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Single point modeling of rotating turbulent flows

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A model for the effects of rotation on turbulence is proposed and tested. These effects which influence mainly the rate of turbulence decay are modeled in a modified turbulent energy dissipation rate equation that has explicit dependence on the mean rotation rate. An appropriate definition of the rotation rate derived from critical point theory and based on the invariants of the deformation tensor is proposed. The modeled dissipation rate equation is numerically well behaved and can be used in conjunction with any level of turbulence closure. The model is applied to the two-equation k-ϵ turbulence model and is used to compute separated flows in a backward-facing step and an axisymmetric swirling coaxial jets into a sudden expansion. In general, the rotation modified dissipation rate model show some improvements over the standard k-ϵ model.

1. Motivation and objectives

The ability to accurately model the effects of rotation on turbulence has a wide variety of important applications in rotating machinery and combustion devices. Many turbulent flows of engineering importance involve combinations of rotational and irrotational strains. However, turbulence models of the eddy viscosity type are oblivious to the presence of rotational strains since they depend only on the mean velocity gradients through their symmetric part (i.e. the mean rate of strain tensor). The rotation rate, for example, does not explicitly enter the equations for the turbulent kinetic energy and its dissipation rate, yet evidence from experiments (Wigeland and Nagib 1978, Jacquin et al. 1990) and from direct numerical simulation (Bardina *et al.* 1985, Speziale *et al.* 1987, Mansour *et al.* 1991) show that the decay rate of turbulence is reduced by the presence of rotation.

The effects of rotation on turbulence are known to be subtle. They are manifested through changes in the spectrum of the turbulence caused by nonlinear interactions. For initially isotropic turbulence, rotation inhibits the cascade of energy from large to small scales. Zeman (1994) proposed a modified energy spectrum that takes into account the effects of rotation at high Reynolds number by introducing a rotation wavenumber, $k_\Omega = \sqrt{\Omega^3/\epsilon}$, below which rotation effects on spectral transfer are important. Much of the application work in simulating rotating flows have been conducted using varieties of eddy viscosity models (k-ϵ or k-l) and second order closure models with modified dissipation rate transport equation to account for

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rotational effects. However, most of these models fail to predict the asymptotic behavior of the turbulence decay rate in the limits of large rotation rate. The objectives of this work are to model the effects of rotation using single-point two equation models and to offer an appropriate definition of the mean rotation rate that is consistent with the fact that spin is the main cause of reduction in the dissipation rate.

2. Accomplishments

For incompressible viscous flow with constant properties, the modeled transport equations for the turbulent kinetic energy, k, and its dissipation rate, ϵ, that are widely used for engineering applications take the form;

$$k_{,t} + U_j k_{,j} = D_k + P_k - \epsilon \tag{1}$$

$$\epsilon_{,t} + U_j \epsilon_{,j} = D_\epsilon + P_\epsilon - \Phi_\epsilon \tag{2}$$

where D_k and D_ϵ are the diffusion terms for k and ϵ respectively and are modeled as

$$D_k = \left[\left(\nu + \frac{\nu_t}{\sigma_k}\right) k_{,j}\right]_{,j}, \qquad D_\epsilon = \left[\left(\nu + \frac{\nu_t}{\sigma_\epsilon}\right) \epsilon_{,j}\right]_{,j},$$

where ν is the laminar viscosity and ν_t is the eddy viscosity $= C_\mu k^2/\epsilon$. σ_k and σ_ϵ are the ratio of Prandtl to Schmidt numbers and are taken as constants. P_k is the production term for k given as $P_k = -\overline{u_i' u_j'} U_{i,j}$, where $\overline{u_i' u_j'}$ is the Reynolds stress term and U_i is the mean velocity in the i-direction.

Assuming that the production of the dissipation rate P_ϵ is proportional to the production of turbulent kinetic energy P_k, i.e $P_\epsilon \sim P_k/T$ where T is the turbulent time scale given by $T = k/\epsilon$. Similarly assume that the destruction rate of dissipation rate Φ_ϵ is proportional to the turbulent energy dissipation rate term, i.e. $\Phi_\epsilon \sim \epsilon/T$. The modeled form of the dissipation rate equation becomes

$$\epsilon_{,t} + U_j \epsilon_{,j} = D_\epsilon + C_1 \frac{\epsilon}{k} P_k - C_2 \frac{\epsilon^2}{k} \tag{3}$$

Due to the symmetry of the Reynolds stress tensor $\overline{u_i' u_j'}$, the kinetic energy production term can be written as $P_k = -\overline{u_i' u_j'} S_{ij}$, where $S_{ij} = (U_{i,j} + U_{j,i})/2$ is the mean rate of strain tensor. Therefor it can be seen that the standard dissipation rate, eq. (3), has no explicit dependence on the mean rotation tensor $\Omega_{ij} = (U_{i,j} - U_{j,i})/2$. It follows that the commonly used modeled dissipation rate equation can only be affected indirectly by rotational strains through the changes that they induce in the Reynolds stress tensor.

In order to sensitize the dissipation rate equation to rotational effects, consider the simple case of isotropic turbulence in a rotating frame. In this case, an initially decaying isotropic turbulence is described by;

$$k_{,t} = -\epsilon \tag{4}$$

$$\epsilon_{,t} = -C_2 \frac{\epsilon^2}{k} \tag{5}$$

Equations (4) and (5) do not distiguish the difference between isotropic turbulence in a rotating frame and in an inertial frame. Models that have a non zero rotational correction have been proposed by Bardina *et al.* (1985), for example, for rotating isotropic turbulence where eq. (5) takes the form

$$\epsilon_{,t} = -C_2 \frac{\epsilon^2}{k} - C_3 \Omega \epsilon \tag{6}$$

with $C_2 = 1.83$ and $C_3 = 0.15$.

The above model is able only to accurately predict the reduction in the decay rate of the turbulent kinetic energy in rotating isotropic turbulence for weak to moderate rotation rates where the effects are small. However, for sufficiently high rotation rates and long enough time, the model drastically underpredicts the decay rate of the turbulent kinetic energy.

Hanjallic and Launder (1980) proposed a model for which the ϵ-transport equation in rotating isotropic turbulence takes the form

$$\epsilon_{,t} = -C_2 \frac{\epsilon^2}{k} - C_3 \Omega^2 k \tag{7}$$

where $C_2 = 1.92$ and $C_3 = 0.27$.

This model predicts unphysical behavior of negative dissipation rate at high rotation rates, thus violating the realizability constraint. Other modifications to the dissipation rate transport equation have been proposed to account for rotational strains, e.g Raj (1975) and Pope (1978). Again they fail in one way or another to account accurately for the rotational effects.

3. Proposed model

In the present work a new model is proposed that accounts for rotational effects and correctly predicts the asymptotic behavior at zero to inifinte rotation rates. Consider the dissipation rate equation in rotating isotropic turbulence

$$\epsilon_{,t} = -\left(1.7 + \frac{5}{6} \frac{\alpha^2}{\alpha^2 + 1}\right) \frac{\epsilon^2}{k} \tag{8}$$

with

$$\alpha = 0.35 Ro^{-1} \tag{9}$$

where Ro is the Rossby number defined as $Ro^{-1} = \Omega k/\epsilon$. For $\Omega >> 1$, $C_2 = 2.5$, which gives a power law exponent $n = 0.6$ (in $k \sim t^{-n}$) maching the power law proposed by Squires *et al.* (1993) for the asymptotic state of rotating homogeneous turbulence at high Reynolds numbers.

The experminental data of Jacquin *et al.* (1990) are used to test the proposed model. Their experiments consisted of measuring the velocity field and characteristic quantities characterizing the fluctuating field downstream of a rotating cylinder containing a honycomb structure and a turbulence producing grid. The coupled differential equations for k and ϵ describing the effects of rotation on an initially isotropic turbulence can be written as

$$k_{,t} = -\epsilon \tag{10}$$

$$\epsilon_{,t} = -\left(C_2 + C_3 \frac{\alpha^2}{\alpha^2+1}\right)\frac{\epsilon^2}{k} \tag{11}$$

These equations were solved numerically using a fourth-order Runge-Kutta integration scheme. The model predictions (with $C_2 = 1.7$ and $C_3 = 5/6$) are compared with the experimental data of Jacquin *et al.* (1990) as shown in Fig. 1a. The model predicts well the evolution of turbulent kinetic energy and its decay rate for a wide range of rotation rates. We have also tested the model for the three Reynolds numbers measured by Jacquin *et al.* (1990), and found similar agreement of the model predictions with the data. We should point out at this point that the value $C_2 = 1.7$, proposed here for zero rotation rate, is lower than the value conventionally used in k-ϵ modeling. We find that with the conventional value of $C_2 = 1.92$ (and $C_3 = 3/5$) the model fails to predict the experimental data (see Fig. 1b)

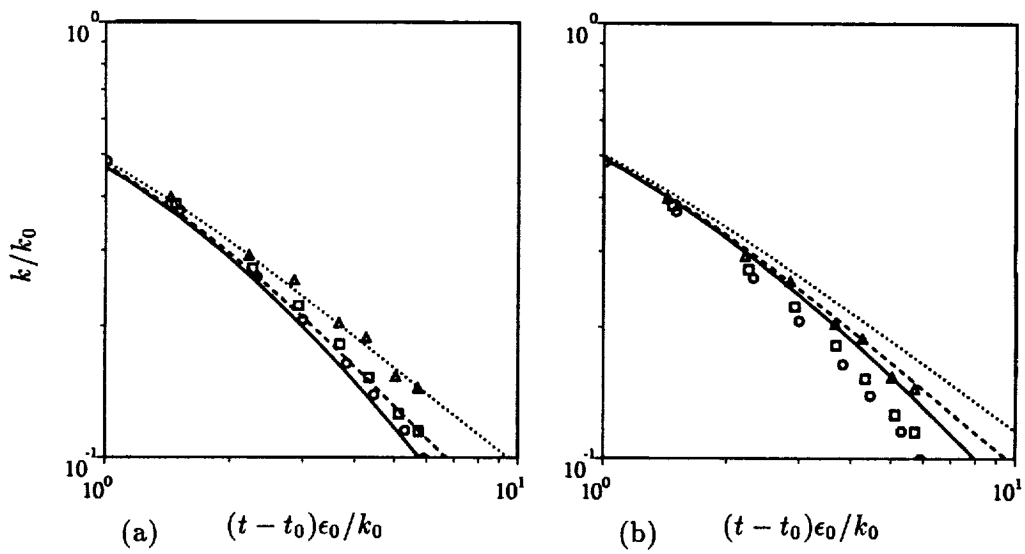

FIGURE 1. Decay of turbulent kinetic energy. Symbols are the data of Jacquin *et al.* (1990), lines are the model predictions. ○ & —— $\Omega = 62.8$ (rad/s), □ & ---- $\Omega = 31.4$ (rad/s), △ & ········ $\Omega = 15.7$. (a) Model predictions with $C_2 = 1.7$ and $C_3 = 5/6$; (b) Model predictions with $C_2 = 1.92$ and $C_3 = 3/5$.

4. Rotation Rate For General Flows

In order to test the rotational correction proposed in eq. (8) to the dissipation rate equation for general flows where the rotation rate is a function of position and in the presence of mean strains, the question arises as to what is the appropriate definition of the rotation rate, Ω?

In most previous studies, the rotation rate or the mean vorticity Ω was replaced by $\sqrt{\Omega_{ij}\Omega_{ij}/2}$, where $\Omega_{ij} = (U_{i,j} - U_{j,i})/2$ is the rotation rate tensor of the mean flow. However, such definition does not distinguish between a vortex sheet and a vortex. A definition of a vortex or a region of vorticity (with spin) was given by Chong *et al.* (1990) -using the arguments of the critical point theory and the invariants of the deformation tensor- as a region in space where the vorticity is sufficiently strong to cause the rate of strain tensor to be dominated by the rotation tensor, i.e. the rate of deformation tensor has complex eigenvalues. This definition satisfies the principle of frame invariance since it depends only on the properties of the deformation tensor. We shall use it because the reduction in the dissipation rate is due mainly to the spin that the mean imposes on the turbulence. Consider the matrix D_{ij} of the elements of the deformation tensor,

$$D_{ij} = U_{i,j} \tag{12}$$

which can be split to

$$D_{ij} = S_{ij} + \Omega_{ij} \tag{13}$$

The complex eigenvalues of D_{ij} are found by solving the characteristic equation $|D_{ij} - \lambda\delta_{ij}| = 0$, where the λ's are the eigenvalues of D_{ij}. For a 3×3 matrix, λ can be found from the solution of

$$\lambda^3 + P\lambda^2 + Q\lambda + R = 0 \tag{14}$$

where P, Q and R are the matrix invariants and are given by

$$P = -U_{i,i} \tag{15}$$

$$Q = \frac{1}{2}(P^2 - S_{ij}S_{ji} - \Omega_{ij}\Omega_{ji}) \tag{16}$$

$$R = \frac{1}{3}\left(-P^3 + 3PQ - S_{ij}S_{jk}S_{ki} - 3\Omega_{ij}\Omega_{jk}S_{ki}\right) \tag{17}$$

For an incompressible flow $P = 0$ from continuity and the characteristic equation becomes

$$\lambda^3 + Q\lambda + R = 0 \tag{18}$$

Now if

$$A = \left[-\frac{R}{2} + \sqrt{\left(\frac{R^2}{4} + \frac{Q^3}{27}\right)}\right]^{1/3}$$

and,

$$B = \left[-\frac{R}{2} - \sqrt{\left(\frac{R^2}{4} + \frac{Q^3}{27}\right)}\right]^{1/3}$$

then the three roots of λ are;

$$\left[A+B, -\frac{A+B}{2}+\mathrm{i}\frac{A-B}{2}\sqrt{3}, -\frac{A+B}{2}-\mathrm{i}\frac{A-B}{2}\sqrt{3}\right]$$

That is λ can have:

(i) all real roots which are distinct when

$$\left[(Q/3)^3+(R/2)^2\right]<0,$$

or

(ii) all real roots where at least two roots are equal when

$$\left[(Q/3)^3+(R/2)^2\right]=0,$$

or

(iii) one real root and a pair of complex conjugate roots when

$$\left[(Q/3)^3+(R/2)^2\right]>0.$$

We shall follow Chong *et al.* (1990) and define the rotation rate

$$\Omega=\Im(\lambda)=\frac{\sqrt{3}}{2}(A-B), \quad \text{when } \left[(Q/3)^3+(R/2)^2\right]>0, \tag{19}$$

$\Omega=0$ otherwise. It is important to note that for two dimensional Cartesian flows, the rotation rate defined by Eq. (19) reduces to $\Omega=\sqrt{|Q|}$, when Q, the determinant of the deformation tensor matrix, is negative. For pure shear the definition, eq. (19) yields $\Omega=0$. Conventional models that are calibrated for shear flows, need not be recalibrated when corrections based on Ω are added to the model.

5. Numerical Procedure

For a two-dimensional, incompressible and steady turbulent flow, the Reynolds averaged momentum, continuity, turbulent kinetic energy and dissipation rate equations can be written in the generalized form;

$$\frac{\partial}{\partial x}(\rho U\Phi)+\frac{1}{r}\frac{\partial}{\partial y}(\rho rV\Phi)=\frac{\partial}{\partial x}\left(\Gamma_{\Phi_x}\frac{\partial\Phi}{\partial x}\right)+\frac{1}{r}\frac{\partial}{\partial y}\left(r\Gamma_{\Phi_r}\frac{\partial\Phi}{\partial y}\right)+S_\Phi \tag{20}$$

Where $r=1$ for Cartesian two-dimensional flow, and $y=r$ for two-dimensional axisymmetric flow. Table 1 gives a summary of the terms in eq. (20) for the dependent variables solved in the code.

Φ	Γ_{Φ_x}	Γ_{Φ_r}	S_Φ
1	0.	0.	0.
U	$2\mu_e$	μ_e	$-\partial P/\partial x + 1/r\partial(\mu_e r\partial V/\partial x)/\partial y$
V	μ_e	$2\mu_e$	$-\partial P/\partial y + \partial(\mu_e \partial U/\partial y)/\partial y$
W	μ_e	μ_e	$-\rho VW/r - W/r^2 \partial(r\mu_e)/\partial r$
k	$\mu + \mu_t/\sigma_k$	$\mu + \mu_t/\sigma_k$	$P_k - \rho\epsilon$
ϵ	$\mu + \mu_t/\sigma_\epsilon$	$\mu + \mu_t/\sigma_k$	$C_1 P_k \epsilon/k - C_2 \rho\epsilon^2/k$

Table 1. Summary of the governing equations. ρ is the density, Γ_{Φ_x} and Γ_{Φ_r} are the exchange coefficients in the axial and radial directions respectively, S_Φ is the source term for the variable Φ. In the table, μ_e is the effective viscosity given as $\mu_e = \mu + \mu_t$, where μ is the laminar viscosity and μ_t is the turbulent viscosity, $\mu_t = C_\mu \rho k^2/\epsilon$.

In the standard k-ϵ turbulence model the constants C_μ, C_1, C_2, σ_k and σ_ϵ have the values 0.09, 1.44, 1.92, 1.0 and 1.0 respectively.

In the rotation modified k-ϵ turbulence model, only C_2 takes the form given by eq. (11) i.e, $C_2 = 1.7 + (5/6)\alpha^2/(\alpha^2+1)$.

The governing transport eq. (20) is solved using the primitive variables on a nonstaggered mesh and converted into a system of algebraic equations by integrating over control volumes defined around each grid point. The SIMPLE pressure-correction scheme (Patankar 1980) is used to couple the pressure and velocities and the resulting algebraic equations are solved iteratively. The convective terms are differenced using a second-order upwind scheme while the diffusion terms are approximated by a central differencing scheme.The physical domain is discretized using a non-uniform mesh where grid points are clustered close to the walls.

6. Model Application

The performance of the present model for complicated recirculating flows is demonstrated through calculations and comparisons with the experimental data of Driver & Seegmiller (1985) for backward-facing step flows and with the experiments of Roback & Johnson (1983) for a confined swirling coaxial jets into a sudden expansion.

Figure 2, shows the streamlines for the backward-facing step using the rotation modified k-ϵ turbulence model. The calculations were performed on a 100x40 grid points. The computational domain had a length of $50H$ (H is the step height) and a width of $9H$. The experimental data were used to specify the inflow conditions for a channel flow calculation where the fully developed profiles at the channel exit were used as the inlet conditions for the backward-facing step calculations. Fully developed flow conditions were imposed at the outflow boundary. The standard wall function approach (Launder & Spalding 1974) was used to bridge the viscous sublayer near the wall.

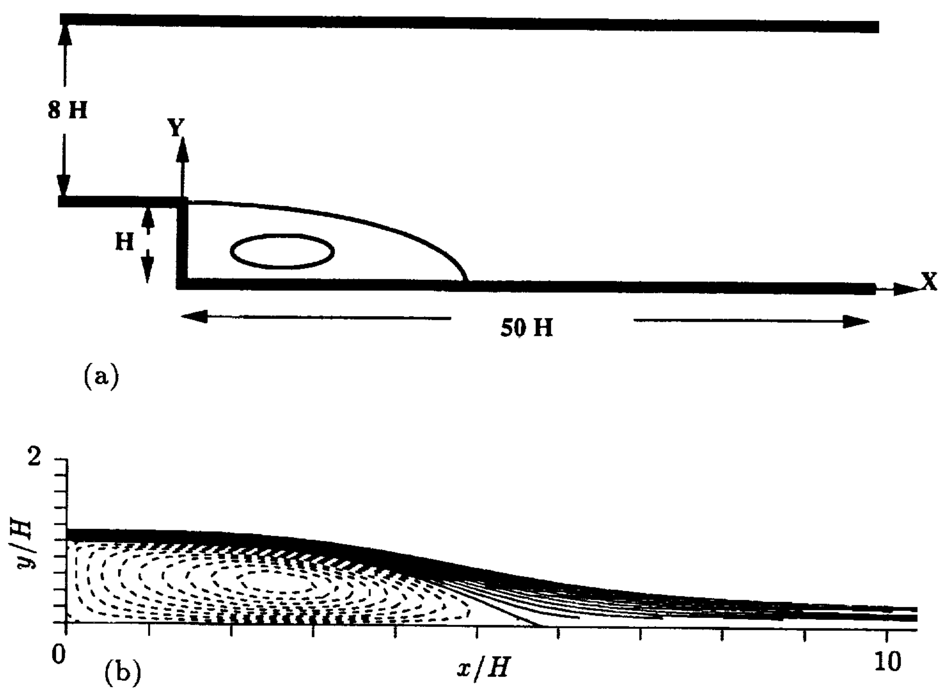

FIGURE 2. Backward-facing step geometry and stream-function contours. The contour levels were set between (-0.1 and 0.1) with an increment level = 0.01. ---- negative values, —— positive values.

The computed reattachment lengths were $5.50H$ using the standard k-ϵ turbulence model and $6.22H$ for the rotation modified k-ϵ turbulence model. The modified k-ϵ model prediction is closer to the experimental value of $6.10H$. While these results are encouraging, they are mainly due to the fact that we have changed the value of C_2 for the non-rotating case. In general, a change in the value of C_2 will result in poor predictions of the mean profiles. The mean velocity profile at three locations downstream are shown on Fig. 3, while the turbulent stress profiles at $X/H = 4$ are shown on Fig. 4. All the quantities were normalized by the step height (H) and the experimental reference free-stream velocity (U_{ref}). It can be seen that the overall performance of the rotation modified dissipation rate equation is better than the standard k-ϵ model especially in the recirculation region (Figs. 3a, and 4). Some improvements are also obtained in the recovery region using the modified k-ϵ model. Figure 5 shows the contours of the effective rotation rate used as defined by Eq. (19).

For the 2D/axisymmetric swirling flow computations, the expressions for the invariants Q and R (Eqs. (16) & (17) respectively) are expanded and Eq. (19) is used to obtain the values of Ω. The model was used to predict the mean profiles for a confined double concentric jets with a swirling outer jet flow into a sudden expansion (Roback & Johnson, 1983, see Fig. 6). Measurements are available for the mean

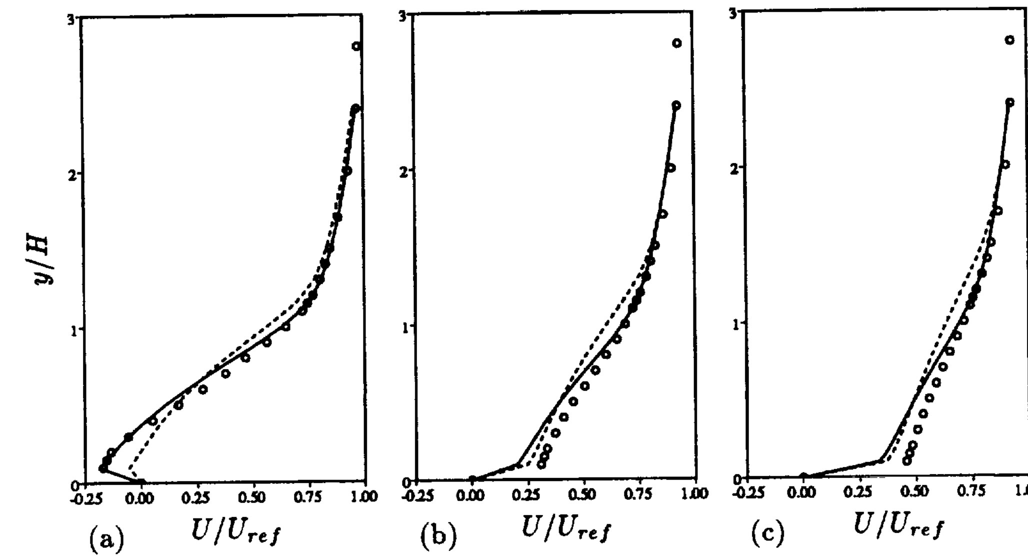

FIGURE 3. Mean axial velocity profiles at different axial locations. o data (Driver & Seegmiller, 1985); —— modified k-ϵ model; ---- standard k-ϵ model. (a) $X/H = 4$, (b) $X/H = 8$, (c) $X/H = 12$.

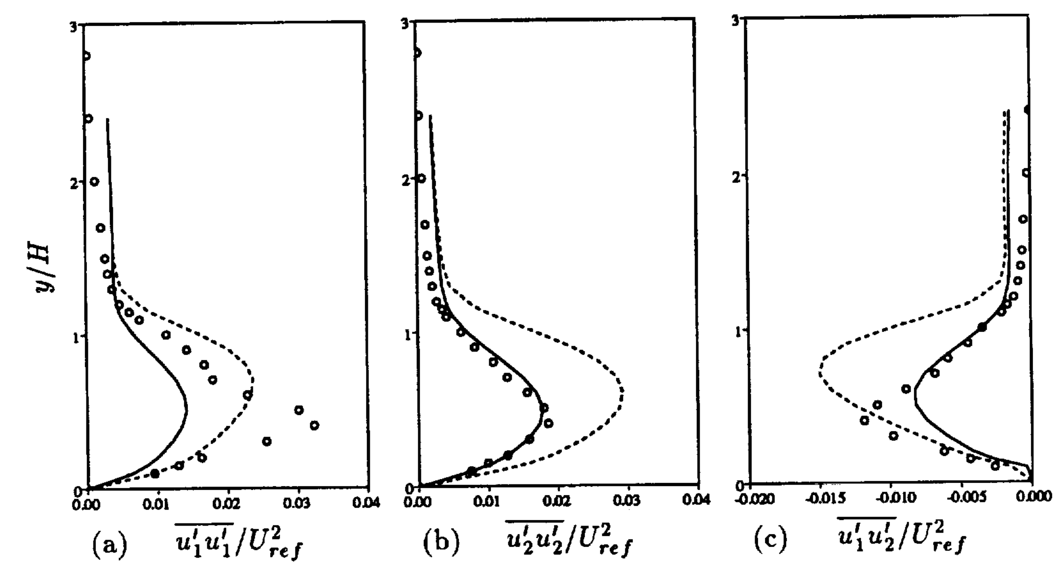

FIGURE 4. Turbulent stress profiles at $X/H = 4$. o data (Driver & Seegmiller, 1985); —— modified k-ϵ model; ---- standard k-ϵ model. (a) $\overline{u_1'u_1'}/U_{ref}^2$, (b) $\overline{u_2'u_2'}/U_{ref}^2$, (c) $\overline{u_1'u_2'}/U_{ref}^2$.

velocity profiles and velocity fluctuations downstream of the expansion. Simulations with a coarse nonuniform grid of 30×20 mesh points were made. However, there is some uncertainty about the inlet conditions to be used since the first velocity

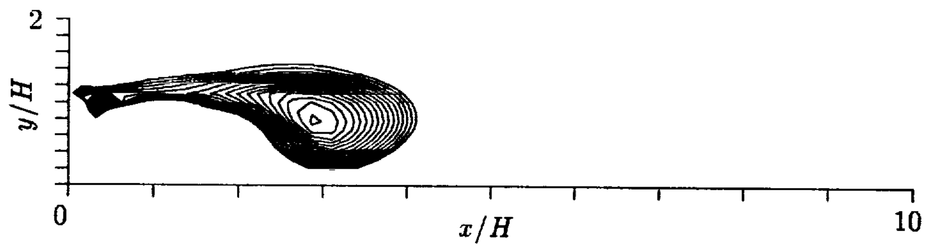

FIGURE 5. Contours of the effective rotation rate, Ω. Contour levels were set between (0.1,1.0) with an increment level = .01. are

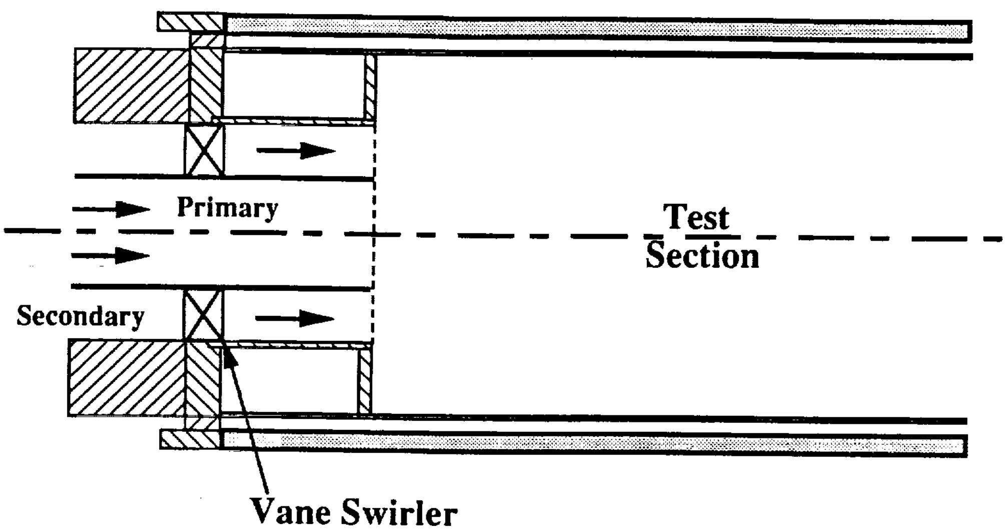

FIGURE 6. Roback & Johnson's swirling coaxial jets discharging into an expanded duct.

profiles measured were 5mm downstream of the expansion.

To predict this flow, the measured profiles at 5mm were adjusted near the edges and were used as inlet conditions at the expansion plane. Preliminary results obtained with the coarse mesh indicate similar trends as the experiment. Figure 7 shows the streamline contours using the standard and the modified k-ϵ turbulence models. The figure shows that a closed internal recirculation zone forms at the center with an additional zone at the corners downstream of the step. This causes a flow diversion outwards with high gradients between these regions. Figure 8 shows the axial and tangential velocity profiles at 25 mm downstream of the expansion using the standard and the modified k-ϵ turbulence models. Results in this case indicate little or no improvements offered using the modified k-ϵ model over the standard k-ϵ model. Finer mesh may improve the results but the uncertainties in the inlet boundary conditions raise the question about the adequacy of using this experiment for validation purposes.

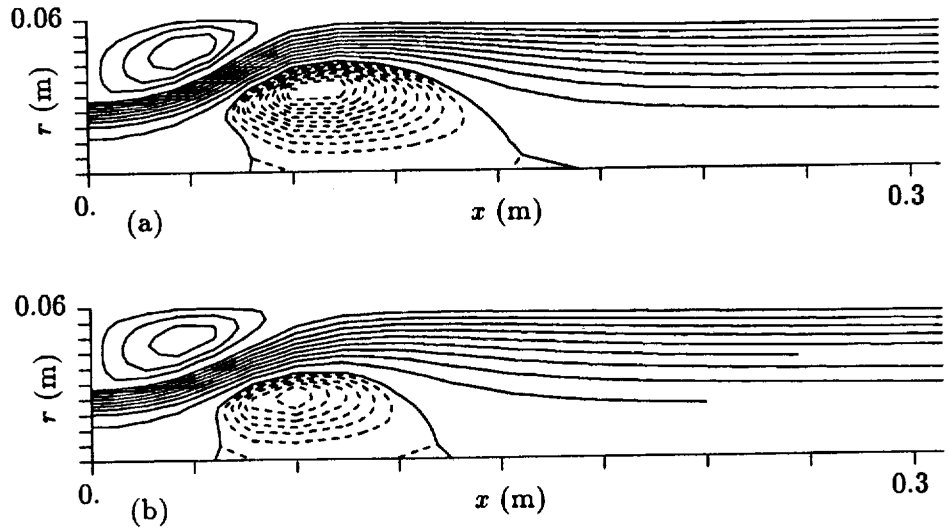

FIGURE 7. Swirling coaxial jets discharging into an expanded duct. Stream-function contour. ---- levels were set between (-0.15,0.) with an increment level = 0.01, —— levels were set between (0.,0.7) with an increment level = 0.05. (a) Standard k-ϵ model, (b) Modified k-ϵ model.

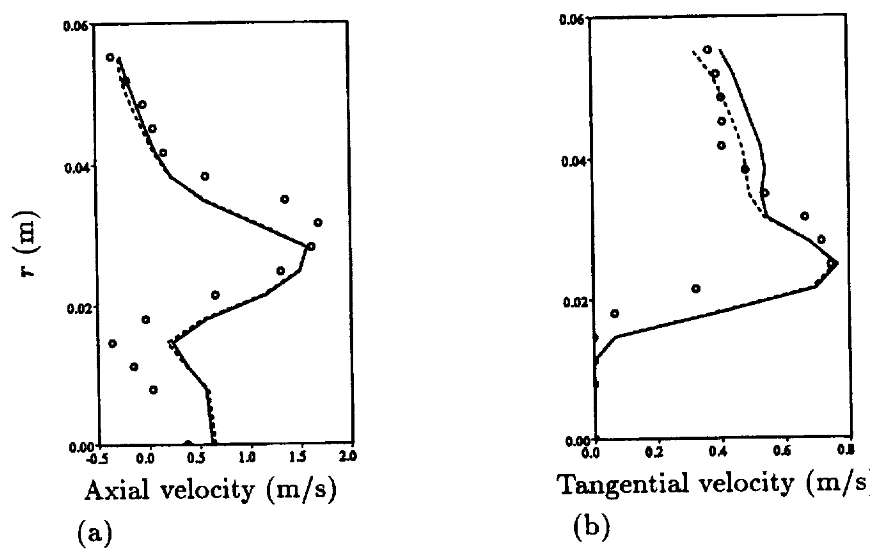

FIGURE 8. Velocity profiles at X = 25 mm. o data (Roback & Johnson, 1983); —— modified k-ϵ model; ---- standard k-ϵ model. (a) Axial Velocity, (b) Tangential velocity.

7. Conclusions

A new simple model for the turbulent energy dissipation rate equation has been proposed to account for the rotational effects on turbulence. A frame invariant definition of the rotation rate proposed by Chong *et al.* (1990) based on the critical point theory was used. The model can be used in conjunction with any level of turbulence closure. It was applied to the two-equation k-ϵ turbulence model and was tested for separted flows in a backward-facing step and for axisymmetric swirling jet into a sudden expansion. The model is numerically stable and showed improvements over the standard k-ϵ turbulence model. It is important to point out that the present study was carried out to roughly evaluate the model, but that a systematic recalibration of the constants in the k-ϵ model is needed before going any further with the proposed model.

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