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# Radial Basis Function Neural Networks Applied to NASA SSME Data Technical Report \# TR154/6/93 ${ }^{1}$ 

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#### Abstract

This paper presents a brief report on the application of Radial Basis Function Neural Networks (RBFNN) to the prediction of sensor values for fault detection and diagnosis of the Space Shuttle's Main Engines (SSME). The location of the Radial Basis Function (RBF) node centers was determined with a K-means clustering algorithm. A neighborhood operation about these center points was used to determine the variances of the individual processing nodes.


## 1 Introduction

In test firing and during on-line operation of the Space Shuttle's Main Engines (SSME), an efficient method of detecting anomalous sensor values is needed for detection and diagnosis of engine and sensor faults. Due to the volume of data acquired during a single test, and the fact that the nominal ranges of sensor values are dependent upon system parameters, the analysis currently consumes many man hours. To automate the analysis of anomalous sensor values, the use of Radial Basis Function Neural Networks (RBFNN) is being investigated. This report shows that by using the last five samples in time of a select group of sensor values, it is possible to have a RBFNN predict the value of a particular sensor at the next discrete instant of time. The predicted value can then be compared to the actual value. If the difference is greater then some threshold (which could be based upon the standard deviation of the data), then an anomalous sensor value has been detected.

A brief review of RBFNNs is given next, after which a description of the SSME data is presented. This is followed by a presentation of the implementations and experiments performed, and suggestions for future work.

## 2 Radial Basis Function Neural Networks

The basic radial basis function neural network contains an input layer for input signal distribution, a single hidden layer of processing units, and an output summation unit as shown in Figure 1.

The input vector $\vec{x}$ with components 1 to n is presented to each processing unit. Each processing unit has a centroid vector $\overrightarrow{c_{i}}$ which determines the location of the center of the radial basis function. The radial basis function is applied to the Euclidean distance between the input vector and its own centroid. The output of each unit is then weighted by $w_{i}$ and summed together by the unit in the third layer:

$$
f(\vec{x})=\sum_{i=1}^{K} w_{i} \phi\left(\left\|\vec{x}-\overrightarrow{c_{i}}\right\|\right)
$$

where $\phi$ is the radial basis function (typically Gaussian), $\vec{c}_{i}$ are the K centers, and $w_{i}$ are the K weighted connections from the units to the third layer.


Figure 1: Radial Basis Function Neural Network

With this basic topology, the system can be represented in state space form:

$$
\vec{y}=(F \cdot \vec{w})
$$

where F is the matrix of activations:

$$
F=\left(\begin{array}{ccc}
f\left(\left\|\overrightarrow{x_{1}}-\overrightarrow{c_{1}}\right\|\right) & \cdots & f\left(\| \overrightarrow{x_{1}}-\overrightarrow{c_{K}}\right) \\
\vdots & \ddots & \vdots \\
f\left(\left\|\overrightarrow{x_{N}}-\overrightarrow{c_{1}}\right\|\right) & \cdots & f\left(\| \overrightarrow{x_{N}}-\overrightarrow{c_{K}}\right)
\end{array}\right)
$$

The weights can be calculated using the Moore-Penrose inverse:

$$
\vec{w}=\left(F^{T} \cdot F+\alpha I\right)^{-1} \cdot F^{T} \cdot \vec{y}
$$

where $\alpha$ is much less than 1 , and $I$ is the identity matrix. $\alpha I$ is added in the event that the square matrix $F^{T} \cdot F$ is close to being singular. This singularity will only occur if redundant data is applied to the input of the neural system. It is also possible to calculate the weights using an iterative gradient descent algorithm.

Internal normalization can be implemented amongst the Gaussian nodes themselves. Without normalization the output of each node is calculated as:

$$
\phi\left(\left\|\vec{x}-\overrightarrow{c_{i}}\right\|\right)
$$

The following equation incorporates normalization across all of the Gaussian nodes upon presentation of each input vector:

$$
\frac{\phi\left(\left\|\vec{x}-\overrightarrow{c_{i}}\right\|\right)}{\sum_{i=1}^{K} \phi\left(\left\|\vec{x}-\overrightarrow{c_{i}}\right\|\right)}
$$

The normalization requires that the sum of the outputs of all nodes be 1 for any input vector. This allows for the output of smaller units to have a greater impact on the overall output. Thus when an input vector falls between two nodes, the system will be able to better interpolate [1].

The parameters which need to be established are the locations of the centroids (means), the weight values, the widths of the radial basis functions (variances), and the type of radial basis function to be used. It may be desirable to alter the topology of the network to include direct weighted connections from input to output, and to change the linear unit in layer three to use a nonlinear function. Although the modifications to the topology may reduce output error, it is no longer possible to use the simple weight calculation mentioned above, so learning time will increase.

The locations and widths of the radial basis functions depends upon the data being presented to the network. One way to find these parameters is to use a clustering algorithm such as K-means on the data. Once the data has been clustered, the centers of these clusters can be used as the centers of the Gaussian nodes. Note that this method requires some a priori information as to how many clusters should be performed. This number is not as important as one might believe. This will be demonstrated when the results of the experiments are presented.

## 3 Description of Data

Three sets of startup data were available, each of which contained a different number of sensor parameters. The parameters used in each list were selected in the work of [2]. The sensors I.D. numbers used in the training and recall

Table 1: Training-based Parameter Lists

| Parameter <br> List | Number <br> of PIDs | Parameters <br> used in training |
| :---: | :---: | :---: |
| 1 | 6 | 21582097349511050 |
| 2 | 7 | 21582093277349511058 |
| 3 | 8 | 2152582093277349511050 |

are depicted in Table 1. Descriptions of the sensor I.D. numbers are given in Table 2. In this table a "T" indicates that a set of vectors was used for training, a "V" indicates a set used only for validation. Only List1 was used in this report. It was the easiest of the three lists to learn in terms of the extent of required generalization necessary for adequate performance on the validation sets. List 1 uses 6 sensor values which, when combined into a single input vector representing the last five sampled sensor values for each sensor, becomes a 30 dimensional vector. The sensor value that was to be predicted for all sets is the SSME's High Pressure Oxidizer Turbine (HPOT) discharge temperature, which has Parameter Identification (PID) number 233. It is important to note that all of the data was transient startup data. This data contained many transients which varied from test firing to test firing. These transients increase the difficulty that any learning paradigm will have in accurately interpolating between the training and validation data sets. The HPOT sensor values for the four training sets are plotted against time in Figure 2, the HPOT values for the validation sets are plotted in Figure 3. From these figures it can be seen that there was a wide range of nominal sensor values.

## 4 Implementations and Experiments

Six different variations of RBFNNs were implemented in C and tested on List 1: RBF1, RBF11, RBF6, RBF8, RBF10 and RBF12. All simulations were executed on a SPARC IPX. These implementations mainly differ in the way that they calculate the variance associated with each Gaussian node. All of implementations used Gaussian based radial basis functions. The means of

Table 2: Parameter Descriptions

| PID | Description |
| :--- | :--- |
| 21 | Main Combustion Chamber Oxidizer Injection Ternperature |
| 40 | Oxidizer Preburner Oxidizer Valve Actuator Position |
| 42 | Fuel Preburner Oxidizer Valve Actuator Position |
| 52 | High Pressure Fuel Pump Discharge Pressure |
| 58 | Fuel Preburner Chamber Pressure |
| 59 | Preburner Boost Pump Discharge Pressure |
| 209 | High Pressure Oxidizer Pump Inlet Pressure |
| 231 | High Pressure Fuel Turbine Discharge Temperature |
| $233 \dagger$ | High Pressure Oxidizer Turbine Discharge Temperature |
| 327 | High Pressure Oxidizer Pump Balance Cavity Pressure |
| 480 | Oxidizer Preburner Chamber Pressure |
| 734 | Low Pressure Oxidizer Pump Shaft Speed |
| 951 | High Pressure Oxidizer Pump Primary Seal Drain Pressure |
| 1050 | Oxidizer Tank Discharge Temperature |
| 1058 | Engine Oxidizer Inlet Temperature |
| 1205 | Facility Fuel Flow |
| 1212 | Facility Oxidizer Flow |
| O/Cs | Dummy Parameter indicating Open/Closed Loop Operation |
| OPBs | Dummy Parameter indicating Oxidizer Preburner Prime Time |

the modeled parameter


Figure 2: P.I.D. 233 Training sets


Figure 3: P.I.D. 233 Validation sets
the functions were chosen by the K -means algorithm. The number of clusters to be formed by this algorithm was selected by the user. It will be shown that the number of nodes selected was not that important as long as it fell within a reasonable range.

In the experiments, the start up data was sampled from 0.4 to 5.96 seconds at 25 hertz. This resulted in 140 vectors per set, or 560 vectors total for the 4 training sets:

$$
(5.96-0.4) * 25 \text { cycles } / \text { sec } .+1=140 \text { vectors }
$$

If one Gaussian node were to be placed on every data vector, this would result in 560 nodes. The following number of nodes were used in the experiments: $28,56,112,140,560$. These correspond to using $5 \%, 10 \%, 20 \%$, and $100 \%$ of all possible nodes.

Note that if 560 nodes were used, then no data clustering was performed. 560 nodes were only used with RBF11, in order to see what error would be associated with Gaussian function interpolation on the data with fixed variance. Training occurred by presenting the four training sets to the system: B1046, B1060, B1070 and B1077. The error statistic were then generated by individually presenting each set and recording the errors. The experimental results are presented in the tables and plots at the end of this text. In the plots, both the output of the network and the desired output are plotted, next to these are plots of the associated error. Note that in the plots the solid line represents the output of the neural network, and the dashed line represents the actual value of the P.I.D. 233 sensor. All the plots used between node normalization unless otherwise noted on the plot.

All of the implementations used an iterative singular value decomposition to invert the matrix in the weight calculation except for RBF6. RBF6 used gradient descent to solve for the weights. This was implemented only for processing time considerations. All of the data had originally been mapped into the range $[-0.5,0.5]$, although it was not necessary to have the data in the $[-0.5,0.5]$ range. The data was then normalized to have zero mean and unit variance before it was presented to the K -means preprocessor. The data was denormalized back to the $[-0.5,0.5]$ range before it was presented to the RBF algorithm. All algorithms randomized the order of the data vectors so that K-means could better cluster the data. The randomization made a dramatic improvement on the uniformity of the center distributions found by K-means.

RBF1 and RBF11 used a global variance parameter, in this case all nodes used a variance of 0.01 . This parameter was chosen heuristically to work best with the given data. Two other forms of RBFs were created to avoid having to choose a global variance parameter, and to allow for the variance to vary with each node. In order to determine the effect of normalizing across the Gaussian nodes, RBF1 was implemented without the between node normalization, and RBF11 was implemented with the between node normalization described previously.

RBFS used the square of the mean of the 50 nearest vectors to a centroid (the mean of a Gaussian node) for the variance of the node which belongs to that centroid.

RBF10 used the square median of the 50 nearest vectors to a centroid as the variance of the node. The size of the neighborhoods used in determining the variance was selected by trial and error. A larger size neighborhood helps reduce the effect of data point outliers. Too large a neighborhood will cause all nodes to have essentially the same variance. The performance of the system was sensitive to the size of the neighborhood.

RBF12 used the kmeans algorithm to calculate the variance of each cluster. Note that a Euclidean distance metric was used in calculating the distance of each vector in a cluster for the centroid of that cluster. Otherwise, it would have been necessary to use a covariance matrix approach.

RBF6 was the same as RBF8 except that the weights were calculated using gradient descent. The error statistics of this method will be as good as those of the algebraic method as long as enough iterations are performed. Thus, an important consideration is the processor time used in convergence.

The tables at the end of this text contain three columns of numbers: the Root Mean Square (RMS) of the error between the output of the network and the actual prediction value, the Normalized Root Mean Square Error (NRMS), and the Maximum percentage of the error (Max \%). The last two columns will be used for comparison between the experiments.

The error statistics for RBF1 with global variance of 0.01 and $25 \%$ nodes (i.e. 140 nodes were used out of 560 possible nodes) without between node normalization is shown in Table 3. The Normalized Root Mean Square Error (NRMS) was always under $6 \%$, and the maximum percent error was as high as $55 \%$. Clearly this is unacceptable for most applications. When between node normalization was added, the greatest maximum percent error dropped to $11 \%$ as shown in Table 4.

All algorithms described from here on will incorporate between node normalization. Table 5 shows the error statistics when a node was located at every data point ( 560 nodes), with a global variance of 0.01 . The errors were higher in this case then when 140 nodes were used. This can be explained by the fact that with a node on every data point, individual variances were needed from node to node. With less nodes than data points, the clusters may overlap without adverse affect on the function approximation capabilities. It should be noted that this conjecture is only valid for the data used in these experiments.

The greatest difficulty with the above method involves the heuristic selection of two parameters: variance, and the number of nodes. In order to avoid having to pick a variance parameter, and to provide for the ability of each cluster to have an individual variance, other methods are being investigated. The next two methods use a neighborhood operation to assign a variance to each cluster. Unfortunately, this still involves heuristic selection of a neighborhood size, and of the number of nodes to be used in the network.

RBF8 used a mean neighborhood operation to determine the variance of each node individually. To determine the variance of a node, the algorithm calculates the mean of the vectors within some neighborhood about the centroid (mean of the Gaussian) of that node. If we let $N$ represent the size of the neighborhood, then this can be written as:

$$
\text { variance }=\frac{1}{N} \sum_{j=1}^{N} \sum_{i=1}^{\operatorname{dim}}\left(x_{i}-c_{i}\right)^{2}
$$

where $x_{i}$ is the $i^{\text {th }}$ component of the input vector $\vec{x}$, and dim refers to the dimension of the input vectors. Thus if $N=50$, this means that the 50 input data vectors with the smallest square distance from the centroid will be used in calculating the variance for that centroid.

Table 6 shows the error statistics for Rbf8 with 140 nodes and a neighborhood size of 50 . This neighborhood size was arbitarily chosen. The effects of this selection will be discussed later. The errors in this table are slightly higher than for the Rbf11 with 140 nodes in Table 4. However, the problem of selecting a suitable global variance has been traded for selecting a suitable neighborhood size. This is only a benefit if the neighborhood size selection is less sensitive than the variance parameter.

Since in Rbf8 a mean neighborhood operation was used, it is only natural
to determine if a median neighborhood operation would produce better results. Rbf10 used the median of the square of the distance to the N nearest input vectors to a centroid, to calculate the variance for that centroid. The error statistics for Rbf10 using 140 nodes with a neighborhood size of 50 is shown in Table 7. Note that the mean and median implementations produce very similar error statistics, with the mean operation being slightly favorable.

The effects of changing the neighborhood size and the effects of changing the number of nodes will now be discussed. Since both the mean and median implementations produce nearly identical results, only the mean implementation RBF8 will be used to show the results. Tables 8,9 , and 10 represent the error statistics for Rbf8 with 140 nodes and neighborhood sizes of: 5, 60, and 95 . This represents $10 \%, 120 \%$, and $190 \%$ of the original value of 50 . Looking at both the NRMS and the maximum \% error, it can be seen that the errors were lower for the training sets and higher for the validation sets. When considering only the validation sets, a neighborhood size of 5 was the best. But even with a $90 \%$ change in the value of the neighborhood size, the errors still remained low except for set B1071 with a neighborhood size of 95 . It should be noted that B1071 requires the most generalization of all of the validation sets. This indicates that the selection of neighborhood size was not critical as long as it fell within some reasonable range. This is only significant if the output of RBF1 is sensitive to similar variations in the value of the global variance parameter.

Tables 11, 12, and 13 show the error statistics for the global variance (RBF11) values of $0.001,0.012$, and 0.19 . This represents a variation of $10 \%, 120 \%$, and $190 \%$. Comparing these tables with Table 4 reveals that the errors don't really change that much even with a $90 \%$ change in the variance parameter.

The bottom line of this is that neither the selection of the neighborhood size nor the global variance parameter are critical as long as they fall within some reasonable range. Thus, there is no advantage to using the neighborhood operation to determine the variance.

The other heuristic to be investigated is the selection of the number of nodes in the RBFNN. Tables 14, 15, and 16 specify the error statistics for RBF8 with 28,56 , and 112 nodes with a neighborhood size of 50 respectively. This represents $5 \%, 10 \%$, and $20 \%$ of the possible nodes. Compare these tables with Table 6 which had 140 nodes ( $25 \%$ ). The error did increase when only $5 \%$ of the nodes were used, especially the maximum percent error
for set B1071. The difference in error between $20 \%$ and $25 \%$ nodes seems insignificant.

Tables 17, 18, and 19 represent the error statistics for RBF11 with global variance parameter of 0.01 with 28,56 , and 112 nodes respectively. As the variance increases the error for the training sets increases, but overall, the errors remain small. This is because as the variance increases, the overlap becomes too great for an individual set to be learned exactly. The RBF1 implementation seems to be a little more sensitive to the selection of the number of nodes than the neighborhood operation RBF8.

Another performance consideration was the computation time required by the various implementations. Since most of the algorithms were virtually the same, only RBF8 and RBF6 will be compared. RBF8 used the weight matrix inverse approach and RBF6 used gradient descent to calculate the weights. RBF8 takes approximately 225 cpu seconds whereas the gradient descent approach RBF6 takes hours when working with 112 out of 560 possible nodes.

The gradient descent approach can be greatly improved by performing only localized weight update. Therefore each epoch will involve only updating a small number of weights directly having influence on the output of the given input vector, rather than updating all of the weights. Since the error statistics for the gradient descent approach change with the number of iterations performed, and those iterations are currently too expensive to justify performing, no error statistics have been generated for the gradient descent approach.

The latest implementation (Rbf12) involved using the kmeans algorithm to determine the variance of each cluster and assigning this value to each node associated with that cluster. Since it is possible that a cluster will only contain one or two vectors, it was necessary to create an artificial lower bound on the calculated variance. In this case the lowest the variance was allowed to be was 0.01 . This value was chosen so that this method could be compared to the the global variance method. In other words, the variance of each node will be 0.01 unless a larger value is required. Since it was desirable to have enough overlap between the nodes to provide for sufficient generlization, all of the calculated variances of this procedure were multiplied by a constant. The error statistics for this method are presented in 20. Plots of this appear at the end. The error statistics for this implementation are similar to those presented for Rbf11. The plots also show that the behavior of the output of the network is oscillatory about the desired value.

Timing comparisons have also been made between the two clustering algorithms: K-means and Kohonen's LVQ [3]. The two took approximately the same amount of C.P.U. time.

## 5 Conclusions and Future Directions

This report has shown that combination of the K-means algorithm with Radial Basis Function Neural Network with variable variance has allowed for successful sensor value prediction.

Work is being done on automating the two main heuristics of this approach: the variance of the processing nodes, and the number of nodes to use on the data. The automation is being done so that this approach may be used as part of an on-line diagnostic, fault detection system. It should be noted that the number of nodes and the value of the variance(s) was not really that important as long as it fell within a reasonable range.

Investigation of the K -means algorithm as a stand-alone predictor using vector component labeling is also proceeding. The purpose of this approach is to determine if the weight calculations can be avoided, and to form a comparison to component labeling with Kohonen's LVQ.

Two approaches that are currently under investigation are the implementation of a regularization network using Radial Basis Functions, and the implementation of a mulitplicative gaussian bar network. All of the approaches described in this paper require that the variance be the same for all of the dimensions of each node. The multiplicative gaussian bar network will allow a varying variance not only for each node, but also for each dimension of each node. It is hoped that this approach will allow the network to have a smoother prediction output. Currently, as can be seen from the plots, the output of the network tends to jump when a different group of nodes becomes active. This is bad when trying to use the system for fault detection because it forces the use of large confidence intervals. Hopefully with variable variance in each dimension, the confidence interval will be very small.

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Table 3: Error Statistics from Parameter List 1 for Rbf1, 140 out of 560 possible nodes, global variance 0.01 , no normalization across nodes

| Test <br> Firing | Training/ <br> Validation | $R M S$ | NRMS | Max. <br> $\%$ Error |
| :---: | :---: | :---: | :---: | :---: |
| B1046 | T | 51.59860 | 0.05187 | 55.43803 |
| B1060 | T | 28.21390 | 0.02770 | 43.82708 |
| B1061 | V | 29.12591 | 0.02831 | 37.07641 |
| B1062 | V | 39.23239 | 0.03863 | 47.86150 |
| B1063 | V | 33.72927 | 0.03320 | 31.26490 |
| B1066 | V | 53.21917 | 0.05209 | 46.77677 |
| B1067 | V | 42.51067 | 0.04179 | 29.82455 |
| B1070 | T | 22.83335 | 0.02182 | 27.27271 |
| B1071 | V | 56.68070 | 0.05435 | 40.25718 |
| B1072 | V | 41.90969 | 0.03953 | 38.49752 |
| B1075 | V | 48.30697 | 0.04708 | 44.38711 |
| B1077 | T | 36.09605 | 0.03450 | 34.11739 |

Table 4: Error Statistics from Parameter List 1 for Rbf11, 140 out of 560 possible nodes, global variance 0.01 , with normalization across nodes.

| Test <br> Firing | Training/ <br> Validation | $R M S$ | NRMS | Max. <br> \% Error |
| :---: | :---: | :---: | :---: | :---: |
| B1046 | T | 9.08039 | 0.00913 | 5.05390 |
| B1060 | T | 15.11922 | 0.01484 | 8.18955 |
| B1061 | V | 16.56278 | 0.01610 | 7.55070 |
| B1062 | V | 23.88363 | 0.02352 | 8.80874 |
| B1063 | V | 24.06748 | 0.02369 | 6.74517 |
| B1066 | V | 28.07834 | 0.02748 | 10.30139 |
| B1067 | V | 30.19188 | 0.02968 | 8.40462 |
| B1070 | T | 7.80751 | 0.00746 | 4.90432 |
| B1071 | V | 36.64726 | 0.03514 | 11.16738 |
| B1072 | V | 22.29507 | 0.02103 | 4.61144 |
| B1075 | V | 24.04426 | 0.02343 | 9.24767 |
| B1077 | T | 10.94992 | 0.01047 | 5.01032 |

Table 5: Error Statistics from Parameter List 1 for Rbf11, 560 out of 560 possible nodes, global variance 0.01 , with normalization across nodes.

| Test <br> Firing | Training/ <br> Validation | RMS | NRMS | Max. <br> \% Error |
| :---: | :---: | :---: | :---: | :---: |
| B1046 | T | 0.00721 | 0.00001 | 0.00164 |
| B1060 | T | 5.83704 | 0.00573 | 0.86627 |
| B1061 | V | 21.17175 | 0.02058 | 8.06793 |
| B1062 | V | 26.14354 | 0.02574 | 8.81908 |
| B1063 | V | 47.42858 | 0.04669 | 19.71690 |
| B1066 | V | 25.07479 | 0.02454 | 10.11135 |
| B 1067 | V | 55.24511 | 0.05431 | 24.98962 |
| B 1070 | T | 4.89017 | 0.00467 | 0.71944 |
| B 1071 | V | 42.73459 | 0.04097 | 13.92102 |
| B 1072 | V | 37.87623 | 0.03572 | 16.95097 |
| B 1075 | V | 27.00886 | 0.02632 | 12.82674 |
| B 1077 | T | 7.48279 | 0.00715 | 0.96961 |

Table 6: Error Statistics from Parameter List 1 for Rbf8, 140 out of 560 possible nodes, neighborhood size of 50 .

| Test <br> Firing | Training/ <br> Validation | $R M S$ | NRMS | Max. <br> $\%$ Error |
| :---: | :---: | :---: | :---: | :---: |
| B1046 | T | 2.38426 | 0.00240 | 1.39597 |
| B1060 | T | 12.59777 | 0.01237 | 8.94835 |
| B1061 | V | 17.01483 | 0.01654 | 10.51706 |
| B1062 | V | 27.73178 | 0.02731 | 7.73836 |
| B1063 | V | 36.67299 | 0.03610 | .8 .43292 |
| B1066 | V | 30.88582 | 0.03023 | 9.61599 |
| B 1067 | V | 35.97727 | 0.03537 | 8.59062 |
| B1070 | T | 7.00646 | 0.00670 | 4.22006 |
| B 1071 | V | 38.58204 | 0.03690 | 13.70739 |
| B 1072 | V | 23.79091 | 0.02244 | 7.77105 |
| B 1075 | V | 30.23295 | 0.02946 | 12.56111 |
| B 1077 | T | 9.82221 | 0.00939 | 7.98302 |

Table 7: Error Statistics from Parameter List 1 for Rbf10, 140 out of 560 possible nodes, neighborhood size of 50 .

| Test <br> Firing | Training/ <br> Validation | RMS | NRMS | Max. <br> $\%$ Error |
| :---: | :---: | :---: | :---: | :---: |
| B1046 | T | 2.83764 | 0.00285 | 2.27082 |
| B1060 | T | 13.01944 | 0.01278 | 10.44688 |
| B1061 | V | 16.99191 | 0.01652 | 8.35701 |
| B1062 | V | 27.63844 | 0.02721 | 10.59826 |
| B1063 | V | 35.93978 | 0.03538 | 7.85795 |
| B1066 | V | 30.68942 | 0.03004 | 8.84875 |
| B1067 | V | 34.05498 | 0.03348 | 7.91770 |
| B1070 | T | 7.27333 | 0.00695 | 4.86389 |
| B1071 | V | 40.26593 | 0.03861 | 12.87269 |
| B1072 | V | 23.99103 | 0.02263 | 7.29484 |
| B1075 | V | 31.83439 | 0.03102 | 14.72255 |
| B1077 | T | 9.86484 | 0.00943 | 6.64400 |

Table 8: Error Statistics from Parameter List 1 for Rbf8, 140 out of 560 possible nodes, neighborhood size of 5

| Test <br> Firing | Training/ <br> Validation | RMS | NRMS | Max. <br> \% Error |
| :---: | :---: | :---: | :---: | :---: |
| B1046 | T | 12.59036 | 0.01266 | 5.91643 |
| B1060 | T | 18.86700 | 0.01852 | 8.26020 |
| B1061 | V | 15.54236 | 0.01511 | 8.09296 |
| B1062 | V | 23.84036 | 0.02347 | 7.95695 |
| B1063 | V | 29.50691 | 0.02905 | 6.01330 |
| B 1066 | V | 27.19326 | 0.02661 | 8.17823 |
| $\mathrm{B1067}$ | V | 27.51375 | 0.02705 | 6.54893 |
| B 1070 | T | 12.13479 | 0.01160 | 4.74145 |
| B 1071 | V | 37.65979 | 0.03611 | 14.24448 |
| B 1072 | V | 20.79559 | 0.01961 | 6.86008 |
| B 1075 | V | 25.46773 | 0.02482 | 9.03917 |
| B 1077 | T | 11.67591 | 0.01116 | 5.18719 |

Table 9: Error Statistics from Parameter List 1 for Rbf8, 140 out of 560 possible nodes, neighborhood size of 60

| Test <br> Firing | Training/ <br> Validation | RMS | NRMS | Max. <br> \% Error |
| :---: | :---: | :---: | :---: | :---: |
| B1046 | T | 1.99468 | 0.00201 | 1.47569 |
| B1060 | T | 12.11123 | 0.01189 | 7.26255 |
| B1061 | V | 16.98131 | 0.01651 | 9.71268 |
| B1062 | V | 26.68269 | 0.02627 | 8.88408 |
| B1063 | V | 35.55722 | 0.03500 | 8.21124 |
| B1066 | V | 31.38819 | 0.03072 | 9.40951 |
| B1067 | V | 39.94933 | 0.03927 | 9.00299 |
| B1070 | T | 6.99072 | 0.00668 | 4.03056 |
| B1071 | V | 39.72683 | 0.03809 | 14.84076 |
| B1072 | V | 25.54100 | 0.02409 | 7.60504 |
| B1075 | V | 27.51913 | 0.02682 | 10.99728 |
| B1077 | T | 0.43180 | 0.00902 | 5.93759 |

Table 10: Error Statistics from Parameter List 1 for Rbf8, 140 out of 560 possible nodes, neighborhood size of 95

| Test <br> Firing | Training/ <br> Validation | RMS | NRMS | Max. <br> \% Error |
| :---: | :---: | :---: | :---: | :---: |
| B1046 | T | 2.52465 | 0.00254 | 1.01239 |
| B1060 | T | 5.89194 | 0.00578 | 2.65420 |
| B1061 | V | 14.11111 | 0.01372 | 8.36682 |
| B1062 | V | 25.79183 | 0.02540 | 14.42955 |
| B1063 | V | 32.26750 | 0.03177 | 14.87512 |
| B1066 | V | 28.31700 | 0.02771 | 15.87984 |
| B1067 | V | 40.62741 | 0.03994 | 11.26816 |
| B1070 | T | 5.18209 | 0.00495 | 2.84313 |
| B1071 | V | 65.97996 | 0.06326 | 32.48063 |
| B1072 | V | 39.38403 | 0.03715 | 21.43067 |
| B1075 | V | 20.39765 | 0.01988 | 9.73600 |
| B1077 | T | 5.62804 | 0.00538 | 2.67773 |

Table 11: Error Statistics from Parameter List 1 for Rbf11, 140 out of 560 possible nodes, global variance 0.001 , with normalization across nodes.

| Test <br> Firing | Training/ <br> Validation | RMS | NRMS | Max. <br> \% Error |
| :---: | :---: | :---: | :---: | :---: |
| B1046 | T | 17.39792 | 0.01749 | 9.41842 |
| B1060 | T | 19.24398 | 0.01889 | 8.72347 |
| B1061 | V | 14.86860 | 0.01445 | 9.14472 |
| B1062 | V | 22.17325 | 0.02183 | 7.69661 |
| B1063 | V | 25.62952 | 0.02523 | 6.37022 |
| B1066 | V | 24.79422 | 0.02427 | 8.00535 |
| B1067 | V | 25.81279 | 0.02538 | 6.63005 |
| B1070 | T | 12.97352 | 0.01240 | 5.38796 |
| B1071 | V | 39.82024 | 0.03818 | 15.11610 |
| B1072 | V | 26.84831 | 0.02532 | 6.29536 |
| B1075 | V | 25.14532 | 0.02450 | 9.98049 |
| B1077 | T | 11.28997 | 0.01079 | 5.32295 |

Table 12: Error Statistics from Parameter List 1 for Rbf11, 140 out of 560 possible nodes, global variance 0.012 , with normalization across nodes.

| Test <br> Firing | Training/ <br> Validation | RMS | NRMS | Max. <br> $\%$ Error |
| :---: | :---: | :---: | :---: | :---: |
| B1046 | T | 7.53711 | 0.00758 | 4.02564 |
| B1060 | T | 14.65475 | 0.01439 | 8.15110 |
| B1061 | V | 16.48162 | 0.01602 | 7.53241 |
| B1062 | V | 24.26593 | 0.02389 | 8.67870 |
| B1063 | V | 25.48573 | 0.02509 | 8.54002 |
| B1066 | V | 27.90006 | 0.02731 | 10.27619 |
| B1067 | V | 31.02996 | 0.03050 | 8.69295 |
| B1070 | T | 7.65893 | 0.00732 | 4.90027 |
| B1071 | V | 36.54959 | 0.03504 | 10.38719 |
| B1072 | V | 23.02699 | 0.02172 | 4.71767 |
| B1075 | V | 23.63481 | 0.02303 | 9.14347 |
| B1077 | T | 10.89041 | 0.01041 | 4.90642 |

Table 13: Error Statistics from Parameter List 1 for Rbf11, 140 out of 560 possible nodes, global variance 0.019 , with normalization across nodes.

| Test <br> Firing | Training/ <br> Validation | RMS | NRMS | Max. <br> \% Error |
| :---: | :---: | :---: | :---: | :---: |
| B1046 | T | 3.77056 | 0.00379 | 1.80409 |
| B1060 | T | 13.21858 | 0.01298 | 7.99314 |
| B1061 | V | 18.48718 | 0.01797 | 7.70497 |
| B1062 | V | 26.25988 | 0.02586 | 8.18643 |
| B1063 | V | 29.54284 | 0.02908 | 11.77689 |
| B 1066 | V | 26.13952 | 0.02558 | 9.94572 |
| B 1067 | V | 31.32031 | 0.03079 | 9.47755 |
| B 1070 | T | 6.90393 | 0.00660 | 4.77293 |
| $\mathrm{B1071}$ | V | 37.59862 | 0.03605 | 9.40809 |
| B 1072 | V | 29.65932 | 0.02797 | 7.29677 |
| B 1075 | V | 24.94946 | 0.02431 | 9.01241 |
| B 1077 | T | 10.57636 | 0.01011 | 5.07506 |

Table 14: Error Statistics from Parameter List 1 for Rbf8, 28 out of 560 possible nodes, neighborhood size of 50 .

| Test <br> Firing | Training/ <br> Valiclation | RMS | NRMS | Max. <br> \% Error |
| :---: | :---: | :---: | :---: | :---: |
| B1046 | T | 14.41434 | 0.01449 | 9.00289 |
| B1060 | T | 28.22631 | 0.02771 | 15.22439 |
| B1061 | V | 20.45557 | 0.01988 | 15.12137 |
| B1062 | V | 27.14355 | 0.02673 | 13.25370 |
| B1063 | V | 31.14893 | 0.03066 | 7.27571 |
| B1066 | V | 25.22222 | 0.02469 | 9.23387 |
| B1067 | V | 31.01706 | 0.03049 | 6.86656 |
| B1070 | T | 18.86116 | 0.01803 | 6.47004 |
| B1071 | V | 42.92448 | 0.04116 | 23.21731 |
| B 1072 | V | 25.89541 | 0.02442 | 7.66308 |
| B1075 | V | 29.25733 | 0.02851 | 14.23219 |
| B1077 | T | 14.43709 | 0.01380 | 7.00140 |

Table 15: Error Statistics from Parameter List 1 for Rbf8, 56 out of 560 possible nodes, neighborhood size of 50

| Test <br> Firing | Training/ <br> Validation | RMS | NRMS | Max. <br> \% Error |
| :---: | :---: | :---: | :---: | :---: |
| B1046 | T | 4.45642 | 0.00448 | 2.73107 |
| B1060 | T | 19.72829 | 0.01937 | 8.06534 |
| B 1061 | V | 13.82834 | 0.01344 | 7.98474 |
| $\mathrm{B1062}$ | V | 27.61247 | 0.02719 | 6.78202 |
| B 1063 | V | 30.51164 | 0.03004 | 5.60983 |
| B 1066 | V | 27.69112 | 0.02710 | 7.22070 |
| B 1067 | V | 28.52224 | 0.02804 | 6.90655 |
| B 1070 | T | 9.74294 | 0.00931 | 4.63169 |
| B 1071 | V | 36.70488 | 0.03519 | 11.93740 |
| B 1072 | V | 19.51650 | 0.01841 | 4.33229 |
| B 1075 | V | 23.80098 | 0.02319 | 8.70032 |
| B 1077 | T | 13.61962 | 0.01302 | 4.74346 |

Table 16: Error Statistics from Parameter List 1 for Rbf8, 112 out of 560 possible nodes, neighborhood size of 50 .

| Test <br> Firing | Training/ <br> Validation | RMS | NRMS | Max. <br> $\%$ Error |
| :---: | :---: | :---: | :---: | :---: |
| B1046 | T | 2.83689 | 0.00285 | 1.87113 |
| B1060 | T | 15.10454 | 0.01483 | 7.26094 |
| B1061 | V | 15.33540 | 0.01491 | 9.99841 |
| B1062 | V | 27.02377 | 0.02661 | 7.66580 |
| B1063 | V | 31.69655 | 0.03120 | 6.39428 |
| B1066 | V | 30.97042 | 0.03031 | 9.15610 |
| B 1067 | V | 30.84572 | 0.03032 | 7.25803 |
| $\mathrm{B1070}$ | T | 7.02387 | 0.00671 | 4.29946 |
| B 1071 | V | 36.32853 | 0.03483 | 11.16077 |
| B 1072 | V | 19.99030 | 0.01885 | 7.18022 |
| B 1075 | V | 20.72743 | 0.02020 | 9.70741 |
| B 1077 | T | 13.02576 | 0.01245 | 7.72375 |

Table 17: Error Statistics from Parameter List 1 for Rbf11, 28 out of 560 possible nodes, global variance 0.01 , with normalization across nodes.

| Test <br> Firing | Training/ <br> Validation | RMS | NRMS | Max. <br> \% Error |
| :---: | :---: | :---: | :---: | :---: |
| B1046 | T | 27.18175 | 0.02732 | 15.67179 |
| B1060 | T | 25.14493 | 0.02468 | 11.02959 |
| B1061 | V | 16.65054 | 0.01619 | 10.56191 |
| B1062 | V | 24.08476 | 0.02372 | 8.69066 |
| B1063 | V | 30.18663 | 0.02972 | 6.02264 |
| B1066 | V | 22.65658 | 0.02217 | 6.00475 |
| B1067 | V | 30.09099 | 0.02958 | 6.73048 |
| B1070 | T | 18.25288 | 0.01745 | 6.61461 |
| B1071 | V | 39.08114 | 0.03747 | 15.32269 |
| B1072 | V | 22.67000 | 0.02138 | 5.39135 |
| B1075 | V | 26.91294 | 0.02623 | 11.02567 |
| B1077 | T | 12.99691 | 0.01242 | 5.55255 |

Table 18: Error Statistics from Parameter List 1 for Rbf11, 56 out of 560 possible nodes, global variance 0.01 , with normalization across nodes.

| Test <br> Firing | Training/ <br> Validation | $R M S$ | NRMS | Max. <br> $\%$ Error |
| :---: | :---: | :---: | :---: | :---: |
| B1046 | T | 16.89184 | 0.01698 | 9.67346 |
| B1060 | T | 22.36876 | 0.02196 | 10.31860 |
| B1061 | V | 14.56266 | 0.01416 | 10.03354 |
| B1062 | V | 20.76527 | 0.02045 | 8.12575 |
| B1063 | V | 23.57223 | 0.02321 | 5.79424 |
| B1066 | V | 21.03067 | 0.02146 | 5.90312 |
| B1067 | V | 25.39526 | 0.02497 | 6.24095 |
| B1070 | T | 15.45565 | 0.01477 | 5.73375 |
| B1071 | V | 40.40978 | 0.03875 | 13.09041 |
| B1072 | V | 23.68318 | 0.02234 | 5.45226 |
| B1075 | V | 24.96156 | 0.02433 | 10.16316 |
| B1077 | T | 11.03639 | 0.01055 | 5.47923 |

Table 19: Error Statistics from Parameter List 1 for Rbf11, 112 out of 560 possible nodes, global variance 0.01 , with normalization across nodes.

| Test <br> Firing | Training/ <br> Validation | RMS | NRMS | Max. <br> \% Error |
| :---: | :---: | :---: | :---: | :---: |
| B1046 | T | 9.19158 | 0.00924 | 5.07136 |
| B1060 | T | 18.42548 | 0.01809 | 8.74580 |
| B1061 | V | 14.63324 | 0.01422 | 7.68326 |
| B1062 | V | 22.38574 | 0.02204 | 7.07681 |
| B1063 | V | 22.37034 | 0.02202 | 5.83572 |
| B1066 | V | 24.51416 | 0.02399 | 7.59422 |
| B1067 | V | 27.92617 | 0.02745 | 7.61870 |
| B1070 | T | 9.62823 | 0.00920 | 5.21438 |
| B1071 | V | 37.67616 | 0.03612 | 11.96297 |
| B1072 | V | 26.63420 | 0.02512 | 4.70912 |
| B1075 | V | 25.96042 | 0.02530 | 9.57053 |
| B1077 | T | 12.05393 | 0.01152 | 4.43430 |

Table 20: Error Statistics from Parameter List 1 for Rbf12, 140 out of 560 possible nodes

| Test <br> Firing | Training/ <br> Validation | RMS | NRMS | Max. <br> \% Error |
| :---: | :---: | :---: | :---: | :---: |
| B1046 | T | 6.01873 | 0.00605 | 3.15011 |
| B1060 | T | 14.86046 | 0.01459 | 7.73349 |
| B1061 | V | 16.48907 | 0.01603 | 7.48909 |
| B1062 | V | 23.49133 | 0.02313 | 8.62899 |
| B 1063 | V | 23.95228 | 0.02358 | 6.73804 |
| B 1066 | V | 28.18141 | 0.02758 | 9.58983 |
| B 1067 | V | 30.06580 | 0.02956 | 8.39829 |
| B 1070 | T | 7.49930 | 0.00717 | 4.29970 |
| B 1071 | V | 36.89098 | 0.03537 | 11.30262 |
| B 1072 | V | 22.69011 | 0.02140 | 4.92904 |
| B 1075 | V | 24.03571 | 0.02342 | 9.16707 |
| B 1077 | T | 11.20852 | 0.01071 | 6.01282 |





























Figure 1: Rbf8 with 140 nodes, neighborhood 5


Figure 2: Rbf8 with 140 nodes, neighborhood 5


Figure 3: Rbf8 with 140 nodes, neighborhood 5


Figure 4: Rbf8 with 140 nodes, neighborhood 5


Figure 5: Rbf8 with 140 nodes, neighborhood 5



















## 11. SUPPLEMENTARY NOTES

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This paper presents a brief report on the application of Radial Basis Function Neural Networks (RBFNN) to the prediction of sensor values for fault detection and diagnosis of the Space Shuttle's Main Engines (SSME). The location of the Radial Basis Function (RBF) node centers was determined with a K-means clustering algorithm. A neighborhood operation about these center points was used to determine the variances of the individual processing nodes.
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