A NEW STOCHASTIC SYSTEMS APPROACH TO STRUCTURAL INTEGRITY

N95- 19474

1B0 58

James W. Provan
University of Victoria
Victoria, BC, Canada

Khalil Farhangdoost
McGill University
Montreal, Quebec, Canada

SUMMARY

This paper develops improved stochastic models for the description of a large variety of fatigue crack growth phenomena that occur in components of considerable importance to the functionality and reliability of complex engineering structures. In essence, the models are based on the McGill-Markov and Closure-Lognormal stochastic processes. Not only do these models have the capability of predicting the statistical dispersion of crack growth rates, they also, by incorporating the concept of crack closure, have the capability of transferring stochastic crack growth properties measured under ideal laboratory conditions to situations of industrial significance, such as those occurring under adverse loading and/or environmental conditions. The primary data required in order to be in a position to estimate the pertinent parameters of these stochastic models are obtained from a statistically significant number of replicate tests. In this paper, both the theory and the experimental technique are illustrated using a Ti-6Al-4V alloy. Finally, important structural integrity, reliability, availability and maintainability concepts are developed and illustrated.

INTRODUCTION

While most industrial failures involve fatigue, the assessment of the fatigue reliability of structural components being subjected to a variety of dynamic loading situations is still one of the most difficult engineering problems that remains to be solved. This is because material property degradation processes due to fatigue are extremely sensitive to material, component geometry, loading, and environmental conditions. To control the failure of components and systems, within specified performance limits and for a specified length of time, the concepts of Reliability, Availability and Maintainability (RAM) have been brought into play.

The study of structural reliability and the scatter in mechanical failure data was placed on a sound footing by Freudenthal (ref. 1) and Weibull (ref. 2). Lately, however, more research has been devoted to the derivation of reliability models based on probabilistic interpretations of the fatigue process. Thus, Birnbaum and Saunders (ref. 3) proposed a life distribution to characterize fatigue crack extension failures and Freudenthal and Shinozuka (ref. 4) presented a similar law substantiated by several sets of fatigue data. Subsequently, Payne (ref. 5) introduced a statistical reliability model for assessing the fatigue strength of aircraft structures by evaluating the random variability in crack propagation rates and residual strengths of crack structures at any stage during their evolution. Provan *et al.* (refs. 6, 7, 8) have derived and experimentally investigated new reliability distributions based on probabilistic micromechanic concepts applied to the fatigue failure of polycrystalline metals. Bogdanoff and Kozin (ref. 9) and Yang *et al.* (ref. 10) have investigated the stochastic fatigue crack growth process based on Markov and lognormal processes, respectively, a view that is shared in the current investigation.

PRECEDING PAGE BLANK NOT FILMED

Turning attention to the fatigue process itself, Elber (ref. 11) observed that fatigue-crack surfaces contact each other even during tension-tension cyclic loading. This simple observation of the crack-closure phenomenon immediately explained many crack growth characteristics that had been troubling researchers during the '60s. Since then, several closure mechanisms have been identified, among them, "plasticity" induced closure. These new closure mechanisms and the influence of the plastic wake on the local crack-tip strain field have greatly advanced an understanding of the fatigue-crack growth and fracture behaviour of metallic materials, under, for example, variable amplitude loading. Furthermore, the occurrence of crack-closure significantly affects the local crack driving force stress-intensity factor and plays a crucial role in quantifying the fatigue crack growth or arrest characteristics of the material in question. Thus, while the qualitative interpretation of the closure phenomena justifies a large number of fatigue crack anomalies, a quantitative knowledge of the crack closure stress intensity level is required to correlate fatigue crack growth data.

Most fatigue design requirements based on damage tolerance concepts assume the existence of flaws in the component either from the initial delivery or from some later stage during the service life. To assure a high level of reliability and before these flaws grow to critical lengths, it is necessary to either repair or replace the component. Thus, in-service inspections are required to detect various sizes and shapes of cracks and other defects. For a reliability analysis of an in-service airframe, for example, fatigue, environment and accidental events are three sources of damage that must be taken into consideration.

Hence, the importance of fatigue; the unifying nature attributed to closure; the need for a stochastic process interpretation of the whole fatigue process, and the need for reliability analyses are clear. While crack closure and stochastic processes are briefly detailed, this paper primarily concerns an investigation of new methods of assessing the fatigue reliability of structures based upon probabilistic approaches. It summarizes: i) the experimental determination of a statistically significant number of crack growth rates for a Ti-6Al-4V titanium alloy; ii) the determination of the Closure-Lognormal, c, m, $\tilde{\sigma}^2$, ξ parameters for Ti-6Al-4V; iii) the simulation of the crack propagation based upon these Closure-Lognormal parameters; iv) the determination of the McGill-Markov λ and κ parameters for the Ti-6Al-4V alloy in question; and v) the RAM assessment of Ti-4Al-6V.

Fatigue Crack Closure

An understanding of crack closure under cyclic tension was developed by Elber (ref. 11) who showed that the occurrence of premature contact between the opposing crack faces during unloading was due to the residual plastic stretch in the crack wake. During the loading portion of a cycle, the elastic constraints acting on the residual material in the wake of an advancing crack, keep the crack tip closed until these constraints are overcome by the externally applied load. The stress intensity factor associated with a fully opened crack, K_{op}, based upon the crack opening load, P_{op}, is necessary for the quantitative knowledge of the effective stress intensity range factor, ΔK_{eff}, which is used in life prediction through $K_{eff} = K_{\max} - K_{op}$ and the well known Paris-Erdogan relation $da / dN = c\Delta K_{eff}{}^{m}$.

ΔK_{eff} is an appropriate field parameter for correlating crack growth under constant-amplitude loading conditions to the influence of a large number of variables, such as an actual load spectrum, load ratio and/or a specific environment, that are known to affect the rate at which cracks grow in any practical situation. As an example, fatigue crack closure effects at different load ratios have been extensively investigated by Ritchie *et al.* (ref. 12), who initially showed a significant difference in the $da / dN - \Delta K$ behaviour due to different load ratios over a wide range of ΔKs, but, when plotted as a function of ΔK_{eff}, were able to eliminate this discrepancy and consolidate the curves into a narrow band.

Stochastic Processes

Stochastic processes deal with statements that can be made concerning a random phenomenon, and a random phenomenon has the property that under a particular set of conditions its observation leads to a multitude of possible outcomes in such a way that statistical regularity can be observed.

The Lognormal Process. The aim of the present study of lognormal process is to consider fatigue crack growth rates as random phenomena. A random variable x has a lognormal distribution if its probability density function is given by:

$$f(x) = \frac{1}{\tilde{\sigma}\sqrt{2\pi x}} \exp\{-\frac{(\ln x - \mu)^2}{2\tilde{\sigma}^2}\} \quad (1)$$

where μ and $\tilde{\sigma}^2$ and the mean and variance of the associated normal distribution.

The validity of lognormal crack growth rate models, including the lognormal random process, white noise, and random variable models along with the general lognormal random process model, have been investigated using extensive fatigue crack growth data gathered from fastener hole specimens by Yang *et al.* (ref. 10, 13, 14, 15). In order to account for the random nature of crack growth rate, use was made of the following model for constant amplitude loading situations:

$$\frac{da}{dN} = X(t)F(\Delta K, K_{\max}, R, S, a), \quad (2)$$

where a(t) is the random crack size, and X(t) is a non-negative random process. Based on extensive experimental data, Yang proposed X(t) as a non-negative, stationary lognormal random process. The lognormal random process, X(t), is defined by the it's logarithm being a normal random process, i.e., Z(t) is a normal random process, where Z(t) = log X(t). The stationary normal random process Z(t) is defined by the mean value μ_z and the autocorrelation function $R_{zz}(\tau)$ given by:

$$R_{zz}(\tau) = E[Z(t)Z(t+\tau)], \quad (3)$$

where E[ ] indicates the expected value.

The mean value, μ_z, of Z(t) is equal to the logarithm of the median value of X(t). Since the median value of X(t) is equal to unity, the mean value μ_z is equal to zero. Hence Z(t) is a stationary normal random process with zero mean, and is completely defined by the autocorrelation function given in eqn(3).

Yang, *et al.* (ref. 10) chose the autocorrelation function to be an exponentially decaying function of time difference τ, as follows:

$$R_{zz}(\tau) = \tilde{\sigma}^2 \exp\{-\xi|\tau|\}, \quad (4)$$

where ξ is the correlation parameter determined from a comparison with experimental test results. This correlation parameter plays a significant role in describing the statistical fatigue crack growth rate and propagation behaviour of materials.

The McGill-Markov Process. In order to further evaluate the stochastic characteristics of fatigue crack propagation processes, it is necessary to consider the concepts behind the McGill-Markov process. Any Markov process is based upon the assumption that the prediction of the process that is going to occur is influenced only by the properties of the current state which the process is in and not by the history that led to its present state. According to Provan and Rodriguez (ref. 16) and the nature of fatigue crack growth, the crack propagation process is a discrete-state continuous-parameter and nonhomogeneous Markov process since the transition probability density is a variable which depends only on the time difference τ. Furthermore, the crack size variable, a(t), can only be measured to within equipment and operator limitations. By considering the observable zones, i, the crack size may be written as:

$$a_i < a(t) < a_i + \Delta a_i, \qquad (5)$$

where i is the state number and Δa_i is the width of a state. A discrete-state and continuous-parameter stochastic process, $\{a(t), t \in \tau\}$, may be described by a one step transition probability of the form:

$$P\{a(t) = j | a(\tau) = i\} = p_{ij}(\tau, t), \qquad 0 \leq \tau \leq t, \qquad (6)$$

where i and j are integer states and τ and t are times. This probability is called the transition probability and is defined as the probability of a transition from state i to state j during the time interval τ to t.

In order to solve the Kolmogorov differential equations which govern p_{ij}, an infinitesimal transition scheme must be specified. After a review of the existing intensity functions, Provan and Rodriquez (ref. 16) developed a new intensity which gives a good description of the time evolution of material property degradation processes. The intensity functions of the McGill-Markov process are:

$$q_j(t) = \lambda_j = \frac{\lambda j(1+\lambda t)}{1+\lambda t^{\kappa}} \qquad for \qquad j = 1,2,..., \qquad (7)$$

$$q_{ij}(t) = \{ \begin{matrix} \lambda_{j-i} = \dfrac{\lambda(j-1)(1+\lambda t)}{1+\lambda t^{\kappa}} & for \quad i = j-1, \\ 0 & otherwise. \end{matrix} \qquad (8)$$

λ and κ are empirical system parameters which are determined by a fit to experimental data. They are positive empirical system parameters that describe all of the various effects that influence the process, such as, temperature, material properties, experimental error, etc. When these parameters are found, the system may be modeled. If the system is changed, new system parameters must be found. These intensity functions are then used in the Kolmogorov forward differential equation, which becomes:

$$\frac{\partial p_{ij}(\tau,t)}{\partial t} = -\lambda_j p_{ij}(\tau,t) + \lambda_{j-1} p_{i,j-1}(\tau,t). \qquad (9)$$

Hence, by solving this Kolmogorov equation governing a linear nonhomogeneous birth process, the transition probabilities $p_{ij}(\tau,t)$ are determined.

Finally, in order to determine the entire history of the crack propagation distribution, the total probability may be continuously monitored via the fundamental absolute probability relation:

$$P_j(t) = \sum_{i=1}^{j} p_{ij}(\tau,t) P_i(\tau), \qquad (10)$$

where $P_j(t)$ is the probability of being in the j-th (later) state at time t; $P_i(\tau)$ is the probability of being in the i-th (earlier) state at time τ; and $p_{ij}(\tau,t)$ are the transition probabilities of a Markov process. Hence, for any McGill-Markov process, it is necessary to specify the initial state and the transition probabilities in terms of λ and κ in order to describe the evolution of the entire process.

Reliability Analyses

One of the main objectives of this paper is to introduce reliability maintenance and inspection/correction procedures via the McGill-Markov interpretation of material property degradation. Inspection/correction failure control systems play a significant role in a reliable repair policy which adheres to well scheduled inspection programs. Reliability is defined as the probability of satisfactory performance of a component for a specified period in a specified environment. Hence, the reliability can be found if the critical crack size is known; it is the probability that the crack does not exceed this critical size. This quantity can be obtained by summing up the probability of a crack being any sub-critical size, i.e.:

$$R(t) = \sum_{j=0}^{j=N_f} P_j(t), \tag{11}$$

where N_f is the identity of the state corresponding to the critical crack size.

Two specific uses of McGill-Markov model are as follows: the first use is for the prediction of repair times in order to maintain a certain level of reliability, and the second is the determination of the optimum time for an inspection/correction procedure.

Reliability Maintenance - Inspection/Correction Process. The operator of a structure will often decide upon a desired level of reliability. Once this level has been determined, perhaps by company policy, standard industry practices or other means, it becomes necessary to determine when to schedule the maintenance procedures that correspond to the desired level of reliability. This may be accomplished by employing the McGill-Markov model to predict when the probability of failure will reach the desired limit and then calling for an inspection/correction procedure. Briefly, inspection/correction processes are summarized as: stopping the degradation process; locating components that pose a risk to structural integrity; and carrying out the necessary maintenance procedure.

Analytically, as a result of the removal and replacement of some components, there are two distinct populations referred to as Population I, which consists of the remaining components from the initial group, and Population II, which is the group of replacement components. For an inspection/correction at time $T_{inspect}$, the fatigue process continues for Population I while for Population II it starts at time T=0 and ends at time $T = T_{final} - T_{inspect}$. As long as the fatigue loading situation remains the same, the system parameters λ and κ can be used for populations, I, II,... Following the development of Rodriguez and Provan (ref. 17), this total probability of failure is a combination of the probabilities of failure of Populations I and II. This process can easily be extended to include as many inspection/correction procedures as desired.

Reliability Maintenance - Inspection Optimization. Another useful form of reliability analysis which can be carried out with the McGill-Markov model is the optimization of the inspection time. As an example, suppose that it is desired to minimize the total probability of failure at a future time, and furthermore there will be only one inspection/correction process in a given interval time. Hence, the question: "what is the optimum time for this procedure ?" is an appropriate one. In order to decrease the probability of failure an inspection/correction procedure will be carried out at some time. An inspection too early in the service life will, on the one hand, remove few components that may subsequently fail while a later inspection may be too late to remove components that will have failed. The optimum time for inspection will depend on several variables such as: critical crack size, repair size, inspection process, and the quality of replacement components. Hence, the McGill-Markov model, used in conjunction with a failure control methodology, can be a useful tool for obtaining valuable reliability information.

EXPERIMENTAL PROCEDURE AND BASIC RESULTS

Exerimental Procedure. A computer controlled, increasing stress intensity factor ΔK test method, according to the American Standard Test Method ASTM E647 (ref. 18), was applied to eighteen standard C(T) specimens manufactured from a forged Ti-6Al-4V jet engine, fan disk grade, titanium alloy. The main objectives of carrying out these experiments were to examine the stochastic properties of crack growth for Ti-6Al-4V, and then to analyze the results in an effort to establish the parameters associated with the Closure-Lognormal and McGill-Markov stochastic processes and in the reliability assessment of components manufactured from this titanium alloy.

The fatigue tests were carried out under the control of the in-house "FATIG" computer program based on a crack closure compliance method. For the compliance calculations, the CMOD measurement was determined using a mechanical clip gage with a maximum resolution of 0.00025 mm, while that of the load cell was 0.005 kN. The FATIG program utilizes three main loops and determines: i) the crack length based on load vs. CMOD data (compliance); ii) the ΔK based upon the ASTM E647 standard; and iii) the ΔK_{eff} based upon ΔK and the closure load. During each "data acquisition" block, the load vs. CMOD curve for 200 individual data points was obtained. The lower limit was a variable such that the nonlinearity was distinguished by FATIG as the closure load. A linear curve fit was made through the remaining points to obtain the normalized compliance, and the relationship between the compliance and crack length was determined by that proposed by Mirzaei and Provan (ref. 19), namely:

$$\frac{EBv}{P} = \frac{3(\alpha+1)(\alpha+2.3)}{\alpha^2 - 2\alpha + 1} + 8(1+v)\alpha; \qquad \alpha = \frac{a}{W}. \tag{12}$$

In eqn(12), α is the normalized crack length, E = 117,000 MPa is the elastic modulus of Ti-Al-4V, v is the CMOD, B the specimen thickness, P the load and W the specimen width. From this information the ΔK_{eff} was determined from the expression given in the ASTM E674 standard, except that $\Delta P = P_{max} - P_{op}$, P_{op} being the closure load.

When a specific test was completed, the stored data was analyzed with the second in-house "FADA" program to obtain a da/dN vs. ΔK_{eff} curve using the incremental polynomial method with n=3 , i.e., 7 successive data points. The FADA program plotted the crack length, a, vs. the number of cycles, N, and the crack growth rate, da/dN vs. ΔK_{eff}.

Experimental Test Results For Ti-6Al-4V. Results were obtained from eighteen standard C(T), Ti-6Al-4V specimens prepared according to conventional procedures. The fatigue crack propagation for all eighteen specimens are shown in fig. 1(a), while the associated da/dN vs. ΔK_{eff} curves are presented in fig. 1(b). Examination of the data revealed a few surprising observations. The most unexpected observation was the change in the growth rate which occurred in almost every test, indicating that fatigue crack propagation is not a stable, smooth, well ordered process. A close observation of the fatigue crack growth rates show two transition points, namely, at $\Delta K_{eff} \cong 8-9MPa$ and at $\Delta K_{eff} \cong 13-14MPa$.

Hence, the fatigue crack growth process is highly complex, especially in the Paris-Ergodan regime and it is reasonable to have different Paris-Erdogan regimes correlated to these experimental results. However, from the viewpoint of reliability analyses the first part of the fatigue crack growth rate curve, i.e., prior to the first transition point, is more important than the second.

THE CLOSURE-LOGNORMAL CHARACTERISTICS OF Ti-6Al-4V.

As introduced above, a lognormal process is defined by the fact that its logarithm is a normal random process. Also, the general form of fatigue crack propagation laws indicates that the crack growth rate is a function of the stress intensity factor, maximum stress intensity factor, stress amplitude, load ratio, and so on. Some commonly used crack growth rate functions, such as the Paris-Erdogan model (ref. 20) are such that c and m are functions of the load condition and environment.

A comprehensive assessment of the closure phenomena necessitates the evaluation of the state of residual stress and strain in the neighbourhood of the crack tip and the extent of crack closure. Crack closure effects are most pronounced at low ΔK levels. This can be seen in fig. 2 which shows the results obtained from the eighteen specimens of Ti-6Al-4V tested as described above. Hence, P_{op} for Ti-6Al-4V as a function of crack length, a, is found by a polynomial curve fit to be as follows:

$$P_{op} = 2.90203 - 0140572a + 0.0021723a^2; \quad (kN;mm). \tag{13}$$

As detailed previously, the crack closure concept may be used to describe the influence of the actual load spectrum, load ratio and/or environmental parameters and, therefore, the quantitative knowledge of the crack closure stress intensity level is required to correlate fatigue crack growth rate data. Using the sense of "closure", in the form of the effective stress intensity range factor for fatigue crack propagation, the following equation, eqn(14), is adopted in such a manner that it is independent of component geometry, loading spectrum or load ratio, environment, etc. In this way, the entire description of the statistical scatter in fatigue crack growth data may be incorporated into the assessment of the reliability of any prospective component prior to its manufacture.

Hence, in order to account for the random nature of the crack growth rate, the following model is suggested:

$$\frac{da}{dt} = X(t)c(\Delta K_{eff})^m, \tag{14}$$

where a(t) is now the random crack size and X(t) is a non-negative random process. In this case, ΔK_{eff} is a function of crack length and closure effects, and X(t) is a lognormal random process. By taking the logarithm of both side of eqn(14), it follows that:

$$\log(\frac{da}{dt}) = \log X(t) + \log c + m\log(\Delta K_{eff}). \tag{15}$$

By substitution $\Psi = \log(\frac{da}{dt})$, $\quad Z(t) = \log X(t)$, $\quad C = \log c$, $\quad \Gamma = \log(\Delta K_{eff})$, eqn(15) may be rearranged into the form:

$$\Psi = m\Gamma + C + Z(t). \tag{16}$$

Z(t) describes the inherent scatter of a specific material. These parameters, as well as the variance $\tilde{\sigma}^2$ and the autocorrelation parameter, ξ, are obtained from the Ti-6Al-4V test results of the crack growth rate vs. the effective stress intensity range in the logarithmic scale.

According to the stochastic model, eqn(14), and the sense of closure for fatigue crack propagation, four important Closure-Lognormal parameters, namely m, c $\tilde{\sigma}^2$ and ξ, may be interpreted as material properties. Table 1 shows these Closure-Lognormal parameters for Ti-6Al-4V. Hence, a complete specification of a material's stochastic fatigue crack growth characteristics are found from the Closure-Lognormal model. These parameters may be interpreted as material properties that are independent of component geometry, loading spectrum or load ratio, environment, etc. In this way, the entire description of the statistical scatter in actual test results or simulated fatigue crack growth data (see the next section), may be incorporated into the assessment of component reliability. Since, by using these parameters and the simulation procedure, a definitive reliability, availability and maintainability procedure may then be carried out using the associated McGill-Markov parameters, a complete specification of a material's stochastic crack growth characteristics based upon the Closure-Lognormal interpretation of scatter is of paramount importance to the assessment of crack growth rates in any practical situation.

SIMULATION OF CRACK PROPAGATION -- Ti-6Al-4V

In order to remove the limitation of a specific crack geometry, loading, or environment, etc., and to be in a position to use both the previous Closure-Lognormal and the following McGill-Markov procedures of describing the statistics associated with the fatigue crack growth process, a simulation procedure plays a crucial role in transferring information from data generated by a standardized Closure-Lognormal procedure to real crack propagation processes as they occur in actual components and under any of a large number of specified situations. The stationary Gaussian random process Z(t), may be simulated using the well-known Fast Fourier Transform (FFT) technique to simulate the two parameters, ξ, and $\tilde{\sigma}^2$.

As an example, the C(T) specimen geometry along with a knowledge of the P_{op} for the Ti-6Al-4V was used to generate the simulated fatigue crack growth characteristics of this material. The results of these analyses are shown in fig. 3, which clearly shows that the two sets of data, one experimentally obtained and the other simulated, superimpose on each other.

ΔK_{eff}, incorporating a quantitative knowledge of the crack opening stress level, now appears as an appropriate field parameter for correlating knowledge of the constant-amplitude crack rates to practical growth rate situations. Specifically, knowing in terms of m, c $\tilde{\sigma}^2$ and ξ, the crack growth rate as a function of ΔK_{eff} for a specific material, such as Ti-6Al-4V, then the influence of an actual load spectrum, load ratio and/or environment may be incorporated during the design stage into an estimate of the fatigue reliability of a specific component. In this way, the simulation process plays an important role in removing the dependency on actual experimental results obtained under specific loading and environmental conditions. Hence, the closure behaviour, either experimentally determined or predicted by a model, expressed in terms of the effective stress intensity factor range expressions found in handbooks and the Closure-Lognormal parameters, constitute the only information required for analysing any particular situation.

THE MCGILL-MARKOV PARAMETERS FOR Ti-6Al-4V

The major objective of this section is the determination of the McGill-Markov stochastic fatigue crack growth properties of Ti-6Al-4V. As was described in the introduction, the two constant parameters λ and κ in the intensity functions q_i and q_{ij}, defined in eqns(7) and (8), are related to the stochastic properties of the material system being investigated. Accordingly, the probability of the crack tip being in state i is increased, state by state, as the process progresses. Essentially, failure means a situation that, in a finite time interval, there is a probability that the state goes past a prescribed limit, which causes a failure of the structure. Hence, the McGill-Markov system parameters control the system failure and play a significant role in both reliability assessment and in inspection/correction procedures. Applying the McGill-Markov model to the Ti-6Al-4V data is an appropriate way to illustrate the capabilities of this approach.

These parameters are determined by a fit to the experimental or simulated data. Several steps, however, must be taken before this iterative process is undertaken. The first step is the normalization of data to an initial crack length of a_0 at time t=0. This is done to eliminate the crack initiation stage. In the next step, the data is discritized into states of width Δa. Using the resulting λ and κ, the probability histograms at future times are generated and the mean and variance. The system parameters, are thus determined by an iterative process of fitting the model predictions to the simulated Ti-6Al-4V data. For this case, a state size of 0.4mm along with a failure state of $N_f = 40$ were chosen this was judged to be a sufficient number for the interval of time t=0 through t=2E+06 cycles. The values of λ and κ which give a good fit to the simulated (or, in this case the experimental) data are:

$$\lambda = 0.14 \quad , \quad \kappa = 0.98. \tag{17}$$

A comparison between the experimental, simulated and McGill-Markov model predictions, shown in fig. 4, shows that the whole concept of treating fatigue crack growth as stochastic processes is a flexible method of predicting crack propagation for industrially significant materials and situations. The fact that the λ and κ parameters are applicable in reliability prediction and inspection/correction procedure development, as detailed in the following section, again allude to the benefits of this approach.

RELIABILITY ANALYSIS OF Ti-6Al-4V

The combination of a failure control procedure with the McGill-Markov technique can be a very powerful tool for practical engineering reliability calculations. Two specific uses of the McGill-Markov approach are detailed. The first application is for the prediction of repair times in order to maintain a certain level of reliability, while the second is the determination of the optimum time for a single inspection/correction procedure. Before these are examined, however, a method that is central to both, the method of predicting reliability at a future time, is presented.

Reliability of Ti-6Al-4V. Reliability, as was discussed earlier, has been defined as the probability a component will perform satisfactorily for a specified period of time. For determining reliability, the first step is to use another in-house program, SOLVE, to generate probability histograms for crack sizes at given future times. The reliability can then be found if the critical crack size is known; it is the probability that the crack does not exceed this critical length. This is illustrated in fig. 5 as a function of time.

RAM for Ti-6Al-4V. By defining the replacement size (state), N_r, to be 25 and the desired reliability to be 0.9999, the total probability of failure for times 0.8E+06to 2.0E+06 cycles were obtained as illustrated in fig. 6(a). The optimum times for the inspection/correction procedures are thus: i) the first inspection time will be at 1.500E+06 cycles, ii) the second time at1.700E+06 and the third at 1.88E+06.

Furthermore, the results for a change in acceptable reliability level from 0.9999 to 0.9995 are presented in fig. 6(b). From a comparison with fig. 6(a), it is apparent that not only will the first maintenance procedure be carried out at a later time but that one fewer procedure will be necessary.

The effect of varying the repair size while maintaining the desired level of reliability at 0.9995 is illustrated in fig. 7. This figure shows how the inspection interval is affected by a change in repair size (state) from $N_r = 20$, through $N_r = 25$, to $N_r = 30$ prior to 2.0E+06 cycles. There are a total of two inspection/corrections for $N_r = 20$, three for $N_r = 25$ and four for $N_r = 30$. These figures illustrate the type of information that can be obtained from this approach.

Inspection Optimization. Finally, in order to carry out an inspection optimization analysis, the design engineer must control such variables as failure state, N_f, repair size (state), N_r, and the desired level of reliability. By using $N_f = 40$ and $N_r = 30$, the solid line in fig. 8 is obtained. From this figure it is apparent that the optimum time for inspection/correction is at 1.700E+06 cycles and that the total probability of failure is decrease by 88% over the no inspection case. By changing repair policy, such that N_r is varied from 30 to 20, different curves, also shown in fig. 8, are obtained. In this way, it is shown that while the optimum inspection time is increased by increasing the repair crack size from 20 to 30, the overall reliability of system is reduced.

CONCLUDING REMARKS

The conversion of ΔK into ΔK_{eff} through the inclusion of closure effects plays a significant role in reliability analysis. By using ΔK_{eff}, it now becomes possible to transfer the stochastic properties of crack growth rates, measured under ideal laboratory conditions, to practical situations. Incorporating the sense of closure into both the fatigue crack growth rate description and the lognormal interpretation of the scatter has led to the development of the Closure-Lognormal model which describes the statistical nature of crack growth rates.

On the other hand, the McGill-Markov process, employing data generated by simulations of the information contained in the Closure-Lognormal interpretation of the basic material's fatigue crack growth characteristics, vary with respect to closure effects that describe the influence of variations in the loading, environment, crack geometry, etc. With the crack propagation characteristics being predictable, the reliability and inspection processes may then be evaluated.

REFERENCES

[1] Freudenthal. A. M.: Safety of Structures, *Trans. ASCE*, vol. 112, pp. 125 - 180, (1947).

[2] Weibull, W.: A Statistical Distribution of Wide Applicability, *Journal of Applied Mechanics*, vol. 18, pp. 293 - 297,(1951).

[3] Birnbaum, Z. W. and Saunders, S. C.: A New family of Life Distributions, and Estimation for a Family of Life Distributions with Applications to Fatigue, *Journal of Applied Probability*, vol. 6, pp. 319 - 347, (1969).

[4] Freudenthal, A. M. and Shinozuka, M.: Structural Safety under Conditions of Ultimate Load-Failure and Fatigue, *WADD Technical Report*, pp. 61 - 77, (1961).

[5] Payne, A. O.: A Reliability Approach to Fatigue for Structures, *Probabilistic Aspects of Fatigue*, ASTM STP 511, pp. 106 - 155, (1972).

[6] Provan, J. W.: Probabilistic Approaches to the Material-Related Reliability of Fracture-Sensitive Structures, *Probabilistic Fracture Mechanics and Reliability*, J.W. Provan (ed), Martinus Nijhoff, pp. 1 - 45, (1987).

[7] Provan, J. W. and Theriault, Y.: An Experimental Investigation of Fatigue Reliability Laws, *Defects, Fracture and Fatigue*, G. C. Sih and J. W. Provan (eds.), Martinus Nijhoff, (1983).

[8] Provan, J. W. and Bohn, S. R.: Stochastic Fatigue Crack Growth and the Reliability of Deteriorating Structures, *Fatigue 90*, H. Kitagawa, and T. Tanaka, Eds., vol. IV, pp. 2259 - 2264 (1990).

[9] Bogdanoff, J. L. and Kozin, F.: *Probabilistic Models of Cumulative Damage*, John Wiley & Sons, (1985).

[10] Yang, J. N., Hsi, W. H. and Manning, S. D.: Stochastic crack growth models for application to aircraft structures, *Probabilistic Fracture Mechanics and Reliability*, J. W. Provan (ed), Martinus Nijhoff, The Hague, pp. 171 - 212, (1987).

[11] Elber, W.: Fatigue Crack Closure Under Cyclic Tension, *Engineering Fracture Mechanics*, vol. 2, pp. 37 - 45, (1970).

[12] Ritchie, R. O., Yu, W., Holm, D. K., and Blom, A. F.: *Mechanics of Fatigue Crack Closure*, ASTM STP 982, J. C. Newman, Jr. and W. Elber, (eds), American Society for Testing and Materials, Philadelphia, pp. 300 - 316, (1988).

[13] Yang, J. N. and Manning, S. D.: Stochastic crack growth analysis methodologies for metallic structures, *Engineering Fracture Mechanics*, vol. 37, No. 5, pp. 1105 - 1124, (1990)

[14] Yang, J. N.: Simulation of Random Envelope Processes, *Journal of Sound and Vibration*, vol. 21, no. 1, pp. 73-85, (1972).

[15] Yang, J. N.: Statistical modeling of fatigue-crack growth in a nickel-base superalloy, *Engineering Fracture Mechanics*, vol. 18, no. 2, pp. 257-270, (1983).

[16] Provan, J. W. and Rodriguez III, E. S.: Part I: Development of a Markov Description of Pitting Corrosion, *Corrosion Science*, vol. 45, no. 3, pp 178 - 192, (1989).

[17] Rodriguez III, E. S. and Provan, J. W.: Part II: Development of a General Failure System for Estimating the Reliability of Deteriorating Structures, *Corrosion Science*, vol. 45, no. 3, pp 193 - 206, (1989).

[18] *Annual Book of ASTM Standards*, Section 3, Vol. 3.01, (1993).

[19] Mirzaei, M. and Provan, J.W.: A New Method for the Analysis and Assessment of fatigue Crack Closure. I: Modeling and II: Experimental Study, *Theoretical and Applied Fracture Mechanics*, vol 18, pp. 47 - 63, (1992).

[20] Paris, P. C., and Erdogan, F.: A Critical Analysis of Crack Propagation Laws, *J. Basic Engng., Trans. ASME, Series D*, vol 85, pp. 528 - 534, (1963).

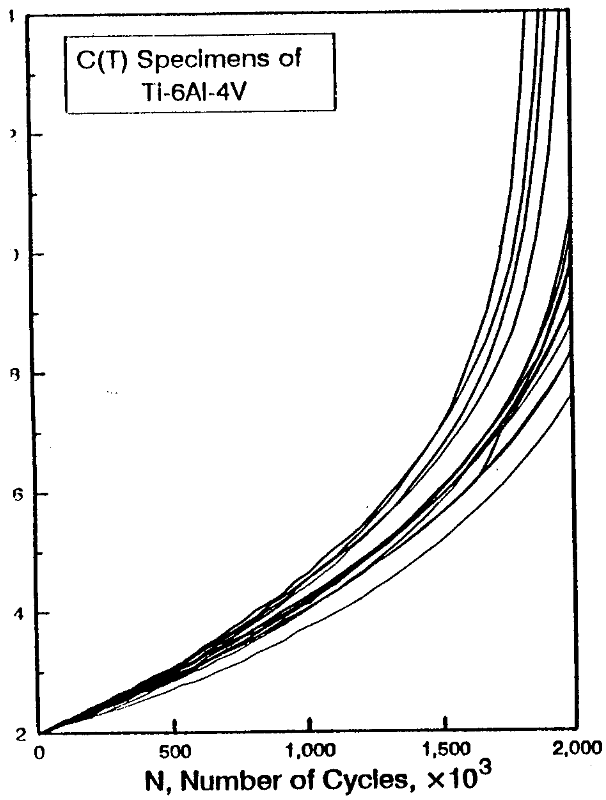


Figure 1(a): a vs N for Ti-6Al-4V

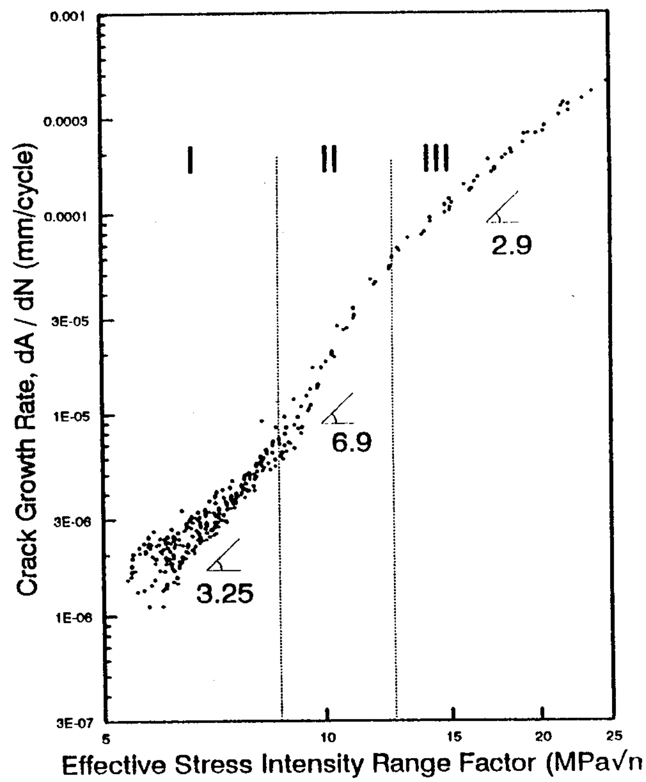


Figure 1(b): da/dN vs ΔK_{eff} for Ti-6Al-4V

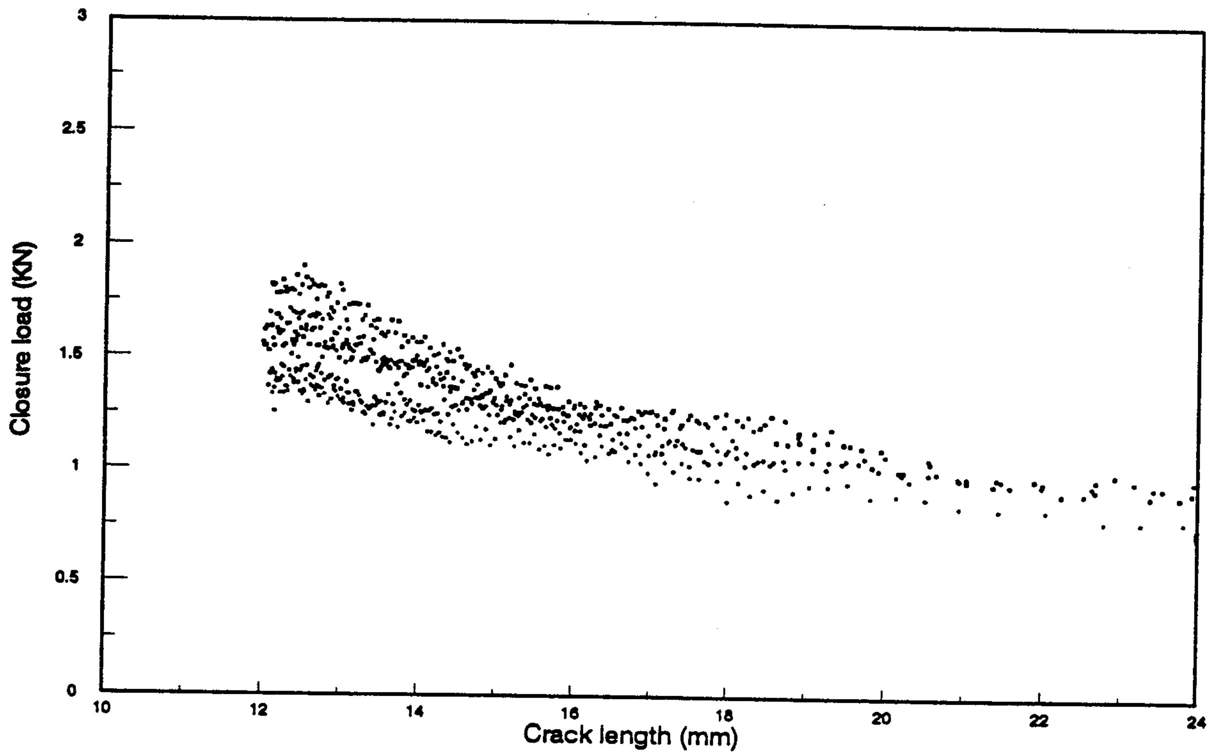


Figure 2: Closure Load vs Crack Length for Ti-6Al-4V

Table 1: Closure-Lognormal Parameters for Ti-6Al-4V

Closure-Lognormal parameters	$\tilde{\sigma}^2$ 10^{-4}	ξ 10^{-4}	m	c 10^{-9}
Ti-6Al-4V	47	1.4	3.25	5.4

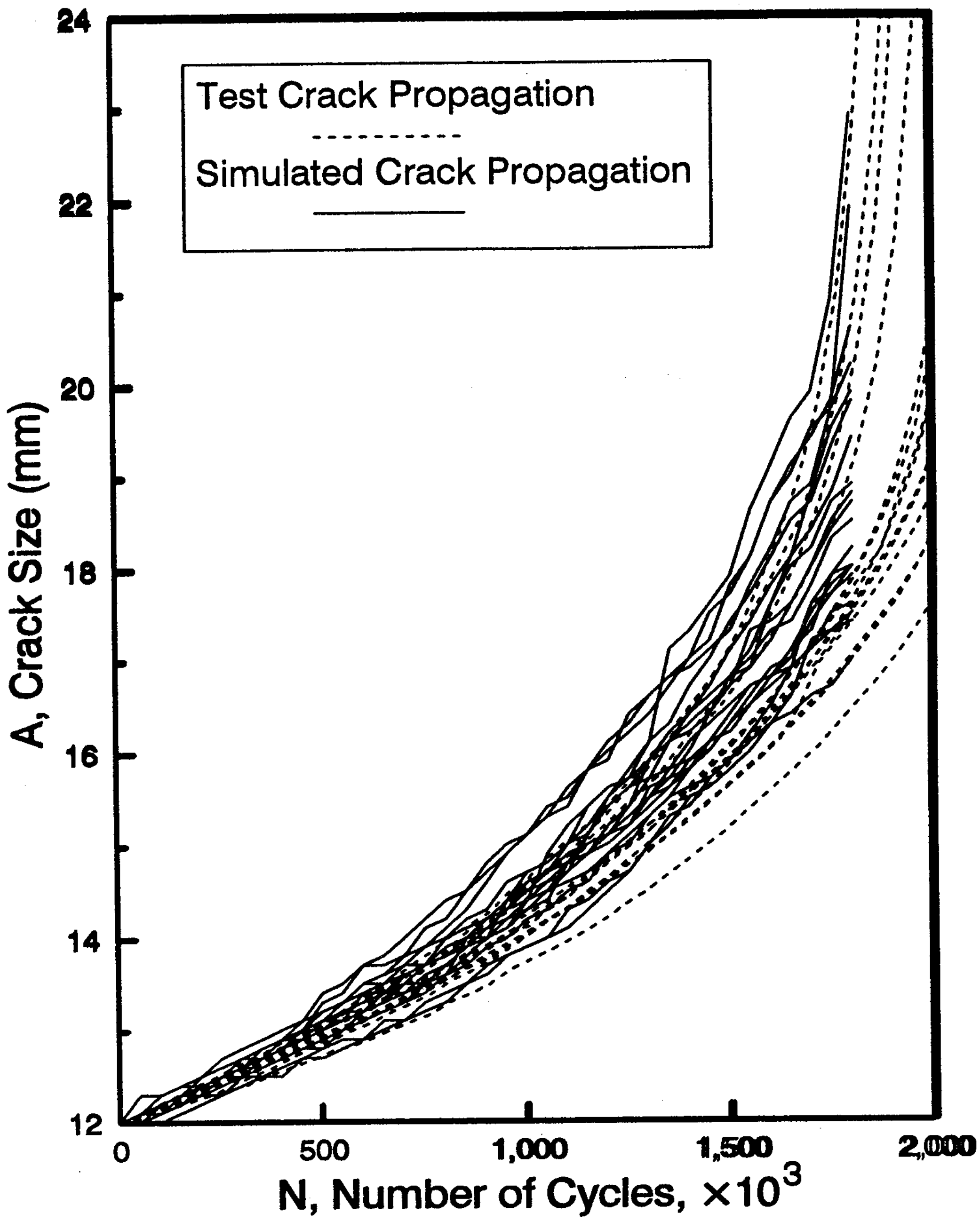


Figure 3: The Simulated and Experimental Crack Propagation of Ti-6Al-4V

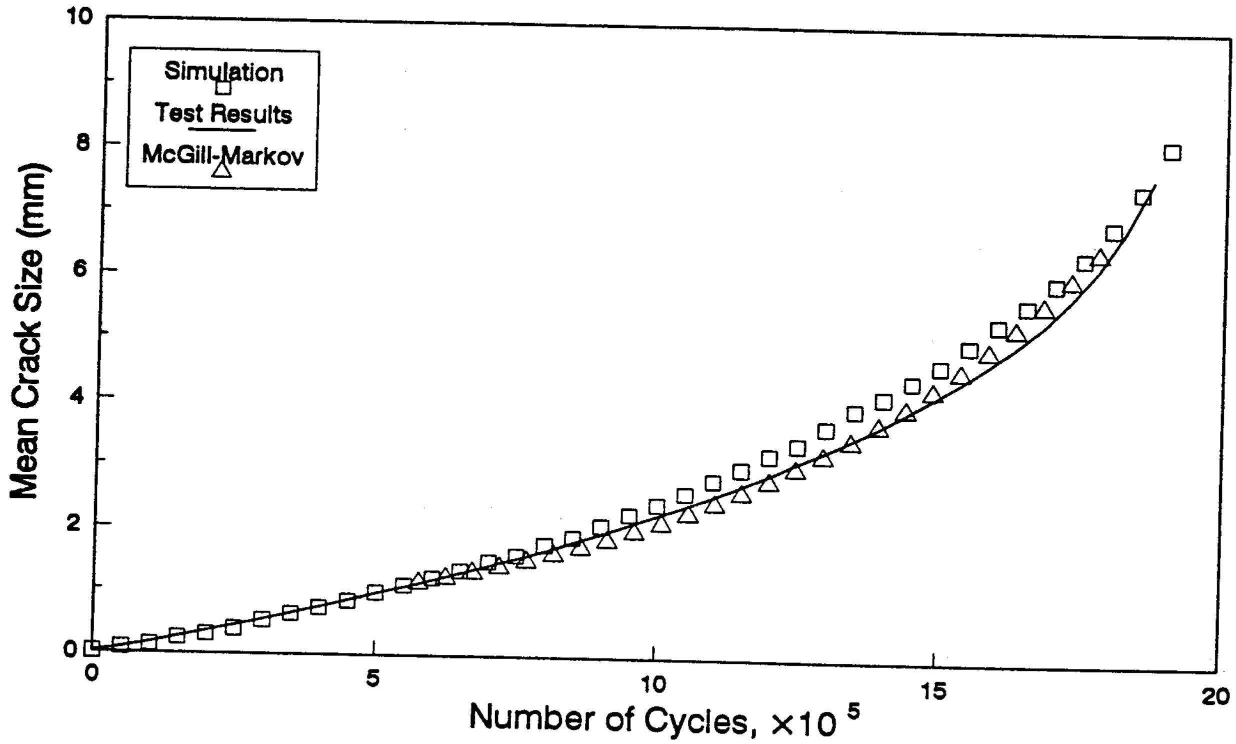


Figure 4(a): Ti-6Al-4V mean crack size, from McGill-Markov, simulated sample, and experimental test crack propagation.

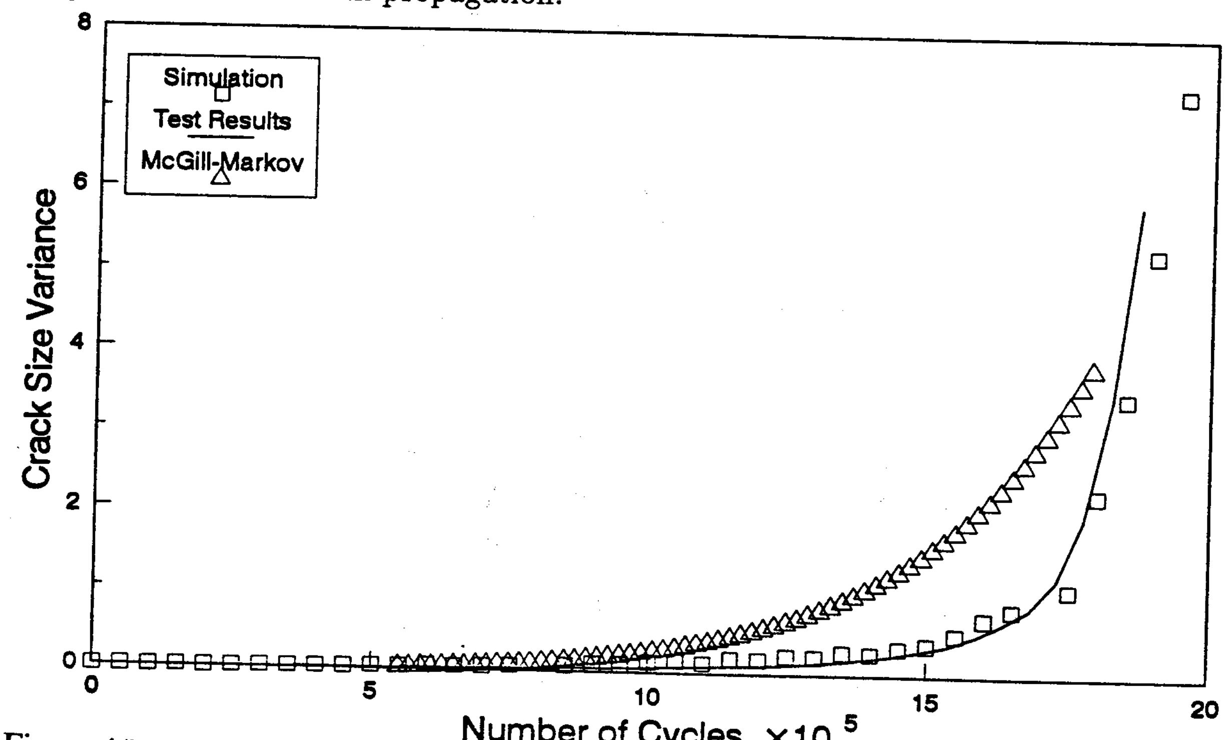


Figure 4(b): Ti-6Al-4V variance crack size, from McGill-Markov, simulated sample, and experimental test crack propagation.

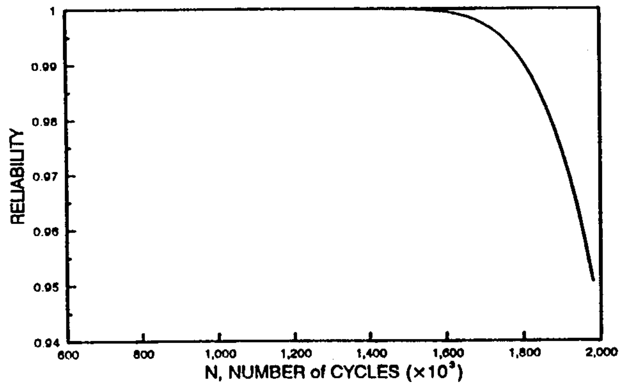
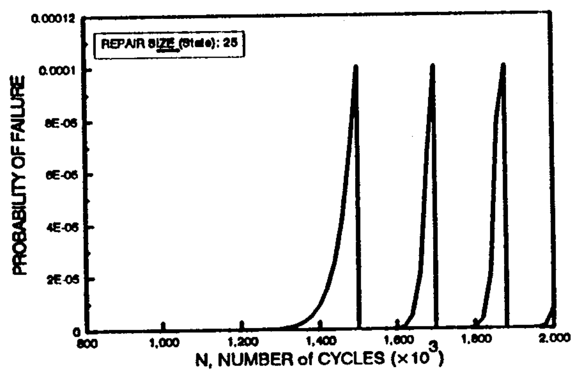
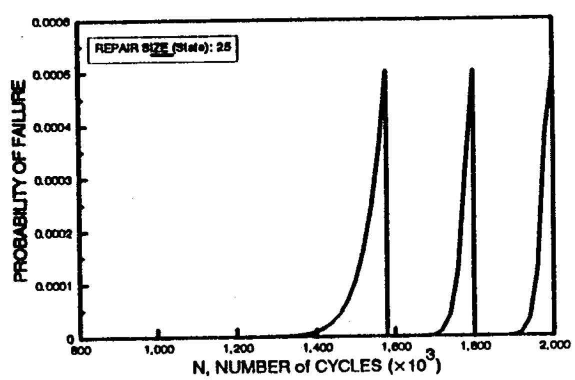


Figure 5: Reliability as a Function of Fatigue Cycles



a) At 0.9999 maintenance reliability



b) At 0.9995 maintenance reliability

Figure 6: Inspection/Correction Procedure for Ti-6Al-4V

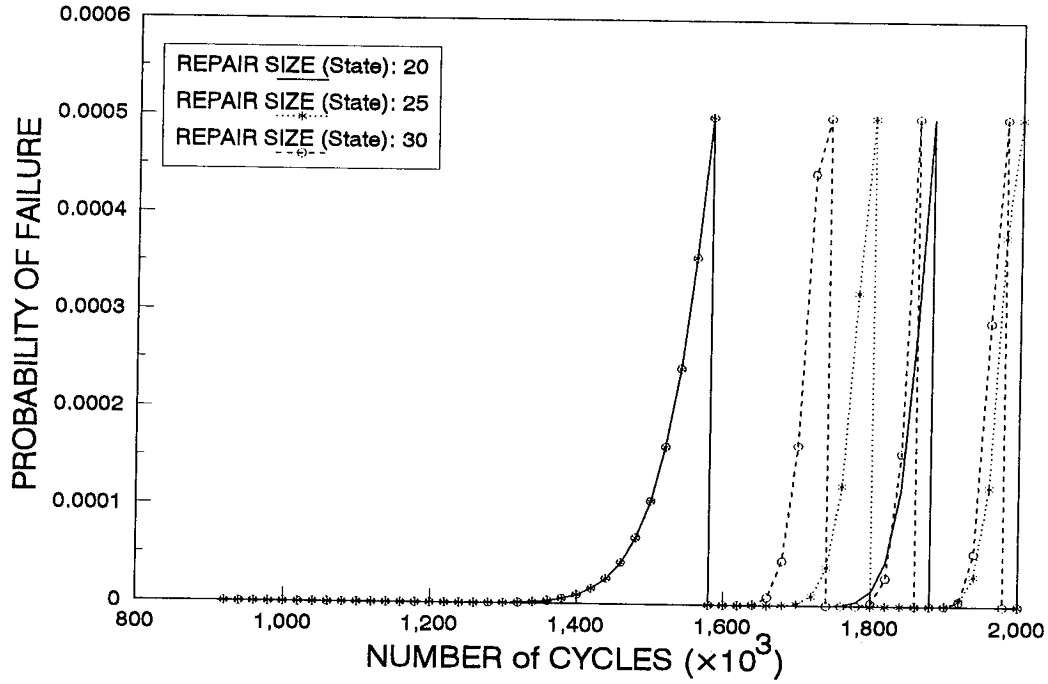


Figure 7: Inspection/Correction under different Repair Sizes

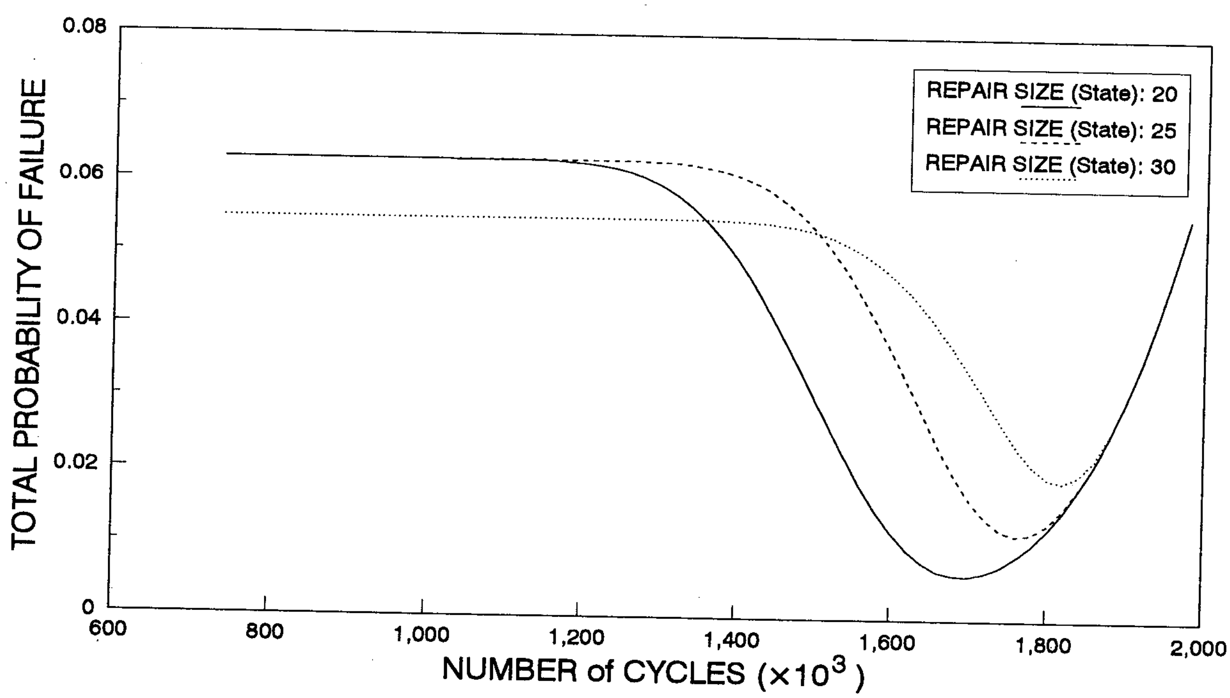


Figure 8: Inspection Optimization for Ti-Al-4V