Quasi–Linear Regime of Gravitational Instability as a Clue to Understanding the Large–Scale Structure in the Universe

Sergei F. Shandarin
Department of Physics and Astronomy
University of Kansas
Lawrence, Kansas 66045, USA

to be published in the Proceedings of the dedication seminar for the Devayni Complex of Institutional Buildings of IUCAA Pune, India, Dec. 29–30, 1992

Quasi–Linear Regime of Gravitational Instability as a Clue to Understanding the Large–Scale Structure in the Universe

Sergei F. Shandarin
Department of Physics and Astronomy
University of Kansas
Lawrence, Kansas 66045, USA

Abstract

In the late seventies, an image of the large–scale structure in the Universe began to emerge as a result of the accumulation of the galaxy redshifts. Most of the galaxies are found to concentrate in large filaments and perhaps sheets leaving most of the volume empty. Similar structures were predicted theoretically in the frame of the adiabatic theory of galaxy formation (Zeldovich) and later in the hot dark matter cosmology. However, both scenarios have been ruled out by the observations. With these sceanrios the dynamical part of the scenario was also erroneously rejected by many as well. In this talk, I derive the Zeldovich approximation from the exact dynamic equations and show that it is always better than the standard linear approximation. The advantage of the Zeldovich approximation is the greatest in the quasi–linear regime when $\delta_{rms} < 1 (\delta \equiv \delta\rho/\rho)$, but the displacement of the matter is essential. The range of scales in the quasi–linear regime depends upon the slope of the initial spectrum and increases with decreasing n, where n is the exponent, if the initial spectrum is approximated by a simple power law $P(k) \propto k^n$.

1. Introduction

The detection of the fluctuations in the MBR temperature (Smoot, *et al*, 1992) provides a solid foundation for the theories of the large-scale structure based on gravitational instability. Although the most popular model of the eighties, the standard CDM (Cold Dark Matter) model, has been unable to reconcile the amplitude of the temperature fluctuations with the level of structure on the scale of tens of Mpc, its combination with the HDM (Hot Dark Matter) model marginally agrees with all current observations (Davis *et al.* 1992; Klypin *et al.* 1993; Pogosyan and Starobinsky, 1993). There are other options as well. For instance, one may change the primeval spectrum in a CDM–dominated universe (the $n = 0.8$ model looks reasonably good, Cen *et al.*, 1992), or postulate a non–zero cosmological term, $\Lambda \approx 0.7 - 0.8$ (Efstathiou *et al.* 1990). Still another option is the hypothesis invoking the co-operative galaxy formation in the CDM scenario (Bowen *et al.* 1992). The common feature of all the above models is the assumption that the present structures in the universe emerged from the small random Gaussian density fluctuations due to gravitational instability in an expanding universe.

The existing structures which are commonly mentioned in a cosmological context range from globular clusters with $M \sim 10^6 M_{\odot}$ to the largest superclusters of galaxies with $M \sim 10^{15} M_{\odot}$. On larger scales the distribution of galaxies becomes gradually more homogeneous with the growth of the scale. The small scale part of this range is a highly nonlinear regime $\delta_{rms} >> 1$, here $\delta \equiv \delta\rho/\bar{\rho}$. The scale where the amplitude of density fluctuations reaches unity, $\delta_{rms} = 1$ defines the scale of nonlinearity which is of the order of a few Mpc depending on a particular definition of δ_{rms}. Practically the evaluation of δ_{rms} involves smoothing of a density or galaxy distribution with a certain window function. Two common windows, the Gaussian and the top–hat windows, result in an approximately two–fold difference in the scale of nonlinearity; $R_{T-H} \approx 2R_G$ almost independently of the initial spectrum (Melott and Shandarin, 1993). It is accepted that the nonlinear scale in the universe $R_{T-H} = 8h^{-1}$Mpc (where $h = H/100$) and this value is often used for the normalization of the initial perturbations.

On larger scales the density perturbations are small, $\delta_{rms}(R > R_{n\ell}) < 1$ and the linear approximation is usually thought to be at least qualitatively correct. However, this contradicts to the assertion that there are structures on a scale of several tens of Mpc: for example, the Great Wall or the Great Attractor.

There are several ways to resolve this contradiction. First, the structures on large scales are not statistically significant and are not real physical objects. They are rather artifacts of the remarkable ability of the human eye to see structures in purely random distributions. The lack of an unambiguous statistic able to characterize the large scale structures quantitatively seems to support this point of view. On the other hand, the N–body simulations of the most successful cosmological models seem to exhibit similar structures in the simulated "galaxy" distributions. Dismissing this result is more difficult, because in the N–body simulations we have a lot more information about the distribution in question and can control the evolution of the structures (Kofman *et al.* 1992, Coles, Melott, and Shandarin 1993).

Secondly, one can say that either the assumption about the Gaussianity of the initial fluctuations or the gravitational instability scenario itself is wrong. However, no good scenario based on the non–Gaussian initial conditions and explaining the large–scale structure has been suggested. The same can be said about nongravitational mechanisms of the formation of the large scale structure.

In this talk I discuss the third option. I assume that the initial fluctuations of Gaussian type are amplified by gravitational instability. The scale of nonlinearity $R_{n\ell}$, defined by the equality $\delta_{rms}(R_{n\ell}) = 1$ corresponds to the measured one, assuming an appropriate smoothing window function. I show that there is a range of scales $R_{n\ell} < R < R_{qn\ell}$ where $\delta_{rms}(R) < 1$ but the density field is non–Gaussian, which can naturally account for the large–scale structure in the universe. It is natural to define this stage of evolution as a quasilinear regime: on the one hand it is not nonlinear since $\delta_{rms}(R) < 1$, but the spatial distribution is very non–Gaussian. The density distribution is also non–Gaussian in the sense of the distribution function. This actually is an additional factor helping to explain the large–scale structure, but I stress the non–Gaussian character of the geometry of the density distribution, which I believe is more important.

2. Linear and Quasi–Linear Solutions

It is useful to begin with a brief summary of the linear theory of gravitational instability (Peebles, 1980). The equations describing the evolution of density perturbations in a dust–like medium is convenient to write down in terms of the comoving coordinates $x_i = r_i/a$, the peculiar velocity $v_i = \dot{D}^{-1} \cdot dx_i/dt = dx_i/dD$ and the density perturbation $\delta = (\rho - \bar{\rho})/\bar{\rho}$

$$\frac{\partial \delta}{\partial D} + \frac{\partial v_i}{\partial x_i} + v_i \frac{\partial \delta}{\partial x_i} + \delta \frac{\partial v_i}{\partial x_i} = 0 \ , \tag{1}$$

$$\frac{\partial v_i}{\partial D} + v_k \frac{\partial v_i}{\partial x_k} = \frac{3}{2} \frac{\Omega}{D \cdot f^2} \left(\frac{\partial \varphi}{\partial x_i} + v_i \right) , \tag{2}$$

$$\frac{\partial^2 \varphi}{\partial x_i^2} = \frac{\delta}{D} , \tag{3}$$

where D is the function of time t or the scale factor $a = a(t)$ describing the growing mode in the linear regime, $\Omega = \bar{\rho}/\bar{\rho}_{cr}$, $f = \dot{D}/(H \cdot D)$, and $H = \dot{a}/a$. The perturbation of the gravitational potential has been normalized in such a way that in the linear regime $v_i = -\partial\varphi/\partial x_i$ and the right hand side of Eq. (2) is zero to the linear order. The last two terms in Eq. (1) and the second term in Eq. (2) are nonlinear and usually are discarded in the linear analysis. Now we can easily obtain the growing linear solution of the gravitational instability

$$\delta(\mathbf{x}, D) = D \cdot \delta_0(\mathbf{x}) , \tag{4}$$

$$v_i(\mathbf{x}, D) = v_{0i}(\mathbf{x}) , \tag{5}$$

where $\delta_0(\mathbf{x})$ and $v_{0i}(\mathbf{x})$ are the initial density and velocity perturbations.

Discarding the nonlinear terms in Eq. (1) and (2) we assume that they are small compared to the corresponding linear terms which are proportional to δ. However, in some cases an additional condition must be satisfied before the nonlinear terms are dropped. Comparing the first two linear terms in Eq. (1) we conclude that $\delta/D \sim v/\ell_v$, which roughly means that δ is small ($\delta < 1$) if $D < \ell_v/v$, where ℓ_v is the characteristic scale of the velocity field and v is the magnitude of the velocity perturbations. Two nonlinear terms in Eq. (1) have the order of $v\delta/\ell_\delta$ and $v\delta/\ell_v$ correspondingly here ℓ_δ is the typical scale of the density fluctuation. The last term is obviously small compared to the linear terms while $\delta < 1$. But the first nonlinear term can be neglected only when $\delta < \ell_\delta/\ell_v$. The ratio of these scales is always less than unity and can be a small number depending on the initial spectrum. In the case of the power law initial spectra

$$P(k) \propto k^n , \; k_1 < k < k_2 , \tag{6}$$

where k_1 and k_2 are the free parameters specifying the longwave and shortwave cutoffs, one can easily show that for $n > -1$ $\ell_v \sim \ell_\delta$ and for $-3 < n < -1$ $\ell_\delta/\ell_v \sim (k_2/k_1)^{\frac{n+1}{2}}$. The last estimate indicates that the standard linear theory can be applied only until $\delta \lesssim (k_2/k_1)^{\frac{n+1}{2}}$ not until $\delta < 1$.

The explanation to this is of course very simple. The linear solution (Eq. (4) and (5)) does not take into consideration the displacement of points from their Lagrangian (*i.e.*, unperturbed positions), and therefore becomes invalid when the rms displacement exceeds the short wavelength cutoff

$$d_{rms} \sim v \cdot D > k_2^{-1} \tag{7}$$

since $v \cdot D \sim \delta \cdot (k_1/k_2)^{(n+1)/2} k_2^{-1}$ for $n < -1$.

Figure 1 shows a simple example illustrating the the above discussion. Three panels show a one–dimensional realization of the initial density, velocity and potential perturbations respectively for the CDM spectrum: $P_{CDM}^{(1D)}(k) = k^2 P_{CDM}^{(3D)}(k)$. The size of the box is 4000 Mpc and the short wave cutoff of the spectrum roughly corresponds to the scale of galactic sizes $\ell_{cutoff} \cong 1Mpc$. Only a small part of the box is shown.

This problem can be easily solved by using Lagrangian coordinates. Retaining the convection terms in Eq. (1) and (2) one can develop the linear theory of gravitational instability in Lagrangian space

$$\frac{d\delta}{dD} = -\left(\frac{\partial v_i}{\partial x_i}\right)_0 \ , \tag{8}$$

$$\frac{dv_i}{dD} = 0 \ , \tag{9}$$

where $-(\partial v_i/\partial x_i)_0 = \delta_0(\mathbf{q})$. The solution to these equations is obvious

$$\delta(\mathbf{q}, D) = D \cdot \delta_0(\mathbf{q}) \ , \tag{10}$$

$$v_i(\mathbf{q}, D) = v_{0i}(\mathbf{q}) \ . \tag{11}$$

The density and velocity distributions in Eulerian space can be obtained via the equation of motion

$$\mathbf{x}(\mathbf{q}, D) = q + D \cdot \mathbf{v}_0(\mathbf{q}) \ , \tag{12}$$

which is the Zeldovich approximation. For $n > -1$ both approaches give similar results, although the Lagrangian one is always more accurate. In this case clumps of a typical mass $M \sim \bar{\rho} k_2^{-3}$ form and they are displaced by the typical distance $d \sim k_2^{-1}$. Usually the Zeldovich approximation is extrapolated until the caustics form. This is a very good approximation when the first generation pancakes form from a continuous median (Shandarin, Zeldovich 1989). However, we are going to apply this approximation to the hierarchical clustering scenario, assuming that small scale fluctuations have already collapsed into gravitationally bound objects. Therefore we can not expect that the medium is continuous, we would rather think of it as consisting of clumps of various sizes.

Equations (10), (11) together with Eq. (12) we call a quasi–linear solution in the contrast to Eq. (4), (5) which are the linear solution to the gravitational instability equations (1)–(3). One may think that there is only a terminological difference between them. In the following sections we show that the difference is much more serious.

3. Perturbations in Quasi–Linear Regime

First of all, it is worth demonstrating that the quasi–linear regime possesses features which distinguish it from the linear regime. However, before doing this we specify the range of scales which are in the quasi–linear regime. For the illustrative purpose we will use an example of a power law intial spectra (Eq. 6), however all the results are easily generalized for an arbitrary spectrum (Shandarin, 1993).

The scale of nonlinearity $k_{n\ell}$ is usually defined by the condition

$$\delta_{rms} = 4\pi \cdot D^2 \int_0^{k_{n\ell}} P(k) k^2 dk = 1 \ . \tag{13}$$

This scale roughly separates perturbations in linear and non–linear regimes:

$$\delta_{rms}(L) < 1 \ \text{ if } \ L > k_{n\ell}^{-1}$$

and

$$\delta_{rms}(L) > 1 \ \text{ if } \ L < k_{n\ell}^{-1}$$

If a nonlinear distribution is smoothed with some window function, then the smoothing scale L_s corresponding to the condition $\tilde{\delta}_{rms}(L_s) = 1$ (where $\tilde{\delta}(L_s)$ is the smoothed density field) roughly equals $k_{n\ell}^{-1}$: $L_s \sim k_{n\ell}^{-1}$ (for details see Melott and Shandarin, 1993).

The perturbations in the quasi–linear regime have scales greater than $k_{n\ell}^{-1}$ and therefore $\tilde{\delta}_{rms}(L_{q\ell}) < 1$. If one looks at the spectrum of the nonlinear distribution he finds very little difference with the linear spectrum if $k < k_{n\ell}$ (see Fig. 2). This means that the evolution of the spectrum in this range of scales is perfectly described by the linear theory. However, the spectrum is not the whole story.

Making gravitationally bound clumps of mass assumes the transport of mass from one place to another. Therefore one can calculate and/or measure the rms displacement of mass in the process of building up the structure. Assuming that the displacements corresponding to the short wave perturbation $k \geq k_{n\ell}$ are averaged out one can give an analytic estimate of the rms displacement when the nonlinearity reaches a scale $k_{n\ell}$ (Shandarin, 1993)

$$d_{rms}^2 = \frac{\int_0^{k_{n\ell}} P(k)dk}{\int_0^{k_{n\ell}} P(k)k^2dk} \quad . \tag{14}$$

For the discussion of this result see Shandarin (1993), however, summarizing one can say that the overall accuracy of the theoretical estimate of d_{rms} is similar to that of $k_{n\ell}$ (Eq. (13)).

In the case of the power law initial spectra (Eq. (6)) one easily finds that

$$d_{rms} \sim k_{n\ell}^{-1}, \text{ if } n > -1 \tag{15}$$

and

$$d_{rms} \sim \left(\frac{k_1}{k_{n\ell}}\right)^{\frac{n+1}{2}} k_{n\ell}^{-1}, \text{ if } -3 < n < -1 \ . \tag{16}$$

In the former case the characteristic displacement of mass roughly corresponds to the characteristic mass of nonlinear clumps $d_{rms} \sim k_{n\ell}^{-1}$, which intuitively is very clear. In the latter case d_{rms} can be easily much greater than $k_{n\ell}^{-1}$: $d_{rms} >> k_{n\ell}^{-1}$, if $k_1 << k_{n\ell}$. If the characteristic displacement d_{rms} is greater than the typical distance between clumps it obviously means that the clumps themselves are displaced coherently by the distance d_{rms}. The perturbations having scales between the scale of nonlinearity $k_{n\ell}^{-1}$ and d_{rms} are in the quasi–linear regime: on the one hand the perturbations are small $\delta(\ell) < 1$ ($k_{n\ell}^{-1} < \ell < d_{rms}$), but on the other hand the spatial structure is different from the initial (Gaussian) field.

As it was already mentioned the power spectrum in the quasi–linear regime can be approximated by linear extrapolation of the initial spectrum

$$P(k, D) = D^2 P_{in}(k), \ k \lesssim k_{n\ell}. \tag{17}$$

Therefore neither spectral analysis nor correlation analysis can detect this regime to the first order in δ. Scherrer *et al.* (1991) indicated that the growth of the perturbations is accompanied by the shift of phases. Ryden and Gramann (1991) found the evolution of the scale $L_\theta = L_\theta(a)$ where the initial phases are significantly disturbed. It is a matter of a simple calculation to show that

$$L_\theta \propto d_{rms} \tag{18}$$

A direct comparison of the density distributions, obtained in 3D N–body simulations, with linear extrapolations from the same initial conditions showed that the 25% agreement could be achieved when they both were smoothed with a scale $L_{25\%} \approx 1.65 d_{rms}$ (Melott and Shandarin, 1993). It is worth stressing that this implies that $L_{25\%}$ does not scale with k_{nl}^{-1} if $n < -1$. Thus, we conclude that the power spectrum of the perturbations in the quasi-linear regime is described by the linear extrapolation of the initial (linear) spectrum and the phases are significantly distributed compared to the initial ones.

4. Distribution Function and Filamentary Structure

We begin with the discussion of smooth initial perturbations, assuming that the initial spectrum is sharply cut off at some scale. In this case the Zeldovich approximation can of course be used. The linear theory in Lagrangian space (Eq. 10) suggests that density perturbations retain all properties of the initial field. In particular, for Gaussian initial fields all statistical properties remain Gaussian to the linear order. On the contrary, the linear evolution of density perturbations ($\delta_{rms} < 1$) described in Eulerian space disturbs some properties of the initial field even to the linear order. As a simple example we consider the density distribution function.

Assuming the initial perturbations to be Gaussian, the density distribution function can be approximated as

$$f_i(\delta) \approx f_G(\delta) \equiv \frac{1}{\sqrt{2\pi}\sigma_\delta} e^{-\delta^2/2\sigma_\delta^2} , \tag{19}$$

where $f_i(\delta)$ is the fraction of Lagrangian volume where the density contrast $\delta = (\rho - \bar{\rho})/\bar{\rho}$ has a value between δ and $\delta + d\delta$; here $\sigma_\delta^2 \equiv \delta_{rms}^2$. Obviously the approximation breaks down at $\delta < -1$, since we know that $\delta > -1$. It also breaks down at large δ, though it is not quite obvious (see Shandarin and Zeldovich 1989 and references therein). Here we shall assume that δ is in the range where Eq. 19 is approximately correct.

In Lagrangian space the growth of perturbations with time does not change the distribution function (to the first order) but reduces its range. On the contrary in Eulerian space the density distribution function ceases to be Gaussian even to the first order in δ. The reason for that is very simple: the volume where $\delta < 0$ shrinks and that where $\delta < 0$ expands. The exact calculation is not quite easy, however the result is simple (assuming both $|\delta| < 1$ and $\delta\sigma_\delta < 1$)

$$f_E(\delta) \approx (1 - \alpha_D\delta) f_G(\delta) , \tag{20}$$

where $f_G(\delta)$ is the Gaussian distribution function (Eq. 19) and α_D is a constant depending on the dimensions: $\alpha_1 = 3$ in 1D, $\alpha_2 = 9/4$ in 2D (follows from the general expression given in Gurbatov *et al.* 1991) and $\alpha_3 = 2$ in 3D (Kofman, 1993).

The density distribution function in Eulerian space (Eq. 20) may explain the formation of structures called filamentary and cellular. By calling a density distribution filamentary we probably assume that the dense regions forming a connected structure occupy a surprisingly small volume. Quantitatively this can be expressed in terms of percolation thresholds (Shandarin and Zeldovich, 1989 and references therein). Let us consider a Gaussian density field. Introducing a biasing parameter b one can study the topology of "overdense" regions with $\delta > b \cdot \sigma_\delta$, and "underdense" regions with $\delta < b \cdot \sigma_\delta$. For example, if $b = 2$ then overdense regions are isolated islands occupying totally about 2.5% of the volume in the ocean formed by the underdense regions. If $b = 0.5$ then both overdense (occupying totally about 31% of the volume) and underdense regions form connected structures with the exception of a few isolated islands. It means that at some intermediate b a topological phase transition happens: overdense regions begin to form a connected structure or percolate. Percolation theory suggests that this happens at $b = 1$, when the overdense regions occupy about 16% of the total volume.

The surfaces of constant b conserve the initial topology to the linear order, however, the total volume within a certain b ($\delta > b\sigma_\delta$) decreases. We probably can see a difference in geometry when the percolating volume ($\delta > \sigma_\delta$) shrinks to roughly half of its initial value of 16%. Making use of Eq. (20) one finds that it happens when $\sigma_\delta = \sigma_{filam} \approx 0.3$.

From the symmetry of a Gaussian distribution one can easily anticipate that voids (*i.e.*, underdense regions) begin to be statistically isolated (= cease to percolate) at $\delta < -\sigma$ or $b < -1$. At the epoch of the formation of the filamentary structure ($\sigma_\delta \approx 0.3$) their total volume is increased from 16% to roughly 24% which does not look as impressive as the decrease of the overdense volume from 16% to 8%. The voids double the volume at a later time when the perturbation amplitude reaches the value of $\sigma_\delta = \sigma_{void} \approx 0.5$.

So far we have discussed the case of smoothed initial conditions. In more realistic scenarios as, for example, the CDM or C+CDM models, the large scale structure forms through a hierarchical clustering: the larger structures are built from smaller clumps. Therefore the clumpiness of the medium is very essential and must be incorporated into the above model.

Recent numerical studies of a bunch of the models with the power law initial spectra (Coles *et al.* 1992, Pellman *et al.* 1993) showed that the Zeldovich approximation works well if the initial spectrum is cut off at $k \sim k_{nl}$. Compared to the results of N-body simulations with similar initial conditions but without the cutoff the Zeldovich approximation predicts much stronger filaments especially when the spectral index n (Eq. 6) becomes greater than

−1. Cutting off the initial spectrum at some scale we eliminate small scale clumpiness. Cutting off the spectrum at larger scales ($k_c < k_{nl}$) and applying the Zeldovich approximation we would produce a quasi–linear density distribution described at the beginning of this section (we will call it a smoothed version of the structure) . Actually this quasilinear structure is always present in the nonlinear distribution but is hidden due to clumpiness. In the case when the scale of a possible quasilinear structure becomes much greater than the scale of clumps ($k_c << k_{nl}$) we may see the structure again because we come closer to the continuous limit.

As was discussed before we may see a filamentary structure when the amplitude of the density perturbations reaches a value of $\sigma_\delta = 0.3(\sigma_{filam}/0.3)$. Correspondingly cutting off the initial spectrum at k_s, satisfying the condition $\delta_{rms}(k = k_{filam}) \approx 0.3$, one can find a typical number of clumps in a randomly placed box of size k_{filam}^{-1}. If this number is small the filamentary structure can not be seen even if present in a smoothed version of the structure, but if it is large enough we may see it. A simple estimate gives $N_{filam} \sim (k_{nl}/k_{filam})^3$.

In the case of the power law initial spectra $k_{nl}/k_{filam} \sim 0.3^{-\frac{2}{n+3}}(\frac{\sigma_{filam}}{0.3})^{-\frac{2}{n+3}}$ and therefore

$$N_{filam} \sim 0.3^{-\frac{6}{n+3}}\left(\frac{\sigma_{filam}}{0.3}\right)^{-\frac{6}{n+3}} \tag{21}$$

If $n = 0$ then $N_{filam} \sim 10(\frac{\sigma_{filam}}{0.3})^{-2}$ and we probably can not see the structure: for smaller σ_s (larger scales) we have more clumps but the density contrast is lower, on the other hand, for greater σ_s the number of clumps is even less. However, if $n = -2$ then $N_{filam} \sim 10^3(\sigma_{filam}/0.3)^{-6}$ and it is probably more than enough for detecting filaments.

Similar arguments can be used in the discussion of voids. However, to detect voids we probably need to deal with greater amplitudes ($\sigma_\delta \approx 0.5$ instead of $\sigma_\delta \approx 0.3$) which considerably reduces the number of clumps per void. This implies that observing voids in a clumpy universe is more problematic.

5. Discussion

To address the problem of large scale structure formation we must study the displacement of mass. This displacement consists of roughly two parts: the displacement towards clumps and the coherent displacement of the clumps themselves.

The former, of course, accounts for the formation of clumps and is taken into consideration by the hierarchical clustering theory (see Peebles, 1980 and references therein). The latter may explain the formation of the large scale structure of the universe. Being coherent on the nonlinearity scale $k_{n\ell}^{-1}$ this part of the displacement is not homogeneous on larger scales and accounts for the large scale density fluctuations. These fluctuations are small ($\delta_{rms} < 1$) but nonGaussian. The nonGaussianity manifests itself by disturbing the initial phases, however, the initial linear spectrum remains almost undisturbed at $k \lesssim k_{n\ell}$. The range of scales where the phases are considerably disturbed is proportional to the characteristic displacement of the mass and can be estimated theoretically (Eq. 14). The range of scales between $k_{n\ell}^{-1}$ and d_{rms} is in the quasilinear regime: $\sigma_\delta < 1$, but the phases are different from the initial ones. For a power law initial spectrum $d_{rms} \sim k_{n\ell}^{-1}$ if $n > -1$ the quasilinear regime practically does not occur.

The formation of the filamentary and cellular structures is generic for gravitational instability in the sense that it is always present in a smoothed version, but in practice it is determined by the interplay of two rival factors: the amplitude of density fluctuations on some scale $\sigma(k_s)$ and the clumpiness of the density distribution. If k_s is not much smaller than $k_{n\ell}$ then the amplitude $\sigma(k_s)$ is large enough but the clumpiness prevents us from seeing filaments. On the other hand, when k_s is much smaller then $k_{n\ell}$ then the clumpiness becomes more or less irrelevant but the amplitude of the density perturbations becomes too small. Thus there must be an optimal range of scales where both factors can "reach a compromise." This strongly depends on the initial spectrum. Quantitative estimates, based on the ideas of percolation theory, show that for a power law initial spectrum with $n \lesssim -1$ the clumpiness is small enough and we can expect the formation of an observable filamentary structure. Incidentally the most popular cosmological scenarios (CDM and C+HDM) have spectra satisfying the above condition in the range of the large scale structure.

Acknowledgements

I am grateful for finanical support from NSF Grant AST-9021414 and NASA Grant NAGW-2923.

References

Bower, R., Coles, P., Frenk, C. and White, S. 1992 preprint.

Cen, R., Gnedin, N., Kofman, L. and Ostriker, J. 1992, Ap.J. 399, L11.

Coles, P., Melott, A. and Shandarin, S. 1993, M.N.R.A.S. 260, 765.

Davis, M., Summers, F. and Schlegel, M. 1992, Nature 359, 393.

Efstathiou, G., Sutherland, W., and Maddox, S. 1990, M.N.R.A.S. 348, 705.

Gurbatov, S.N,., Malahov, A.N. and Saichev, A.I. 1991, Nonlinear Random Waves and Turbulence in Nondispersive Media. Waves, Rays, and Particles (Manchester University Press).

Klypin, A., Holtzman, J., Primack, J. and Regos, E. 1992 preprint.

Kofman, L. 1993, private communication.

Kofman, L., Pogosyan, D., Shandarin, S. and Melott, A. 1992, Ap.J. 393, 437.

Melott, A. and Shandarin, S. 1993, Ap.J. July 10 issue.

Peebles, P.J.E. 1980, The Large Scale Structure of the Universe (Princeton University Press).

Pellman, T., Melott, A. and Shandarin, S. 1993 in preparation.

Pogosyan, D., and Starobinsky 1993, preprint.

Ryden, B. and Grammann, M. 1991, Ap.J. 383, L33.

Scherrer, R., Melott, A. and Shandarin, S. 1991, Ap.J. 377, 29.

Shandarin, S. 1993, in preparation.

Shandarin, S. and Zeldovich, Ya.B. 1989, Rev. Mod. Phys. 61, 185.

Smoot, G. *et al.* 1992, Ap.J. 396, L1.

Zeldovich, Ya.B. 1970, Astron. Astrophys. 5, 84.

Figure Captions

Figure 1. A one–dimensional realization of the initial density, velocity and potential perturbations is shown. The initial spectrum is a one–dimensional analog of the CDM spectrum $P_{CDM}^{(1D)}(k) = k^2 P_{CDM}^{(3D)}(k)$.

Figure 2. Two dashed lines show the initial and linearly extrapolated spectra $P(k) \propto k^0$. The solid line shows the spectrum calculated in the N–body simulation. The short vertical line marks the nonlinear scale (Eq. 13).

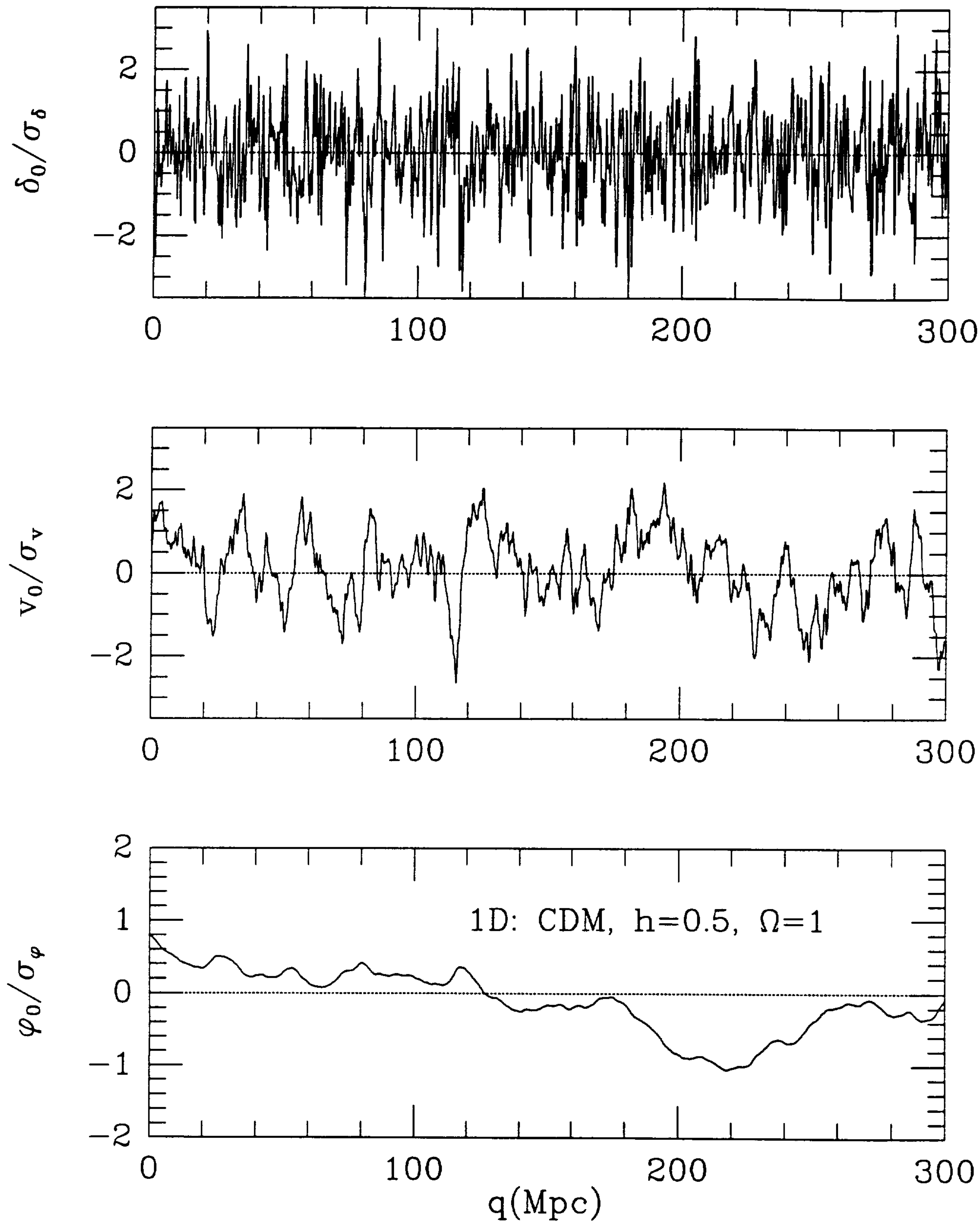
δ_0/σ_δ
v_0/σ_v
φ_0/σ_φ
1D: CDM, h=0.5, Ω=1
q(Mpc)

Figure 1

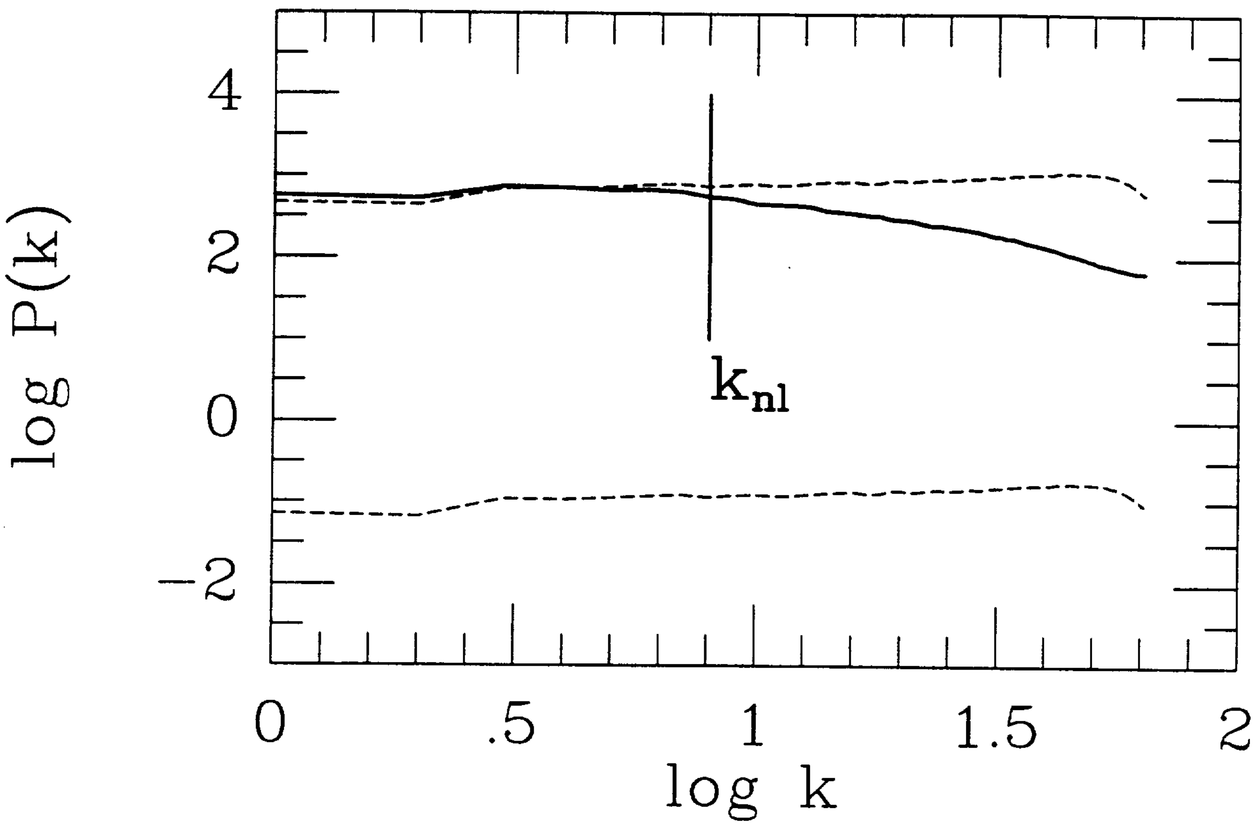

Figure 2