] = I^{-1}\vec{Y} \times I^{-1}\vec{Z}$$

instead of the correct expression

$$I^{-1}[\vec{Y} \times \vec{Z}] \equiv I\vec{Y} \times I\vec{Z} / \det I$$

where I is the inertia tensor of the spacecraft.

two roots, $\omega_1(\Phi)$ is substituted into the second equation. [The selected root $\omega_1(\Phi)$ must turn into the solution of the linear equation in ω_1 , which arises in the limit $\dot{\vec{\omega}}^A \rightarrow \vec{0}$ (Natanson et al., 1990).] Finally, the resultant transcendental equation is numerically solved with respect to Φ .

REAL-TIME SEQUENTIAL FILTER

The RTSF's state vector \vec{X} comprises the four components of the attitude quaternion, \vec{q} , and the three components of the rate correction, \vec{b} , to $\vec{\omega}^A$:

$$\vec{X} = [\vec{q}^T \vec{b}^T]^T \quad (4)$$

The RTSF uses sensor data to estimate \vec{q} as well as \vec{b} , with \vec{b} being estimated kinematically in the same manner as gyro biases for a gyro-based spacecraft, i.e., by attributing differences between the measured and propagated attitudes to errors in $\vec{\omega}^A$. The \vec{b} estimates are then used to correct $\vec{\omega}^A$, and these corrected rates are used as initial conditions to propagate Euler's equation to the next measurement time. The propagation of \vec{b} is modeled via a first-order Markov model:

$$\frac{d\vec{b}}{dt} = -\tau^{-1}\vec{b} + \vec{\eta}_b, \quad (5)$$

where $\vec{\eta}_b$ is a white noise vector, and τ is a finite time constant. The novel feature of the RTSF is that, since \vec{b} represents rate errors accumulated between measurements, the optimum value for τ is the data period: 5 seconds for the SAMPEX data used here. (In contrast, the same model, when used for gyro bias estimation, requires τ of several hours.)

BRIEF DESCRIPTION OF SAMPEX

SAMPEX is the first of the Small Explorer satellites and is designed to study elemental and isotopic composition of energetic particles of solar and cosmic origin. It has a 550 × 675-km orbit with an 82-deg inclination. SAMPEX nominally is Sun-pointing and has a rate of one rotation per orbit (RPO) about the spacecraft-to-Sun vector. The

attitude accuracy requirement of 2 deg is achieved using a fine Sun sensor (FSS), and a TAM. The control hardware consists of a momentum wheel and a magnetic torquer assembly (MTA). During eclipse periods, the MTA is turned off, and attitude control is performed by only the momentum wheel under the assumption that the spin axis remains directed along the Sun vector.

The wheel momentum, \vec{h} , is directed along the body y axis, which is also the FSS boresight. The SAMPEX mass distribution is approximately symmetric about this axis. The body z axis is directed along the boresights of the science instruments.

ATTITUDE CONVENTIONS

In following Crouse (1991), the Sun-pointing orbital coordinate system (OCS) used here has its z axis directed along the target vector as it was initially defined by Flatley et al. (1990). Later McCullough et al. (1992) modified the control law, and as a result, the nominal direction of the body z axis in space differs slightly from the direction of the OCS z axis. The roll, pitch, and yaw angles are defined as the 1-2-3 decomposition of the matrix transformation from the OCS to the body frame. During the nominal 1-RPO mode, the roll and yaw angles are both close to 0, and $\vec{\omega}^A \sim (0, 0.06, 0)^T$ deg/sec, while the pitch angle may deviate from zero by a few degrees for the reason mentioned above.

The present work also uses the 2-3-2 Euler decomposition of the matrix transformation from GCI to the body frame. The advantage of this attitude parametrization during eclipse is that the third Euler angle directly reflects the 1-RPO rate of the spacecraft, while the other two angles are very nearly constant because no external control torque exists, and environmental torques acting on the SAMPEX are negligibly small.

The tests discussed below were performed using SAMPEX telemetry data for an eclipse on July 12, 1992. The truth model here is the attitude solutions from the single-frame TRIAD algorithm (Wertz, 1984), which are computed using the onboard al-

gorithm; i.e., assuming that the Sun vector remains unchanged during eclipse.

RESULTS

Figures 1(a) and 1(b) present the first and third Euler angles for the 2-3-2 decomposition of the GCI-to-body attitude matrix, respectively. Except for the region between 400 and 700 seconds (discussed below), only two solutions are obtained, which significantly differ from each other. If attitude control is performed solely with the momentum wheel and environmental torques are negligibly small, one can use conservation of the angular momentum to select the physical solution (Natanson, 1992). In the absence of spacecraft nutation, this implies that the first two Euler angles must remain unchanged. In fact, the first Euler angle depicted in Figure 1(a) remains unchanged for one of the two deterministic solutions and significantly varies for another. Except for the region of multiple solutions, the physical solution closely follows the straight lines of the TRIAD solution.

A similar conclusion can be drawn from an analysis of Figures 2(a) and 2(b) presenting the x and y body components of the angular velocity vector. Note that the DADMOM solutions presented here were obtained assuming constant wheel speed equal to the nominal value. Taking into account actual values from telemetry did not result in any noticeable gain in the accuracy.

More than two solutions appear when \vec{B} becomes perpendicular to the pitch axis about 400 seconds after the beginning of the eclipse. Before this occurred, the vector functions $\vec{\Lambda}_0(\Phi)$ and $\vec{\Lambda}_1(\Phi)$ in Equation (3) could be roughly approximated as:

$$\vec{\Lambda}_0(\Phi) \approx \vec{\Lambda}_0^0(\Phi) \equiv \vec{\Omega}_0^0(\Phi) \times \vec{B}^A$$

$$= \frac{[I \vec{\omega}_\perp(\Phi) (\vec{h} \cdot I \vec{B}^A) - I \vec{h} (\vec{\omega}_\perp(\Phi) \cdot I \vec{B}^A)]}{\det I} \quad (6a)$$

$$\vec{\Lambda}_1(\Phi) \approx \vec{\Lambda}_1^0 \equiv \vec{\Omega}_1^0 \times \vec{B}^A$$

$$= \frac{[I \vec{B}^A (\vec{h} \cdot I \vec{B}^A) - I \vec{h} (\vec{B}^A \cdot I \vec{B}^A)]}{\det I} \quad (6b)$$

where

$$\vec{\Omega}_0^0(\Phi) \equiv I^{-1} [\vec{h} \times \vec{\omega}_\perp(\Phi)],$$

$$\vec{\Omega}_1^0 \equiv I^{-1} [\vec{h} \times \hat{B}^A].$$

The approximation can be understood easily by taking into account that the magnitude of the vector $I\vec{\omega}^A$ is generally much smaller than wheel momentum. For the same reason, one can neglect the quadratic term in Equation (2). By projecting the resultant equation onto the vector $\vec{B}^A \times \vec{\Lambda}_1^0$, one then obtains the following quadratic equation:

$$a_0 + a_1 \tan \frac{\Phi}{2} + a_2 \tan^2 \frac{\Phi}{2} = 0 \quad (7)$$

which is analogous to that derived by Natanson et al. (1990) for the constant-angular-velocity limit. Obviously, this equation may not have more than two solutions. (Another advantage of this approximation is that one needs only the *first* derivatives of \vec{B} with respect to time, which can be evaluated relatively accurately from a 30-second batch of magnetometer measurements.) However, the approximation made to derive Equation (7) fails if $\vec{h} \cdot I\vec{B}^A$ goes to zero, so that the vectors $\vec{\Lambda}_0^0(\Phi)$ and $\vec{\Lambda}_1^0$ become parallel regardless of the particular value of Φ . Because SAMPEX is very nearly symmetric about the pitch axis, the relation

$$\vec{h} \cdot I\vec{B}^A \approx 0$$

is satisfied in the region where \vec{B} becomes perpendicular to the pitch axis. In addition, in this region

$$\vec{\omega}_\perp(\Phi) \cdot I\vec{B}^A \approx 0$$

regardless of the particular value of Φ . Consequently, $\vec{\Lambda}_0^0(\Phi)$ vanishes at any Φ , which implies that the coefficients of quadratic Equation (7) are all equal to zero. Therefore, when \vec{B} is perpendicular to the pitch axis, one cannot disregard the contribution from the vector $I\vec{\omega}^A$ to $\vec{\Lambda}_0^0(\Phi)$. The coefficients of quadratic Equation (7) remain small for some time, making its solution completely unreliable.

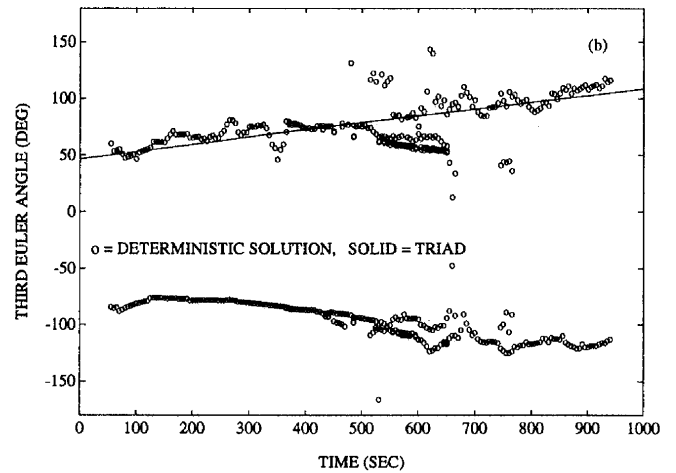
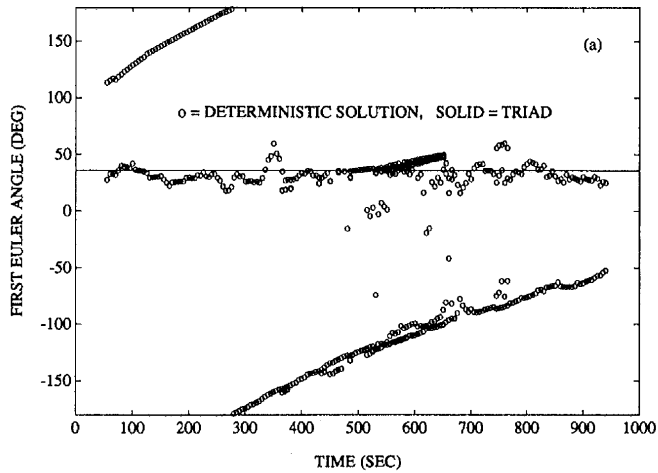
Figure 3 compares the RTSF roll and pitch angle results obtained after initializing the filter with two different schemes: the inertial initial conditions mentioned in the introduction to this paper, and the correct DADMODO solution from Figures 1 and 2. For both starting conditions, the roll angle results of Figure 3a reflect oscillations with the spacecraft's nutational period of 120 sec. The amplitude of the oscillations is a measure of the magnitude of the transverse component of $\vec{\omega}^A$ at $t = 0$. The true nutational amplitudes, however, are negligible for this data span (Natanson et al., 1993). Thus, the amplitude of the oscillations is RTSF errors and is a direct consequence of the initial rate errors.

Although the filter's rate-corrections feature enables it to converge (not shown here) after 2500 sec to within 0.01 deg/sec of the true rates even with the inertial initial conditions, it is clear that the DADMODO reduces the initial errors, as well as the convergence time, by an order of magnitude. More important, the correct DADMODO solution, by providing starting attitude and rates close to the true values, nearly eliminates the possibility of filter divergence.

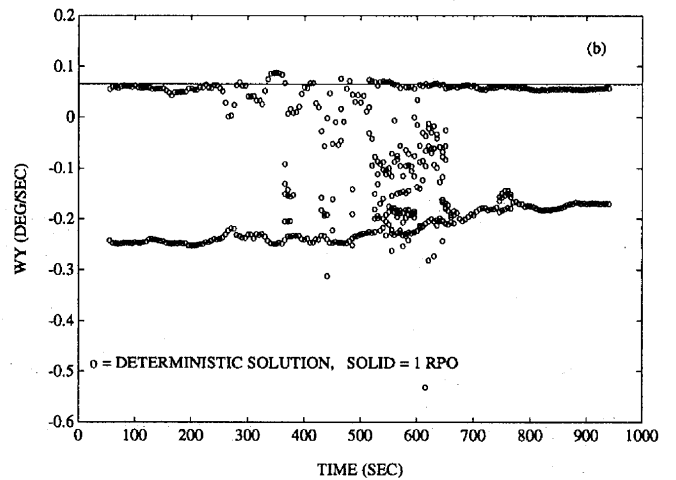
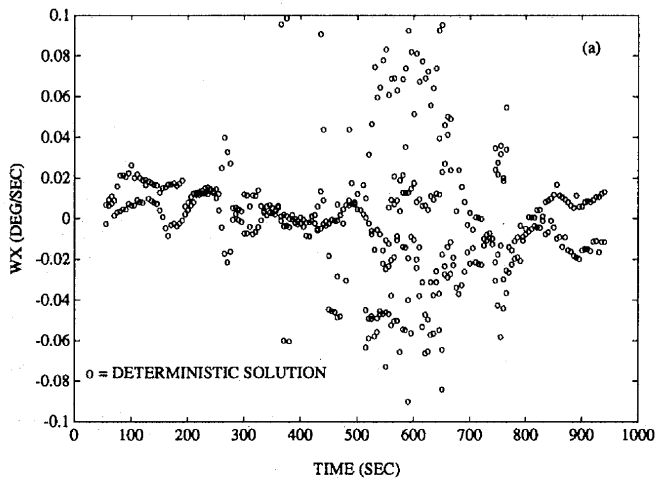
CONCLUSIONS

We find that, using only magnetic field data and no a priori information, the RTSF determines the attitude to within SAMPEX mission requirements of 2 deg and rates to within 0.01 deg/sec, respectively. Using the DADMODO to initialize the RTSF reduces the a priori errors and the RTSF's convergence time by an order of magnitude (to within a few hundred seconds) and also reduces the possibility of divergences.

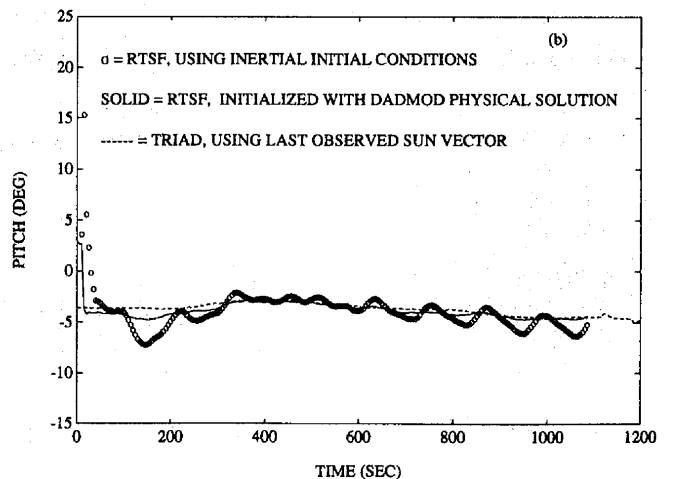
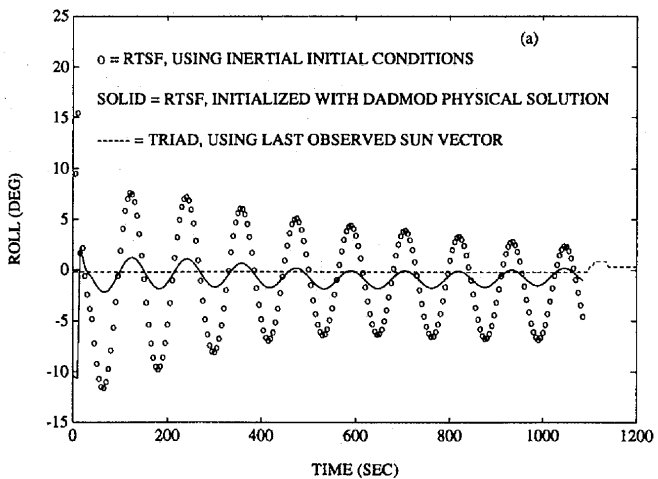
The DADMODO allows one to find the TAM-only attitude solution with an accuracy of 10–15 deg, unless the spacecraft passes through a region where \vec{B} is perpendicular to the wheel momentum. The DADMODO results are consistent with those reported for the RME satellite (Natanson, 1992), where the onboard conditions after the failure of the Earth sensor are similar to those used here.



Figures-1(a) and (b). Attitude Solutions Generated by DADMOD



Figures-2(a) and (b). Rate Solutions Generated by DADMOD



Figures-3(a) and (b). RTSF Results Showing Faster Convergence Using the Correct Solutions for Figures 1 and 2

The current presentation has been deliberately limited to the case with no external torques so that the choice between physical and spurious deterministic solutions can be made by analyzing changes in the direction of the total angular momentum in space. It should, therefore, be noted that the inertial initial conditions enable the RTSF to converge in more severe conditions such as SAMPEX's Sun-acquisition mode, where the magnetic torquers are used to vary ω_x and ω_z rapidly, with amplitudes up to 0.6 deg/sec. This is shown in Figure 4 where the telemetered data span the transition (at about 2000 sec) from SAMPEX's Sun acquisition mode to the 1-RPO mode. Here, the TRIAD attitude solutions are obtained using both Sun and magnetic field data, and these are differenced to produce the TRIAD rate solutions. These TRIAD results serve as the truth model for evaluating the RTSF, which used only the magnetic field data. Despite a priori errors of up to 90 deg in attitude and 10 RPO in rates, the RTSF attitude and rate estimates converge to within 2 deg and 0.01 deg/sec, respectively, in about 1200 sec.

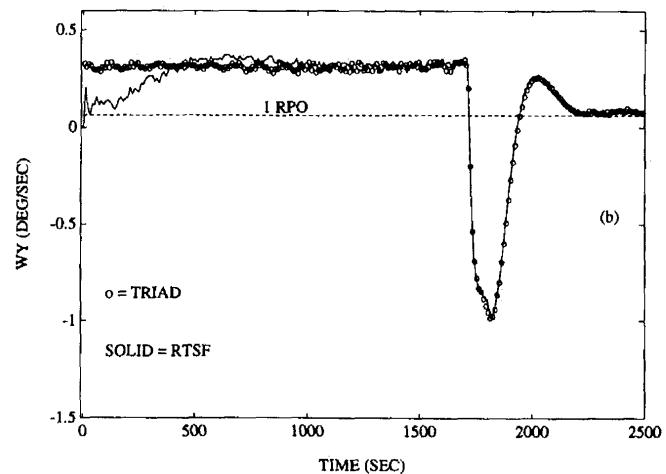
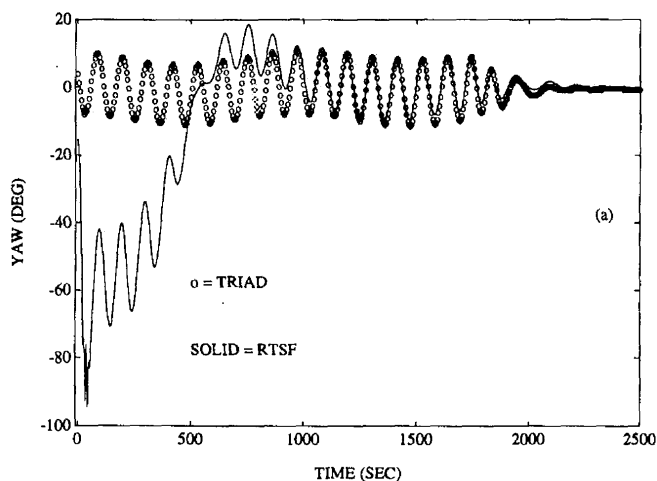
Therefore, the RTSF can also be used for TAM-only attitude determination in the magnetic despin mode using the magnetic field solely for the attitude control. This mode has been successfully used, for example, to despin ERBS during the control anomaly mentioned previously in this paper.

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Figures-4(a) and (b). RTSF TAM-Only Results Using Inertial Initial Conditions and SAMPEX Sun Acquisition Mode Data

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